Consumer and Speculator Market Dynamics - Estimation

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1 Estimation

1.1 Consumer-only Model Estimation

We estimate the structural parameter vector $\boldsymbol{\theta} = \begin{pmatrix} \sigma_\mu^2 & \sigma_\xi^2 & \rho_\eta & \rho_\gamma \end{pmatrix}'$ of our model by expressing the common structural errors μ_t as functions of observed prices using the equilibrium price equation we derived earlier. Isolating these errors would allow us to use GMM directly to estimate the model; however, the Monte Carlo experiments suggest that the small sample properties of a GMM estimator are quite poor even when the number of moments is large. Instead, we utilize a maximum likelihood estimator that uses a Kalman filter to construct a sequence $\{\hat{\mu}_t\}_{t=1}^{\infty}$ of estimated errors which we use to proxy for the true μ_t values in our likelihood function. To keep things simple for now, we first derive the estimator for the model without speculators and then adapt it to the full model.

To implement a Kalman filter for our model, we begin by defining

$$
\begin{aligned}
\boldsymbol{s}_t &= \begin{pmatrix} \eta_t & \bar{\delta}_{t-1} & \mu_t \end{pmatrix}' \\
\boldsymbol{v}_t &= \begin{pmatrix} \mu_t & 0 & \mu_t \end{pmatrix}' \\
\boldsymbol{F} &= \begin{pmatrix} \rho_\eta & 0 & 0 \\ 0 & c_{2\delta} & c_{2\mu} \\ 0 & 0 & 0 \end{pmatrix} \\
\boldsymbol{\Psi} &= \begin{pmatrix} \frac{1}{1-\beta\rho_\eta} & c_{1\delta} & c_{1\mu} \end{pmatrix}' \\
\boldsymbol{Q} &= \begin{pmatrix} \sigma_\mu^2 & 0 & \sigma_\mu^2 \\ 0 & 0 & 0 \\ \sigma_\mu^2 & 0 & \sigma_\mu^2 \end{pmatrix}
\end{aligned}
$$

Using these, we can write the state and observation equation of our system respectively as

$$\boldsymbol{s}_t = \mathbf{F}\boldsymbol{s}_{t-1} + \mathbf{v}_t \tag{1}$$

$$p_t = \frac{\bar{x} + b^*(0)}{1-\beta} + \boldsymbol{\Psi}'\boldsymbol{s}_t \tag{2}$$

For Gaussian disturbances, the Kalman filter starts by using the unconditional mean and variance of $\boldsymbol{s}_1$ and iteratively solves forward for the mean and variance of $\boldsymbol{s}_t$ in each period t given the history of observations from $t-1$ and earlier. For our system, we start by setting the initial mean and variance $\hat{\boldsymbol{s}}_{1|0}$ and $\boldsymbol{P}_{1|0}$ equal to their unconditional values[1]

$$\begin{aligned}
\hat{\boldsymbol{s}}_{1|0} &= E[\boldsymbol{s}_1] \\
&= \mathbf{0} \\
\boldsymbol{P}_{1|0} &= E\left[\boldsymbol{s}_1 \boldsymbol{s}_1'\right] \\
&= \begin{pmatrix} \frac{\sigma_\mu^2}{1-\rho_\eta^2} & \frac{c_{2\mu}\rho_\eta\sigma_\mu^2}{1-\rho_\eta c_{2\delta}} & \sigma_\mu^2 \\ \frac{c_{2\mu}\rho_\eta\sigma_\mu^2}{1-\rho_\eta c_{2\delta}} & \frac{c_{2\mu}^2\sigma_\mu^2}{1-c_{2\delta}^2} & 0 \\ \sigma_\mu^2 & 0 & \sigma_\mu^2 \end{pmatrix}
\end{aligned}$$

and iterating on the Kalman filter updating equations:

$$\hat{\boldsymbol{s}}_{t|t} = \hat{\boldsymbol{s}}_{t|t-1} + \boldsymbol{P}_{t|t-1}\boldsymbol{\Psi}\left(\boldsymbol{\Psi}'\boldsymbol{P}_{t|t-1}\boldsymbol{\Psi}\right)^{-1}\left(p_t - \frac{\bar{x}+b^*(0)}{1-\beta} - \boldsymbol{\Psi}'\boldsymbol{s}_t\right) \tag{3}$$

$$\hat{\boldsymbol{s}}_{t+1|t} = \boldsymbol{F}\hat{\boldsymbol{s}}_{t|t} \tag{4}$$

$$\boldsymbol{P}_{t|t} = \boldsymbol{P}_{t|t-1} - \boldsymbol{P}_{t|t-1}\boldsymbol{\Psi}\left(\boldsymbol{\Psi}'\boldsymbol{P}_{t|t-1}\boldsymbol{\Psi}\right)^{-1}\boldsymbol{\Psi}'\boldsymbol{P}_{t|t-1} \tag{5}$$

$$\boldsymbol{P}_{t+1|t} = \boldsymbol{F}\boldsymbol{P}_{t|t}\boldsymbol{F}' + \boldsymbol{Q} \tag{6}$$

