MAX PLANCK INSTITUTE
FOR DEMOGRAPHIC
RESEARCH

Time spent and left of transient states in stationary populations

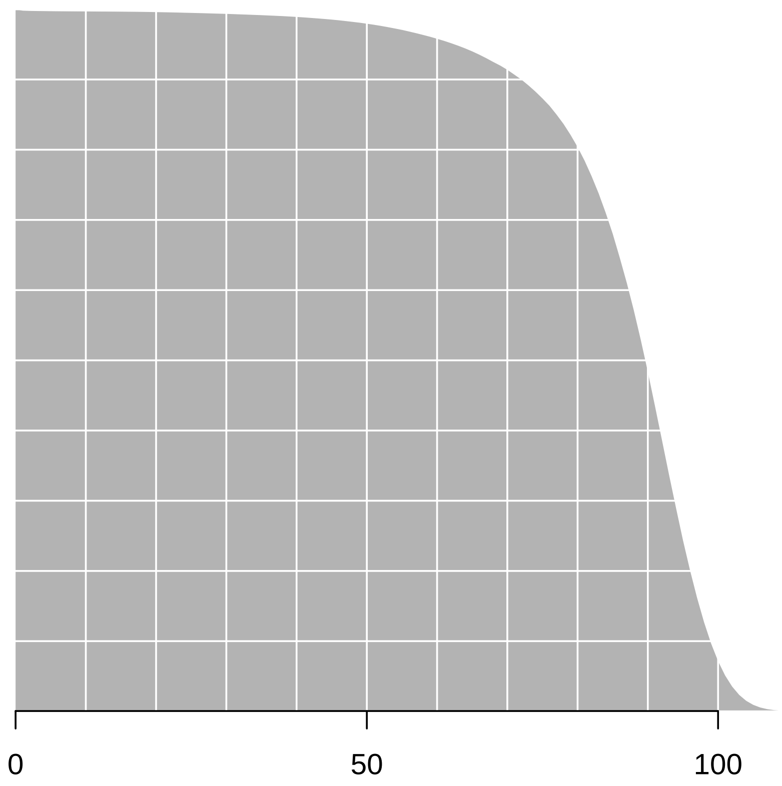
Tim Riffe & Francisco Villavicencio & Nicolas Brouard

Brouard-Carey equality

Under stationarity, the population aged x equals the population with x life left to live.

(Brouard, 1989; Vaupel, 2009; Villavicencio & Riffe, 2016)

A visual explanation



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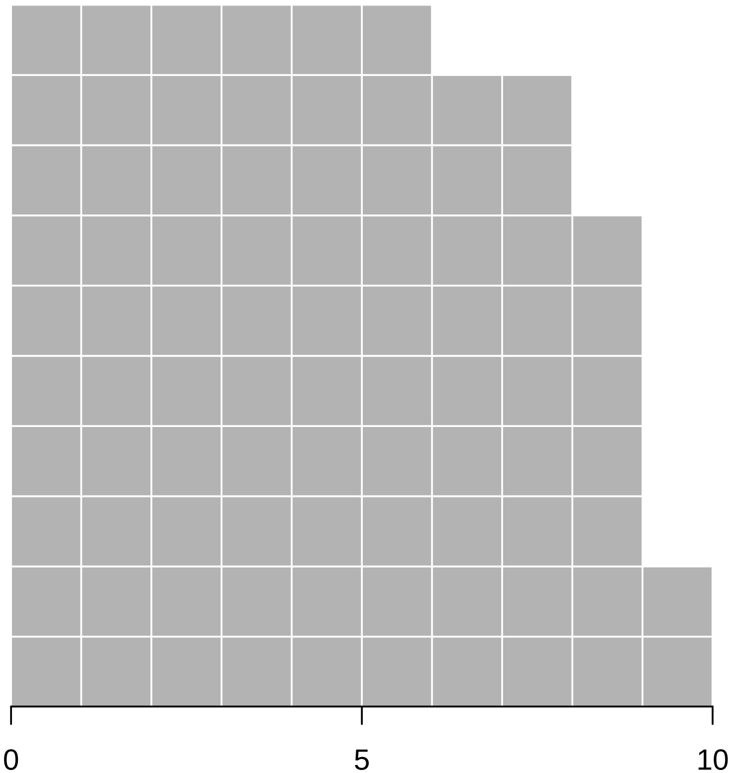

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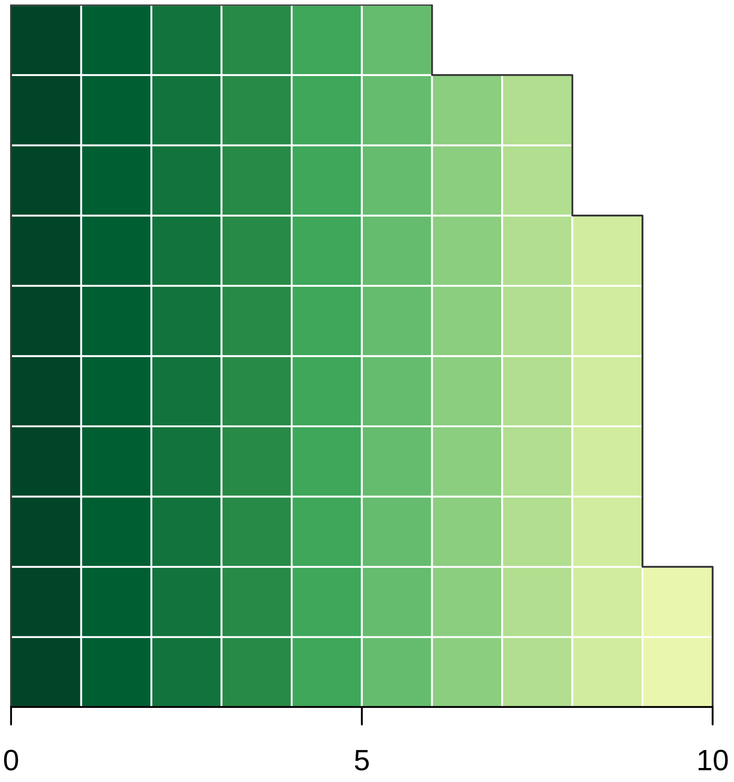

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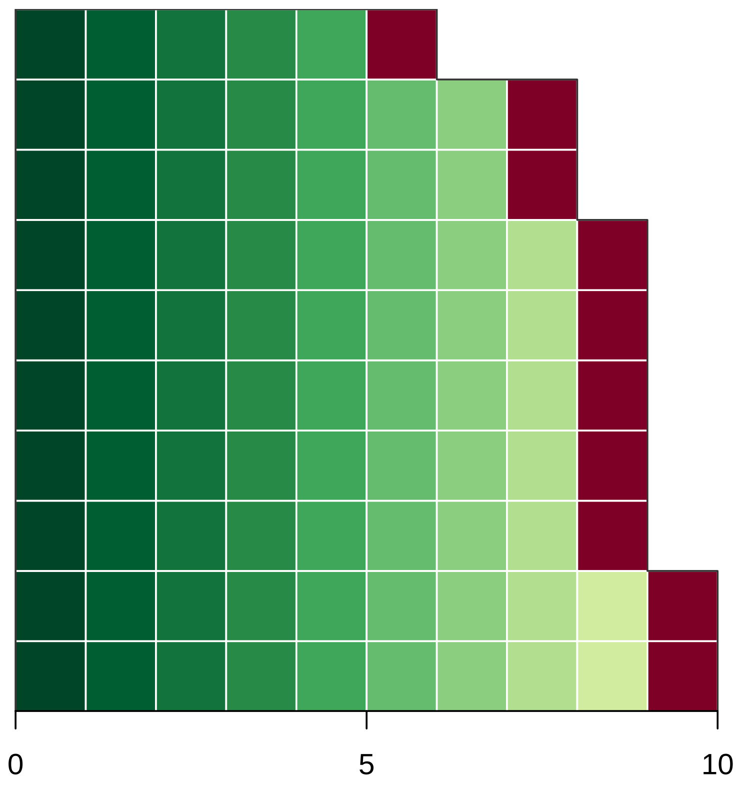

