Econ 828 Term Project:
How much is too much?
Beliefs about perceived inequality in a coordination game

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3.1 Payoffs

The underlying model of the coordination game in this paper is that of a stag hunt with the added dimension of a third party whose decisions potentially affect the payoffs of the other two players who try to coordinate. In the context of inequality, the poor are the parties that play the traditional stag hunt game while the rich person has influence over the exact form of the game by deciding whether to purchase insurance. The resulting payoffs for poor players are shown in Table 1.

In order to correctly represent the classical stag hunt game, it has to be the case that the high payoff for successfully coordinating on the payoff dominant strategy is greater than the

Table 1: General Payoffs of Poor

Under `no insurance` (n)

	revolt	do nothing
revolt	h_n, h_n	l_n, m_n
do nothing	m_n, l_n	m_n, m_n

Under `insurance` (i)

	revolt	do nothing
revolt	h_i, h_i	l_i, m_i
do nothing	m_i, l_i	m_i, m_i

Payoffs of poor players depending on the other poor player's decision and depending on whether the rich person opted for insurance.

medium payoff for playing from playing the risk dominant strategy, which in turn is greater than the payoff from unsuccessfully playing the risky strategy. That is, $h_j > m_j > l_j$ in both the `no insurance` and the `insurance` case (i.e. $\forall j \in \{n, i\}$). Additionally, successful coordination on the risky action results in the same payoff regardless of the actions of the rich player ($h_n = h_i = h$) and not revolting is risk free not only within a game but also across the two different games resulting from the rich's decisions ($m_n = m_i = m$). Lastly, revolting unsuccessfully is worse if the rich player takes out insurance ($l_i < l_n$) and can be seen as the rich person "fighting back". These assumptions yield a simplified payoff table for poor players shown in Table 2.

Table 2: Simplified Payoffs of Poor

	revolt	do nothing
revolt	h, h	l_j, m
do nothing	m, l_j	m, m

Where $j \in \{i, n\}$ and $l_n > l_i$.

The decision made by the rich player is that of purchasing insurance against a possible revolution. His payoffs depend on whether a revolution was attempted and, if so, whether it was successful. Since a revolutions is only successful if both poor players decide to play the risky action of "revolt" , this case corresponds to the outcome with two revolutionaries. An outcome with only one revolutionary represents an unsuccessful revolution and zero revolutionaries mean nobody tried to revolt.

If a revolutions is successful, having insurance has no effect on the rich's payoffs. That is, in terms of Table 3, $y_{2i} = y2n = y_2$. Instead, insurance is only effective in the case of no or failed revolutions (here denoted by the subscript f): $y_{0i} = y_{1i} = yfi$. This is better than the outcome y_{1n}, which represents an attempted but unsuccessful revolutions and can be thought of as being damaging to the rich nonetheless. The best possible outcome for the rich player is not buying insurance and none of the poor attempting a revolution resulting in y_{0n}.[1] Purchasing insurance can therefore be thought of as costing $y_{0n} - y_{fi}$ and paying out $y_{fi} - y_{1n}$ in the case of a failed revolution attempt.

Table 3: Payoffs of Rich

General Form

number of revolutionaries	insurance	no insurance
0	y_{0i}	y_{0n}
1	y_{1i}	y_{1n}
2	y_{2i}	y_{2n}

Simplified Form

number of revolutionaries	insurance	no insurance
0	y_{fi}	y_{0n}
1	y_{fi}	y_{1n}
2	y_2	y_2

3.2 Decision Making Process

Given these payoffs, a rational poor player will choose to revolt conditional on beliefs about what the other poor player will do and whether the rich person will buy insurance. Let a poor player believe that the other poor player chooses to revolt with probability γ and that the rich player purchases insurance with probability δ. Then a payoff maximizing poor person will revolt if

$$\gamma h + (1 - \gamma) \left[(1 - \delta) l_n + \delta l_i \right] \geq m$$

A rich person will decide to purchase insurance based on his beliefs about how many poor players will choose to revolt. In the most general case, let a rich player believe that there are zero revolutionaries with probability α, one revolutionary with probability β and

[1] In summary: $y_{0n} > y_{0i} = y_{1i} = y_{fi} > y_{1n} > y_{2i} = y_{2n} = y_2$

two revolutionaries with probability $1 - \alpha - \beta$. Then a rich player will purchase insurance if

$$(\alpha + \beta)Y_{fi} + (1 - \alpha - \beta)y_2 \geq \alpha y_{0n} + \beta y_{1n} + (1 - \alpha - \beta)y_2$$
$$(\alpha + \beta)Y_{fi} \geq \alpha y_{0n} + \beta y_{1n}$$

