MAX PLANCK INSTITUTE
FOR DEMOGRAPHIC
RESEARCH

# Time spent and left of transient states in stationary populations

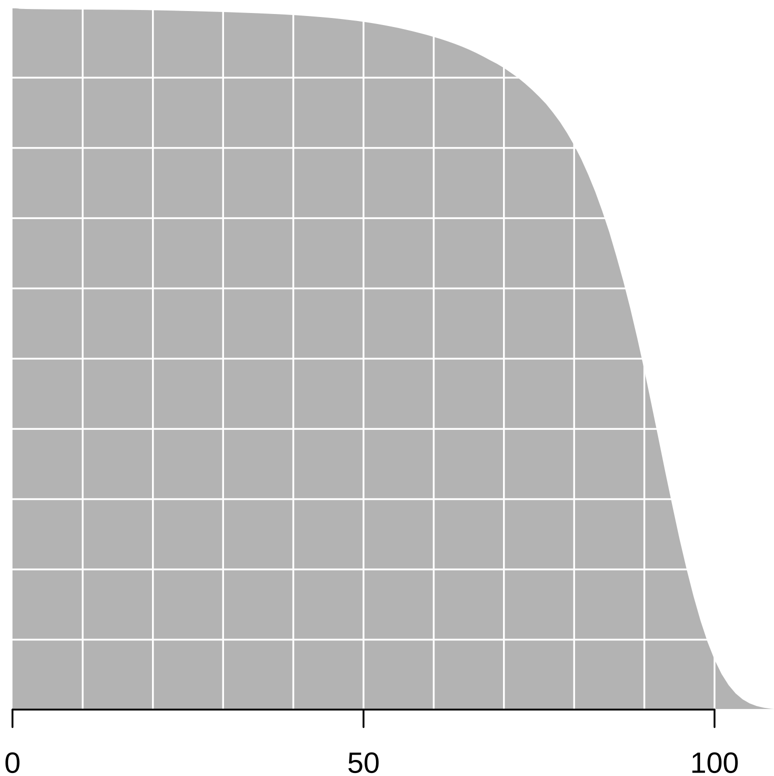
Tim Riffe & Francisco Villavicencio

**Brouard-Carey equality**

Under stationarity, the population aged $x$ equals the population with $x$ life left to live.

(Brouard, 1989; Vaupel, 2009; Villavicencio & Riffe, 2016)

# A visual explanation



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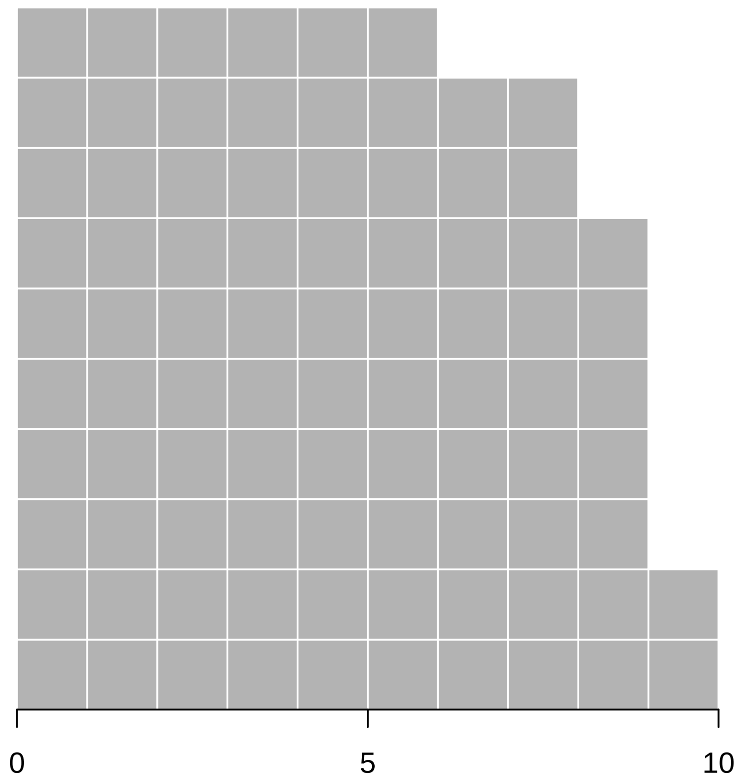

