Galois Transformers and Modular Abstract Interpreters

Reusable Metatheory for Program Analysis

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Program Analysis

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- Lots of choices when designing a program analysis

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- Choices make tradeoffs between *precision* and *performance*

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- **Galois Transformers**:

  Reusable components for building program analyzers

Program Analysis

- Lots of choices when designing a program analysis
- Choices make tradeoffs between *precision* and *performance*
- Implementations are brittle and difficult to change
- **Galois Transformers**:

  Reusable components for building program analyzers
- **Bonus**:

  Variations in path/flow sensitivity of your analyzer for free

Let's Design an Analysis

(in the paradigm of abstract interpretation)

Let's Design an Analysis

Program

```
int x y; // global state
void safe_fun(int N) {
  if (N≠0) {x := 0;}
  else     {x := 1;}
  if (N≠0) {y := 100/N;}
  else     {y := 100/x;}}
```

Let's Design an Analysis

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void safe_fun(int N) {
 if (N≠0) {x := 0;}
 else
 if (N≠0)
 else
```

Analysis Property

$x/0$

Let's Design an Analysis

Abstract Values

$$\mathbb{Z} \sqsubseteq \{-, 0, +\}$$

Let's Design an Analysis

Program | Analysis Property | Abstract Values

Implement

```
analyze : exp → results
analyze(x := æ) :=
    .. x .. æ ..
analyze(IF(æ){e1}{e2}) :=
    .. æ .. e1 .. e2 ..
```

Let's Design an Analysis

Program | Analysis Property | Abstract Values

Get Results

```
N ∈ {-,0,+}
x ∈ {0,+}
y ∈ {-,0,+}

UNSAFE: {100/N}
UNSAFE: {100/x}
```

Let's Design an Analysis

Prove Correct

$$[\![e]\!] \in [\![\mathrm{analyze}(e)]\!]$$

Let's Design an Analysis

Program

```
int x y; // global state
void safe_fun(int N) {
 if (N≠0) {x := 0;}
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Analysis Property

```
x/0
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Abstract Values

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Implement

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analyze : exp → results
analyze(x := æ) :=
    .. x .. æ ..
analyze(IF(æ){e₁}{e₂}) :=
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```

Get Results

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N ∈ {-,0,+}
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UNSAFE: {100/N}
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Prove Correct

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Let's Design an Analysis

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```
N ∈ {-,0,+}
x ∈ {0,+}
y ∈ {-,0,+}

UNSAFE: {100/N}
UNSAFE: {100/x}
```

Flow-insensitive

results :
var ↦ $\mathcal{P}$({-,0,+})

Let's Design an Analysis

```
int x y; // global state
void safe_fun(int N) {
 if (N≠0) {x := 0;}
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 else     {y := 100/x;}}
```

Flow-sensitive

results :
loc ↦ (var ↦ $\mathcal{P}$({-,0,+}))

Let's Design an Analysis

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0: int x y; // global state
1: void safe_fun(int N) {
2:  if (N≠0) {x := 0;}
3:  else     {x := 1;}
4:  if (N≠0) {y := 100/N;}
5:  else     {y := 100/x;}}
```

```
4:   x ∈ {0,+}
4.T: N ∈ {-,+}
5.F: x ∈ {0,+}

N,y ∈ {-,0,+}

UNSAFE: {100/x}
```

Flow-sensitive

results :
loc ↦ (var ↦ $\mathcal{P}$({-,0,+}))

Let's Design an Analysis

```
int x y; // global state
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```

Path-sensitive

results :
$\mathrm{loc} \mapsto \mathcal{P}(\mathrm{var} \mapsto \mathcal{P}(\{-, 0, +\}))$

Let's Design an Analysis

```
0: int x y; // global state
1: void safe_fun(int N) {
2:  if (N≠0) {x := 0;}
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4:  if (N≠0) {y := 100/N;}
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```

```
4: N∈{-,+},x∈{0}
4: N∈{0},x∈{+}

N∈{-,+},y∈{-,0,+}
N∈{0},y∈{0,+}
```

SAFE

Path-sensitive

results :
loc ↦ $\mathcal{P}$(var ↦ $\mathcal{P}$({-,0,+}))

Let's Design an Analysis

Program

```
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1: void safe_fun(int N) {
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```

Analysis Property

x/0

Abstract Values

$\mathbb{Z} \sqsubseteq \{-,0,+\}$

Implement

```
analyze : exp → results
analyze(x := æ) :=
    .. x .. æ ..
analyze(IF(æ){e₁}{e₂}) :=
    .. æ .. e₁ .. e₂ ..
```

Get Results

```
4: N∈{-,+},y∈{0}
4: N∈{0},x∈{+}

N∈{-,+},y∈{-,0,+}
N∈{0},y∈{0,+}

SAFE
```

?

Prove Correct

$[\![e]\!] \in [\![analyze(e)]\!]$

Let's Design an Analysis

Program

```
0: int x y; // global state
1: void safe_fun(int N) {
2:  if (N≠0) {x := 0;}
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```

Analysis Property

$x/0$

Abstract Values

$\mathbb{Z} \sqsubseteq \{-,0,+\}$

Implement

```
analyze : exp → results
analyze(x : τ) :=
    .. x .. τ ..
analyze(IF(æ){e1}{e2}) :=
    .. æ .. e1 .. e2 ..
```

Get Results

```
4: N∈{-,+}, x∈{0}
4: N∈{0}, x∈{+}

N∈{-,+}, y∈{-,0,+}
N∈{0}, y∈{0,+}
```

SAFE

Prove Correct

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Let's Design an Analysis

Program

safe_fun.js ?

Analysis Property

x/0

Abstract Values

$\mathbb{Z} \sqsubseteq \{-,0,+\}$

Implement

```
analyze : exp → results
analyze(x := æ) :=
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Get Results

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4: N∈{-,+},x∈{0}
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N∈{-,+},y∈{-,0,+}
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SAFE

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Let's Design an Analysis

Program

```
safe_fun.js
```

Analysis Property

```
x/0
```

Abstract Values

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Implement

```
analyze : exp → results
analyze(x : ...) :=
    .. x .. ..
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4: N∈{-,+},x∈{0}
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N∈{-,+},y∈{-,0,+}
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SAFE
```

Prove Correct

$[\![e]\!] \in [\![\mathrm{analyze}(e)]\!]$

Problems Worth Solving

- How to change path/flow sensitivity without redesigning from scratch?
- How to reuse machinery between analyzers for different languages?
- How to translate proofs between different analysis designs?

Solution

Monad Transformers + Galois Connections = Galois Transformers

Compositional interpreters

Compositional abstractions

Compositional abstract interpreters

Galois Transformers

- What's a Monad?
- What are Transformers?
- What are Galois Connections?

Galois Transformers

- What's a Monad?
- What are Transformers?
- What are Galois Connections?

