MAX PLANCK INSTITUTE
FOR DEMOGRAPHIC
RESEARCH

Time spent and left of transient states in stationary populations

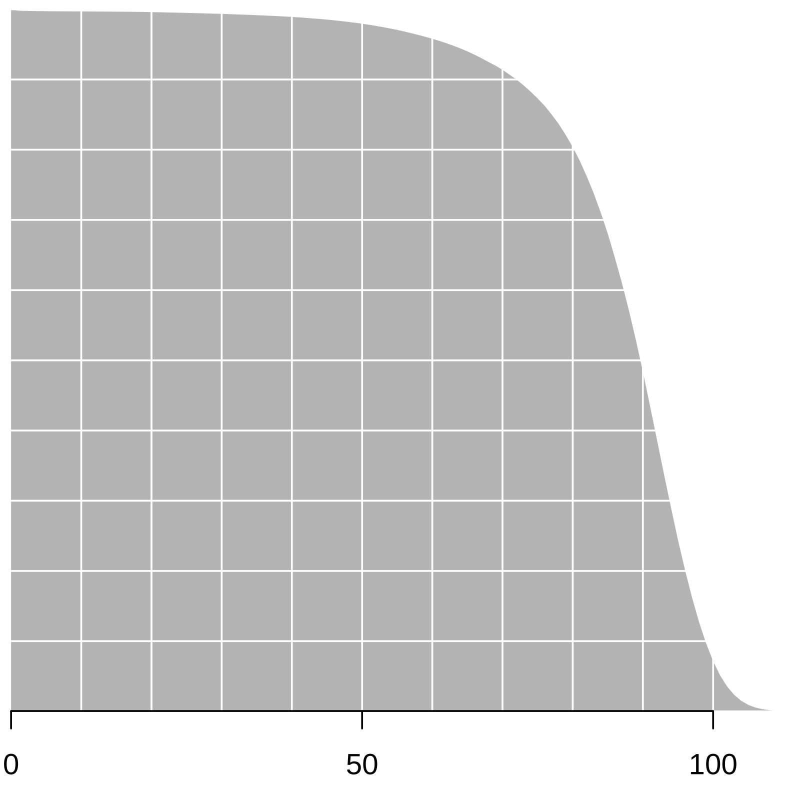
Tim Riffe & Francisco Villavicencio & Nicolas Brouard

Brouard-Carey equality

Under stationarity, the population aged x equals the population with x life left to live.

(Brouard, 1989; Vaupel, 2009; Villavicencio & Riffe, 2016)

A visual explanation



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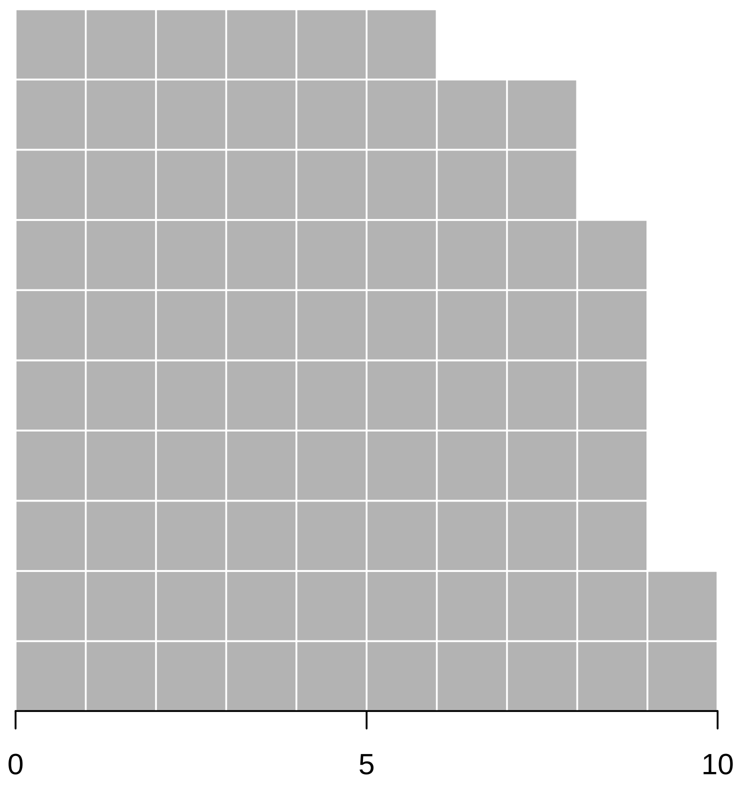

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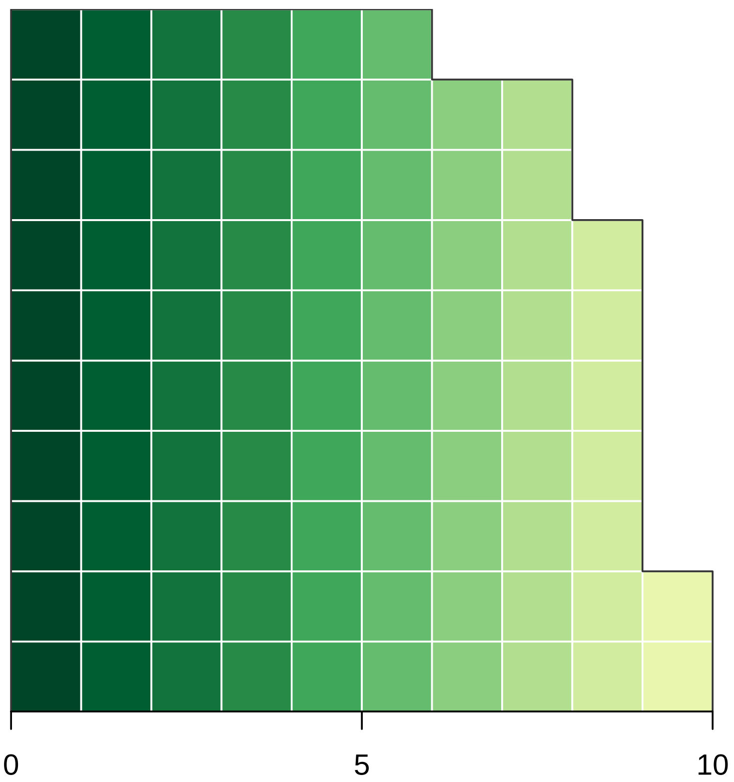

