Edge-Based Adaptive Distributed Method for Synchronization of Intermittently Coupled Spatiotemporal Networks

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***Abstract*—In biology networks, social networks, mobile robots, and many real networks, spatial factors are crucial to the evolution of networks and individual interactions are not always continuous. In this article, a kind of intermittently coupled spatiotemporal networks (ICSNs) is proposed, where the couplings among nodes are intermittent and the coupling strengths are related to both time and space. By constructing piecewise auxiliary functions and developing a direct error method, a distributed intermittent adaptive protocol and its pinning form are designed to determine the space-time dependent weights of edges to realize synchronization of ICSNs. Lastly, the theoretical analysis is supported by means of a numerical example.**

***Index Terms*—Complex network, distributed adaptive control, intermittent coupling, pinning strategy, synchronization.**

I. Introduction

Complex networks are pervasive and numerous practical systems or problems can be depicted and studied by complex networks [1], [2]. As an emergence property of complex networks [3], synchronization has already aroused much concern in view of its great applicable value in opinion consensus of social networks [4], flocking of mobile agents [5], coordination control of agents [6]. Besides, numerous complex networks including food webs, navigation networks, and social networks depend on not only the time but also the spatial distances [7], [8]. For example, as a kind of distance metric, friendship hop is usually considered in social networks and the spreading of network information is modeled as spatiotemporal systems described by partial differential equations [8]. Spatiotemporal system has also a wide range of engineering applications and has been widely utilized in artificial neural networks [9], thermostatically controlled loads in smart grids [10], flexible manipulators [11], mining cable elevators [12], and vibration suppression in deep-sea construction [13]. Until now, numerous excellent results have been obtained to address the synchronization of spatiotemporal networks [14]–[17].

In view of its unique characteristics of adaptation for system evolution and external disturbance [18], adaptive control has been broadly used in synchronization of coupled systems to reduce control gains [19], [20] and determine time-varying coupling weights [21], [22].

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Especially, several distributed adaptive schemes based on edge-weights have been proposed to explore coordination control of complex dynamic models [23]–[26] and synchronization of coupled neural networks with reaction–diffusion [27]–[30], where the adaptive weights of edges are irrelevant to spatial positions and the spatial effect on edge weights is ignored.

On the other side, note that the most results on synchronization of spatiotemporal networks such as [27]–[31] were derived under the assumption that the individual interactions are constantly continuous. Nevertheless, the couplings among nodes in some practical applications may be not always continuous but intermittent. For instance, the intermittent dispersal between two patches in biology networks [32], the intermittent information exchange mode among individuals in networks of mobile robots and multiagent networks [33]–[36], in which each unit exchanges information with its neighbors during specific time periods and disconnects from each other for the rest of time so as to have adequate time to adjust its attitude. Evidently, intermittent coupling is a discontinuous form of communication and has greater flexibility for nodes in comparison with common continuous coupling since they are not subject to communication requirements during the decoupling period. However, it is also full of challenges to investigate intermittent connection. Primarily, intermittent coupling inevitably leads to the difference of dynamics of nodes in coupling time and decoupling period, so this first challenge is how to establish mathematical models to integrate the difference and accurately describe the mechanism of intermittent communication. Additionally, since the coupling among nodes is intermittent, the adaptive design for edge weights should be also intermittent. This means that the traditional continuous design cannot be immediately referred and some innovative intermittent adaptive protocols are required to be developed. Besides, for the established coupled systems, which are probably some hybrid models with high dimensions, it is extremely difficult to analyze the synchronization of hybrid systems based on traditional analytical methods.

Inspired by the aforementioned analysis, under the edge-based weight framework in this article, a class of intermittently coupled spatiotemporal networks (ICSNs) is established and the synchronization is explored via developing edge-based distributed intermittent adaptive schemes and a direct error approach. The innovative contents include the following aspects.