A Monad

```
type M(t)

op x ← e₁ ; e₂
op return(e)

op get
op put(e)
op fail
op ...
```

- A module with:
  - a type operator *M*
  - a semicolon operator (bind)
  - effect operation
- *M*`(t)`:
  - "A computation that performs some effects, then returns `t`"

A Monadic Interpreter

Program

```
int x y; // global state
void safe_fun(int N) {
 if (N≠0) {x := 0;}
 else     {x := 1;}
 if (N≠0) {y := 100/N;}
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```

Analysis Property

x/0

Abstract Domain

$\mathbb{Z} \sqsubseteq \{-,0,+\}$

Implement

```
analyze : exp → results
analyze(x := æ) :=
   .. x .. æ ..
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$$\mathtt{value} := \mathbb{Z} \cup \mathbb{B}$$

$$\rho \in \mathtt{env} := \mathtt{var} \mapsto \mathtt{value}$$

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value := $\mathbb{Z} \cup \mathbb{B}$
$\rho \in$ env := var $\mapsto$ value

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type M(t)

op x ← e₁ ; e₂
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op getEnv
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A Monadic Interpreter

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```
value := ℤ ∪ 𝔹
ρ ∈ env := var ↦ value
```

```
step : exp → M(exp)
```

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ρ ∈ env := var ↦ value

⟦_⟧ : atom → M(value)
```

```
step : exp → M(exp)
step(x := æ) := do
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  ρ ← getEnv
  putEnv(ρ[x↦v])
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step(IF(æ){e₁}{e₂}):= do
  v ← ⟦æ⟧
  case v of
    True → return(e₁)
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Abstractify

```
int x y; // global state
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```
step : exp → M♯(exp)
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```

$\texttt{value}^\sharp := \mathcal{P}(\{-,0,+\}) \cup \mathcal{P}(\mathbb{B})$
$\rho \in \texttt{env}^\sharp := \texttt{var} \mapsto \texttt{value}^\sharp$

$[\![\_]\!]^\sharp : \texttt{atom} \to M^\sharp(\texttt{value}^\sharp)$

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step : exp → M♯(exp)
step(x := æ) := do
  v ← ⟦æ⟧♯
  ρ ← getEnv
  putEnv(ρ ⊔ [x↦v])
  return(SKIP)
step(IF(æ){e₁}{e₂}):= do
  v ← ⟦æ⟧♯
  b ← chooseBool(v)
  case b of
    True → return(e₁)
    False → return(e₂)
```

$$\mathrm{value}^\sharp := \mathcal{P}(\{-,0,+\}) \cup \mathcal{P}(\mathbb{B})$$
$$\rho \in \mathrm{env}^\sharp := \mathrm{var} \mapsto \mathrm{value}^\sharp$$

$$[\![\_]\!]^\sharp : \mathrm{atom} \to M^\sharp(\mathrm{value}^\sharp)$$
$$\mathrm{chooseBool} : \mathrm{value}^\sharp \to M^\sharp(\mathbb{B})$$

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type M♯(t)

op x ← e₁ ; e₂
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Abstractify

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$\mathsf{value}^\sharp := \mathcal{P}(\{-,0,+\}) \cup \mathcal{P}(\mathbb{B})$

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$[\![\_]\!]^\sharp : \mathsf{atom} \to M^\sharp(\mathsf{value}^\sharp)$

$\mathsf{chooseBool} : \mathsf{value}^\sharp \to M^\sharp(\mathbb{B})$

```
type M♯(t)

op x ← e₁ ; e₂
op return(e)

op getEnv
op putEnv(e)

op fail/e₁⊞e₂
```

Monadic Abs. Interpreters

- Start with a *concrete* monadic interpreter
- Abstract value space (`value`$^\sharp$, $[\![\_]\!]^\sharp$)
- Join results when updating `env`$^\sharp$ ($\_\sqcup\_$)
- Branch nondeterministically (`chooseBool`)

Why Monads

- A monadic interpreter can be simpler than a state machine or constraint system
- Two effects, `State[`$\mathcal{s}$`]` and `Nondet`
  - Encode arbitrary small-step state machine relations
- Don't commit to a single implementation of $M^\sharp$
  - Different choices for $M^\sharp$ yield different analyses

Galois Transformers

- What's a Monad?
- What are Transformers?
- What are Galois Connections?

Galois Transformers

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```
type M(t)

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Galois Transformers

- What's a Monad?
- What are Transformers?
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```
type M(t)

op x ← e1 ; e2
op return(e)
```

Why Monads

- A monadic interpreter can be simpler than a state machine or constraint system
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Monad Transformers