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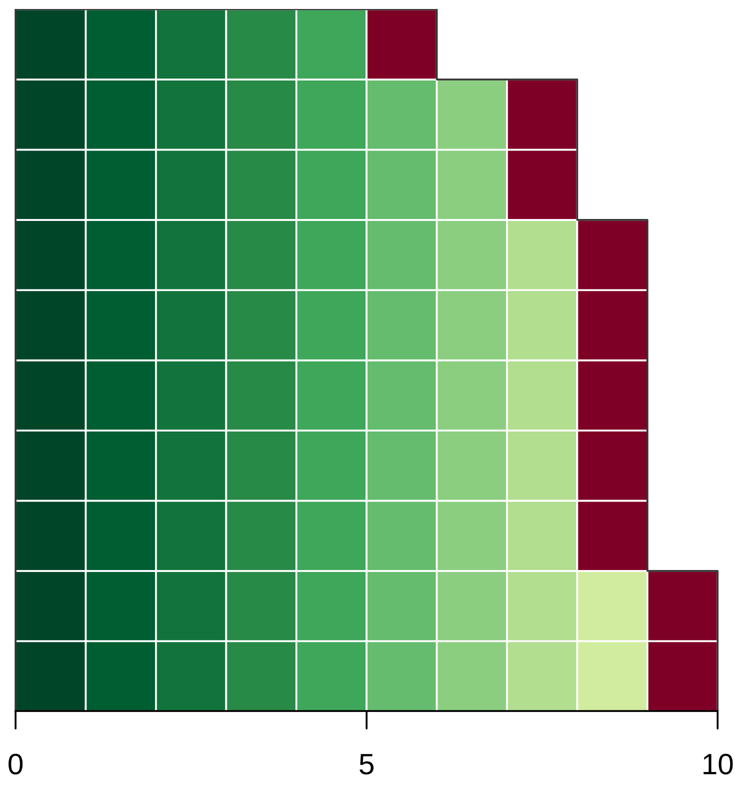

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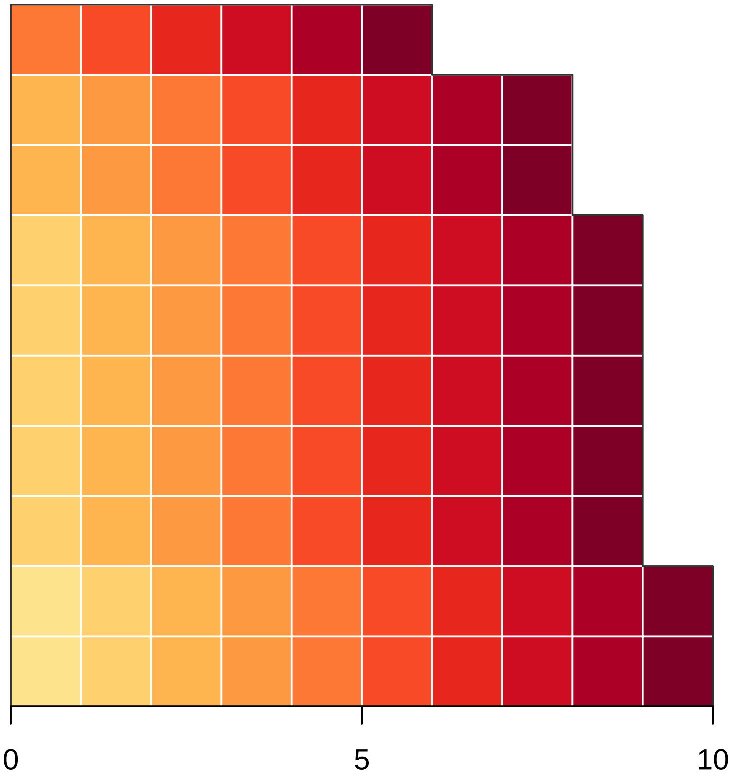

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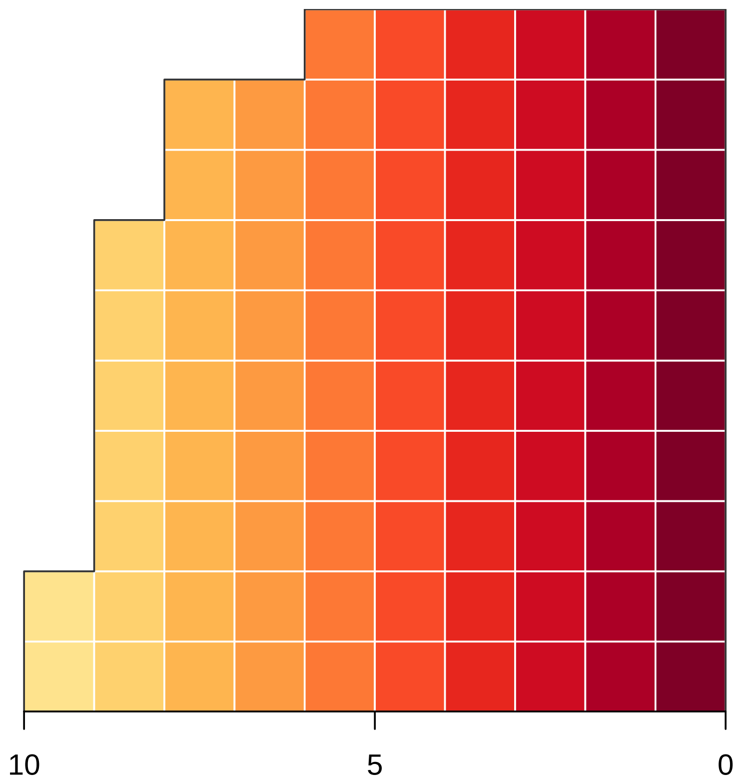

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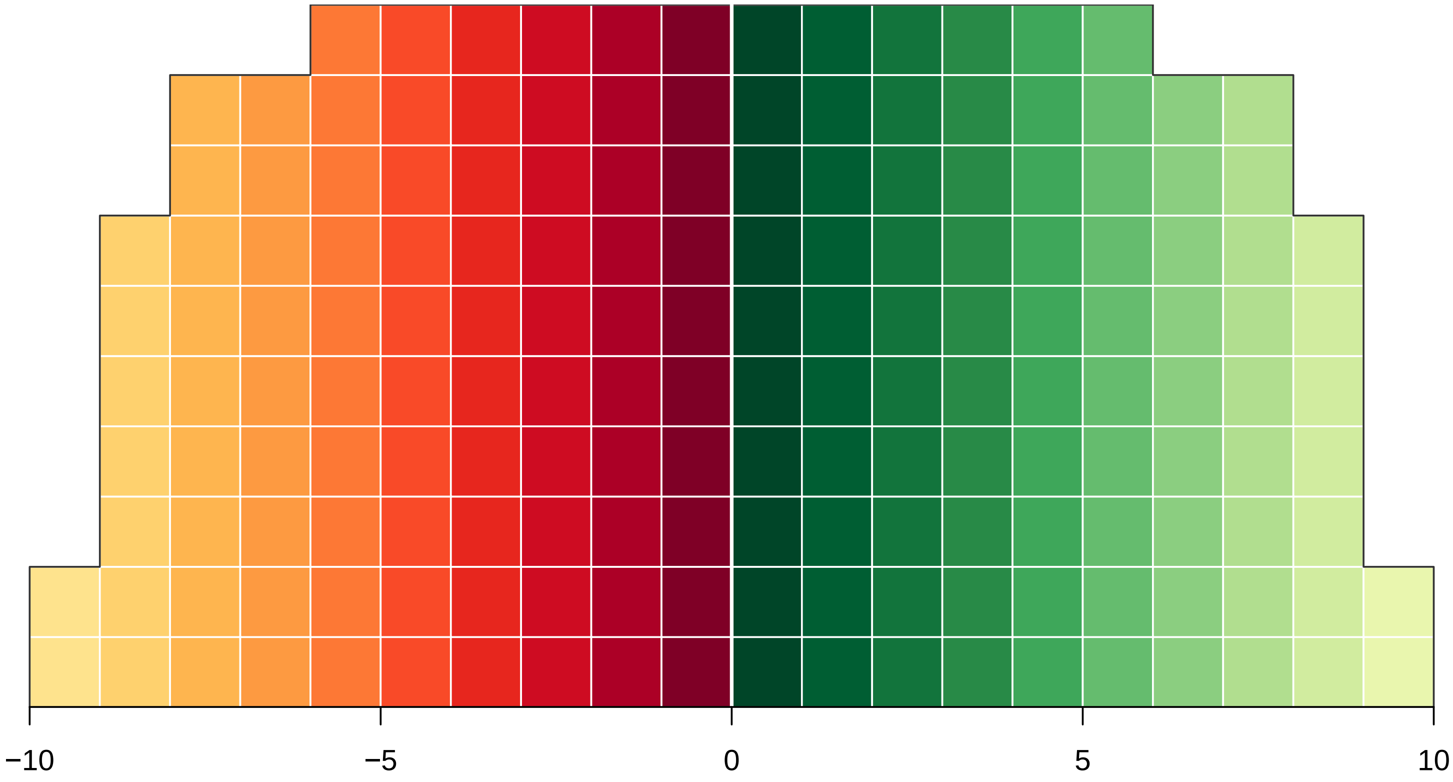

