A1)

a) $$V(s) = \max_{a'} \sum_{s',r} p(s',r|s,a')\left[r + \gamma v(s')\right]$$

$$\leq \underbrace{\max_{a'} \sum_{s',r} p(s',r|s,a')}_{=1} r_{max}$$

$$+ \gamma \underbrace{\max_{a'} \sum_{s',r} p(s',r|s,a')\, v(s')}_{\leq V(s)\ \forall r \leq r_{max}}$$

$$\leq r_{max} + \gamma V(s)$$

$$\Rightarrow V(s) \leq \frac{r_{max}}{1-\gamma}$$

Same proof with $\frac{r_{min}}{1-\gamma} \leq V(s)$

$$\frac{r_{min}}{1-\gamma} \leq V(s) \leq \frac{r_{max}}{1-\gamma} \qquad \square$$

$$|v(s) - v(s')| \leq \frac{r_{max} - r_{min}}{1-\gamma} \leq \underbrace{V_{max} - V_{min}}$$

max. diff. from one state to the another.

11 a) $$\underbrace{\| (Tv)(s) - (Tv')(s) \|_\infty}_{} = \gamma \| v - v' \|_\infty$$

$$\max_a \left| \sum_{s',r} p(s',r|s,a)\left[r + \gamma v(s')\right] - \sum_{s',r} p(s',r|s,a)\left[r + \gamma v'(s')\right] \right|$$

$$= \max_a \left| \sum_{s',r} p(s',r|s,a)\left[r + \gamma v(s') - \left(\cancel{\gamma} r + \gamma v'(s')\right)\right] \right|$$

$$= \gamma \max_a \left| \sum_{s',r} p(s',r|s,a)\left[v(s') - v'(s')\right] \right|$$

$$\leq \gamma \underbrace{\max_a \left| \sum_{s',r} p(s',r|s,a)\right.}_{=1} \max_s \left[v(s) - v'(s)\right]$$

$$= \gamma \max_s |v(s) - v'(s)| = \gamma \| v - v' \|_\infty$$