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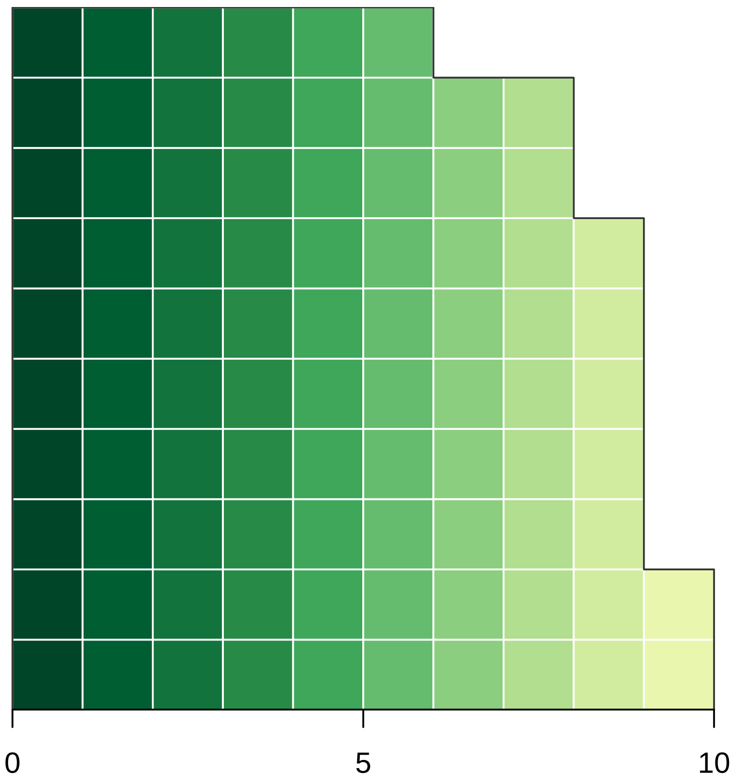

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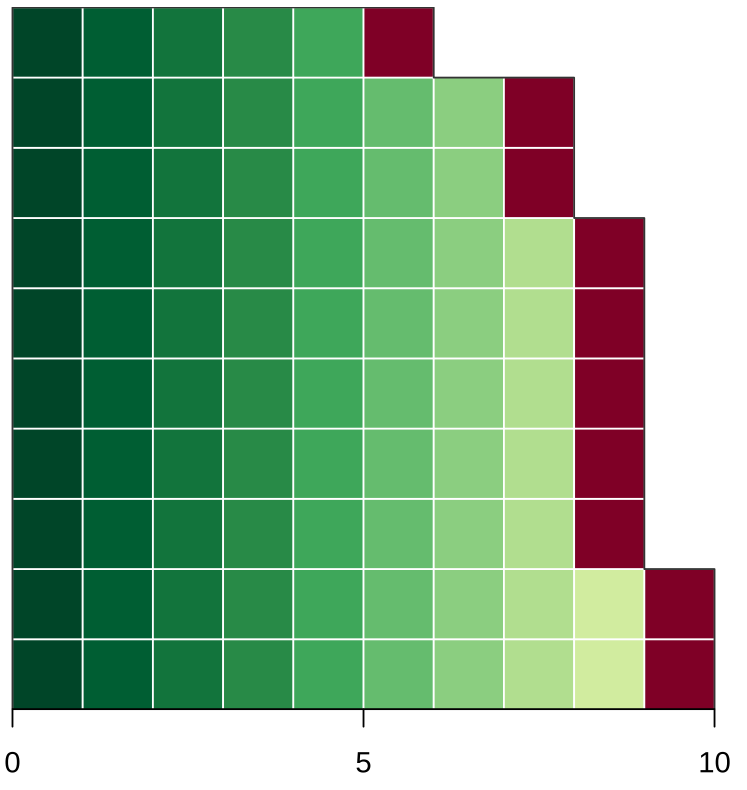