`State[`$\mathcal{S}$`]`

$\texttt{get} : M(\mathcal{S})$

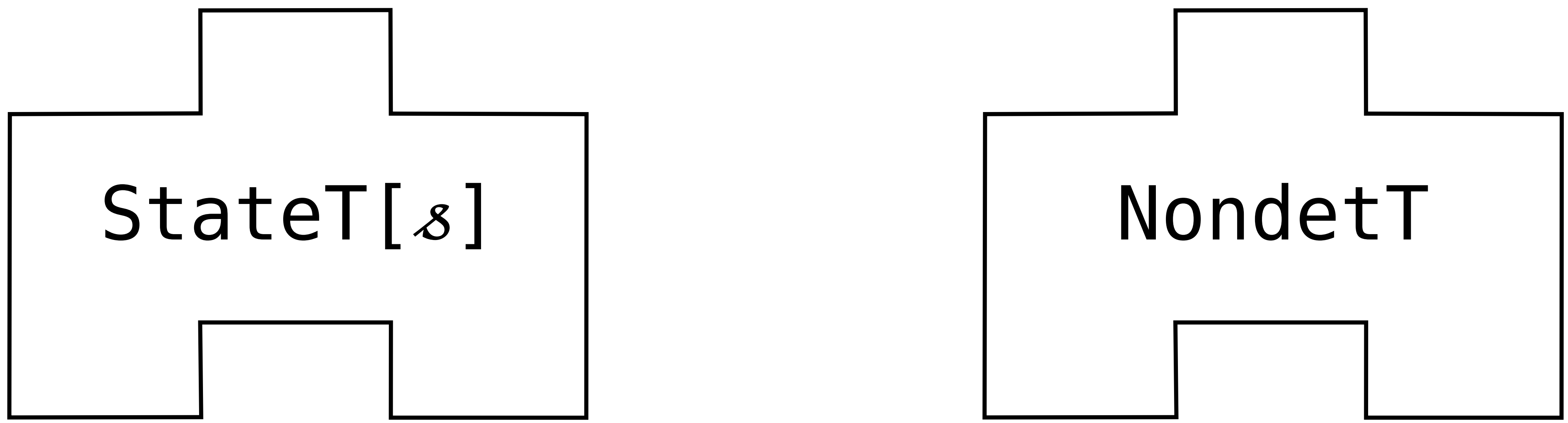
$\texttt{put} : \mathcal{S} \to M(1)$

`Nondet`

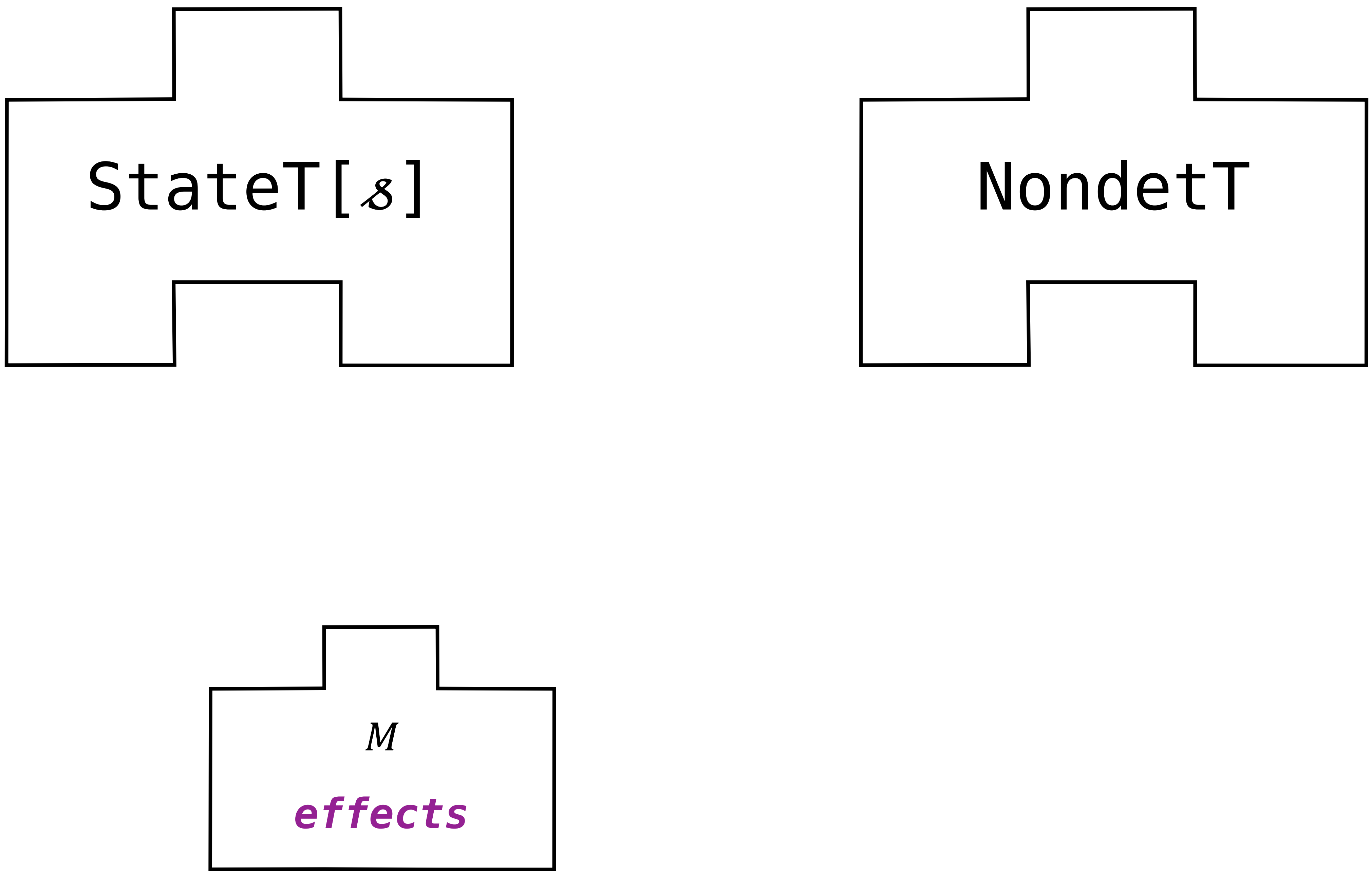
$\texttt{fail} : \forall\, \mathtt{A}.\ M(\mathtt{A})$

$\texttt{\_}\boxplus\texttt{\_} : \forall\, \mathtt{A}.\ M(\mathtt{A}) \times M(\mathtt{A}) \to M(\mathtt{A})$