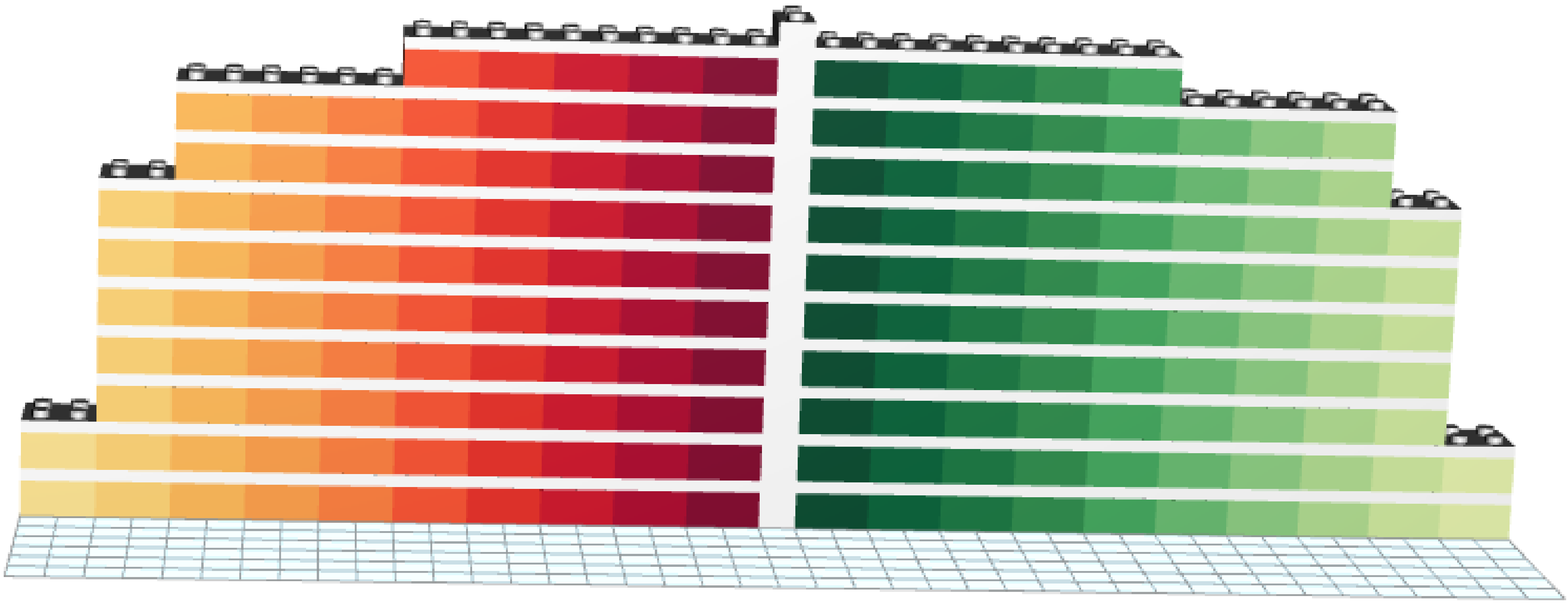

Brouard-Carey Lego design here:
http://www.publishyourdesign.com/design/70442

Brouard-Carey Symmetry

The age distribution is identical to / symmetrical with the time-to-death distribution.

Transient Symmetry

Within a given state, the time-spent distribution is equal to the time-to-exit distribution.

Transient equality

Under stationarity, the probability that a randomly selected individual is in state s and entered s x years ago is equal to the probability of being in state s and exiting in x years.

Requisites:

- All vital and state transition schedules fixed.
- No growth (births = deaths).

Probabilistic result:

- The expected age-state structure is frozen.
- Each potential discrete state trajectory has a fixed probability of occurring.
- Same for past and future cohorts.

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Deterministic result (problematic):

- The age-state structure is frozen.
- Each discrete state trajectory occurs for a fixed fraction of a birth cohort.
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Deterministic result (friendly):

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- The same finite set of discrete state trajectories.
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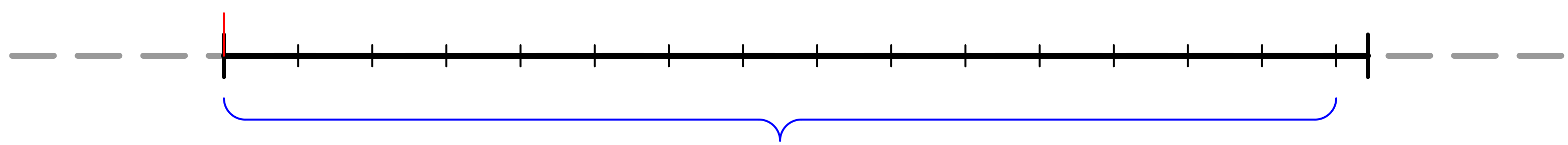
Prove for friendly case, generalize to probability case.

