

Flybys and Foci Scientific Appendix

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1 Appendix

Throughout the story I have done my best to keep the plot scientifically accurate and plausible. This section gives a more thorough explanation of some parts of the story that the reader may find interesting. The appendix is written assuming the reader has some basic knowledge of physics, to the level that can be found on Wikipedia.

1.1 Appendix A: Diving Towards the Sun

In Chapter 8, *Einstein* is put on a trajectory that passes close to the sun before performing its main burn to exit the solar system. This may seem unintuitive - the spacecraft first heads towards the sun to make it to a point very distant to the sun. *Einstein* is taking advantage of the Oberth Effect, where the most efficient place to perform an engine burn is deep in a gravity well.

The total energy of a body in orbit is the sum of the kinetic and potential energy:

$$E = \frac{1}{2}mv^2 - \frac{mMG}{r} = -\frac{mMG}{2a}$$

Where:

m = spacecraft mass

v = velocity

M = solar mass

G = universal gravitational constant

r = distance to the sun

a = semi-major axis

This expression is commonly divided by mass to get the *specific orbital energy*:

$$= \frac{1}{2}v^2 - \frac{MG}{r} = -\frac{MG}{2a}$$

For *Einstein* the higher the specific orbital energy the faster it will reach the sun's focal point. A more tangible measure is v_∞ , which is the spacecraft's velocity when it is so far away from the sun that its potential energy is negligible. As a spacecraft climbs out of the sun's gravity well it sheds velocity and the potential energy goes to 0 (in orbital mechanics potential energy is 0 at infinity and increasingly negative closer to the sun).

$$\begin{aligned} \frac{1}{2}v_\infty^2 &= \frac{1}{2}v^2 - \frac{MG}{r} = -\frac{MG}{2a} \\ v_\infty &= \sqrt{v^2 - \frac{2MG}{r}} \end{aligned}$$

Where:

r = spacecraft distance to the sun

v = velocity at the given r

We can use this to calculate v_∞ for *Einstein* if it had burned directly on a hyperbolic escape trajectory and compare it to v_∞ from its trajectory that took it close to the sun.

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Scenario 1: Direct escape burn

Assume that *Einstein* begins in a heliocentric, circular orbit the same distance from the sun as Jupiter. In the story **Einstein** must first escape from Jupiter's gravity well but for simplicity we'll assume this doesn't require much Δv .

$$\frac{1}{2}v_{\infty}^2 = \frac{1}{2}(v_0 + \Delta v)^2 - \frac{MG}{r_0}$$

$$v_{\infty} = \sqrt{(v_0 + \Delta v)^2 - \frac{2MG}{r_0}}$$

Where: v_0 : starting velocity

Δv : change in velocity supplied by *Sheridan* drive

r_0 : starting distance from the sun - Jupiter's orbit

Scenario 2: Oberth effect

Again we assume that *Einstein* begins in a heliocentric, circular orbit the same distance from the sun as Jupiter. *Einstein* then performs two burns - one retrograde (opposite the direction of orbit) to bring down the perihelion (the point in the orbit nearest the sun) and a second burn near the sun to escape the solar system. In the story the first burn is combined with the maneuver to escape Jupiter, again is taking advantage of the Oberth effect!

The hyperbolic excess velocity (Δv_{∞}) following the second burn is:

$$v_{\infty} = \sqrt{(v_p + \Delta v_2)^2 - \frac{2MG}{r_p}}$$

Where:

v_p : velocity at perihelion prior to the burn

r_p : perihelion distance

Δv_2 : change in velocity from the second burn

The velocity and perihelion of *Einstein* depends on the first burn. To solve for the perihelion:

$$r_p + r_0 = 2a$$

$$a = \left(\frac{2}{r_0} - \frac{(v_0 - \Delta v_1)^2}{GM} \right)^{-1}$$

Where: r_p : perihelion distance

r_0 : starting distance from the sun - Jupiter's orbit

Δv_1 : change in velocity of the first burn

a : semi-major axis

Note that Δv_1 subtracts from v_0 because the burn is opposite the direction of orbit to lower the perihelion. The second equation is a rearrangement of the equation for specific orbital energy. This equation gives us the perihelion distance. The velocity at the perihelion v_p can be calculated from conservation of angular momentum:

$$r_p \times v_p = r_0 \times v_0$$

To summarize, v_∞ depends on two factors:

1. The burn at perihelion
2. How low the perihelion is, which in turn determined by the first burn.

Note that there is a limit to how low the perihelion can be - *Einstein* can only fly so close to the sun. Figure 1 plots the hyperbolic excess velocity as a function of these two burns.