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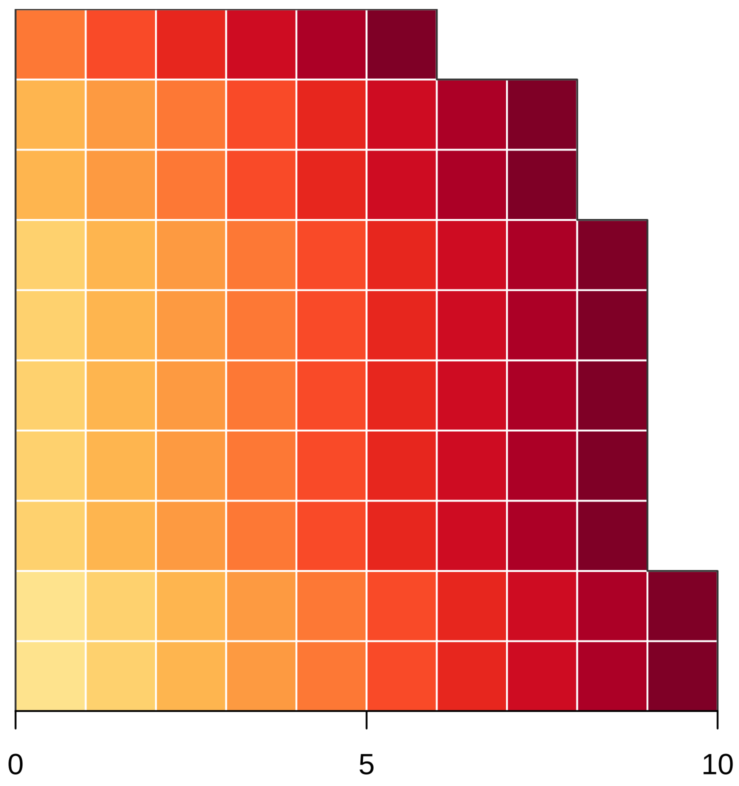

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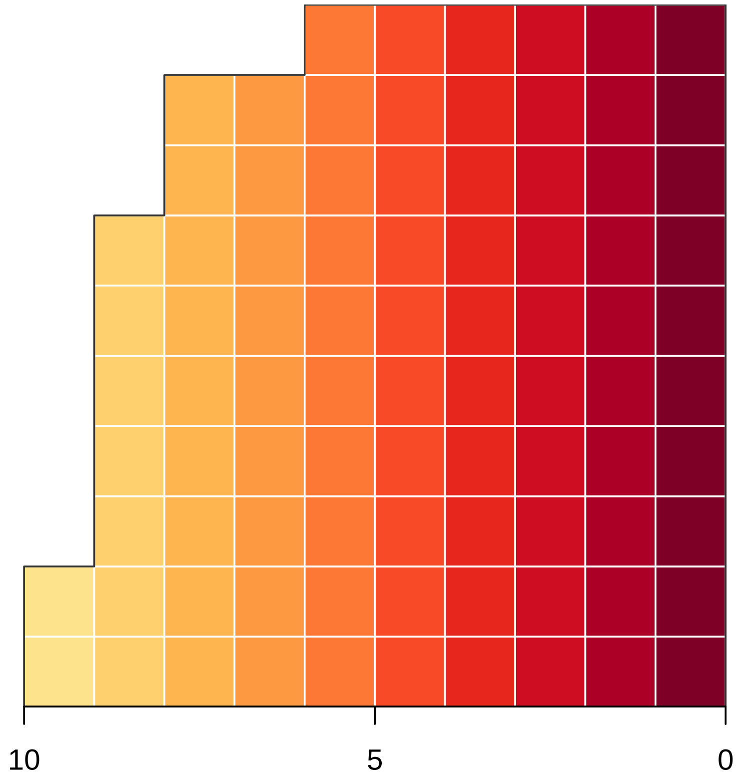

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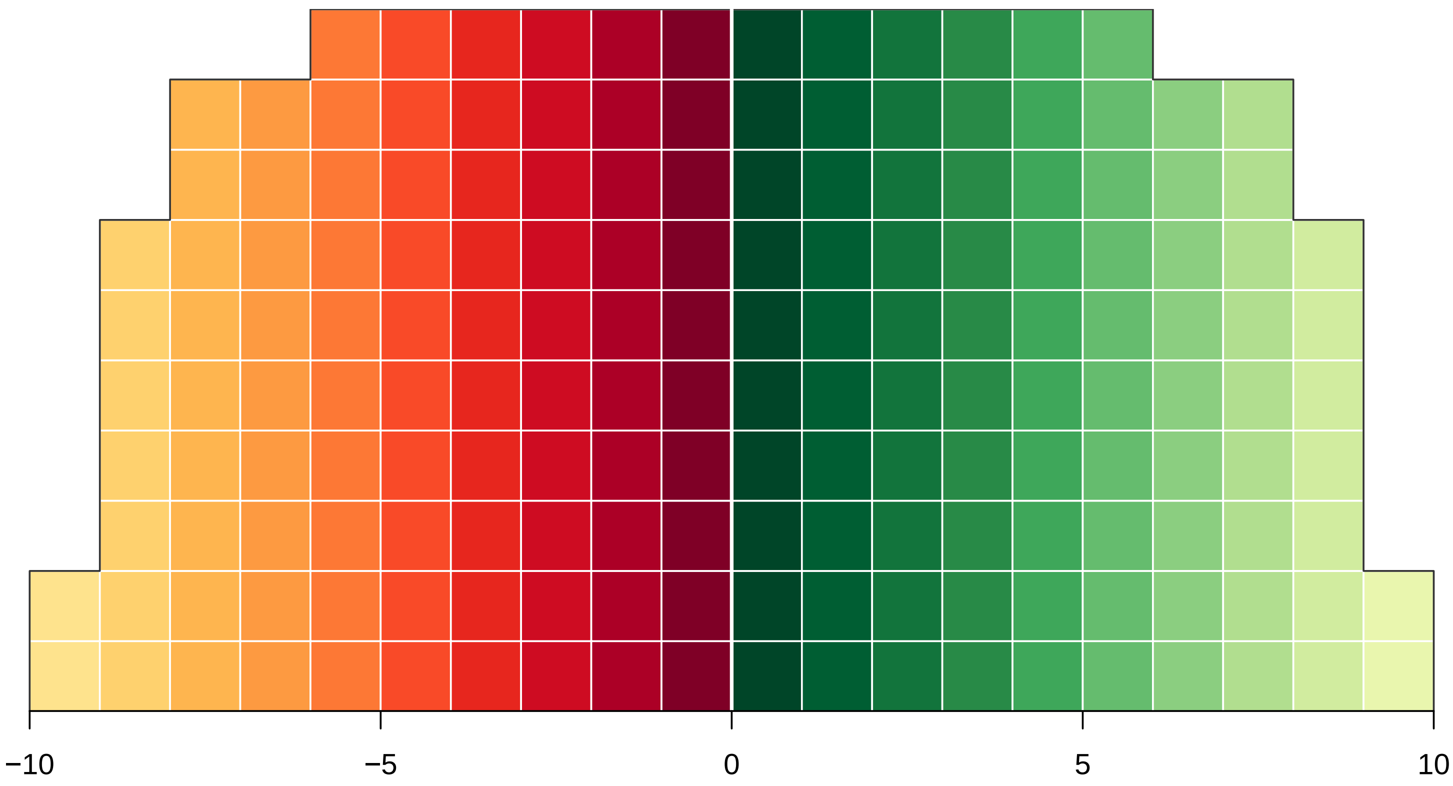