Step 1

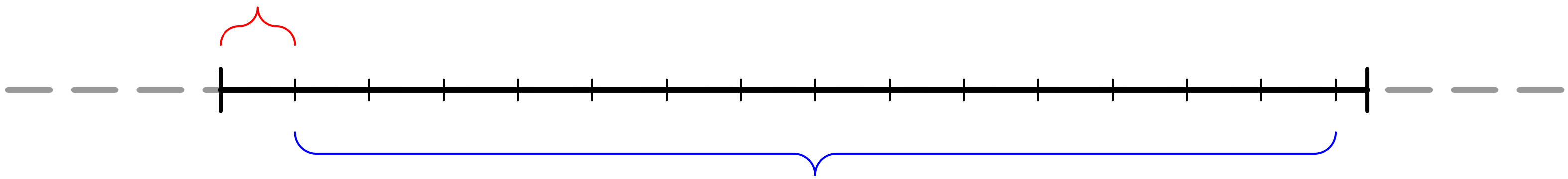
$$A^{(i)} = \{0\}$$



$$T^{(i)} = \{\tau_1\}$$

Step 1

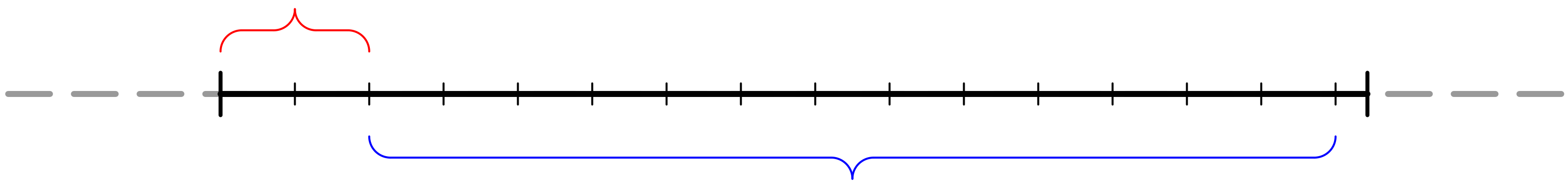
$$A^{(i)} = \{0, a_1\}$$



$$T^{(i)} = \{\tau_1\}$$

Step 1

$$A^{(i)} = \{0, a_1, a_2\}$$



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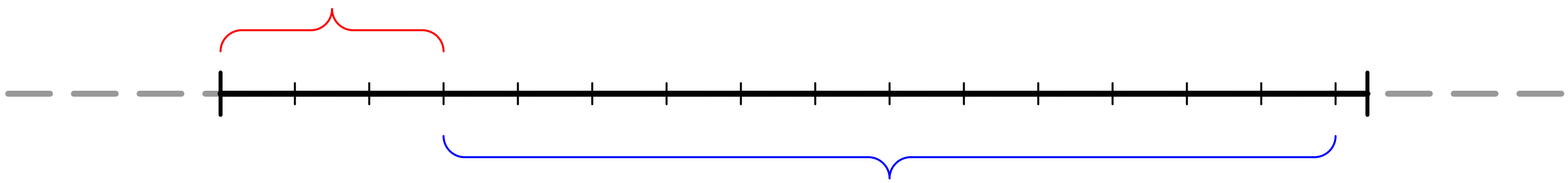
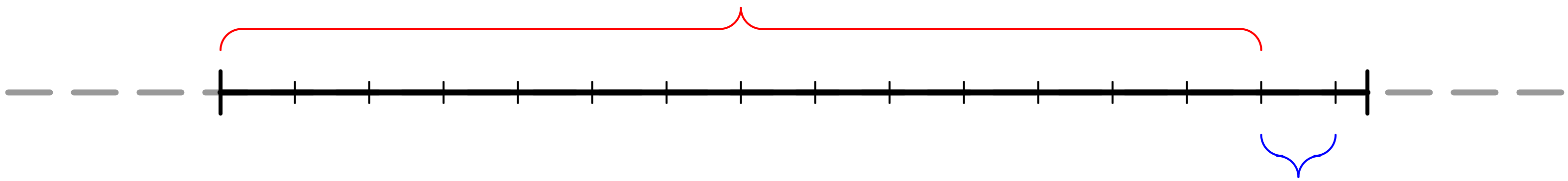


$$T^{(i)} = \{\tau_1, \tau_2, \tau_3\}$$

Step 1

$$A^{(i)} = \{0, a_1, a_2, a_3, \ldots, a_{K-1}\}$$



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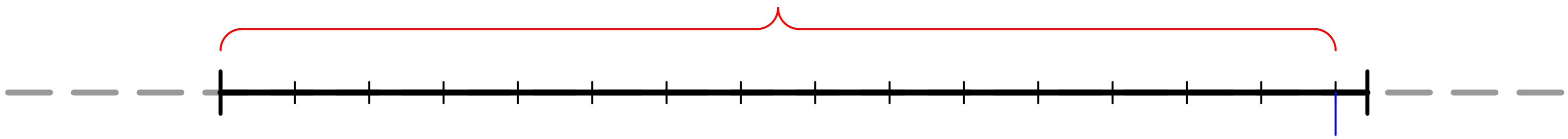

$$T^{(i)} = \{\tau_1, \tau_2, \tau_3, \ldots, \tau_{K-1}, 0\}$$

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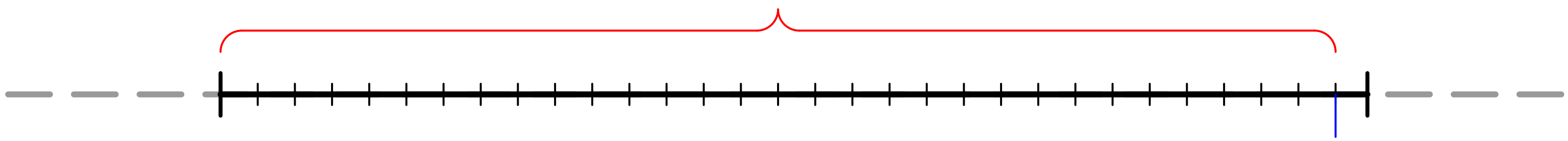


$$T^{(i)} = \{\tau_1, \tau_2, \tau_3, \ldots, \tau_{K-1}, 0\}$$

Complementarity:

Within an individual over time

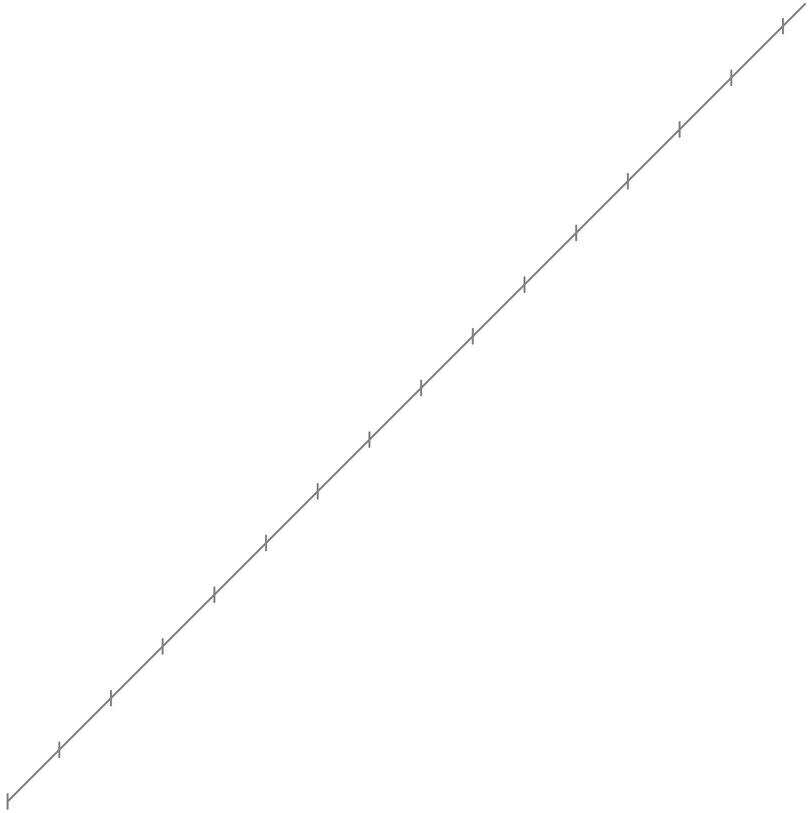
$$A^{(i)} = T^{(i)}$$

Complementarity:

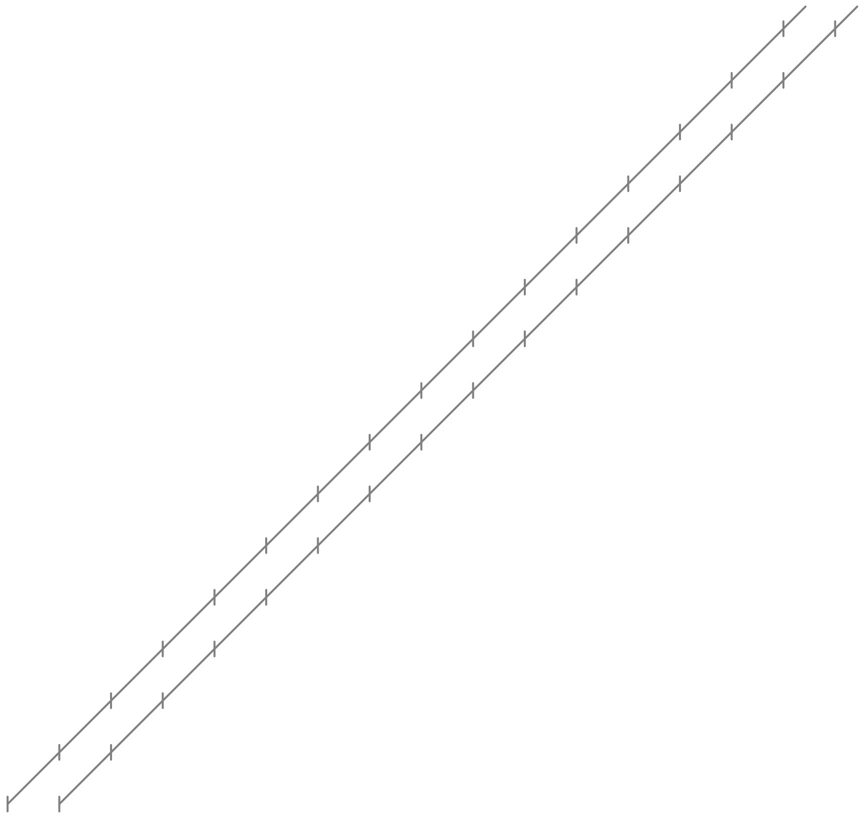
for the union of two individuals

$$\left\{A^1, A^2\right\} = \left\{T^1, T^2\right\}$$

Move to Lexis diagonal



A clone is born every Δ time step



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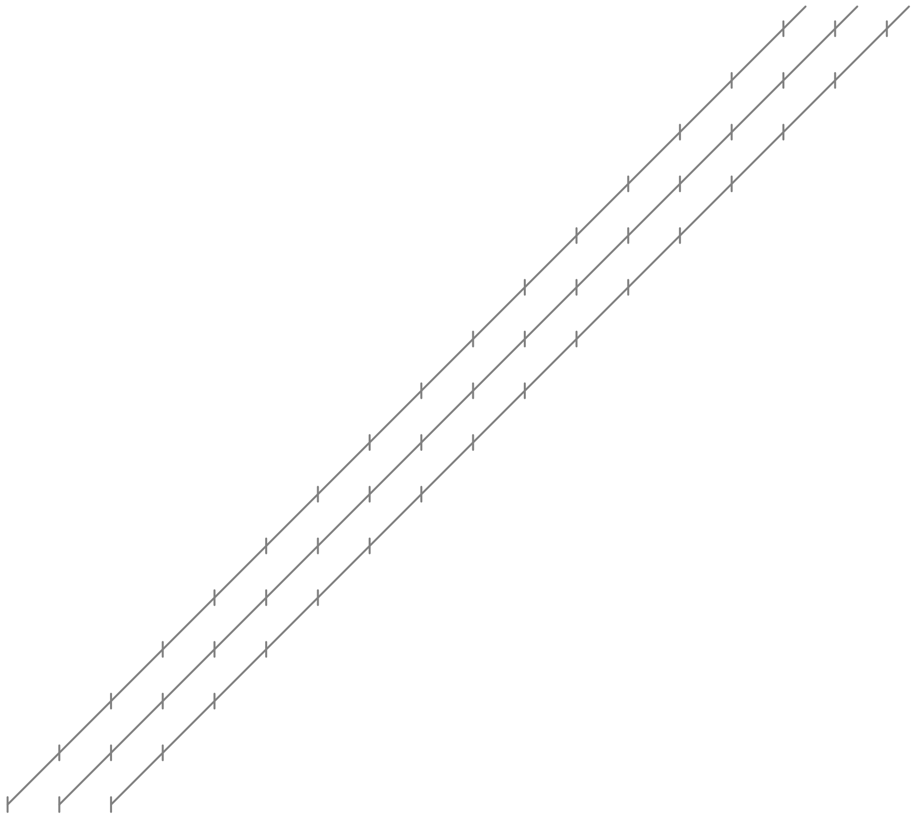
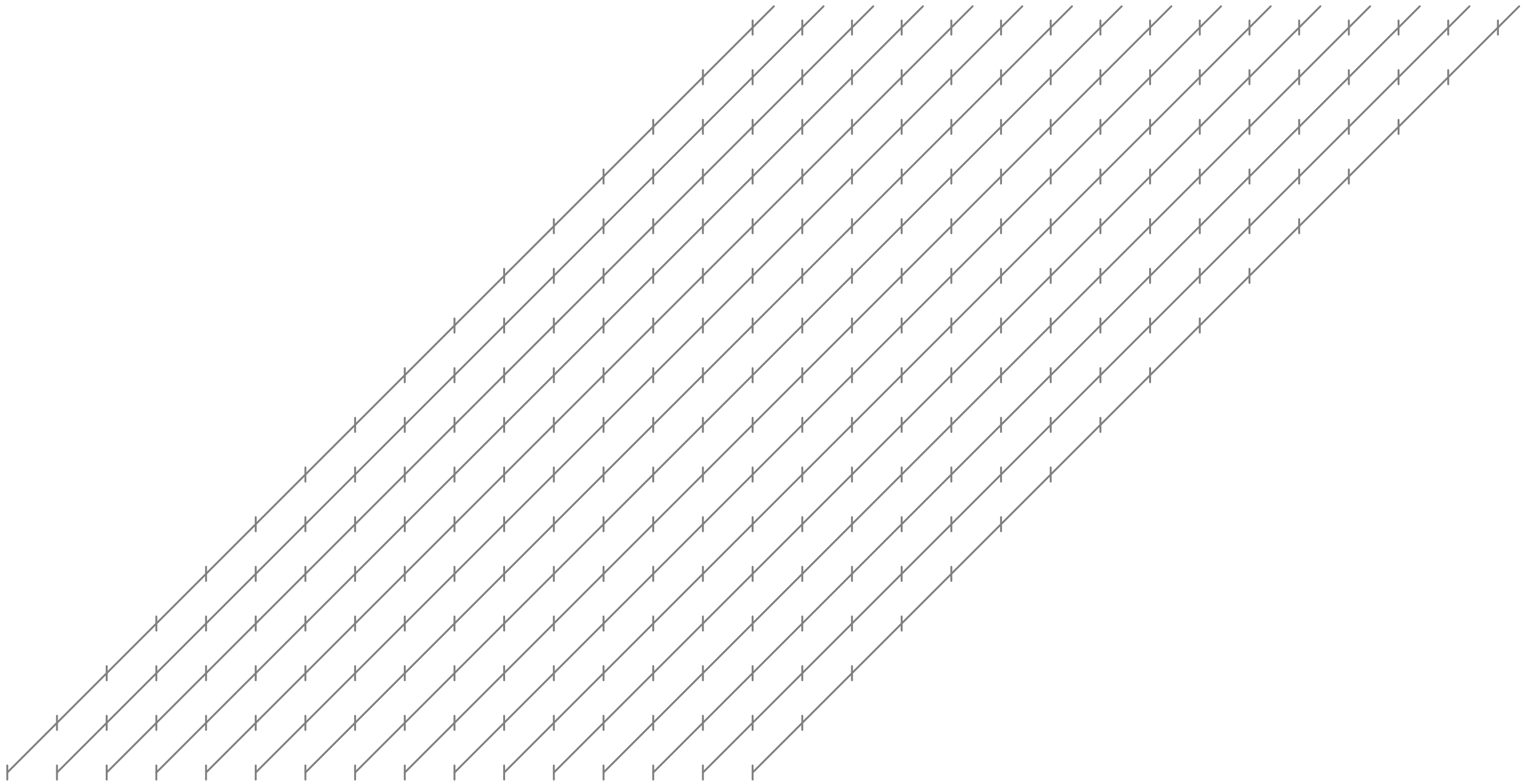
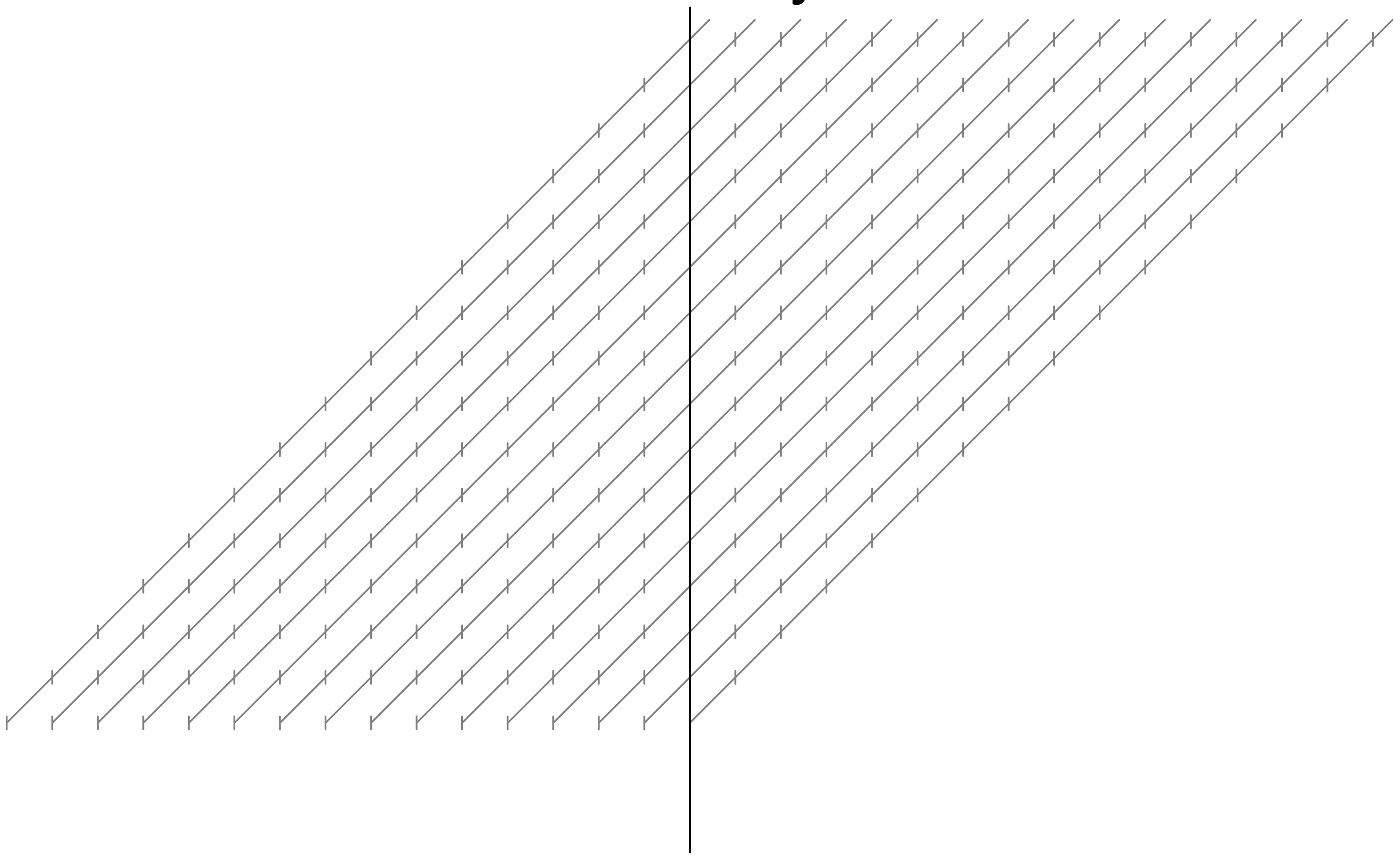