1) Under the edge-based weight framework, a type of ICSNs is proposed by means of the introduction of an index function of intermittent coupling and partial differential equations, where the couplings among nodes are intermittent and the time-varying weights are dependent of time and space, these are different utterly from the traditional continuous couplings [21], [23]–[25] and the space-independent weights discussed in [27]–[30].
2) Unlike the central error technique and continuous Lyapunov construction employed in [21], [23]–[25], and [28]–[31], by constructing a piecewise auxiliary function and developing a direct error approach, a distributed intermittent adaptive scheme is designed to update the space-time dependent weight of each edge to achieve

synchronization. Note that the adaptive strategy designed here is intermittently updated on time, which is a significant difference from the previous time-dependent continuous adaptive schemes [21]–[25], [27]–[30].

3) An edge-based pinning intermittent adaptive scheme is proposed, which indicates that the synchronization of ICSNs can be reached by simply tuning the space-time dependent strengths of edges within a spanning tree. Especially, some previous results in [28] and [29] can be regarded as natural consequences of our results.

Notation: $\mathbb{R}=(-\infty,+\infty)$, $\mathbb{R}^+=[0,+\infty)$, $\mathbb{R}^m$ is the real space with m-dimension, $\mathbb{R}^{n\times m}$ is the real matrix space with $n\times m$ dimension. $\mathscr{N}=\{1,2,\ldots,N\}$, Z^+ consists of all nonnegative integers. For a function $W(t)$, its left and right limits at time t are denoted as $W(t^-)$ and $W(t^+)$. Let $A\in\mathbb{R}^{n\times n}$, $A>0$ $(A<0)$ implies that it is positive (negative) definite. I_n is the n-dimensional unit matrix. For a square matrix H, H^T is its transpose, $\lambda_m(H)$ and $\lambda_M(H)$ are the minimum and the maximum eigenvalues of H. $\Omega=\{z=(z_1,z_2,\ldots,z_q)^T\in\mathbb{R}^q:|z_s|<\gamma_s, s=1,2,\ldots,q\}$ and $\partial\Omega$ denotes its boundary, $\overline{\Omega}=\Omega\cup\partial\Omega$. For a scalar function $u:\Omega\to R$, $\Delta u=\sum_{k=1}^{q}\frac{\partial^2 u}{\partial x_k^2}$ denotes the Laplace operator of u on Ω, $\text{grad}(u)=(\frac{\partial u}{\partial x_1},\ldots,\frac{\partial u}{\partial x_q})^T$ represents the gradient of u. For a vector function $g:R^m\to R^m$, $\text{div}(g)=\sum_{r=1}^{m}\frac{\partial g_r}{\partial x_r}$ is the divergence of g, $\Delta g\triangleq(\Delta g_1,\ldots,\Delta g_m)^T$. The norm $\|F(x,t)\|=(\int_\Omega\sum_{\tau=1}^{m}F_\tau^2(x,t)dx)^{\frac{1}{2}}$ for $F(x,t)=(F_1(x,t),F_2(x,t),\ldots,F_m(x,t))^T\in\mathbb{R}^m$ with $(x,t)\in\Omega\times\mathbb{R}^+$. $\mathcal{G}=\{\mathcal{V},E\}$ denotes an undirected graph, here $\mathcal{V}=\{1,2,\ldots,N\}$ and E are the sets of nodes and edges. $C=(c_{\tau\varrho})_{N\times N}$ is the topological connection matrix of the graph, here $c_{\tau\varrho}=c_{\varrho\tau}=1$ $(\varrho\neq\tau)$ if the τth node is connected with the ϱth node, otherwise $c_{\tau\varrho}=c_{\varrho\tau}=0$ $(\varrho\neq\tau)$, and $c_{\tau\tau}=-\sum_{\varrho=1,\varrho\neq\tau}^{N}c_{\tau\varrho}$ for $\tau\in\mathscr{N}$. In this article, the networks are supposed to be connected during each coupling time.