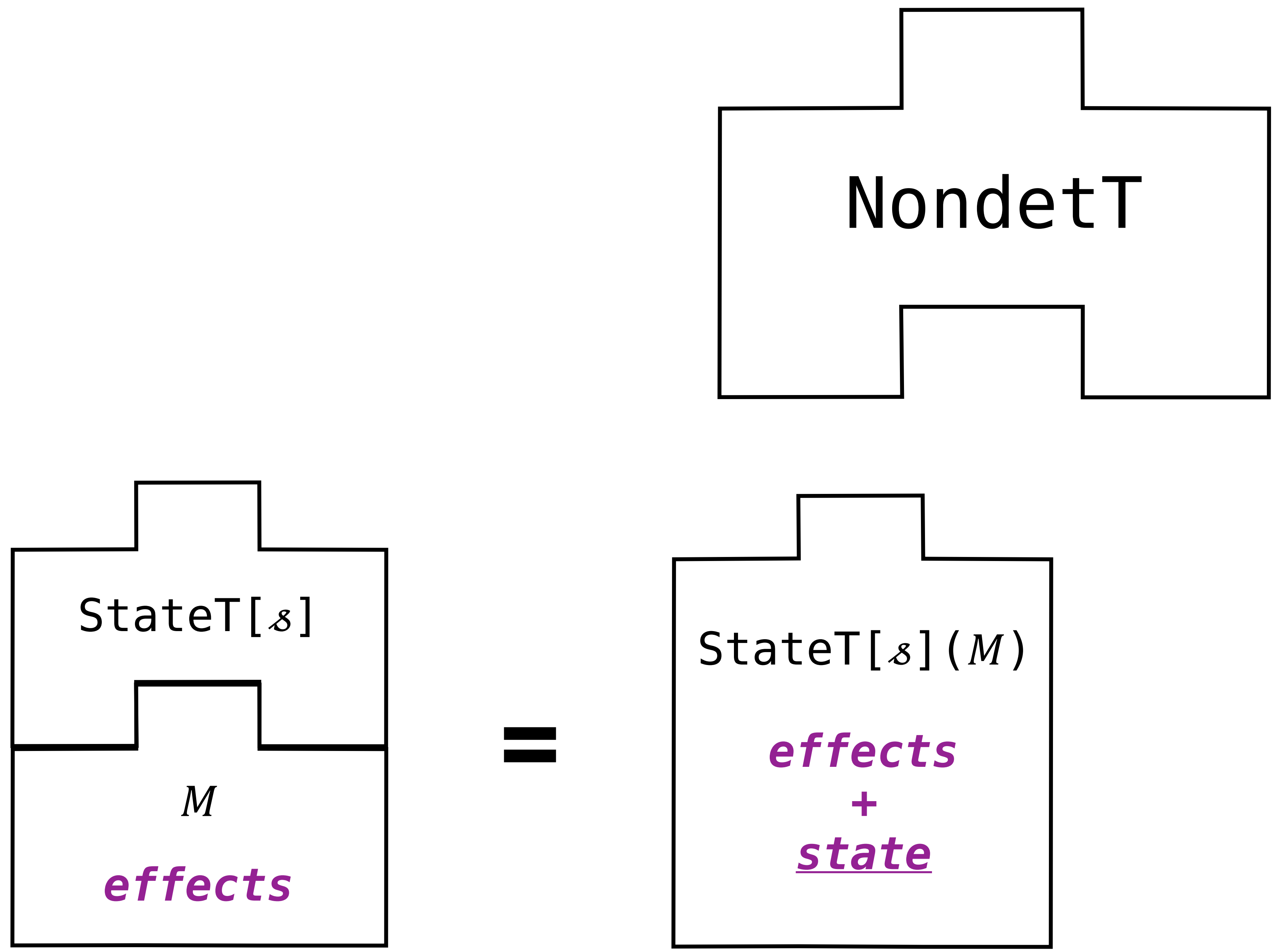
Monad Transformers



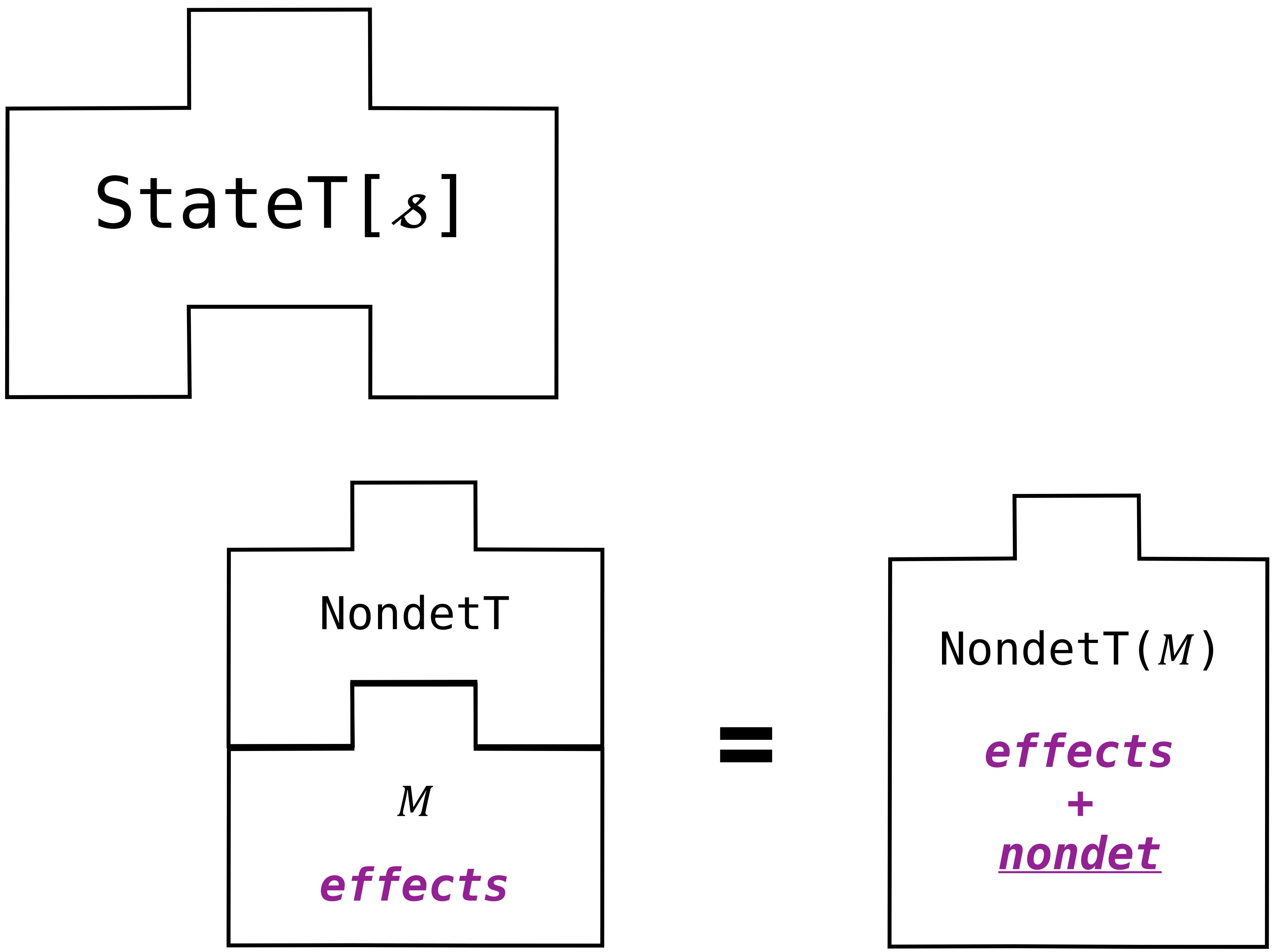
Monad Transformers



Monad Transformers



Monad Transformers



Monad Transformers

```
type M(t)

op x ← e1 ; e2
op return(e)

op getEnv
op putEnv(e)

op fail/e1⊞e2
```

Monad Transformers

ID

=

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type M(t)

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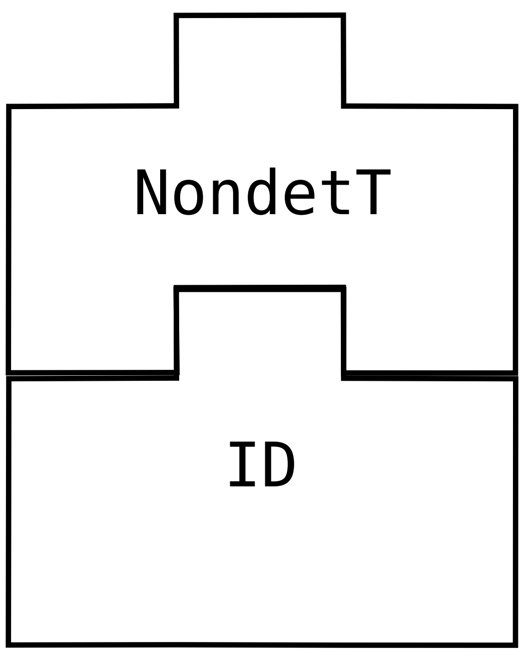
Monad Transformers