Assuming independent and identical poor players, $\alpha = (1-\gamma)^2$, $\beta = 2\gamma(1-\gamma)$ and $(1 - \alpha - \beta) = \gamma^2$. Using this simplification, rich players buy insurance if

$$\left[(1-\gamma)^2 + 2\gamma(1-\gamma)\right] y_{fi} \geq (1-\gamma)^2 y_{0n} + 2\gamma(1-\gamma)y_{1n}$$

which simplifies to

$$\gamma \geq \frac{y_{0n} - y_{fi}}{y_{fi} + y_{0n} - 2y_{1n}} \equiv \gamma_1. \tag{I}$$

For the decision making process of poor players, this means that if their belief of the other poor player's probability of revolting satisfies condition (I), they believe that the rich player will purchase insurance for sure (i.e. $\delta = 1$). Considering these second order beliefs, given that $\gamma > \gamma_1$, the poor player chooses to revolt if

$$\gamma h + (1-\gamma)l_i \geq m$$
$$\gamma \geq \frac{m - l_i}{h - l_i} \equiv \gamma_2. \tag{RI}$$

If the poor player believes the other poor player to be sufficiently unlikely to revolt so that the rich player does not purchase insurance (condition (I) is not satisfied and $\gamma < \gamma 1$), he will only revolt if

$$\gamma h + (1-\gamma) l_n \geq m$$
$$\gamma \geq \frac{m - l_n}{h - l_n} \equiv \gamma_3. \tag{RN}$$

Combining conditions (I), (RI) and (RN), poor players will

1. revolt under insurance if $\gamma \geq \max(\gamma_1, \gamma_2)$
2. not revolt due to the presence of insurance if $\gamma_1 \leq \gamma < \gamma_2$
3. revolt under no insurance if $\gamma_3 \leq \gamma < \gamma_1$
4. not revolt under no insurance if $\gamma < \min(\gamma_1, \gamma_3)$

3.3 Possible Ranges of Outcomes

Since none of the conditions (I), (RI) and (RN) contradict each other directly, the ranges of γ described above can be arranged in various ways by choosing the payoffs of the rich and poor players appropriately. Depending on payoff, some of the cases above can be ruled out from occurring. Possible (rational) outcomes are

I. $0 \leq \gamma_3 < \gamma_1 < \gamma_2 \leq 1$

   This is the only arrangement in which all four cases could potentially be observed. With low beliefs about the probability of revolution ($\gamma < \gamma_3$), the rich person does not purchase insurance and poor players do not risk revolting. For a slightly higher perceived probability ($\gamma_3 \leq \gamma < \gamma_1$), it is still not worth purchasing insurance for the rich player, but poor players still try to revolt. For $\gamma_1 \leq \gamma < \gamma_2$, the rich player buys insurance which in turn deters poor players from attempting a revolution as the consequences of failure are now graver than they were without insurance. If a revolution is believed to be likely ($\gamma \geq \gamma_2$), the rich player still purchases insurance on the off-chance that the revolution fails (if the revolution is successful, the outcome for the rich person is the same regardless of insurance) and poor players decide to revolt.

II. $0 \leq \gamma_3 < \gamma_2 < \gamma_1 \leq 1$

The rich player does not purchase insurance for values of γ below γ_1. If poor players also believe $\gamma < \gamma_3$, they will not revolt even in the absence of insurance. For $\gamma_3 \leq \gamma_1$, poor revolt without facing the rich's insurance. If the beliefs exceed γ_1, the rich player buys insurance and the poor revolt.

III. $0 \leq \gamma_2 < \gamma_3 < \gamma_1 \leq 1$

This case allows for identical outcomes as the one above since γ_2 is meaningless if $\gamma_2 < \gamma_1$.

IV. $0 \leq \gamma_1 < \gamma_3 < \gamma_2 \leq 1$

There will be no insurance as long as players believe the probability of one poor player to revolt to be less than γ_1. For values above γ_1, the rich player buys insurance which deters poor players from revolting as long as beliefs about γ are such that $\gamma < \gamma_2$. If the latter condition is not satisfied, the outcome will be a revolution under insurance.

V. $0 \leq \gamma_1 < \gamma_2 < \gamma_3 \leq 1$

This is, again, equivalent to the case above as γ_3 carries no meaning if $\gamma_3 > \gamma_1$.

VI. $0 \leq \gamma_2 < \gamma_1 < \gamma_3 \leq 1$

This last case only allows for two possible outcomes: No insurance and no revolution if $\gamma < \gamma_1$ or revolution in the presence of insurance if the opposite is believed to be true.

4 Experimental Design

The experiment follows a 2×2 design. First, subjects are randomly assigned either the role of a poor or a rich player. Second, payoffs are designed to have either high or low variation in payoffs for each of the players. Lastly, payoffs are arranged such that the inequality between possible outcomes for poor and rich players is either high, medium or low.