II. Model Description and Preliminaries

In consideration of the objective existence of the intermittent communication and the practical significance of spatial position in engineering applications [7]–[13], the following spatiotemporal coupled system is introduced:

$$\frac{\partial y_\tau(x,t)}{\partial t}=D\Delta y_\tau(x,t)+f(y_\tau(x,t))+\sum_{k=0}^{+\infty}\sum_{\varrho=1}^{N}c_{\tau\varrho}\sigma_{\tau\varrho}(x,t)Hy_\varrho(x,t)\delta_k(t),\ \tau\in\mathscr{N}\tag{1}$$

where $y_\tau=(y_\tau^1,y_\tau^2,\ldots,y_\tau^n)^T$ is the τth node's state variable, $D=\text{diag}(d_1,\ldots,d_n)>0$ is the matrix of diffusion coefficient, $f:R^n\to R^n$ is a nonlinear vector field, $H=\text{diag}(h_1,\ldots,h_n)>0$, $\sigma_{\tau\varrho}(x,t)=\sigma_{\varrho\tau}(x,t)\geq 0$ $(\varrho\neq\tau)$ represents the weight of the edge (τ,ϱ), $c_{\tau\tau}\sigma_{\tau\tau}(x,t)=-\sum_{\varrho=1,\varrho\neq\tau}^{N}c_{\tau\varrho}\sigma_{\tau\varrho}(x,t)$, $\delta_k(t)$ for any $k\in Z^+$ is an index function of intermittent coupling expressed as

$$\delta_k(t)=\begin{cases}1, & \zeta_k\leq t\leq\vartheta_k\\ 0, & \text{otherwise}\end{cases}$$

in which $\zeta_0=0$, $\zeta_k<\vartheta_k<\zeta_{k+1}$ for all $k\in Z^+$ and $\lim_{k\to+\infty}\zeta_k=+\infty$, the time duration $[\zeta_k,\vartheta_k]$ is the coupling time and the rest of time is the decoupling period.

The initial and boundary values of the coupled model (1) are provided by

$$y_\tau(x,0)=\varphi_\tau(x),\quad x\in\Omega$$
$$y_\tau(x,t)=0,\ (x,t)\in\partial\Omega\times\mathbb{R}^+$$

in which φ_τ is continuous and bounded on Ω, $\tau\in\mathscr{N}$.

Remark 1: Evidently, if there is no decoupling time, namely, $\vartheta_k=\zeta_{k+1}$ with $k\in Z^+$, the intermittently coupled network (1) is degenerated as the following continuously coupled system:

$$\frac{\partial y_\tau(x,t)}{\partial t}=D\Delta y_\tau(x,t)+f(y_\tau(x,t))+\sum_{\varrho=1}^{N}c_{\tau\varrho}\sigma_{\tau\varrho}(x,t)Hy_\varrho(x,t),\ \tau\in\mathscr{N}.\tag{2}$$

Definition 1: The spatiotemporal coupled model (1) is called to reach synchronization if

$$\lim_{t\to+\infty}\|y_\varrho(x,t)-y_\tau(x,t)\|=0,\quad\tau,\varrho\in\mathscr{N}.$$

Assumption 1: There exists a real number F such that for all κ, $\mu\in R^n$

$$(\kappa-\mu)^T(f(\kappa)-f(\mu))\leq F(\kappa-\mu)^T(\kappa-\mu).$$

Assumption 2: There exist positive numbers T and β satisfying $T\geq\beta$ such that

$$\inf_{k\in Z^+}\{\vartheta_k-\zeta_k\}=\beta,\quad\sup_{k\in Z^+}\{\zeta_{k+1}-\zeta_k\}=T.$$

Definition 2 (see[37]): For a connected graph $\mathcal{G}$, an acyclic connected subgraph containing all vertices is called to a spanning tree of it.

Lemma 1 (see[38]): For a column vector $u=(u_1^T,\ldots,u_n^T)^T\in R^{nN}$ satisfying $\sum_{\tau=1}^{N}u_\tau=0$ with $u_\tau\in R^n$ and a positive semidefinite matrix $\Xi\in R^{n\times n}$, one has

$$u^T(L\otimes\Xi)u\geq\lambda_2(L)u^T(I_N\otimes\Xi)u$$

where L is the Laplacian matrix of an undirected connected network, $\lambda_2(L)>0$ is the second smallest eigenvalue of it.