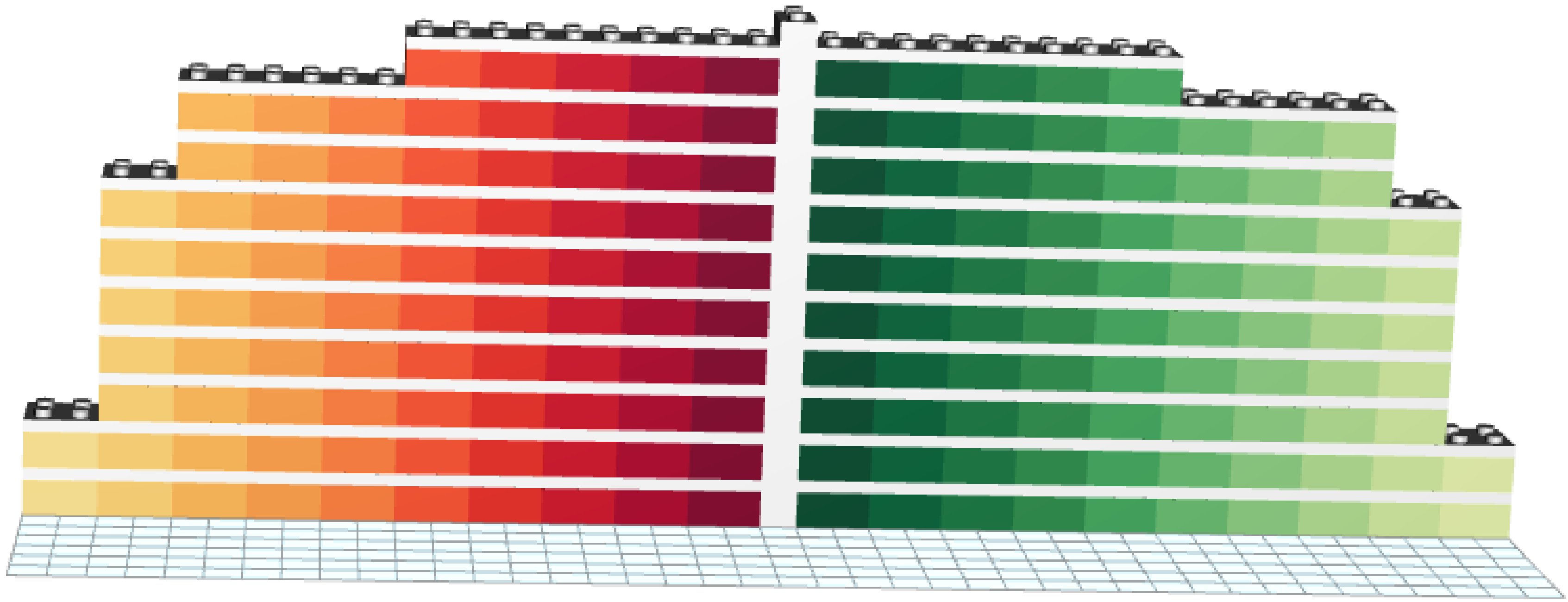

Brouard-Carey Lego design here:
http://www.publishyourdesign.com/design/70442

Brouard-Carey Symmetry

The age distribution is identical to / symmetrical with the time-to-death distribution.

Transient Symmetry

Within a given state, the time-spent distribution is equal to the time-to-exit distribution

Transient equality

Under stationarity, the probability that a randomly selected individual is in state s and entered s x years ago is equal to the probability of being in state s and exiting in x years.

Requisites:

- all vital and state transition schedules fixed
- no growth (births = deaths)

Probabilistic result:

- The expected age-state structure is frozen.
- Each potential discrete state trajectory has a fixed probability of occurring.
- Same for past and future cohorts.

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Deterministic result (friendly):

- The age-state structure is frozen.
- The same finite set of discrete state trajectories
- Same for past and future cohorts (all clones).

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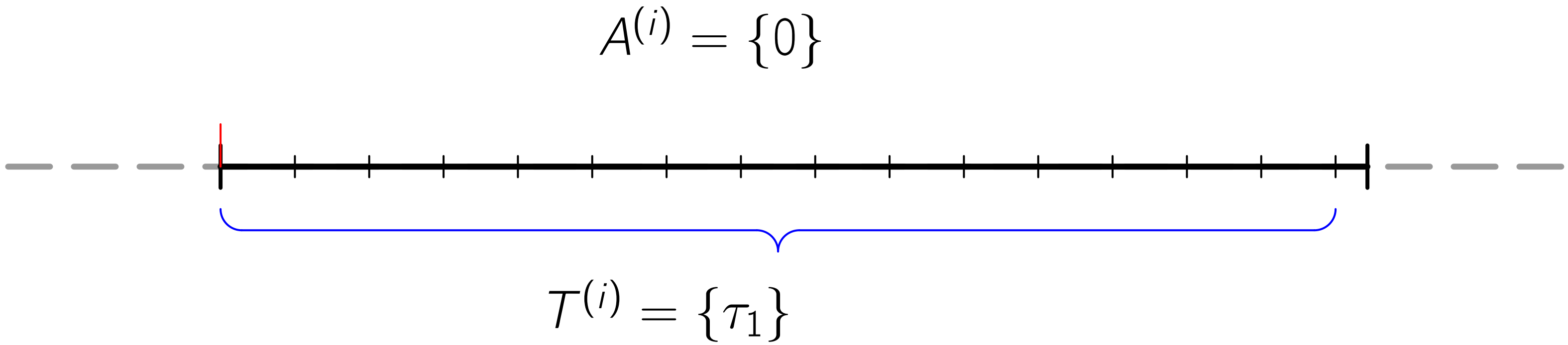
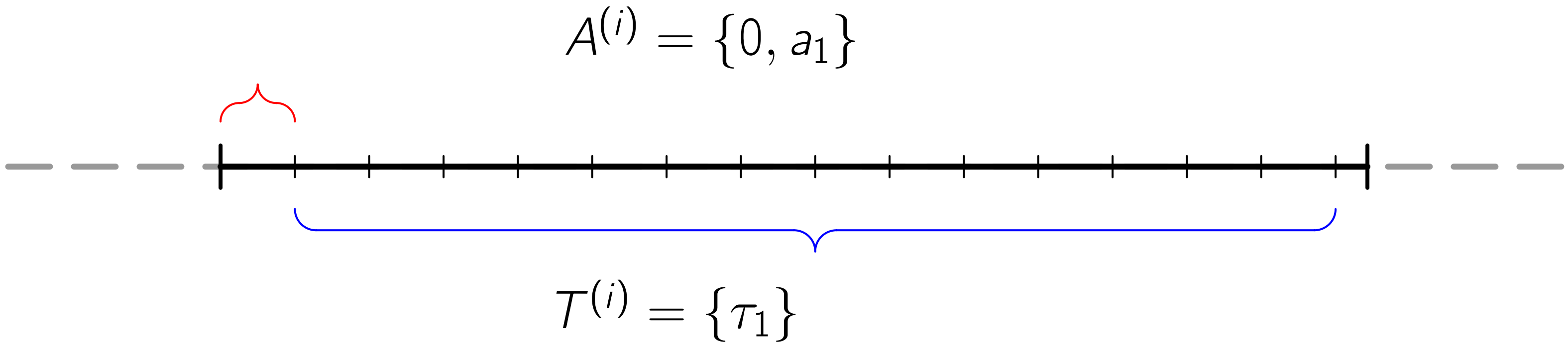