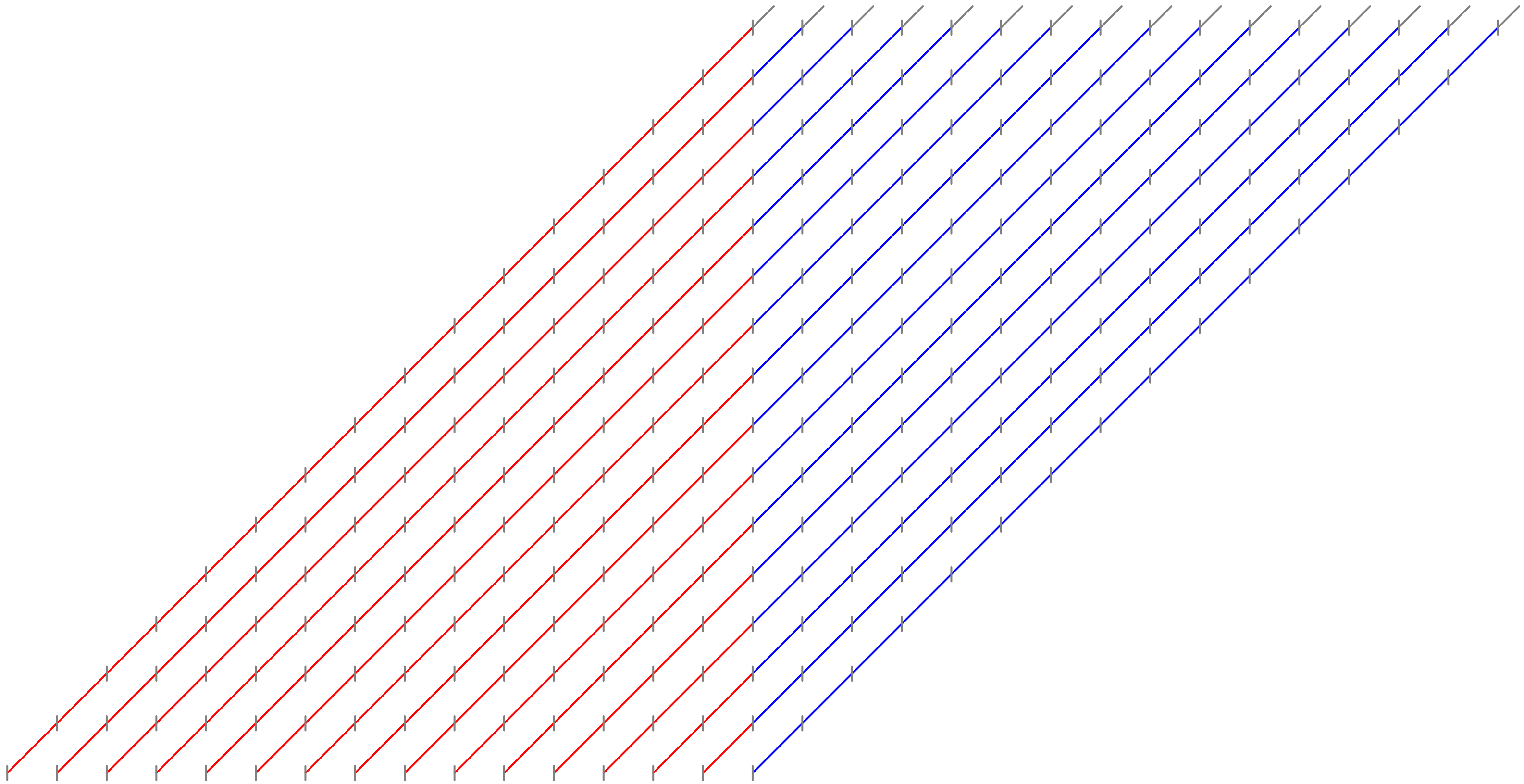
A stationary series of clones



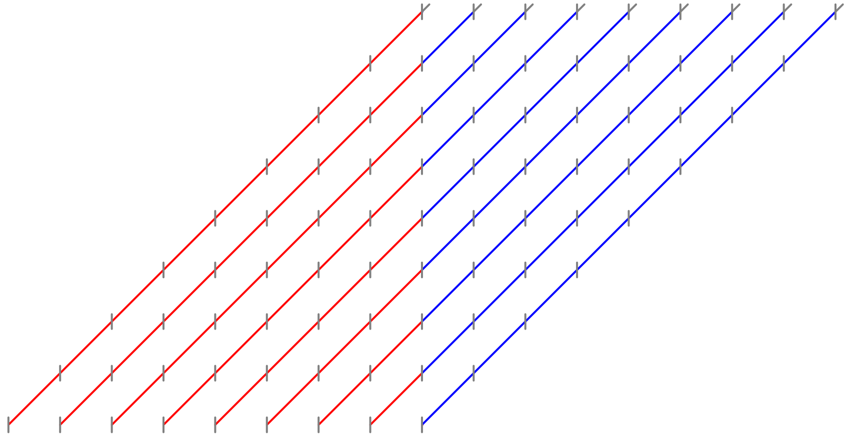
A census in stationary series



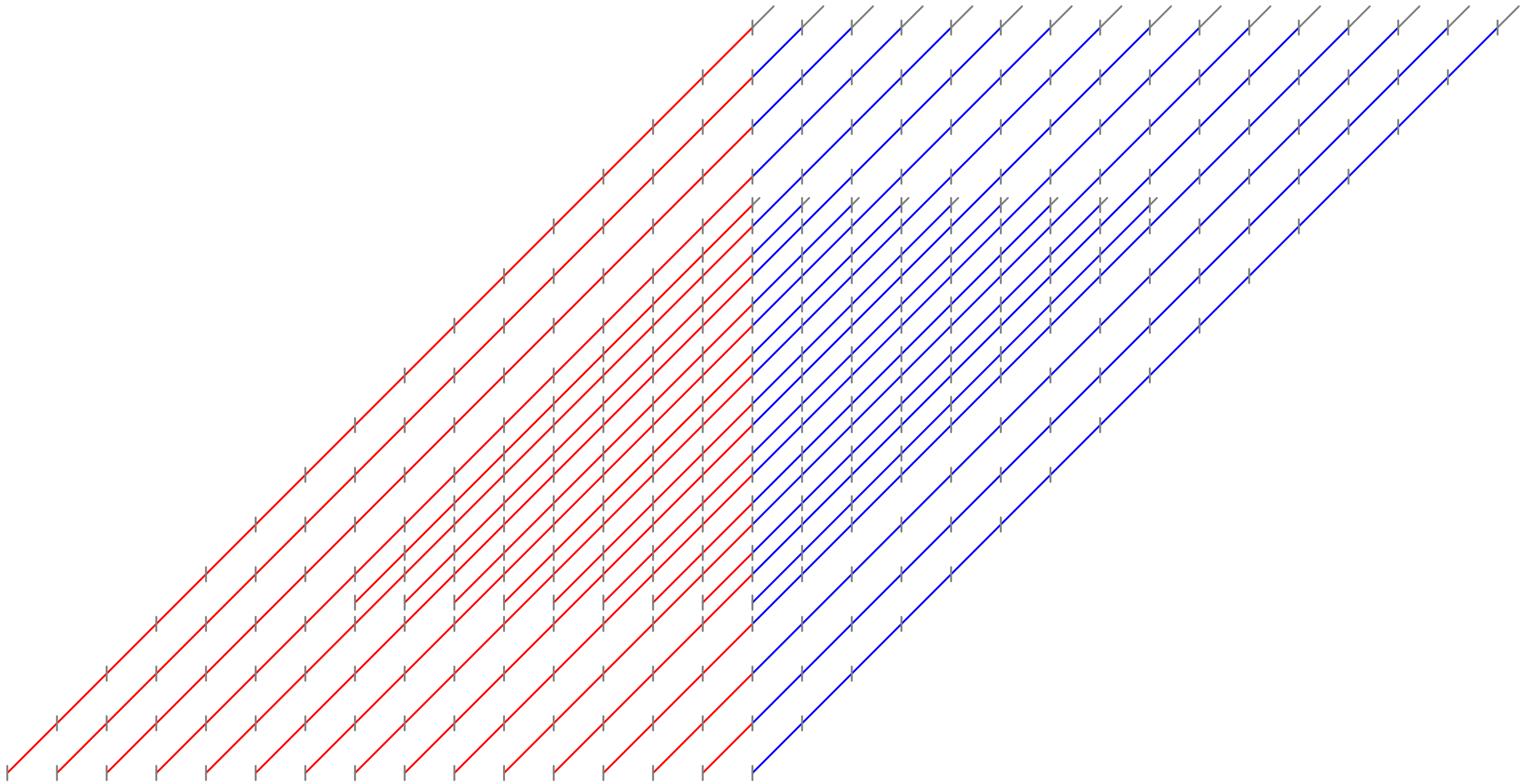
Complementarity in “clone” stationarity



Complementarity for some other episode



Complementarity for set unions



Proof.

By induction, complimentarity for all episodes. This implies equal sets of time spent and left values, a.k.a. equal time spent and left distributions.

- Make Δ smaller if you want.
- Replace “clones” w “fixed fraction”.
- Replace fraction w “fixed probability”.

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Implications:

- Equal time spent-left distributions within states.
- Within episode *order*.
- Cumulative over episodes.
- Conditional on anything (age, TTD).