Lemma 2 (see[39]): Assume that $g(x):\overline{\Omega}\to\mathbb{R}$ is continuous and differentiable on Ω, and $g(x)|_{\partial\Omega}=0$, then

$$\int_\Omega g^2(x)dx\leq\left(\frac{2\gamma_k}{\pi}\right)^2\int_\Omega\left(\frac{\partial g}{\partial x_k}\right)^2dx,\quad k=1,2,\ldots,q.$$

Lemma 3 (Gauss's Divergence Theorem [40]): Assume that V is a volume bounded by a simple closed surface ∂V and g is a differentiable vector field defined in V and on ∂V, then

$$\int_{\partial V}g\cdot d\vec{S}=\int_V\text{div}(g)dV$$

where $d\vec{S}$ is the outward drawn vector element of area.

For any $\tau,\varrho\in\mathscr{N}$, let $e_{\tau\varrho}(x,t)=y_\varrho(x,t)-y_\tau(x,t)$, then the error system can be derived as

$$\frac{\partial e_{\tau\varrho}(x,t)}{\partial t}=\begin{cases}D\Delta e_{\tau\varrho}(x,t)+\tilde{f}(e_{\tau\varrho}(x,t))\\ \quad+\sum_{p=1}^{N}c_{\varrho p}\sigma_{\varrho p}(x,t)He_{\varrho p}(x,t)\\ \quad-\sum_{p=1}^{N}c_{\tau p}\sigma_{\tau p}(x,t)He_{\tau p}(x,t)\\ \qquad\qquad\zeta_k\leq t\leq\vartheta_k\\ D\Delta e_{\tau\varrho}(x,t)+\tilde{f}(e_{\tau\varrho}(x,t))\\ \qquad\qquad\vartheta_k<t<\zeta_{k+1}\end{cases}\tag{3}$$

where $\tilde{f}(e_{\tau\varrho}(x,t))=f(y_\varrho(x,t))-f(y_\tau(x,t))$ and $\tau,\varrho\in\mathscr{N}$.

The following result is crucial to develop a new direct error approach to investigate the synchronization of network (1).

Lemma 4: For the synchronization state error, the following equality holds:

$$\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}\sum_{p=1}^{N}e_{\tau\varrho}^{T}(x,t)\left(c_{\varrho p}\sigma_{\varrho p}(x,t)He_{\varrho p}(x,t)\right.$$
$$\left.-c_{\tau p}\sigma_{\tau p}(x,t)He_{\tau p}(x,t)\right)$$
$$=-N\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}c_{\tau\varrho}\sigma_{\tau\varrho}(x,t)e_{\tau\varrho}^{T}(x,t)He_{\tau\varrho}(x,t). \quad (4)$$

Proof: First, $e_{\tau\varrho}(x,t)$ is simply denoted as $e_{\tau\varrho}$ for convenience. From $e_{\tau\varrho}=e_{\tau p}-e_{\varrho p}$

$$\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}\sum_{p=1}^{N}e_{\tau\varrho}^{T}\left(c_{\varrho p}\sigma_{\varrho p}He_{\varrho p}-c_{\tau p}\sigma_{\tau p}He_{\tau p}\right)$$
$$=\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}\sum_{p=1}^{N}c_{\varrho p}\sigma_{\varrho p}\left(-e_{\varrho p}^{T}He_{\varrho p}\right.$$
$$\left.+e_{\tau p}^{T}He_{\tau p}-e_{\tau\varrho}^{T}He_{\tau\varrho}\right)$$
$$=-(N-1)\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}c_{\tau\varrho}\sigma_{\tau\varrho}e_{\tau\varrho}^{T}He_{\tau\varrho}$$
$$+\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}\sum_{p=1}^{N}c_{\varrho p}\sigma_{\varrho p}e_{\tau p}^{T}He_{\tau p}. \quad (5)$$

Note that $\sum_{p=1}^{N}c_{p\varrho}\sigma_{p\varrho}=0$, one has

$$\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}\sum_{p=1}^{N}c_{\varrho p}\sigma_{\varrho p}e_{\tau p}^{T}He_{\tau p}$$
$$=-\sum_{\tau=1}^{N}\sum_{p=1}^{N}c_{\tau p}\sigma_{\tau p}e_{\tau p}^{T}He_{\tau p}.$$

Submitting it into (5), the equality (4) is obtained. ∎

III. Main Results

The synchronization of ICSNs (1) is investigated in this part via designing some intermittently spatiotemporal adaptive protocols for weights of all or partial edges.