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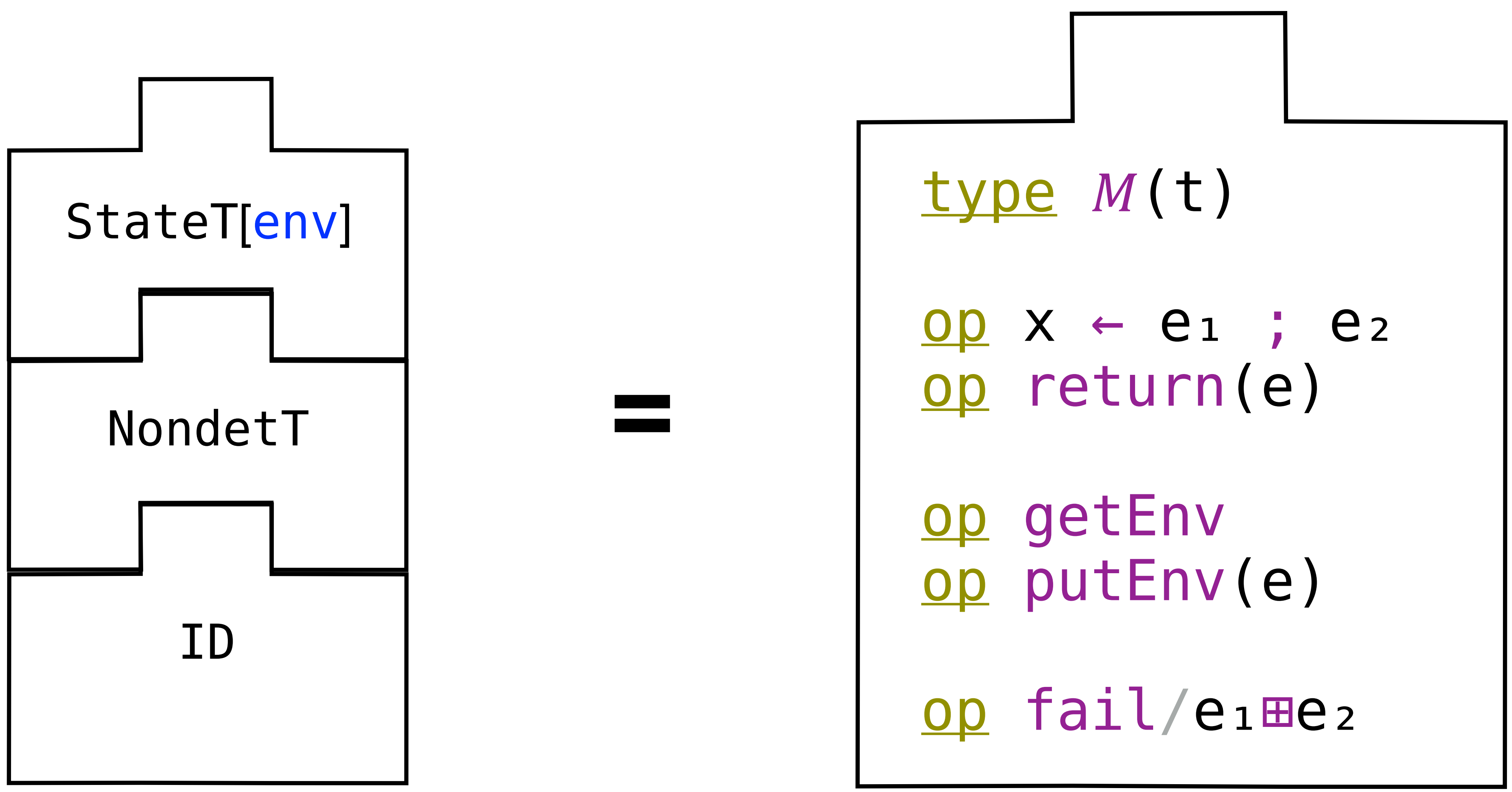
Monad Transformers



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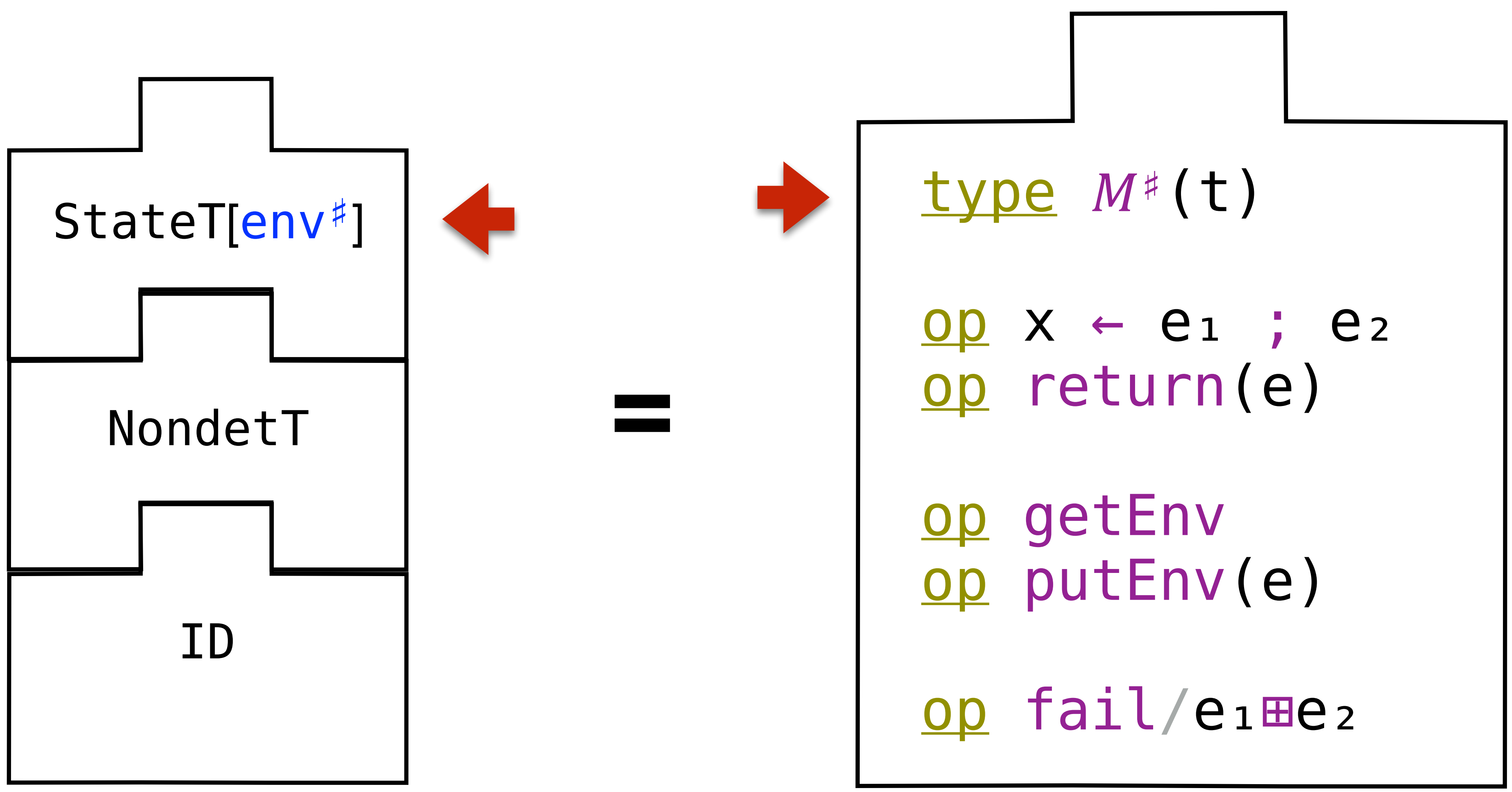

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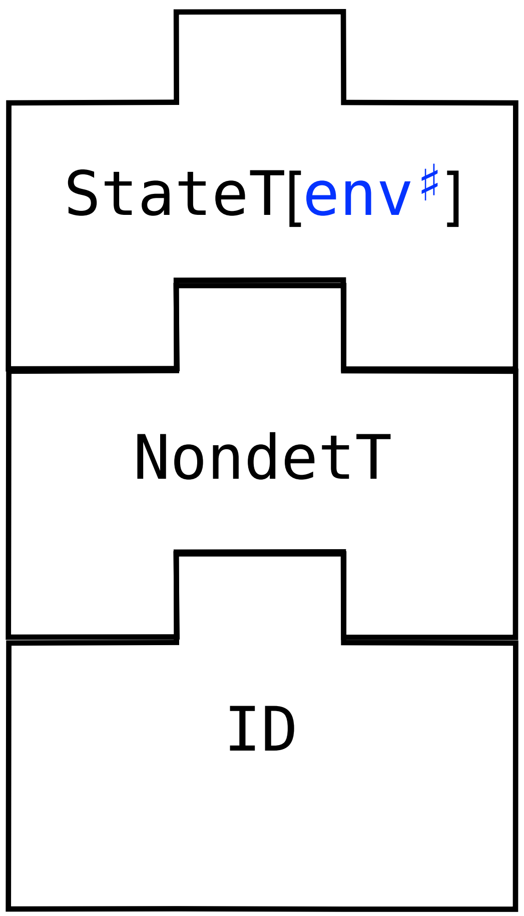