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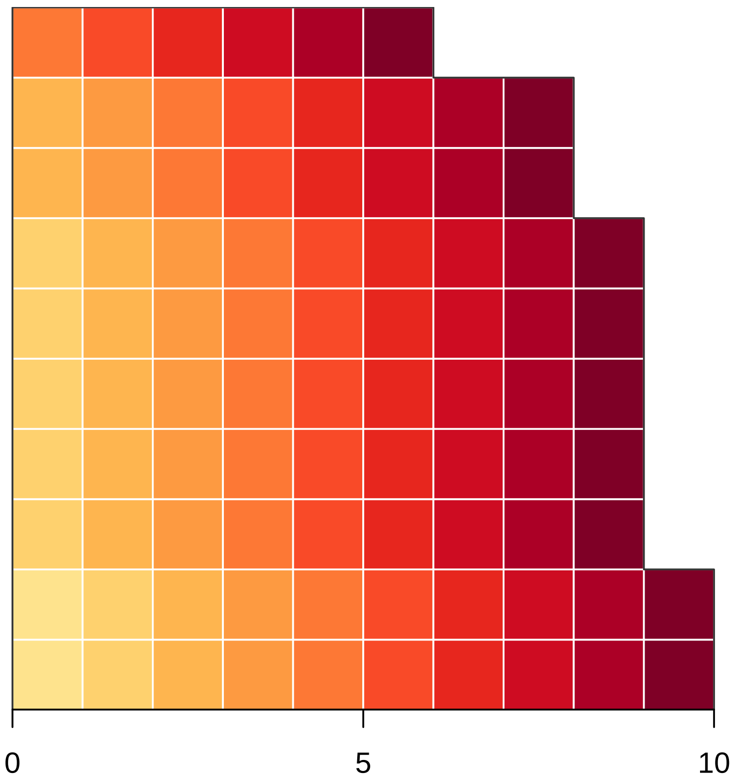

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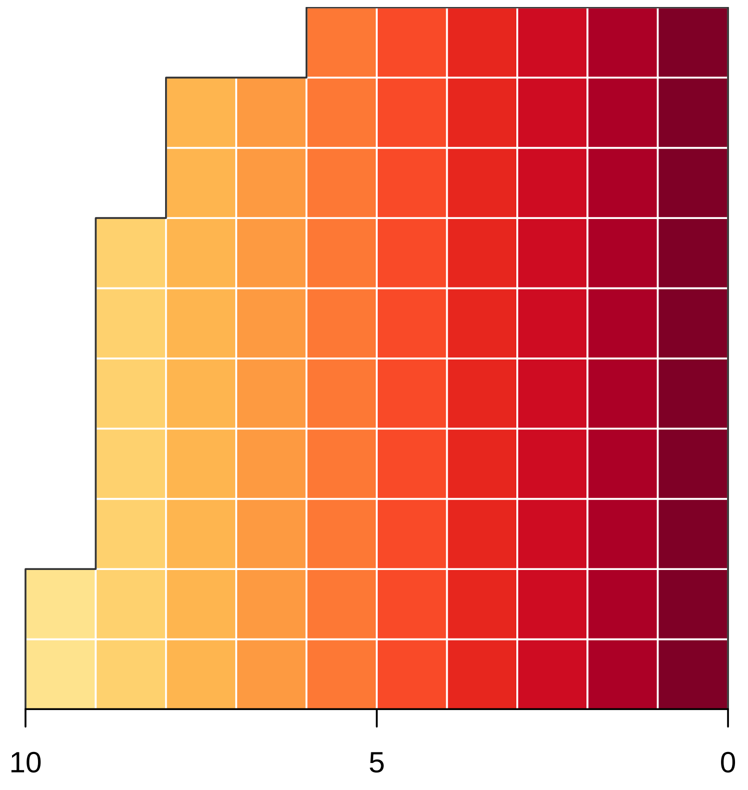