Theorem 1: Under Assumptions 1 and 2, the synchronization of coupled model (1) is realized based on the following distributed intermittent adaptive protocol:

$$\begin{cases}\dfrac{\partial\sigma_{\tau\varrho}(x,t)}{\partial t}=\alpha_{\tau\varrho}c_{\tau\varrho}\left(y_{\varrho}(x,t)-y_{\tau}(x,t)\right)^{T}\\ \qquad\times H\left(y_{\varrho}(x,t)-y_{\tau}(x,t)\right),\ \zeta_k\leq t\leq\vartheta_k\\ \sigma_{\tau\varrho}(x,\zeta_{k+1})=\sigma_{\tau\varrho}(x,\vartheta_k)\\ \sigma_{\tau\varrho}(x,t)=0,\ \vartheta_k<t<\zeta_{k+1},(\tau,\varrho)\in E\end{cases} \quad (6)$$

where the initial values of weights $\sigma_{\tau\varrho}(x,0)$ are continuous and nonnegative for $x\in\overline{\Omega}$, $\alpha_{\tau\varrho}=\alpha_{\varrho\tau}>0$, $(\tau,\varrho)\in E$.

Proof: Evidently, a suitable number $\mu>0$ can be chosen such that $\varepsilon=\mu\beta-2F(T-\beta)>0$. Constructing the following piecewise function:

$$W_1(t)=\begin{cases}\dfrac{N}{2}e^{-\mu(t-\zeta_k)}\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}\dfrac{1}{\alpha_{\tau\varrho}}\\ \times\displaystyle\int_{\Omega}\left(\omega^{*}-\sigma_{\tau\varrho}(x,t)\right)^{2}dx,\ t\in[\zeta_k,\vartheta_k]\\ \dfrac{N}{2}e^{2F(t-\vartheta_k)-\mu(\vartheta_k-\zeta_k)}\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}\dfrac{1}{\alpha_{\tau\varrho}}\\ \times\displaystyle\int_{\Omega}\left(\omega^{*}-\sigma_{\tau\varrho}(x,\vartheta_k)\right)^{2}dx,\ t\in(\vartheta_k,\zeta_{k+1})\end{cases}$$

where $k\in Z^{+}$, ω^{*} is a positive number. It is easy that $W_1(t)$ is continuous except on $t=\zeta_{k+1}$ and

$$W_1(\zeta_{k+1})=W_1(\zeta_{k+1}^{+})$$
$$=e^{\mu(\vartheta_k-\zeta_k)-2F(\zeta_{k+1}-\vartheta_k)}W_1(\zeta_{k+1}^{-}). \quad (7)$$

Construct the following Lyapunov-like function:

$$V_1(t)=U(t)+W_1(t) \quad (8)$$

here

$$U(t)=\frac{1}{2}\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}\int_{\Omega}e_{\tau\varrho}^{T}(x,t)e_{\tau\varrho}(x,t)dx.$$

When $\zeta_k\leq t\leq\vartheta_k$, by Assumptions 1 and 2 and Lemma 4

$$D^{+}V_1(t)\leq\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}\int_{\Omega}e_{\tau\varrho}^{T}D\Delta e_{\tau\varrho}(x,t)dx$$
$$+\left(F+\frac{\mu}{2}\right)\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}\int_{\Omega}e_{\tau\varrho}^{T}(x,t)e_{\tau\varrho}(x,t)dx$$
$$-Ne^{-\mu T}\omega^{*}\int_{\Omega}\sum_{\tau=1}^{N}\sum_{\varrho=1,\varrho\neq\tau}^{N}c_{\tau\varrho}e_{\tau\varrho}^{T}(x,t)$$
$$\times He_{\tau\varrho}(x,t)dx-\mu V_1(t). \quad (9)$$