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type M♯(t)

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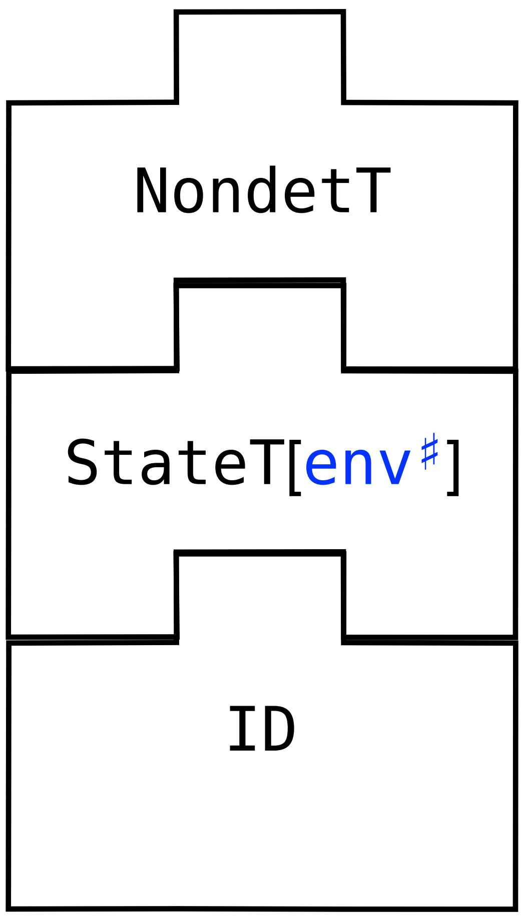
```

Path-sensitive

Monad Transformers



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op return(e)

op getEnv
op putEnv(e)

op fail/e₁⊞e₂
```
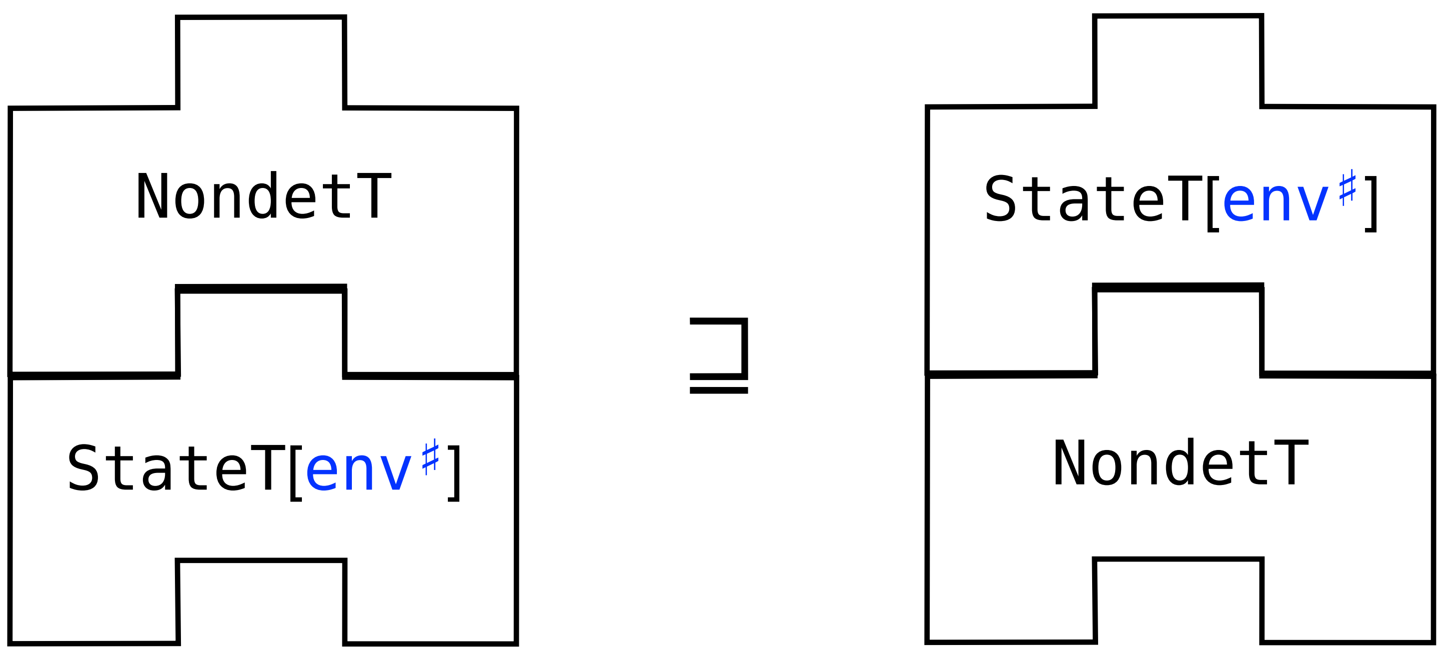
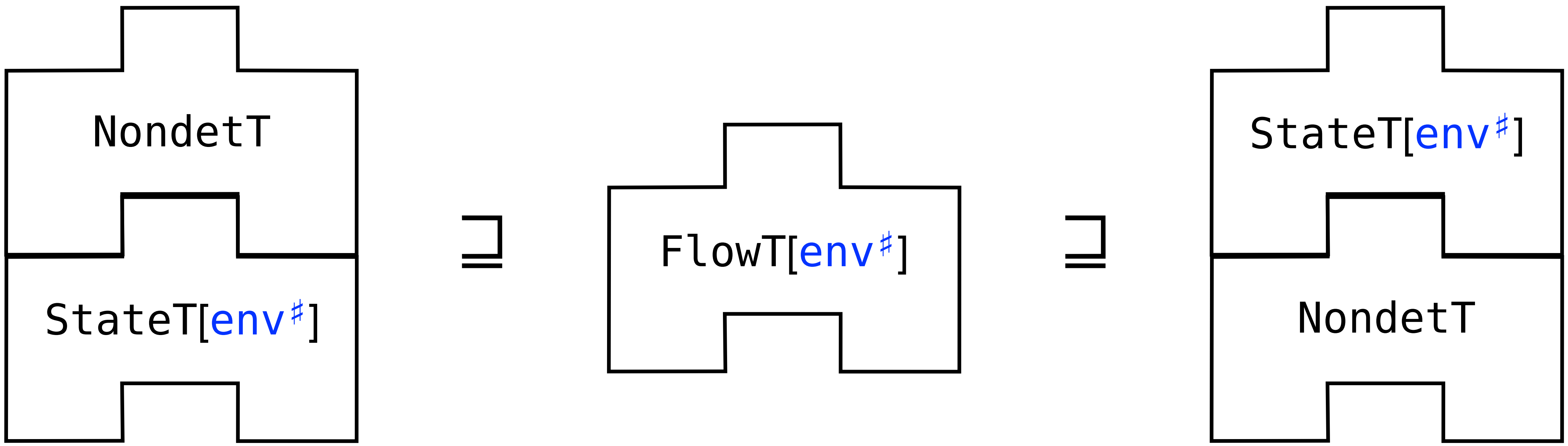