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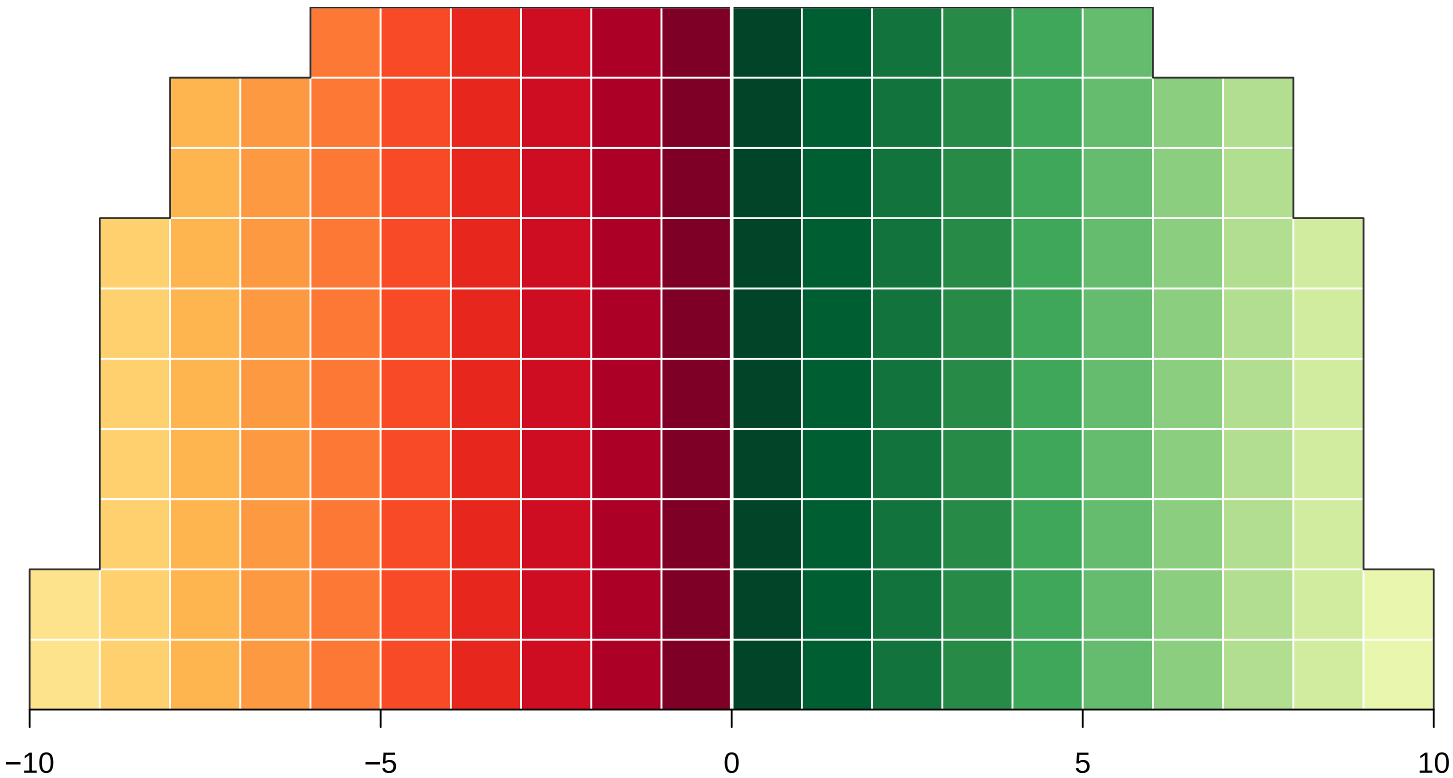
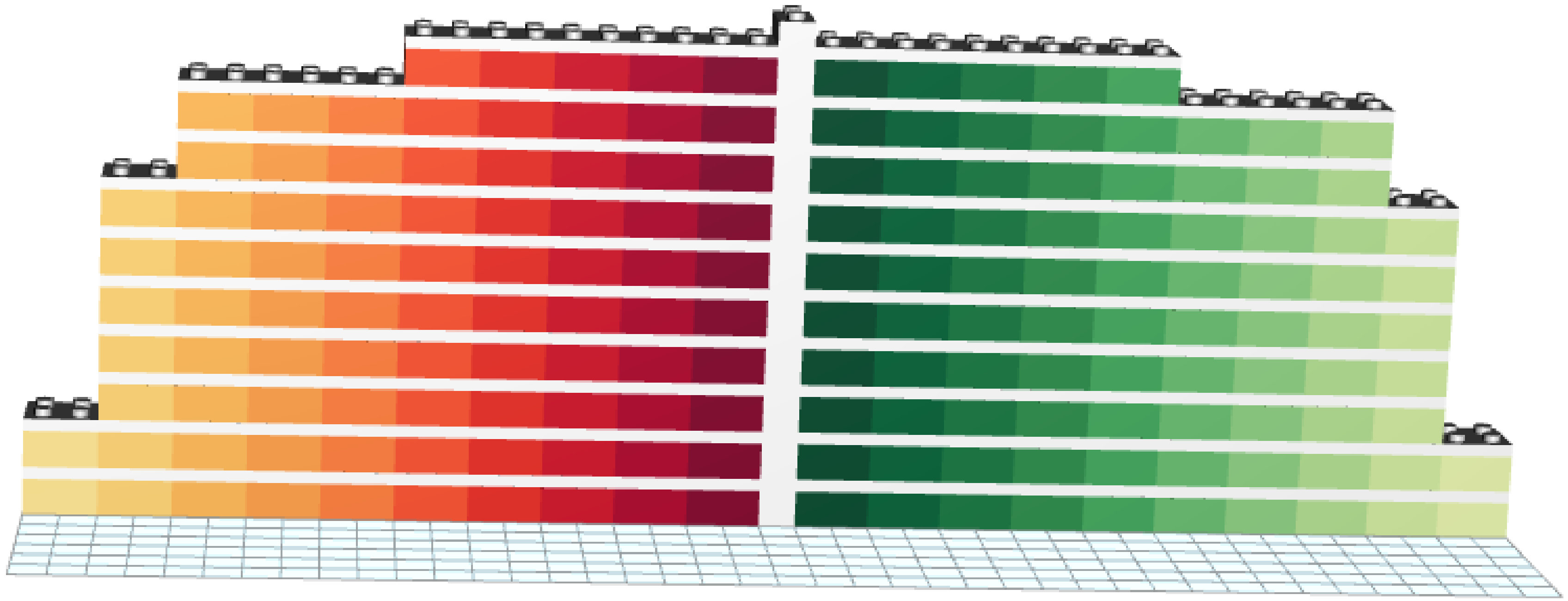