In order to allow for all possible rational action for poor players as discussed in section 3.2, payoffs are chosen such that they meet the requirements of case I. in section 3.3. These payoffs can be seen in Table 4.

Table 4: Payoff Treatments

Treatment		Payoffs						
Risk	Inequality	l_i	l_n	m	h	y_{1n}	y_{fi}	y_{on}
low	low	130	280	440	750	490	1100	1820
low	medium	130	280	440	750	980	2200	3640
low	high	130	280	440	750	1960	4400	7280
high	low	75	130	320	750	500	2250	4000
high	medium	75	130	320	750	1000	4500	8000
high	high	75	130	320	750	2000	9000	16000

Subject are not informed about the existence of these six treatments. Instead, they only know that there are rich and poor players assigned randomly and that two poor players are randomly paired with one rich player into one group.

All players are shown the payoffs selected for the current round. The values from Table 4 are filled into Table 2 and into the right side of Table 3. All players see both tables. That way, poor players not only know the risk that they (and the other poor player) face but also how unequal their payoffs are relative to the rich player. Additionally, poor players can see how risky the rich player's possible payoffs are. Based on this information, poor players form beliefs about the the likelihood of the other poor player revolting and the rich player purchasing insurance (which determines which payoff table is relevant for poor players).

In a similar manner, rich players see their own payoffs and the range between highest and lowest possible outcome on top of how much more they can expect to earn relative to the two poor players in the group and how risky the latter's payoffs are.

Next, poor (rich) players choose whether to revolt (buy insurance). Additionally, rich players are asked to guess whether a revolution will be successful. They are told that guessing correctly is rewarded with 100 points. Afterwards, all players are informed about how many poor players decided to revolt[2] and if insurance was purchased, and all players are told their payoffs from that round.

At the end of a round, each subject is randomly assigned to the role of either a poor or rich player and is furthermore randomly assigned to a new group. Randomizing the order in which subjects play rich or poor players, are faced with high or low risk payoffs, and observe high, medium or low inequality, allows me to ignore any possible order effects.

At the conclusion of the experiment, subjects are paid for three randomly selected periods. Monetary earnings are calculated as $\frac{1}{1000}$ times the points earned in the experiment plus a show-up fee of \$7.

5 Analysis

The focus of this experiment is the subject's belief over the likelihood of successful coordination in response to observed inequality. In the case of poor players, these beliefs are directly observable: A subject should revolt if and only if the other poor player is believed to choose revolt. Therefore, observing a subject's frequency of choosing to revolt, directly represents their belief about the choice of the other poor player. In the case of rich players, this analysis is somewhat less straight-forward. Instead of directly observing the frequency of revolutions, the rich person only indicates whether he believes in a successful revolution. This, however, does not translate directly into γ as the probability of both poor players choosing to revolt

[2]From this, poor players can infer if the other poor player revolted and learn about the likelihood of a revolution based on observed payoffs.

is γ^2 as stated previously. Assuming that the rich player believes revolutions to be successful with probability τ, the variable of interest is $\sqrt{\tau} = \gamma$. In addition, it is possible to identify a range that the rational rich player must have believed the probability of a revolution to fall in. Using condition (I), it is possible to conclude that the rich player must think that $\gamma > \gamma_1$ if he is purchased insurance while the opposite must be true if he did not purchase insurance. This range can be used to verify the validity of using $\sqrt{\tau}$ as the dependent variable in the analysis.

5.1 Regression Analysis

After observing γ and $\sqrt{\tau} = \gamma$ for each subject (the former for in cases when subjects were in the role of a poor player and the latter in the role of the rich player) and treatment,[3] the goal is to estimate how inequality affects a subject's belief over the probability of a revolution while controlling for risk in the payoffs. The second effect of interest is how perspective changes beliefs or the perception of inequality. This can be quantified as the effect of being assigned either role.

An appropriate dummy variable model is therefore

$$\begin{aligned}\gamma_{it} = b_0 &+ b_1 \mathbb{1}\{inequality_t = medium\} \\ &+ b_2 \mathbb{1}\{inequality_t = high\} \\ &+ b_3 \mathbb{1}\{role_t = rich\} \\ &+ b_4 \mathbb{1}\{risk_t = high\} + \kappa_i + \epsilon_{it}\end{aligned}$$

where b_0 is the average belief of a poor person playing the game in a low inequality and

[3] Assigning the value 1 whenever a poor (rich) person chooses to revolt (buy insurance) and 0 otherwise, the probabilites are the means within a treatment for each subject. This results in six observations per subject.

low risk setup. b_1 and b_2 identify the effects of of medium and high inequality, respectively. b_3 measures how a subject's perspective changes as he switches from the role of a poor player to that of a rich player. The dummy variable whose coefficient is b_4 controls for possible risk preferences and κ_i captures unobserved heterogeneity among subjects. For example, some subjects might be biased towards thinking that inequality is inherently bad and are therefore more likely to choose to revolt.