$A^{(i)} = \{0\}$
$T^{(i)} = \{\tau_1\}$


$A^{(i)} = \{0, a_1\}$
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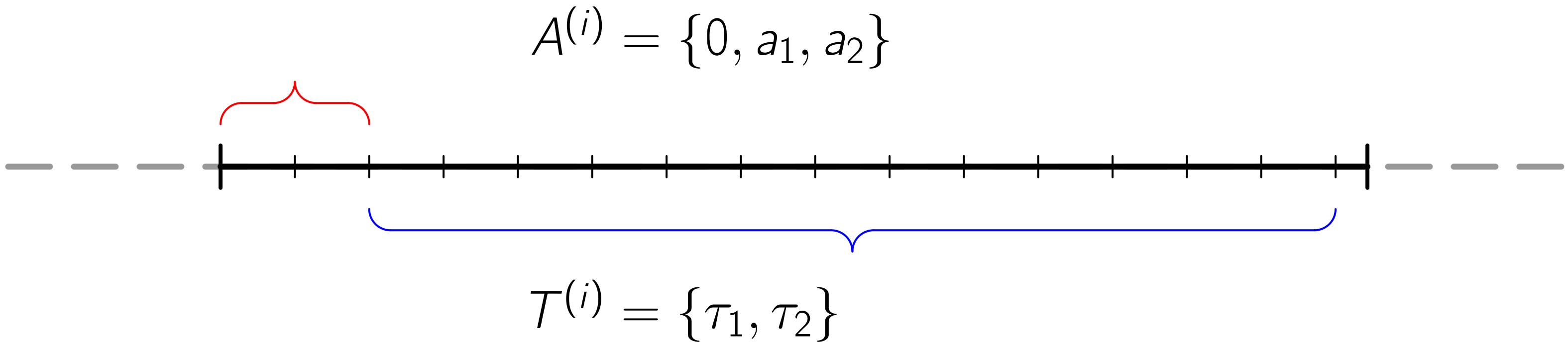
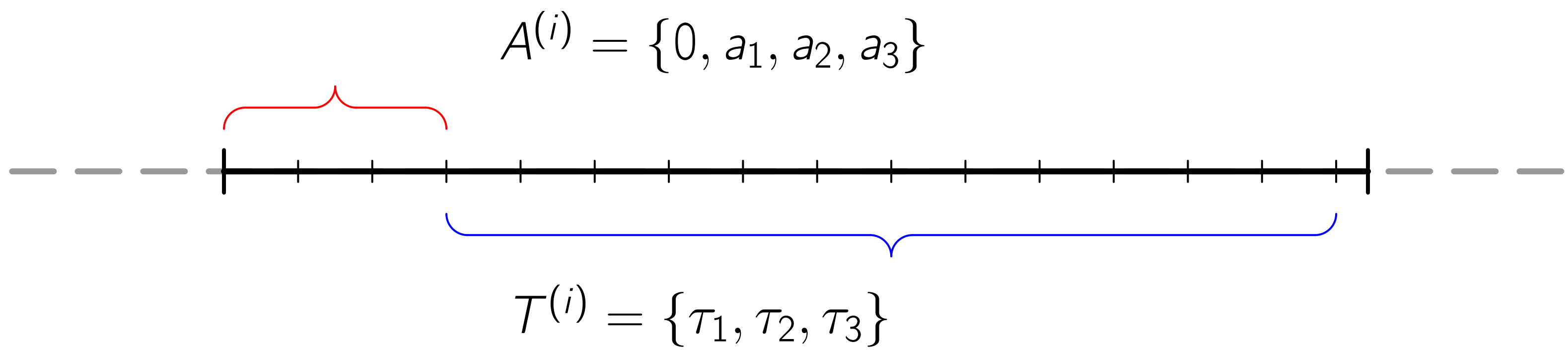
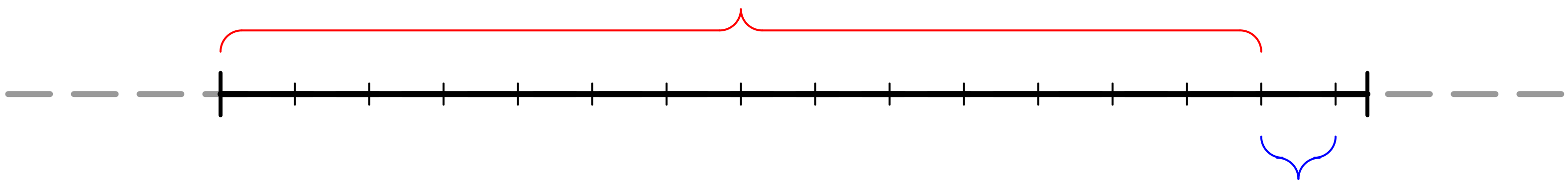

$A^{(i)} = \{0, a_1, a_2\}$
$T^{(i)} = \{\tau_1, \tau_2\}$

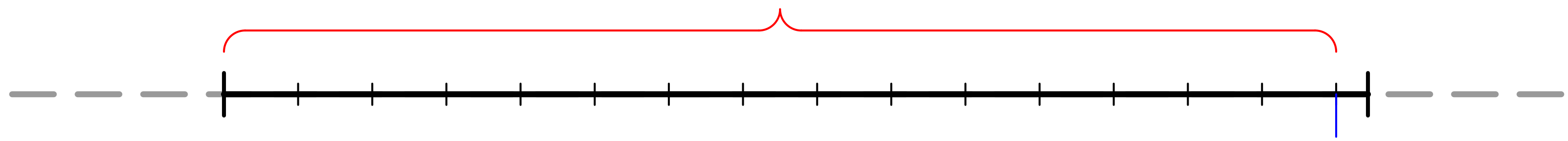

$A^{(i)} = \{0, a_1, a_2, a_3\}$
$T^{(i)} = \{\tau_1, \tau_2, \tau_3\}$

$$A^{(i)} = \{0, a_1, a_2, a_3, \ldots, a_{K-1}\}$$



$$T^{(i)} = \{\tau_1, \tau_2, \tau_3, \ldots, \tau_{K-1}\}$$

$$A^{(i)} = \{0, a_1, a_2, a_3, \ldots, a_{K-1}, a_K\}$$



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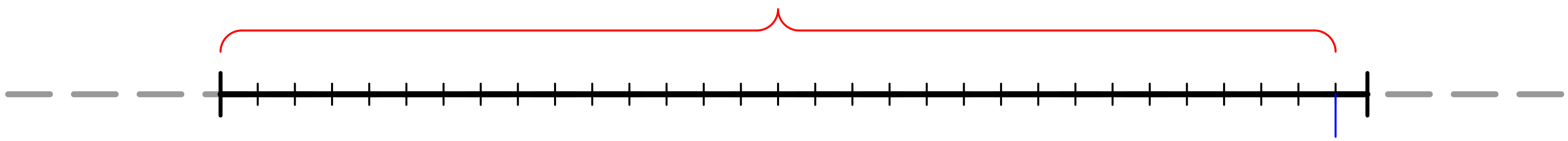


$$T^{(i)} = \{\tau_1, \tau_2, \tau_3, \ldots, \tau_{K-1}, 0\}$$

Complementarity:

Within an individual over time

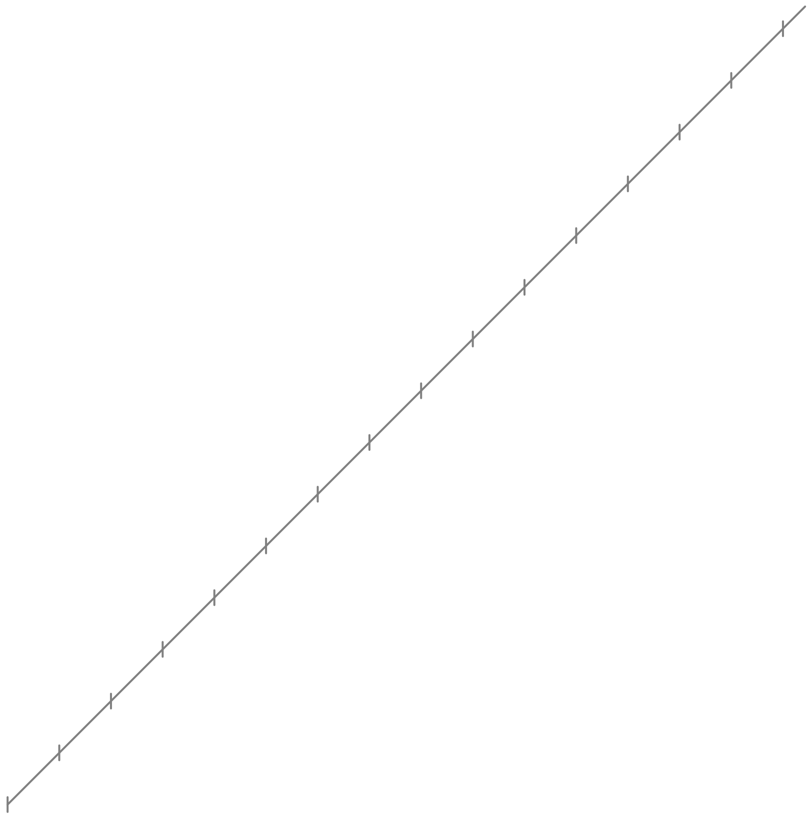
$$A^{(i)} = T^{(i)}$$

Complementarity:

for the union of two individuals

$$\{A^1, A^2\} = \{T^1, T^2\}$$

A clone is born every Δ time step



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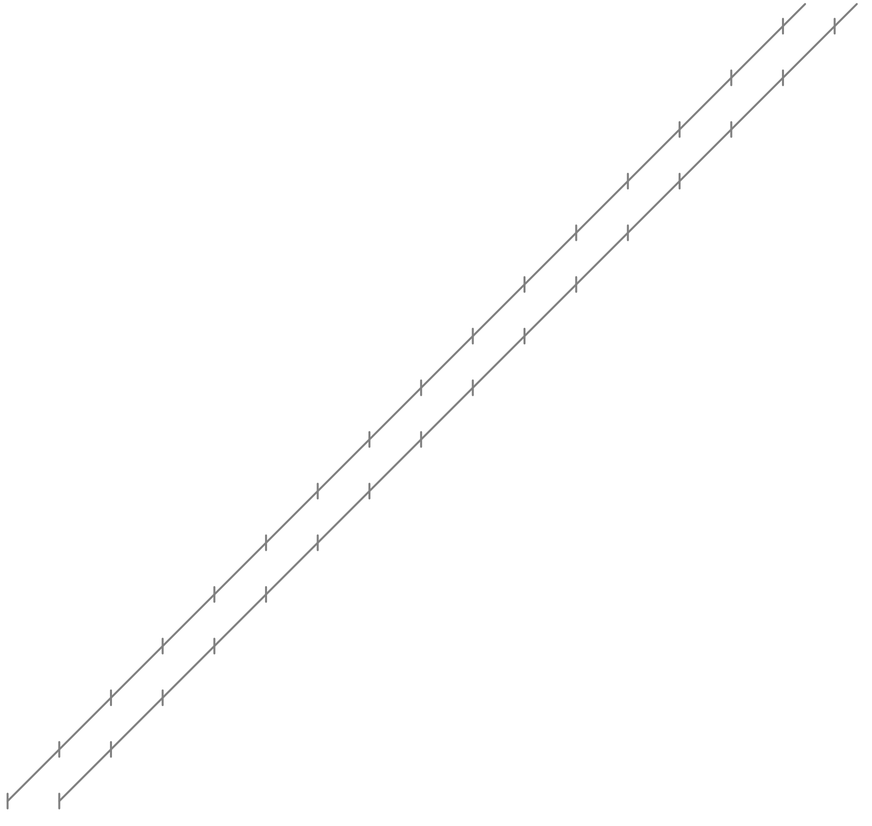