Brouard-Carey Lego design here:
http://www.publishyourdesign.com/design/70442

**Brouard-Carey Symmetry**

The age distribution is identical to / symmetrical with the time-to-death distribution.

**Transient Symmetry**

Within a given state, the time-spent distribution is equal to the time-to-exit distribution

**Transient equality**

Under stationarity, the probability that a randomly selected individual is in state $s$ and entered $s$ $x$ years ago is equal to the probability of being in state $s$ and exiting in $x$ years.

Requisites:

- all vital and state transition schedules fixed
- no growth (births = deaths)

Probabilistic result:

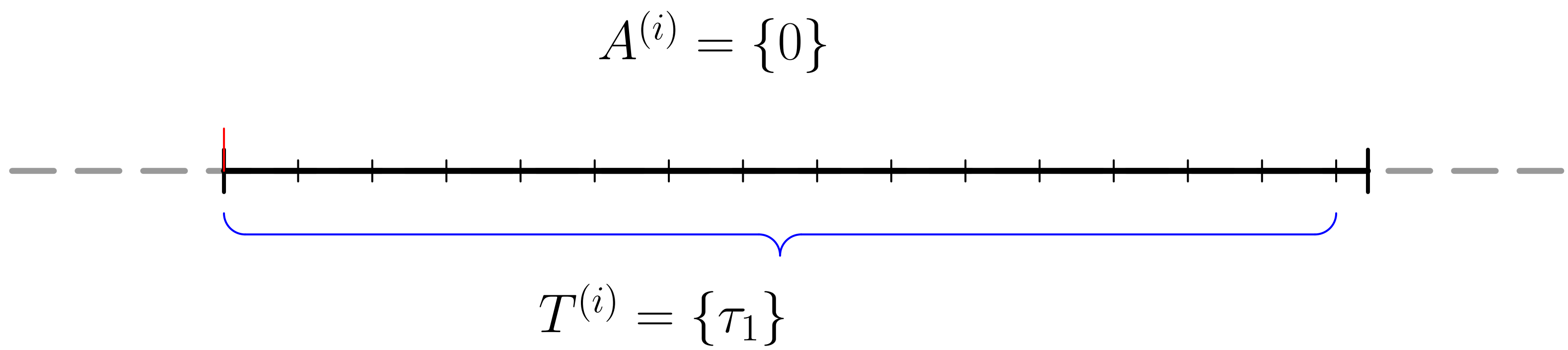
- The expected age-state structure is frozen.
- Each potential discrete state trajectory has a fixed probability of occurring.
- Same for past and future cohorts.

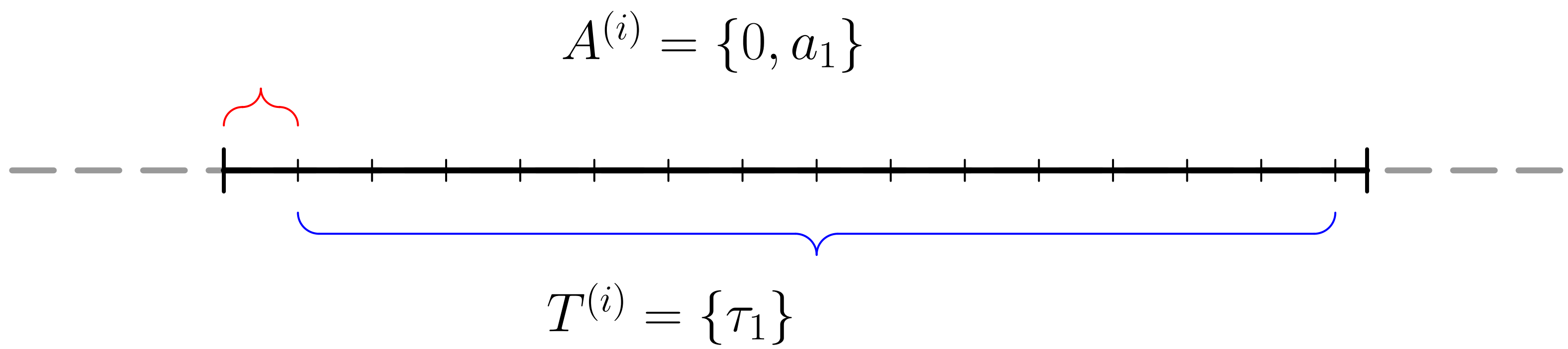
Deterministic result (problematic):