Preferably, this model would be estimated using OLS in order to identify linear effects. However, should predicted values of γ_{it} fall outside of $[0, 1]$, a more appropriate Logit or Probit model would have to be used.

5.2 Initial Hypothesis

A strong positive effect of higher inequality should be observable. Since poor people need to coordinate on one actions, anything from history to social norms suggests that subjects believe that the other subject is more likely to revolt if inequality is greater. Therefore, a reasonable assumption would be that $0 < b_1 < b_2$.

Next, I would expect rich players to perceive inequality as less pressing than if they observed the same payoff possibilities as a poor person, meaning $b_3 < 0$. Though theory does not directly predict it, I would also expect b_3 to be smaller in magnitude than b_2.

Lastly, while the level of risk only serves as a control variable here, a brief discussion of its effect is in order. Risk, as measured in this experiment, only concerns the relative difference between the best and worst possible payoff for a poor player. It is therefore unclear if risk is perceived as the size of a possible loss ($m - l_i$ if insurance is purchased or $m - l_n$ otherwise) or the size of possible gains ($h - m$). The direction of this effect is therefore not clear. The reason risk is a treatment variable in this experiment, is not to measure risk preferences, but rather to control for any possible reaction to risk left after (technically) inducing risk-neutrality by choosing a random round for payment and by doing so to get the least polluted measure of the effect of inequality.

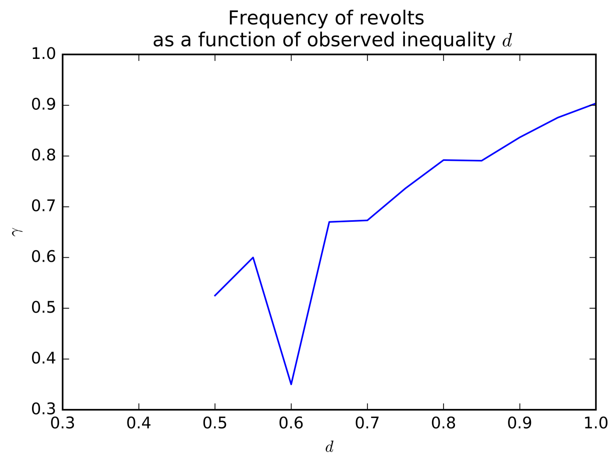
6 Simulation Exercise

A crude implementation in a Monte Carlo simulation of the basics of the model presented here verifies the relevance of the proposed research question. The simulation varies from the experiment in a few important aspects. First, "subjects" (or in this case class instances) do not switch between the roles of poor and rich players. Instead, any initial assignments persist throughout the simulation. Second, the six remaining treatments are not directly implemented. Instead, there is a (near) continuum of treatments. At the beginning of the simulation, 10,000 payoff tables are randomly generated. Of these 10,000, 1,707 satisfied condition the condition that $0 \leq \gamma_3 < \gamma_1 < \gamma_2 \leq 1$, which is necessary to rationally allow for all possible combinations of revolt $\times$ insurance.

Throughout 500 rounds, 30 subjects learned and formed beliefs about the probability of a revolution in response to a variable d, which measures, in very simple terms, the degree of inequality and is defined as one minus the ratio of a poor person's payoff relative to that of a rich person if the status quo remained (i.e. nobody revolts and the rich person does not purchase insurance). That is $d = 1 - \frac{m}{y_{0n}}$. In each round, subjects were randomly assigned into groups of two poor and one rich player. Then, all players were informed about the degree of inequality d in response to which poor players predicted γ as a linear function of d and decided to revolt with that probability γ. Rich players were simulated to purchase insurance if they predicted (based on the Random Forest machine learning algorithm) at least one poor player to revolt. At the end of a round, all simulated players were informed about how many poor players revolted and whether insurance was purchased.

The results of the simulation can be seen in Figure 1. The graph depicts the observed frequency of poor players choosing to revolt in response to the simplified measure of inequality

Figure 1: Simulation Graph



d. As predicted by theory, there is a generally positive relationship between inequality and a poor player's propensity to choose to revolt. The notable exception is the decrease for observations with $d \approx 0.6$. This, again, corresponds to theory in the sense that theory predicts a range in which inequality is high enough that rich players purchase insurance, which in turn deters poor players from revolting. Only once inequality keeps increasing, poor players can again coordinate on the risky action of revolting.