Iterating over this system gives yields two series, $\{\boldsymbol{s}_t\}_{t=1}^T$ and $\{\boldsymbol{P}_{t|t-1}\}_{t=1}^T$, that give the conditional means and variances of the unobserved state $\boldsymbol{s}_t$. By construction, each p_t is conditionally distributed as:

$$\left(p_t \mid \{p_s\}_{s=1}^{t-1}\right) \sim N\left(\frac{\bar{x}+b^*(0)}{1-\beta} + \boldsymbol{\Psi}'\hat{\boldsymbol{s}}_{t|t-1}, \boldsymbol{\Psi}'\boldsymbol{P}_{t|t-1}\boldsymbol{\Psi}\right)$$

from which we get the log-likelihood we use to estimate the model

$$\mathcal{L}(\boldsymbol{\theta}, \{p_t\}_{t=1}^T) = \sum_{t=1}^T \log\left(f\left(p_t - \frac{x+b^*(0)}{1-\beta} \mid \{p_s\}_{s=1}^{t-1}; \boldsymbol{\theta}\right)\right) \tag{7}$$

1.2 Model with Speculators

Things get more complicated when speculators enter the model due to the fact that the nonlinearity of the speculator's policy function with respect to the state vector $\boldsymbol{s}_t$ and the fact that speculator holdings themselves must become part of this state vector introduce nonlinearity into the state space representation above that make the standard Kalman filter approach unworkable.

In the case where speculato

To implement a Kalman filter for our model, we begin by defining

$$\begin{aligned}
\boldsymbol{s}_t &= \begin{pmatrix} \eta_t & \bar{\delta}_{t-1} & \mu_t & h_{t-1} \end{pmatrix}' \\
\boldsymbol{v}_t &= \begin{pmatrix} \mu_t & 0 & \mu_t & 0 \end{pmatrix}' \\
\boldsymbol{Q} &= \begin{pmatrix} \sigma_\mu^2 & 0 & \sigma_\mu^2 & 0 \\ 0 & 0 & 0 & 0 \\ \sigma_\mu^2 & 0 & \sigma_\mu^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\end{aligned}$$

[1]The $x|y$ subscript denotes the expected value of the variable from time x conditional on all observed data up to and including time y.

$$\boldsymbol{s}_t = f_s(\boldsymbol{s}_{t-1}) + \mathbf{v}_t \tag{8}$$

$$p_t = \frac{\bar{x}}{1-\beta} + f_p(\boldsymbol{s}_t) \tag{9}$$

where

$$f_s(\boldsymbol{s}_{t-1}) = \begin{pmatrix} \rho_\eta \eta_{t-1} & c_{2\delta}\delta_{t-2} + c_{2\mu}\mu_{t-1} + c_{2b}g(h_{t-2}) & 0 & h_{t-1}(\boldsymbol{s}_{t-1}) \end{pmatrix}' \tag{10}$$

$$f_p(\boldsymbol{s}_t) = B^*(h_t(\boldsymbol{s}_t), h_{t-1}) + \frac{\eta_t}{1-\beta\rho_\eta} + c_{1\delta}\delta_{t-1} + c_{1\mu}\mu_t \tag{11}$$

2 Monte Carlo Analysis of Estimators

2.1 Consumer-Only Estimates

Estimates from Monte Carlo simulations of the consumer-only model are presented in Table 1. Generically, the results indicate that the parameters are identified, though it looks the variance estimates are really noisy in small samples and probably biased.

	σ^2_μ	σ^2_γ	ρ_η	ρ_γ
True Parameter Value	0.500	1.000	0.980	0.500
ML Estimate	0.516	1.034	0.980	0.505
Difference	-0.016	-0.034	0.000	-0.005
Std. Error	(0.089)	(0.209)	(0.002)	(0.033)
Trials	98			
Obs. per Trial	10000			

(a) Large Sample Results

	σ^2_μ	σ^2_γ	ρ_η	ρ_γ
True Parameter Value	0.500	1.000	0.980	0.500
ML Estimate	0.676	1.436	0.976	0.514
Difference	-0.176	-0.436	0.004	-0.014
Std. Error	(0.378)	(1.017)	(0.008)	(0.109)
Trials	99			
Obs. per Trial	1000			

(b) Small Sample Results

	σ^2_μ	σ^2_γ	ρ_η	ρ_γ
True Parameter Value	0.500	1.000	0.980	0.500
ML Estimate	1.299	3.184	0.967	0.516
Difference	-0.799	-2.184	0.013	-0.016
Std. Error	(1.173)	(3.212)	(0.018)	(0.230)
Trials	99			
Obs. per Trial	250			

(c) Very Small Sample Results

Table 1: Monte Carlo Simulation Results, Consumer-Only Model