# Accounting for temporal variation in morbidity measurement and projections

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**Abstract**

This is important stuff!

## Introduction

Age standardization is an essential tool for the contemporary practice of demography. Demographers age-standardize in order to assess trends and intensities in rates that vary in regular ways over age free from distortion in population structure. Without age standardization or its many cognates, we would judge trends and magnitudes based on crude rates, which are now understood not to carry the same predictive utility as rates that have been purged of structure. This is a cornerstone tenet of contemporary demography. Typically, in order to drive the point home, instructors find or concoct an example where a comparison of age-standardized rates leads to the opposite conclusion as crude rates suggest. This is quite motivating for the pupil, and it soon become second nature. This is the point we wish to remake with respect to unaccounted-for temporal variation in rates that are not vital rates.

Some processes vary over the life course, that is to say, within and over the lives of individuals. If members of the same birth cohort are thought to have something in common, it will surely be the case that members of the same birth cohort that also end up dying in the same year share even more features in their life course: Different aspects of their lives will on average align in empirically regular ways. In general, persons dying

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in the same year probably share many characteristics in the time prior to death, especially persons that die of intrisic or unavoidable mortality. This empirical alignment will in some cases hold, even if the persons are not from proximate birth cohorts. That is to say, for some conditions, temporal variation in terms of remaining years of life or completed lifespan provides sharper and more regular relief than variation in terms of chronological age. In such instances, age standardization of the common variety does not provide the degree of control or precision that is often assumed – it is misapplied. By extension, chronological age patterns of characteristics that do not vary primarily as a function of chronological age are misleading and accidental.

In this paper, we provide an empirical example of a health condition that appears to have a chronological age pattern in the margin, but has none when disagregated. We show how even short term projections of the condition into the future imply great differences in prevalence— changes not only of different magnitude but of different sign— with respect to a projection based only on an apparent marginal age pattern. The empirical excercise is relatively modest, given the available data, but we do demonstrate a vulnerability in the methodological status quo that has consequences for how we measure health at the population level. We hope that this basic observation will motivate the development of new extensions of standardization methods, as the standardization we propose is more amenable to "clean" count data that are not readily available in most instances. We hope to evoke the same feeling you got when you first understood how age standardization works.

## Motivation

Some words from John

## Formal relationships

Say we want to measure a health condition, $G$, something bad and degenerative that usually sets in late in life, but with onset spread out over a wide range of ages. For simplicity, imagine that $G$ is a binary condition: either you have it or you don't. The same excercise could be carried out with adjustments for intensity, but this complicates the measurement of "how much" condition $G$ there is, and the implied methodlogical details might obfuscate the point we wish to make. For a binary condition, a straightforward calculation is a proportion of individuals with $G$. Say then that the number of people with condition $G$ at chronological age $a$, $G(a)$, is simple the people in age $a$ times the probability of having $G$ in that age, $g'(a)$:

$$G(a) = P(a)g'(a) \qquad . \tag{1}$$

(1) holds tautologically in the given year of observation, but it may not hold in the future if the essential variation in $G$ in fact varies by remaining years of life, $y$, one's thanatological age. In this case, $g'(a)$ is itself an aggregate that depends on the real underlying thantological function of $G$, $g(y)$, and the population structure classified by both chronolgical and thanatological age:

$$g'(a) = \frac{\int_0^\omega g(y)P(y,a)\ \mathrm{d}y}{P(a)} \tag{2}$$

$$= \frac{\int_0^\omega g(y)P(a)\mu(a+y)\frac{l(a+y)}{l(a)}\ \mathrm{d}y}{P(a)} \tag{3}$$

$$= \int_0^\omega g(y)\mu(a+y)\frac{l(a+y)}{l(a)}\ \mathrm{d}y \tag{4}$$

$$= \int_0^\omega g(y)f(y|a)\ \mathrm{d}y \qquad , \tag{5}$$

where $\omega$ is the highest age a person can survive to, $\mu(a)$ is the force of mortality at age $a$, and $l(a)$ is the survivor function at age $a$. (2) means that $g'(a)$ is not independent of mortality in the case that it is sufficiently described by $g(y)$. This conclusion is rather intuitive in any case, since for a condition to vary as a function of thanatological age means that it is bound up with variation in lifespan itself. In this case, (1) is better rewritten as:

$$G(a) = P(a)\int_0^\omega g(y)f(y|a)\ \mathrm{d}y \qquad . \tag{6}$$

There may be a sort-of-empirically-regular pattern to $g'(a)$, but it is accidental. $g'(a)$ will move more from year to year than will $g(y)$, simply because mortality also changes. Likewise, the mix of ages in a particular year are subject to different mortality schedules, as they belong to different birth cohorts, each with its own history and future of mortality. The function $\mu(a)$ may change more over time than $g(y)$, ergo heterogeneity between cohorts with respect to $\mu(a)$ may be greater than heterogeity with respect to $g(y)$, and this is a unique source of compositional distortion that affects the measurement of $G$.