Some properties of life left pyramids

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Let us consider a state s for a multistate population, and a being the age in last state s or time spent the last entry in state s.

1. The population density $p(t, a)$ at time t and age a can be defined as the population that, at time t, has an age between a and $a + \mathrm{d}a$. Prove that this number is also equal to the population that has reached age a between time t and $t - \mathrm{d}a$ [**?**].

2. We are now considering a closed population which is defined by the absence of migration. $P(t) = \int_0^\infty p(t, u)\,\mathrm{d}u)$, is the size of the total population at a census occurring at exact time t and let us consider its sub-population $P(t, a) = \int_0^a p(t, u)\,\mathrm{d}u$ under age a. Let us suppose that the next census is at time $t + a$ (see Fig. 1), and consider the sub-population $P(t + a, a_+)$ of more than a years at the new census, $P(t + a, a_+) = \int_a^\infty p(t + a, u)\,\mathrm{d}u)$. Deduce that the difference between both populations $P(t, a) - P(t + a, a_+)$ corresponds to the number of exits from state s occurring in population $P(t)$ between the two censuses.

3. We are now considering a stationary multistate population: a population with constant force of interaction between states over time and constant number of entries in each states by time unit. Let us denote n for the state s considered. Its population "density" at time t and age a is then $p(t, a)\,\mathrm{d}a = nl(a)\,\mathrm{d}a$ and is independent of time t where $l(a)$ is the survivorship in state s. We are neglecting stochastic variations by considering huge populations.

4. Deduce from previous result that in a multistate stationary population, there are exactly the same number of people having lived a years in a state than people having a years to live in this state. And therefore prove that both, pyramids by time spent in a state and by time to live in this state, are identical.

5. Stable multistationary state: suppose now than n is increasing or decreasing exponentially: $n(t) = n_0 \exp(\rho t)$, etc.

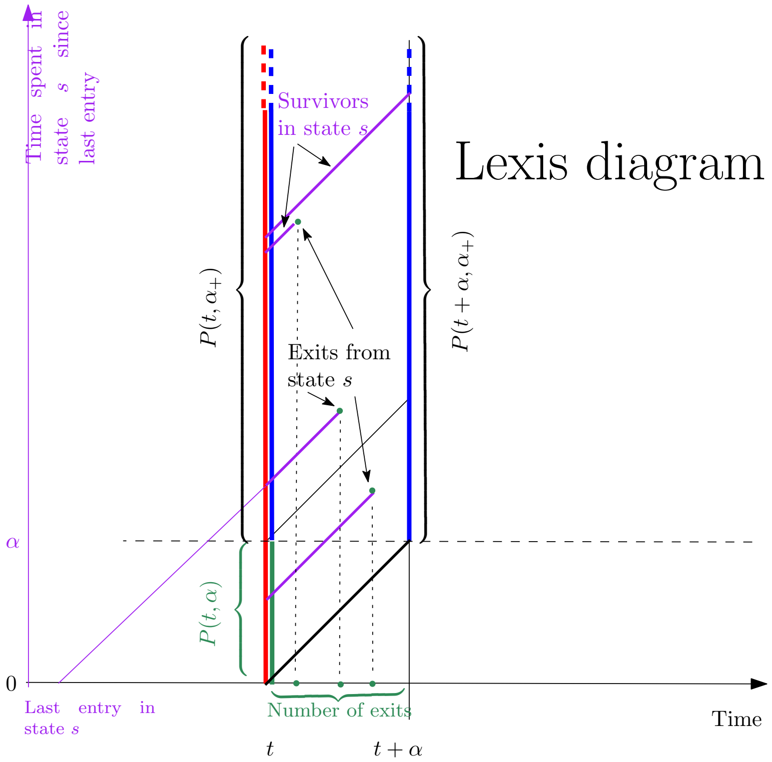

Figure 1: Lexis diagram showing the population at two censuses spaced at α years and sub-populations of less than α years and more than α years.