

With the new revision, the paper has notably improved, but I am still a little bit puzzled with the final steps of the demonstration. The rest of the paper looks fine to me.

On my first revision I made some math and notation suggestions that were accepted by the authors. However, my main concern was regarding the definition of $l^*(y)$. The authors have changed this definition—see current Eq. (5)—but from my understanding it is still not correct. From current Eqs. (4) and (5), it can be inferred that

$$l^*(y) = \int_{\omega}^y f(t)dt = - \int_y^{\omega} f(t)dt = -l(y) \quad (1)$$

contrary to what it is stated on the first line of page 5. I don't think the equality $l^*(y) = -l(y)$ makes sense given that the survival function is always positive, and I believe that this is not a result the authors wanted to obtain. Furthermore, the current definition of $l^*(y)$ is somehow strange as it includes an integral whose lower limit (ω) is bigger than the upper limit (y).

Nonetheless, I would like to suggest an alternative way to define $l^*(y)$:

$$l^*(y) = \int_{-\omega}^0 f(y-t)dt \quad (2)$$

Equation 2 can be interpreted as follows: $l^*(y)$ are the individuals who were born during the past ω years and who still have exactly y remaining years of life. Note that for those born in the current year ($t = 0$) their contribution is $f(y-t) = f(y)$, for those born in the previous year ($t = -1$) their contribution is $f(y-t) = f(y+1)$, etc. Moreover, $l^*(0) = 1$ as expected. Using this definition,

$$\begin{aligned} l^*(y) &= \int_{-\omega}^0 f(y-t)dt = \int_{-\omega}^{-y} f(-t)dt = \\ &= - \int_{\omega}^y f(t)dt = \int_y^{\omega} f(t)dt = l(y) \end{aligned} \quad (3)$$

(In Eq. 3 I use the propriety that $\forall c \in R, \int_{ca}^{cb} f(t)dt = c \int_a^b f(ct)dt$, with $c = -1$). This new definition of $l^*(y)$ and its subsequent interpretation seem clearer and better to me. Moreover, it validates the equality $l^*(y) = l(a)$ if $a = y$. I would be glad to know what the authors think about.

I have a final suggestion regarding the last steps of the demonstration. The equality $l(a) = l^*(a)$ shown in the first line of page 5 can lead to confusion because it can be interpreted as if the functions l and l^* are the

same, which is not accurate because one function refers to chronological age and the other to thanatological age. I believe it would be more appropriate to write something like “and since $l^*(y) = l(a)$ for $a = y$ ”, as in equations (1) and (3) of the paper. In Eq. (7) the y 's should be replaced by a 's