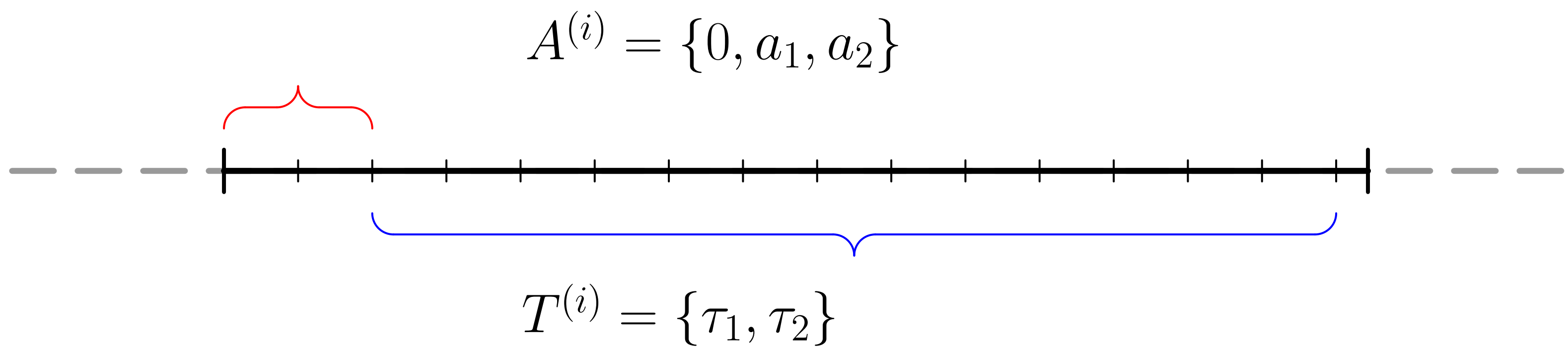
- The age-state structure is frozen.
- Each discrete state trajectory occurs for a fixed fraction of a birth cohort.
- Same for past and future cohorts.

Deterministic result (friendly):

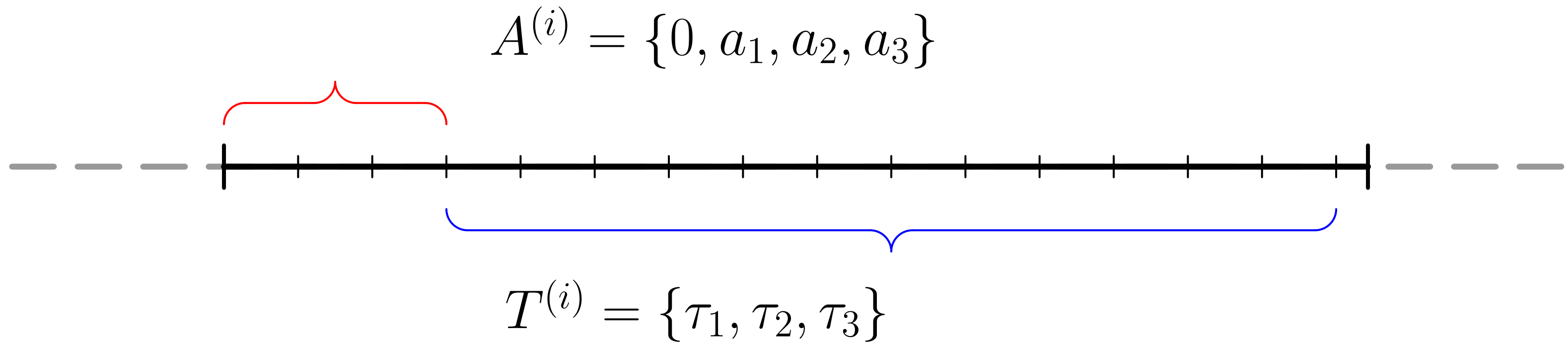
- The age-state structure is frozen.
- The same finite set of discrete state trajectories
- Same for past and future cohorts (all clones).