By using operation laws of divergence

$$\int_{\Omega}e_{\tau\varrho}^{T}(x,t)D\Delta e_{\tau\varrho}(x,t)dx$$
$$=\int_{\Omega}\sum_{r=1}^{n}e_{\tau\varrho}^{r}(x,t)d_r\mathrm{div}\left(\mathrm{grad}(e_{\tau\varrho}^{r}(x,t))\right)dx$$
$$=\int_{\Omega}\sum_{r=1}^{n}\mathrm{div}\left(e_{\tau\varrho}^{r}(x,t)d_r\mathrm{grad}(e_{\tau\varrho}^{r}(x,t))\right)dx$$
$$-\int_{\Omega}\sum_{r=1}^{n}\left(\mathrm{grad}(e_{\tau\varrho}^{r}(x,t))\right)^{T}d_r\mathrm{grad}(e_{\tau\varrho}^{r}(x,t))dx. \quad (10)$$

Note that $e_{\tau\varrho}(x,t)=0$ for $(x,t)\in\partial\Omega\times\mathbb{R}^{+}$, which combines with Lemma 3, one has

$$\sum_{r=1}^{n}\int_{\Omega}\mathrm{div}\left(e_{\tau\varrho}^{r}(x,t)d_r\mathrm{grad}(e_{\tau\varrho}^{r}(x,t))\right)dx$$
$$=\sum_{r=1}^{n}\int_{\partial\Omega}e_{\tau\varrho}^{r}(x,t)d_r\mathrm{grad}(e_{\tau\varrho}^{r}(x,t))\cdot d\vec{S}=0.$$

Besides, from Lemma 2

$$
\begin{aligned}
&-\int_\Omega \sum_{r=1}^n \left(\operatorname{grad}(e^r_{\tau\varrho}(x,t))\right)^T d_r \operatorname{grad}(e^r_{\tau\varrho}(x,t))dx \\
&= -\sum_{r=1}^n\sum_{k=1}^q \int_\Omega d_r \left(\frac{\partial e^r_{\tau\varrho}(x,t)}{\partial x_k}\right)^2 dx \\
&\le -\left(\frac{\pi}{2}\right)^2 \sum_{r=1}^n\sum_{k=1}^q \frac{d_r}{\gamma_k^2}\int_\Omega \left(e^r_{\tau\varrho}(x,t)\right)^2 dx \\
&\le -\xi \int_\Omega e^T_{\tau\varrho}(x,t)e_{\tau\varrho}(x,t)dx
\end{aligned}
$$

where $\xi = (\frac{\pi}{2})^2 \sum_{k=1}^q \frac{\min\{d_r\}}{\gamma_k^2}$.

Submitting the above-mentioned two results into the equality (10)

$$
\begin{aligned}
&\int_\Omega e^T_{\tau\varrho}(x,t)D\Delta e_{\tau\varrho}(x,t)dx \\
&\le -\xi \int_\Omega e^T_{\tau\varrho}(x,t)e_{\tau\varrho}(x,t)dx.
\end{aligned} \tag{11}
$$

Let $\hat{e}(x,t) = (\hat{e}_1^T(x,t), \hat{e}_2^T(x,t)\cdots, \hat{e}_N^T(x,t))^T$ and

$$
\hat{e}_\tau(x,t) = y_\tau(x,t) - \frac{1}{N}\sum_{k=1}^N y_k(x,t), \quad \tau\in\mathscr{N}
$$