Flow-insensitive

Monad Transformers



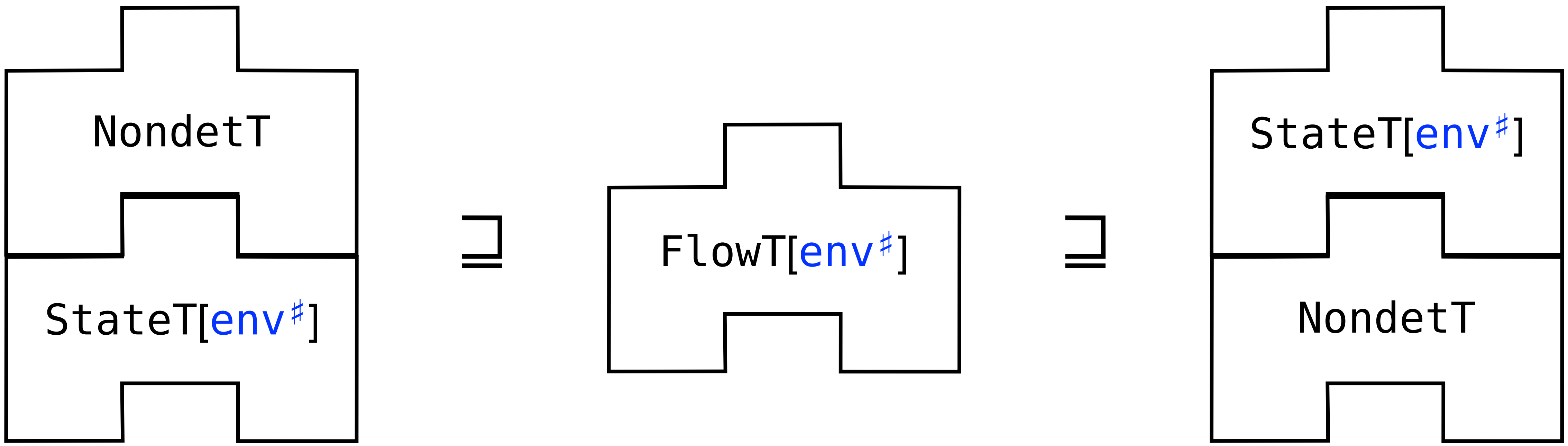
Flow-insensitive *Path-sensitive*

Monad Transformers



Flow-insensitive *Flow-sensitive* *Path-sensitive*

Monad Transformers



Flow-insensitive	*Flow-sensitive*	*Path-sensitive*
$\mathcal{P}(\mathtt{exp}) \times \mathtt{env}^{\sharp}$	$\mathtt{exp} \mapsto \mathtt{env}^{\sharp}$	$\mathtt{exp} \mapsto \mathcal{P}(\mathtt{env}^{\sharp})$

Monad Transformers

Flow-insensitive	*Flow-sensitive*	*Path-sensitive*
$\mathcal{P}(\text{exp}) \times \text{env}^\sharp$	$\text{exp} \mapsto \text{env}^\sharp$	$\text{exp} \mapsto \mathcal{P}(\text{env}^\sharp)$

```
N ∈ {-,0,+}
x ∈ {0,+}
y ∈ {-,0,+}

UNSAFE: {100/N}
UNSAFE: {100/x}
```

```
4:    x ∈ {0,+}
4.T: N ∈ {-,+}
5.F: x ∈ {0,+}

N,y ∈ {-,0,+}

UNSAFE: {100/x}
```

```
4: N∈{-,+},x∈{0}
4: N∈{0},x∈{+}

N∈{-,+},y∈{-,0,+}
N∈{0},y∈{0,+}

SAFE
```

Building Monads

- Construct a monad using `StateT[`$\mathcal{S}$`]`, `FlowT[`$\mathcal{S}$`]` and `NondetT`
- Order matters, yielding different analyses
- Rapidly prototype precision performance tradeoffs

Why Transformers

- Semantics independent building blocks for writing interpreters—also apply to abstract interpreters!
- Reuse of analysis machinery
  - Different abs. interpreters use the same transformers
- Variations in analysis
  - Different transformer stacks fit into the same interpreter

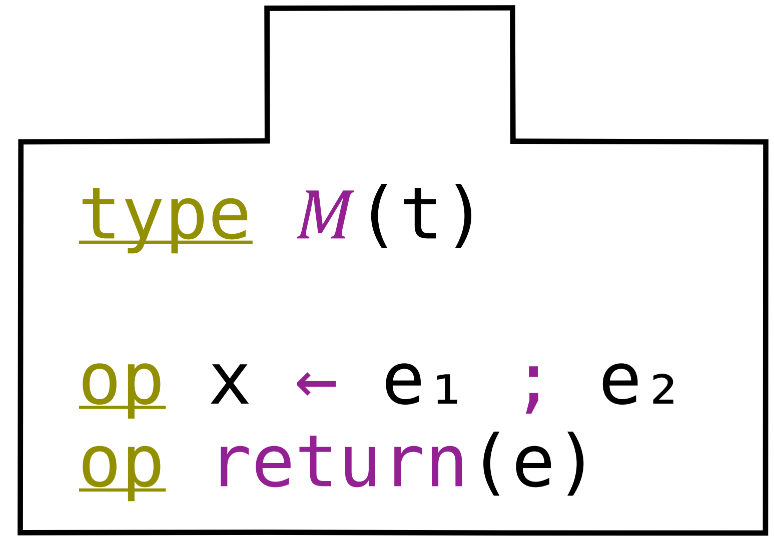
Galois Transformers

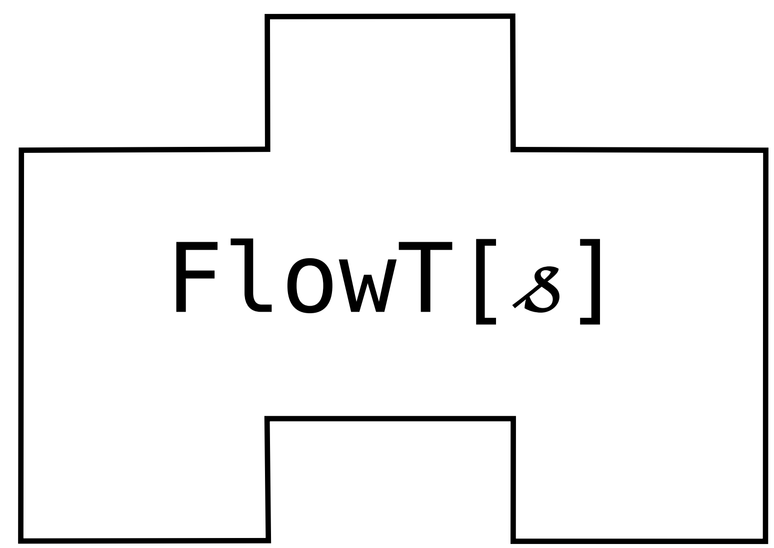
- What's a Monad?
- What are Transformers?
- What are Galois Connections?

```
type M(t)

op x ← e₁ ; e₂
op return(e)
```

Galois Transformers

- What's a Monad?
- What are Transformers?
- What are Galois Connections?