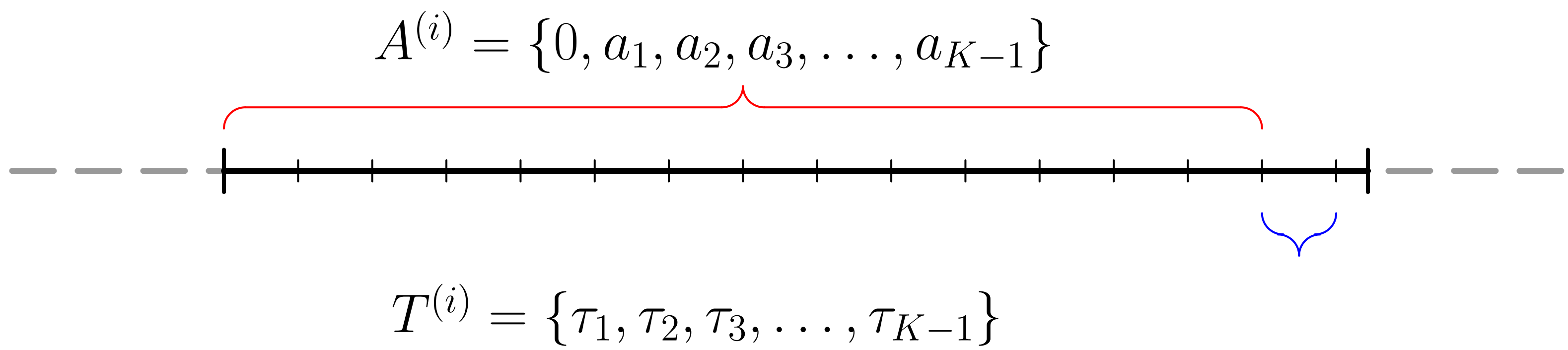

$A^{(i)} = \{0\}$
$T^{(i)} = \{\tau_1\}$


$A^{(i)} = \{0, a_1\}$
$T^{(i)} = \{\tau_1\}$

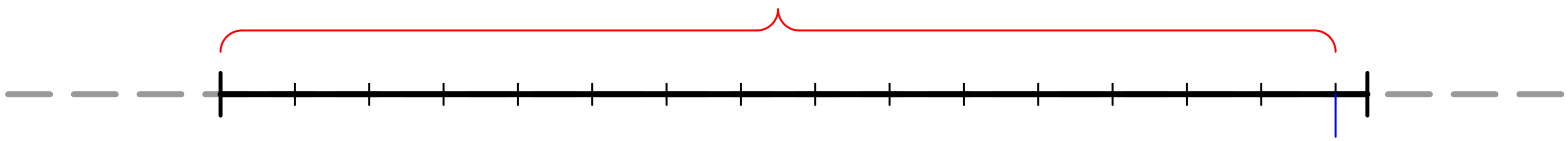


$A^{(i)} = \{0, a_1, a_2\}$
$T^{(i)} = \{\tau_1, \tau_2\}$


$A^{(i)} = \{0, a_1, a_2, a_3\}$
$T^{(i)} = \{\tau_1, \tau_2, \tau_3\}$

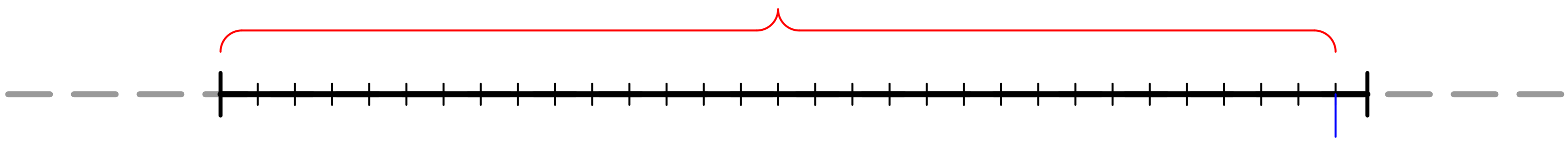

$A^{(i)} = \{0, a_1, a_2, a_3, \dots, a_{K-1}\}$
$T^{(i)} = \{\tau_1, \tau_2, \tau_3, \dots, \tau_{K-1}\}$

$$A^{(i)} = \{0, a_1, a_2, a_3, \ldots, a_{K-1}, a - K\}$$



$$T^{(i)} = \{\tau_1, \tau_2, \tau_3, \ldots, \tau_{K-1}, 0\}$$

$$A^{(i)} = \{0, a_1, a_2, a_3, \ldots, a_{K-1}, a - K\}$$



$$T^{(i)} = \{\tau_1, \tau_2, \tau_3, \ldots, \tau_{K-1}, 0\}$$

Complementarity:

Within an individual over time

$$A^{(i)} = T^{(i)}$$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