it follows from Lemma 1 that

$$
\begin{aligned}
&\sum_{\tau=1}^N \sum_{\varrho=1,\varrho\ne\tau}^N c_{\tau\varrho} e^T_{\tau\varrho}(x,t) H e_{\tau\varrho}(x,t) \\
&= 2\sum_{\tau=1}^N\sum_{\varrho=1}^N l_{\tau\varrho}\hat{e}^T_\tau(x,t)H\hat{e}_\varrho(x,t) \\
&= 2\hat{e}^T(x,t)(L\otimes H)\hat{e}(x,t) \\
&\ge 2\lambda_2(L)\hat{e}^T(x,t)(I_N\otimes H)\hat{e}(x,t)
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
&\sum_{\tau=1}^N \sum_{\varrho=1,\varrho\ne\tau}^N e^T_{\tau\varrho}(x,t) e_{\tau\varrho}(x,t) \\
&= 2\sum_{\tau=1}^N\sum_{\varrho=1}^N m_{\tau\varrho}\hat{e}^T_\tau(x,t)\hat{e}_\varrho(x,t) \\
&= 2\hat{e}^T(x,t)(M\otimes I_n)\hat{e}(x,t)
\end{aligned} \tag{13}
$$

where $M = (m_{\tau\varrho})_{N\times N}$, $m_{\tau\varrho} = -1$ for $\tau\ne\varrho$ and $m_{\tau\tau} = N-1$.

Since $\lambda_2(L) > 0$, a suitable constant $\omega^* > 0$ can be selected such that

$$
\left(F + \frac{\mu}{2} - \xi\right)(M\otimes I_n) - \omega^* N e^{-\mu T}\lambda_2(L)(I_N\otimes H) \le 0
$$

then from (9) to (13)

$$
D^+V_1(t) \le -\mu V_1(t), \quad \zeta_k \le t \le \vartheta_k. \tag{14}
$$

In addition, it is obvious to derive that

$$
D^+V_1(t) \le 2FV_1(t), \quad \vartheta_k < t < \zeta_{k+1}. \tag{15}
$$

It follows from (7) and (8) that $V_1(t)$ is right continuous at the set $\{\zeta_k\}$. According to (14) and (15)

$$
V_1(t) \le V_1(\zeta_k)e^{-\mu(t-\zeta_k)}, \quad \zeta_k\le t\le\vartheta_k,\ k\in Z^+
$$

$$
V_1(t) \le V_1(\vartheta_k)e^{2F(t-\vartheta_k)}, \quad \vartheta_k < t < \zeta_{k+1},\ k\in Z^+
$$

which implies that

$$
V_1(\zeta^-_{k+1}) \le e^{2F(\zeta_{k+1}-\vartheta_k)}e^{-\mu(\vartheta_k-\zeta_k)}V_1(\zeta_k)
$$

this combines with the definition (8), Assumption 2 and the equality (7)

$$
\begin{aligned}
V_1(\zeta_{k+1}) &= U(\zeta_{k+1}) + W_1(\zeta_{k+1}) \\
&= U(\zeta_{k+1}) + e^{\mu(\vartheta_k-\zeta_k)-2F(\zeta_{k+1}-\vartheta_k)}W_1(\zeta^-_{k+1}) \\
&= e^{\mu(\vartheta_k-\zeta_k)-2F(\zeta_{k+1}-\vartheta_k)}V_1^-(\zeta_{k+1}) \\
&\quad + \left[1 - e^{\mu(\vartheta_k-\zeta_k)-2F(\zeta_{k+1}-\vartheta_k)}\right]U(\zeta_{k+1}) \\
&\le V_1(\zeta_k) + (1-e^\varepsilon)U(\zeta_{k+1})
\end{aligned}
$$

which shows that

$$
V_1(\zeta_{k+1}) - V_1(\zeta_k) \le (1-e^\varepsilon)U(\zeta_{k+1}), \quad k\in Z^+. \tag{16}
$$

