Econ 828 Term Project: How much is too much? Beliefs about perceived inequality in a coordination game

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3.1 Payoffs

The underlying model of the coordination game in this paper is that of a stag hunt with the added dimension of a third party whose decisions potentially affect the payoffs of the other two players who try to coordinate. In the context of inequality, the poor are the parties that play the traditional stag hunt game while the rich person has influence over the exact form of the game by deciding whether to purchase insurance. The resulting payoffs for poor players are shown in Table 1.

In order to correctly represent the classical stag hunt game, it has to be the case that the high payoff for successfully coordinating on the payoff dominant strategy is greater than the

Table 1: General Payoffs of Poor

Under `no insurance` (n)

	revolt	do nothing
revolt	h_n, h_n	l_n, m_n
do nothing	m_n, l_n	m_n, m_n

Under `insurance` (i)

	revolt	do nothing
revolt	h_i, h_i	l_i, m_i
do nothing	m_i, l_i	m_i, m_i

Payoffs of poor players depending on the other poor player's decision and depending on whether the rich person opted for insurance.

medium payoff for playing from playing the risk dominant strategy, which in turn is greater than the payoff from unsuccessfully playing the risky strategy. That is, $h_j > m_j > l_j$ in both the `no insurance` and the `insurance` case (i.e. $\forall j \in \{n, i\}$). Additionally, successful coordination on the risky action results in the same payoff regardless of the actions of the rich player ($h_n = h_i = h$) and not revolting is risk free not only within a game but also across the two different games resulting from the rich's decisions ($m_n = m_i = m$). Lastly, revolting unsuccessfully is worse if the rich player takes out insurance ($l_i < l_n$) and can be seen as the rich person "fighting back". These assumptions yield a simplified payoff table for poor players shown in Table 2.

Table 2: Simplified Payoffs of Poor

	revolt	do nothing
revolt	h, h	l_j, m
do nothing	m, l_j	m, m

Where $j \in \{i, n\}$ and $l_n > l_i$.

The decision made by the rich player is that of purchasing insurance against a possible revolution. His payoffs depend on whether a revolution was attempted and, if so, whether it was successful. Since a revolutions is only successful if both poor players decide to play the risky action of "revolt" , this case corresponds to the outcome with two revolutionaries. An outcome with only one revolutionary represents an unsuccessful revolution and zero revolutionaries mean nobody tried to revolt.

If a revolutions is successful, having insurance has no effect on the rich's payoffs. That is, in terms of Table 3, $y_{2i} = y2n = y_2$. Instead, insurance is only effective in the case of no or failed revolutions (here denoted by the subscript f): $y_{0i} = y_{1i} = yfi$. This is better than the outcome y_{1n}, which represents an attempted but unsuccessful revolutions and can be thought of as being damaging to the rich nonetheless. The best possible outcome for the rich player is not buying insurance and none of the poor attempting a revolution resulting in y_{0n}.[1] Purchasing insurance can therefore be thought of as costing $y_{0n} - y_{fi}$ and paying out $y_{fi} - y_{1n}$ in the case of a failed revolution attempt.

Table 3: Payoffs of Rich

General Form

number of revolutionaries	insurance	no insurance
0	y_{0i}	y_{0n}
1	y_{1i}	y_{1n}
2	y_{2i}	y_{2n}

Simplified Form

number of revolutionaries	insurance	no insurance
0	y_{fi}	y_{0n}
1	y_{fi}	y_{1n}
2	y_2	y_2

3.2 Decision Making Process

Given these payoffs, a rational poor player will choose to revolt conditional on beliefs about what the other poor player will do and whether the rich person will buy insurance. Let a poor player believe that the other poor player chooses to revolt with probability γ and that the rich player purchases insurance with probability δ. Then a payoff maximizing poor person will revolt if

$$\gamma h + (1-\gamma)\left[(1-\delta)l_n + \delta l_i\right] \geq m$$

A rich person will decide to purchase insurance based on his beliefs about how many poor players will choose to revolt. In the most general case, let a rich player believe that there are zero revolutionaries with probability α, one revolutionary with probability β and

[1] In summary: $y_{0n} > y_{0i} = y_{1i} = y_{fi} > y_{1n} > y_{2i} = y_{2n} = y_2$

two revolutionaries with probability $1-\alpha-\beta$. Then a rich player will purchase insurance if

$$(\alpha+\beta)Y_{fi}+(1-\alpha-\beta)y_2 \geq \alpha y_{0n}+\beta y_{1n}+(1-\alpha-\beta)y_2$$
$$(\alpha+\beta)Y_{fi} \geq \alpha y_{0n}+\beta y_{1n}$$