Galois Transformers

- What's a Monad?
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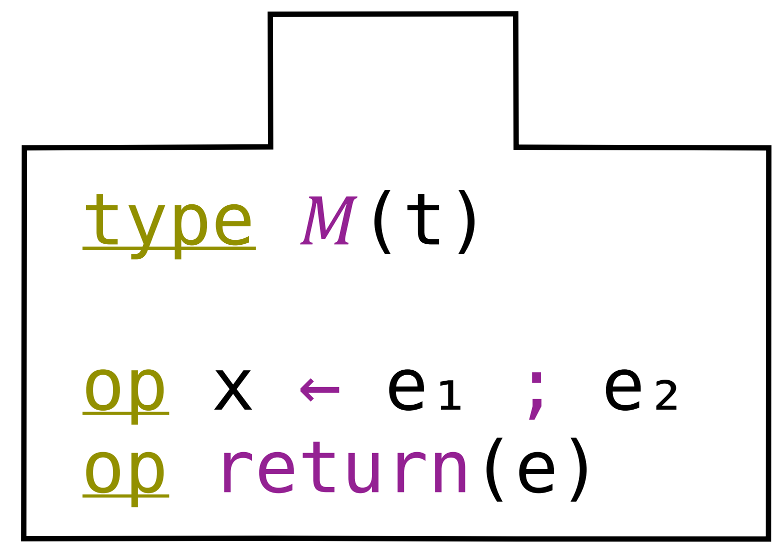
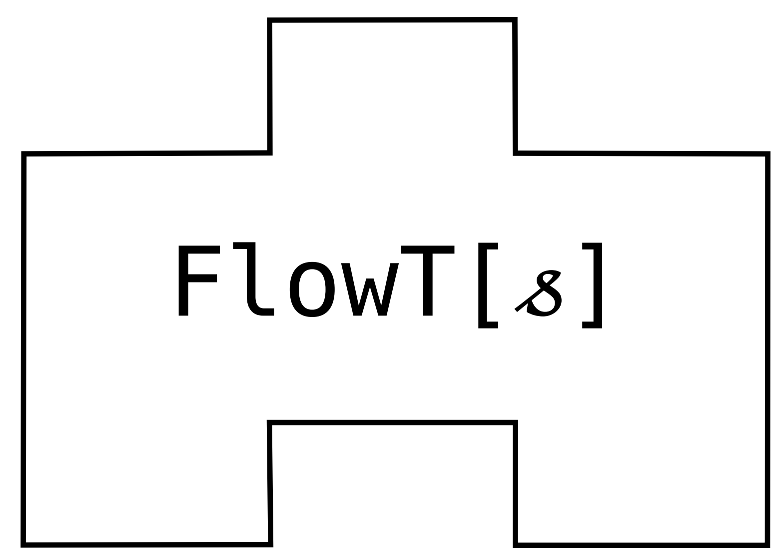
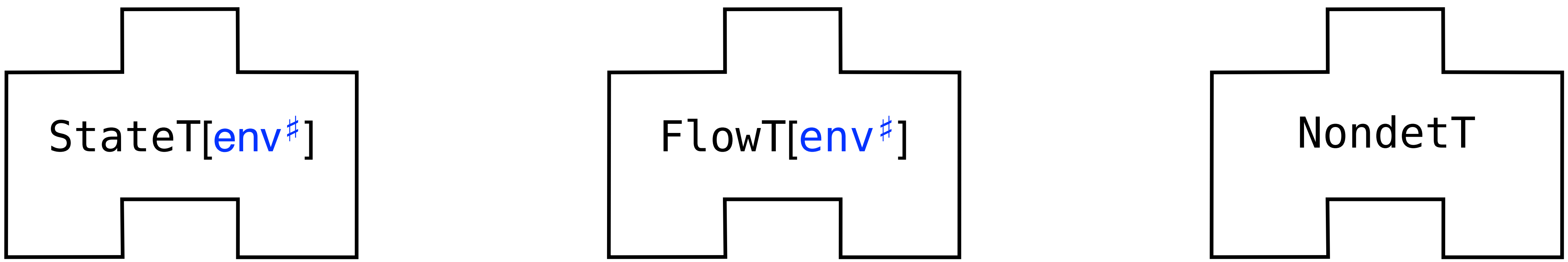




Galois Connections

- Compositional framework for proving correctness
- We build two sets of GCs alongside transformers
- **Code**: Enables execution of monadic analyzers
- **Proofs**: Large number of proofs built automatically
- (See the paper)

Galois Transformers



- GTs = Monad Transformers + Galois connections
- Galois connections are necessary for execution and proof of correctness for abstract interpreter

Putting it All Together

- You design a monadic abstract interpreter
- Instantiate with monad transformers
- Change underlying monad to change results
- Galois connections synthesized for free:
    - **Code**: Execution engine for running the analysis
    - **Proofs**: Large part of correctness argument

Implementation

- Haskell package: `cabal install maam`
- Galois Transformers are implemented as a semantics independent library
- Haskell's support for monadic programming was helpful, but not necessary

Let's Design an Analysis

Program

```
0: int x y; // global state
1: void safe_fun(int N) {
2:  if (N≠0) {x := 0;}
3:  else      {x := 1;}
4:  if (N≠0) {y := 100/N;}
5:  else      {y := 100/x;}}
```

Analysis Property

```
x/0
```

Abstract Values

$\mathbb{Z} \sqsubseteq \{-,0,+\}$

Implement

```
analyze : exp → results
analyze(x := æ) :=
    .. x .. æ ..
analyze(I {e₁}{e₂}) :=
    .. æ .. e₁ .. e₂ ..
```

Get Results

```
4: N∈{-,+},y∈{0}
4: N∈{0},x∈{+}

N∈{-,+},y∈{-,0,+}
N∈{0},y∈{0,+}
```

SAFE

Prove Correct

$[\![e]\!] \in [\![\mathrm{analyze}(e)]\!]$

Let's Design an Analysis

Program

```
safe_fun.js
```

Analysis Property

```
x/0
```

Abstract Values

$\mathbb{Z} \sqsubseteq \{-,0,+\}$

Implement

```
analyze : exp → results
analyze(x := æ ... :=
    .. x ... æ ..
analyze(I... {e₁}{e₂}) :=
    .. æ .. e₁ .. e₂ ..
```

Get Results

```
4: N∈{-,+},x∈{0}
4: N∈{0},x∈{+}

N∈{-,+},y∈{-,0,+}
N∈{0},y∈{0,+}
```

SAFE

Prove Correct

$[\![e]\!] \in [\![\mathrm{analyze}(e)]\!]$

Future Work

- Benchmark interaction between flow sensitivity and other design choices, like context or object sensitivity
- Explore uses of `NondetT` and `FlowT[`$\mathcal{s}$`]` outside analysis
- Other methods for executing monadic abstract interpreters; might relate to pushdown analysis
- Steps toward modular *verified* abstract interpreters in Coq or Agda using Galois Transformer proof framework
  - First step, mechanizing Galois connections
  - Draft: *Mechanically Verified Calculational Abstract Interpretation* (w/Van Horn)