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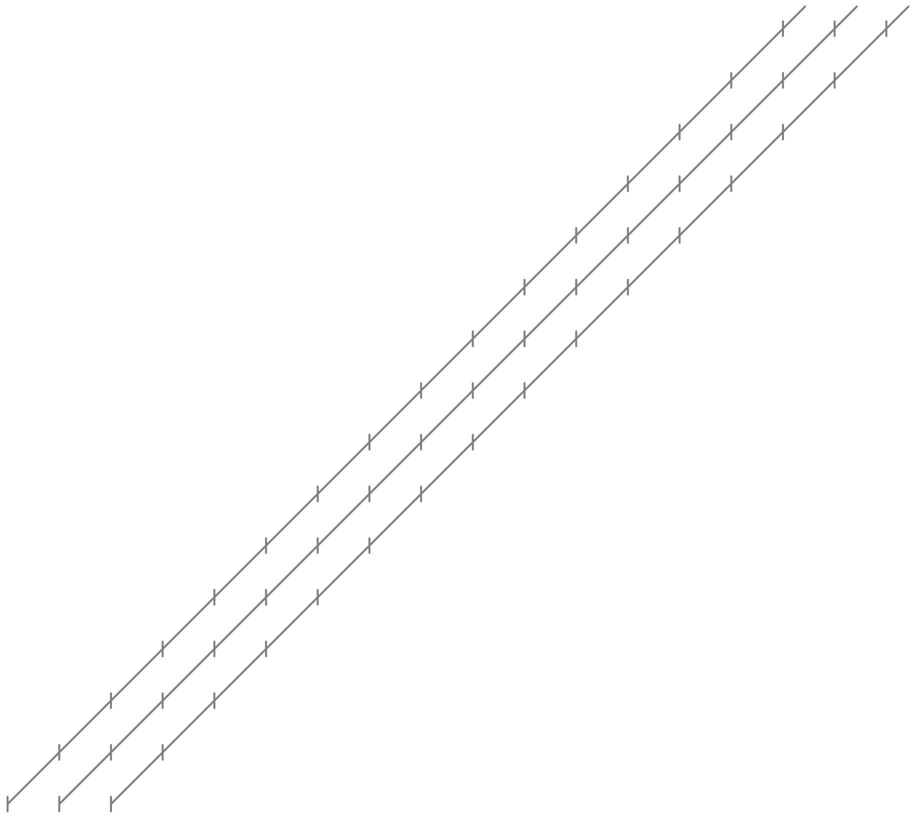
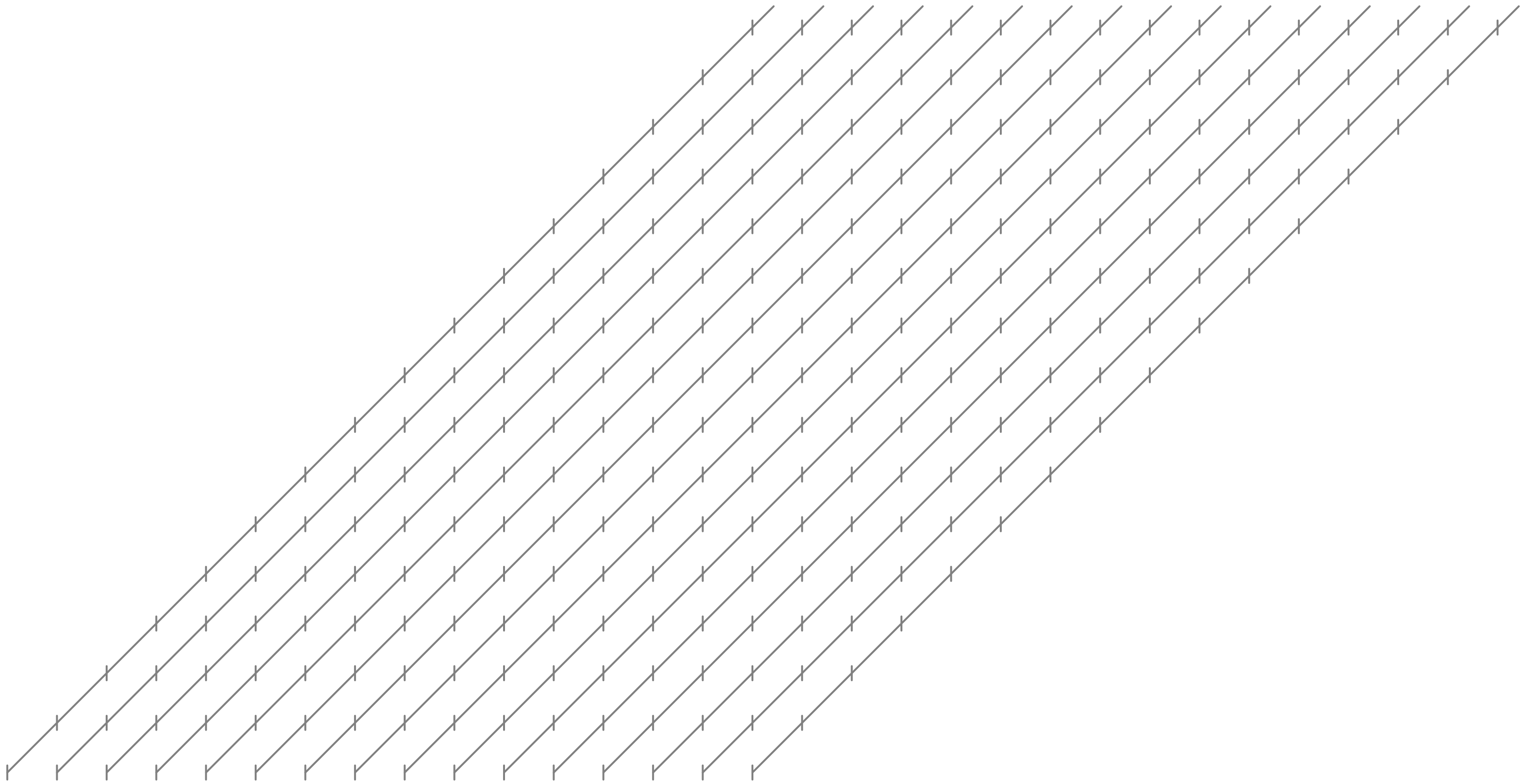
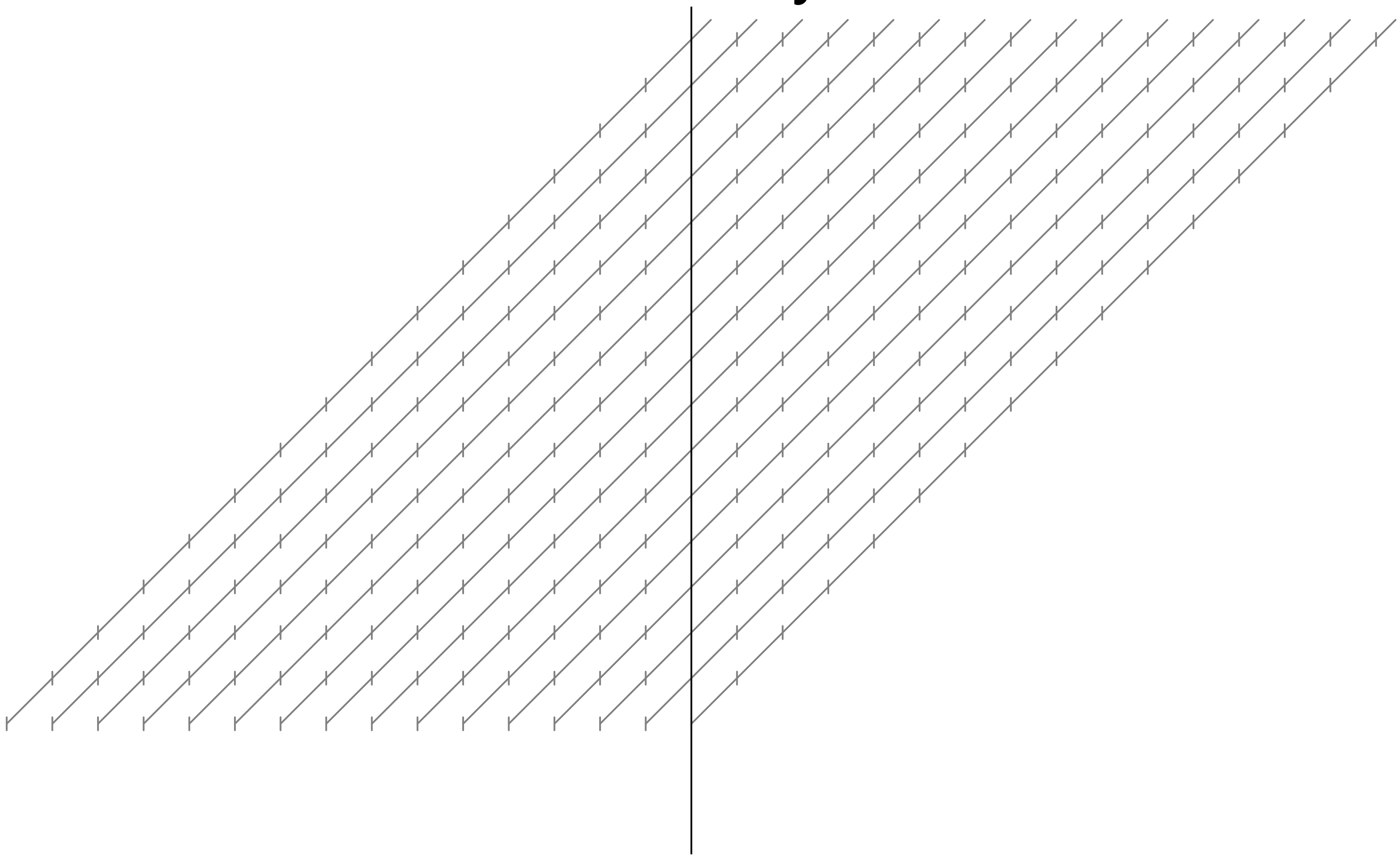