- Brouard-Carey is degenerate case.

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Photo by Eberhard Grossgasteiger on Unsplash

Simulate

- Take transition matrix from Dudel & Myrsklä (2017)
- Simulate trajectories; `rmarkovchain()` in `markovchain` package.
- Take census in stationary series.
- Assume observation at half interval.
- Tabulate time spent and left in sampled episodes.
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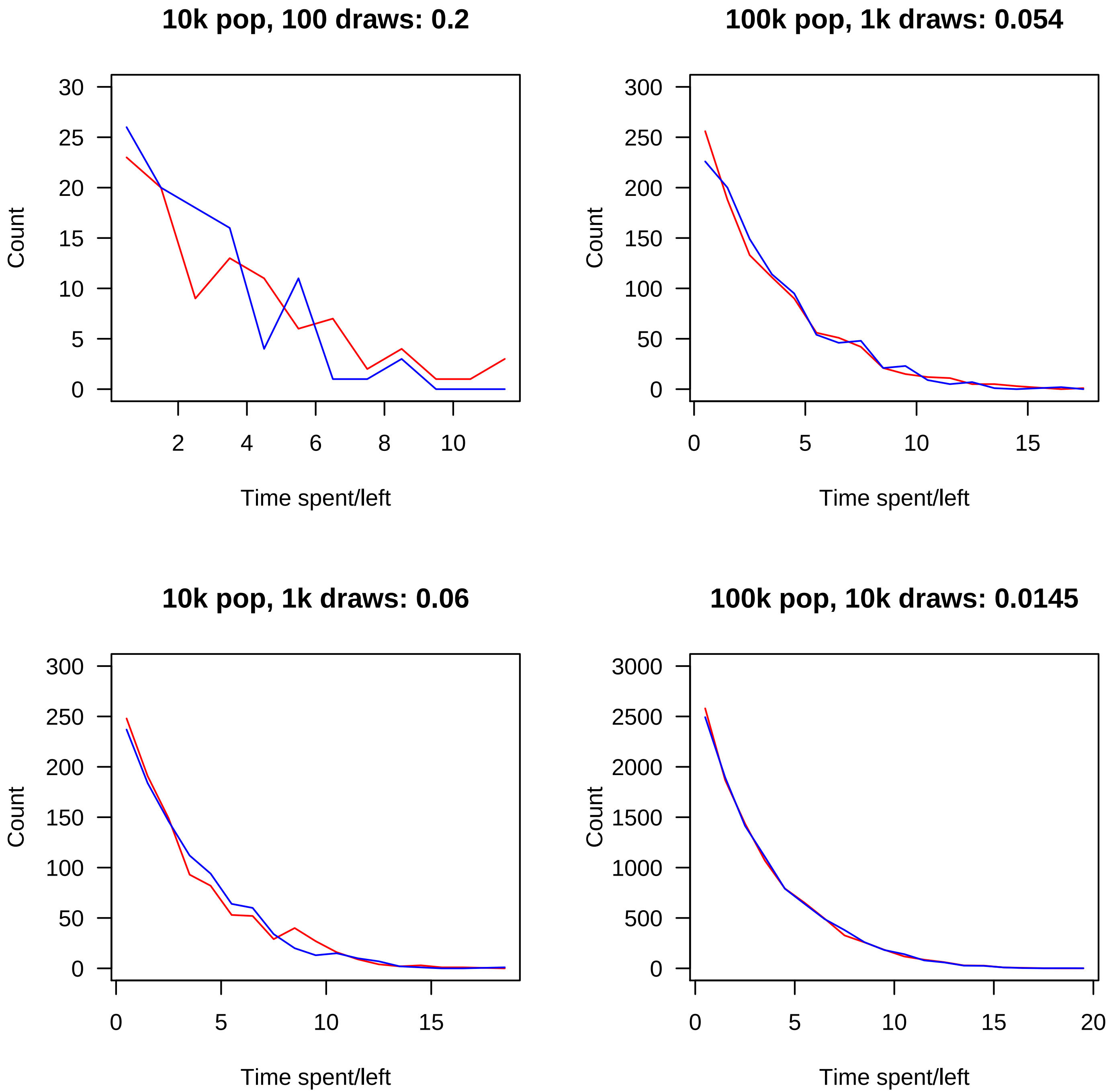
10k pop, 100 draws: 0.2
100k pop, 1k draws: 0.054
10k pop, 1k draws: 0.06
100k pop, 10k draws: 0.0145
Count
Time spent/left
0
5
10
15
20
25
30
2
4
6
8
10
50
100
150
200
250
300
500
1000
1500
2000
2500
3000


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Estimate from the reflection!
Thanks!
riffe@demogr.mpg.de