Assuming independent and identical poor players, $\alpha=(1-\gamma)^2$, $\beta=2\gamma(1-\gamma)$ and $(1-\alpha-\beta)=\gamma^2$. Using this simplification, rich players buy insurance if

$$\left[(1-\gamma)^2+2\gamma(1-\gamma)\right]y_{fi} \geq (1-\gamma)^2 y_{0n}+2\gamma(1-\gamma)y_{1n}$$

which simplifies to

$$\gamma \geq \frac{y_{0n}-y_{fi}}{y_{fi}+y_{0n}-2y_{1n}} \equiv \gamma_1. \tag{I}$$

For the decision making process of poor players, this means that if their belief of the other poor player's probability of revolting satisfies condition (I), they believe that the rich player will purchase insurance for sure (i.e. $\delta=1$). Considering these second order beliefs, given that $\gamma>\gamma_1$, the poor player chooses to revolt if

$$\gamma h+(1-\gamma)l_i \geq m$$
$$\gamma \geq \frac{m-l_i}{h-l_i} \equiv \gamma_2. \tag{RI}$$

If the poor player believes the other poor player to be sufficiently unlikely to revolt so that the rich player does not purchase insurance (condition (I) is not satisfied and $\gamma<\gamma 1$), he will only revolt if

$$\gamma h_(1-\gamma)l_n \geq m$$
$$\gamma \geq \frac{m-l_n}{h-l_n} \equiv \gamma_3. \tag{RN}$$

Combining conditions (I), (RI) and (RN), poor players will

1. revolt under insurance if $\gamma \geq \max(\gamma_1, \gamma_2)$
2. not revolt due to the presence of insurance if $\gamma_1 \leq \gamma < \gamma_2$
3. revolt under no insurance if $\gamma_3 \leq \gamma < \gamma_1$
4. not revolt under no insurance if $\gamma < \min(\gamma_1, \gamma_3)$

3.3 Possible Ranges of Outcomes

Since none of the conditions (I), (RI) and (RN) contradict each other directly, the ranges of γ described above can be arranged in various ways by choosing the payoffs of the rich and poor players appropriately. Depending on payoff, some of the cases above can be ruled out from occurring. Possible (rational) outcomes are

I. $0 \leq \gamma_3 < \gamma_1 < \gamma_2 \leq 0$

   This is the only arrangement in which all four cases could potentially be observed. With low beliefs about the probability of revolution ($\gamma < \gamma_3$), the rich person does not purchase insurance and poor players do not risk revolting. For a slightly higher perceived probability ($\gamma_3 \leq \gamma < \gamma_1$), it is still not worth purchasing insurance for the rich player, but poor players still try to revolt. For $\gamma_1 \leq \gamma < \gamma_2$, the rich player buys insurance which in turn deters poor players from attempting a revolution as the consequences of failure are now graver than they were without insurance. If a revolution is believed to be likely ($\gamma \geq \gamma_2$), the rich player still purchases insurance on the off-chance that the revolution fails (if the revolution is successful, the outcome for the rich person is the same regardless of insurance) and poor players decide to revolt.

II. $0 \leq \gamma_3 < \gamma_2 < \gamma_1 \leq 1$

The rich player does not purchase insurance for values of γ below γ_1. If poor players also believe $\gamma < \gamma_3$, they will not revolt even in the absence of insurance. For $\gamma_3 \leq \gamma_1$, poor revolt without facing the rich's insurance. If the beliefs exceed γ_1, the rich player buys insurance and the poor revolt.

III. $0 \leq \gamma_2 < \gamma_3 < \gamma_1 \leq 1$

This case allows for identical outcomes as the one above since γ_2 is meaningless if $\gamma_2 < \gamma_1$.

IV. $0 \leq \gamma_1 < \gamma_3 < \gamma_2 \leq 1$

There will be no insurance as long as players believe the probability of one poor player to revolt to be less than γ_1. For values above γ_1, the rich player buys insurance which deters poor players from revolting as long as beliefs about γ are such that $\gamma < \gamma_2$. If the latter condition is not satisfied, the outcome will be a revolution under insurance.

V. $0 \leq \gamma_1 < \gamma_2 < \gamma_3 \leq 1$

This is, again, equivalent to the case above as γ_3 carries no meaning if $\gamma_3 > \gamma_1$.

VI. $0 \leq \gamma_2 < \gamma_1 < \gamma_3 \leq 1$

This last case only allows for two possible outcomes: No insurance and no revolution if $\gamma < \gamma_1$ or revolution in the presence of insurance if the opposite is believed to be true.