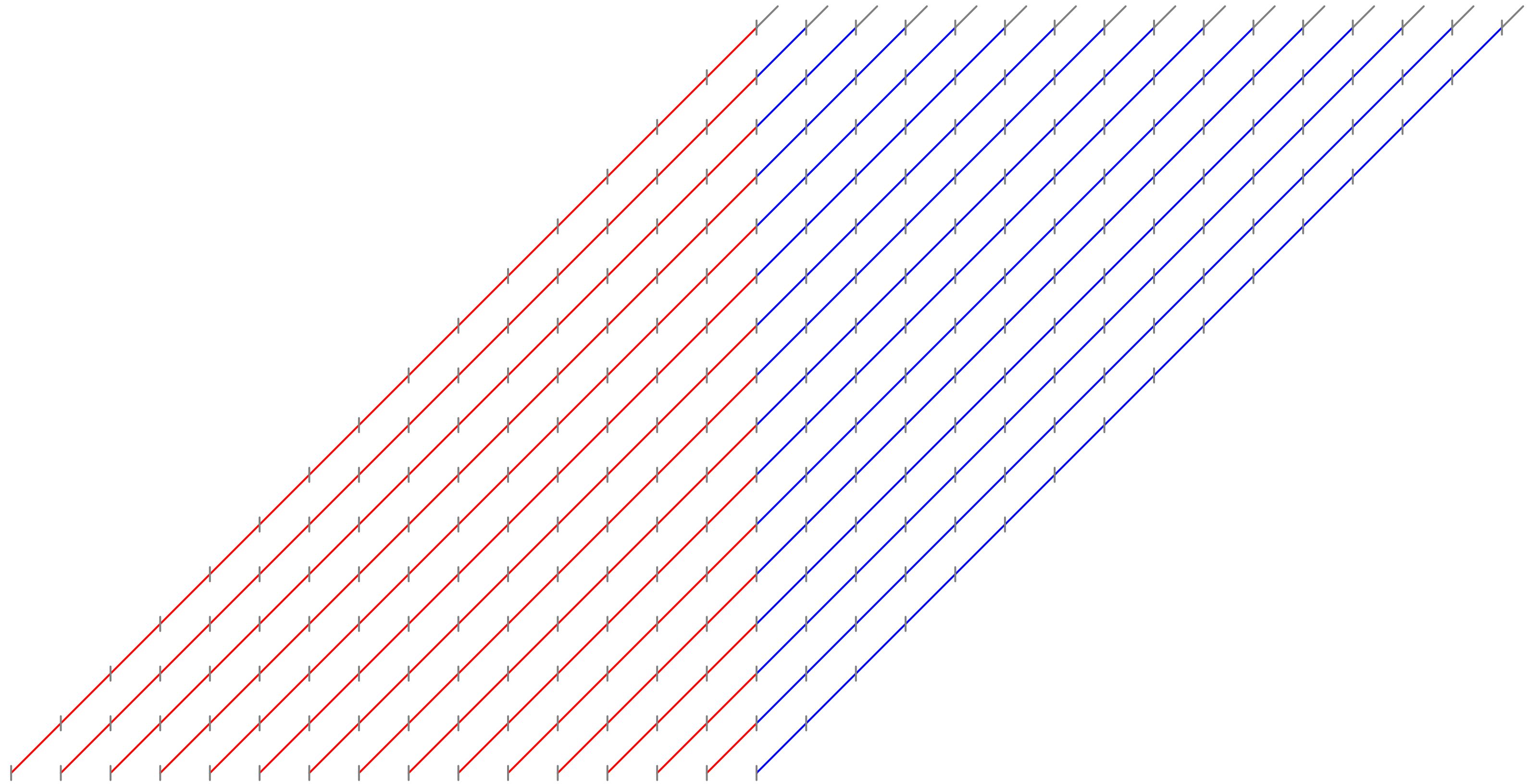
A stationary series of clones



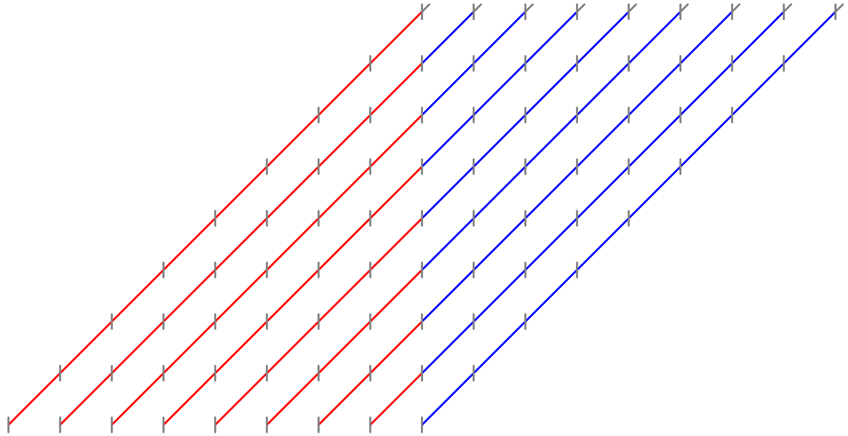
A census in stationary series



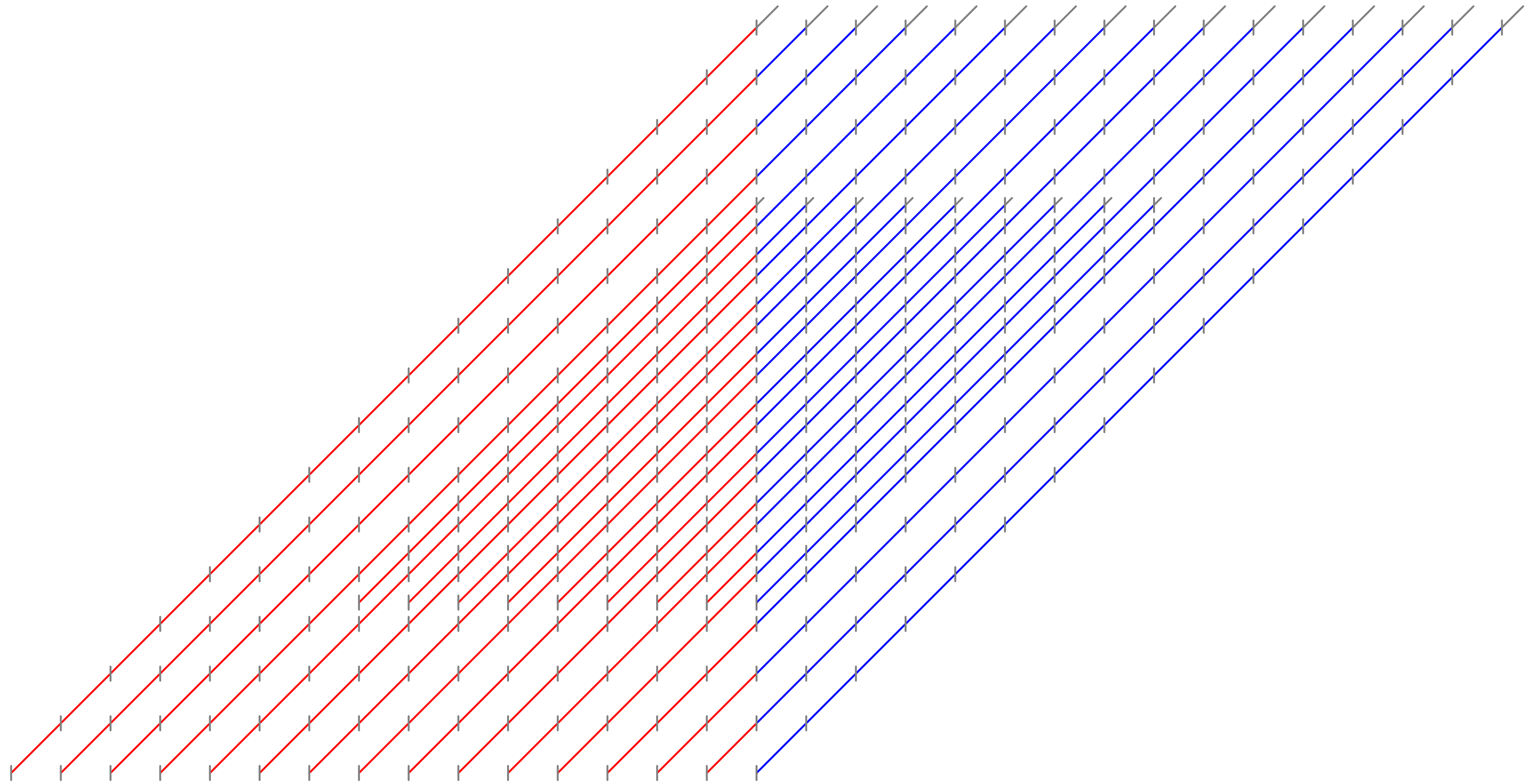
Complementarity in “clone” stationarity



Complementarity for some other episode



Complementarity for set unions



By induction, complimentarity for all episodes ■

- Make Δ smaller if you want
- Replace “clones” w “fixed fraction”
- Replace fraction w “fixed probability”

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Implications:

- Equal time spent-left distributions within states
- Cumulative over episodes
- Within episode order
- Conditional on anything (age, TTD)
- Brouard-Carey is degenerate case

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Photo by Eberhard Grossgasteiger on Unsplash

Simulate

- Take transition matrix from Dudel & Myrsklä (2017)
- Simulate trajectories; `rmarkovchain()` in `markovchain` package.
- Take census in stationary series.
- Assume observation at half interval.
- Tabulate time spent and left in sampled episodes.
- Compare distributions.

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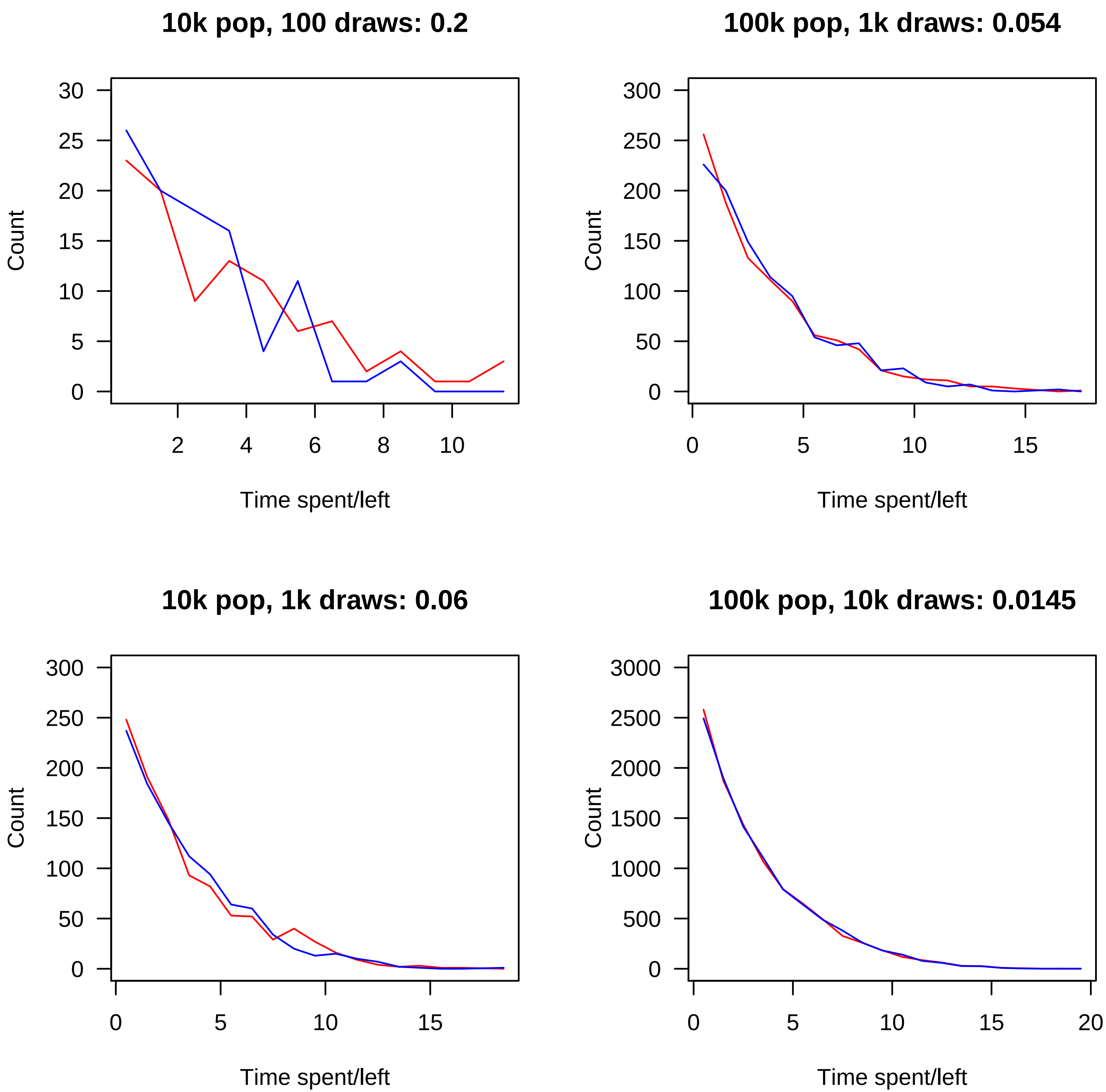
10k pop, 100 draws: 0.2
100k pop, 1k draws: 0.054
10k pop, 1k draws: 0.06
100k pop, 10k draws: 0.0145
Count
Time spent/left
0
5
10
15
20
25
30
2
4
6
8
10
50
100
150
200
250
300
500
1000
1500
2000
2500
3000

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Estimate from the reflection!

Thanks!

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