Note that $\varepsilon > 0$ and $V_1(t)\ge 0$, it follows from (16) that

$$
\sum_{\tau=1}^{+\infty} U(t_\tau) \le \frac{V_1(0)}{e^\varepsilon - 1}.
$$

By means of the property of convergent series

$$
\lim_{k\to+\infty} U(\zeta_k) = 0. \tag{17}
$$

Since $w_{\tau\varrho}(x,t)\ge 0$ for $\tau,\varrho\in Z^+$ and $t\ge 0$, by Lemma 4

$$
D^+U(t) \le 2(F-\xi)U(t), \quad \zeta_k\le t<\zeta_{k+1}
$$

and

$$
U(t) \le \Theta U(\zeta_k), \quad \zeta_k \le t < \zeta_{k+1}
$$

here $\Theta = \max\{1, e^{2(F-\xi)T}\}$. This combines with (17), $\lim_{t\to+\infty} U(t) = 0$ and the network (1) is synchronized. ■

In Theorem 1, the weights of all edges are updated based on the distributed intermittent adaptive control (6). In what follows, an edge-based pinning adaptive control will be proposed to achieve synchronization.

Denote $\hat{\mathcal{G}} = (\mathcal{V}, \hat{E})$ as a spinning tree of the graph associated with the coupled model (1), its existence is ensured by the connectivity of the model (1) in coupling time. Denote $\hat{L}$ be the Laplacian matrix of the subgraph $\hat{\mathcal{G}}$. Obviously, the elements of it can be expressed by

$$
\hat{l}_{\tau\varrho} = \begin{cases} -c_{\tau\varrho}, & \text{if } (\tau,\varrho)\in\hat{E} \\ -\sum_{\varrho=1,\varrho\ne\tau}^N \hat{l}_{\tau\varrho}, & \text{if } \tau=\varrho \\ 0, & \text{otherwise.}\end{cases}
$$

Denote $\chi_{\tau\varrho} = \inf_{x\in\overline{\Omega}}\sigma_{\tau\varrho}(x,0)$, $\hat{\Omega} = (\hat{\omega}_{\tau\varrho})_{N\times N}$, and

$$
\hat{\omega}_{\tau\varrho} = \begin{cases} -c_{\tau\varrho}\chi_{\tau\varrho}, & \text{if } (\tau,\varrho)\in E\setminus\hat{E} \\ -\sum_{\varrho=1,\varrho\ne\tau}^N \hat{\omega}_{\tau\varrho}, & \text{if } \tau=\varrho \\ 0, & \text{otherwise.}\end{cases}
$$

Theorem 2: Under Assumptions 1 and 2, the coupled system (1) achieves synchronization according to the following intermittent space-time dependent adaptive pinning protocol:

$$\begin{cases} \dfrac{\partial \sigma_{\tau\varrho}(x,t)}{\partial t} = \alpha_{\tau\varrho} c_{\tau\varrho} \left(y_\varrho(x,t) - y_\tau(x,t)\right)^T \\ \qquad \times H\left(y_\varrho(x,t) - y_\tau(x,t)\right),\ \zeta_k \le t \le \vartheta_k \\ \sigma_{\tau\varrho}(x, \zeta_{k+1}) = \sigma_{\tau\varrho}(x, \vartheta_k) \\ \sigma_{\tau\varrho}(x,t) = 0,\ \vartheta_k < t < \zeta_{k+1}, (\tau,\varrho) \in \hat{E} \end{cases} \tag{18}$$

in which $\sigma_{\tau\varrho}(x,0)$ are continuous and nonnegative for $x \in \overline{\Omega}$, $\alpha_{\tau\varrho} = \alpha_{\varrho\tau} > 0$ with $(\tau,\varrho) \in \hat{E}$.

Proof: Define a Lyapunov function

$$V_2(t) = U(t) + W_2(t)$$

here

$$W_2(t) = \begin{cases} \dfrac{N}{2} e^{-\mu(t-\zeta_k)} \sum_{\tau=1}^{N} \sum_{(\tau,\varrho)\in\hat{E}} \dfrac{1}{\alpha_{\tau\varrho}} \\ \quad \times \displaystyle\int_\Omega \left(\hat{\omega}^* - \sigma_{\tau\varrho}(x,t)\right)^2 dx,\ t \in [\zeta_k, \vartheta_k] \\ \dfrac{N}{2} e^{2F(t-\vartheta_k) - \mu(\vartheta_k - \zeta_k)} \sum_{\tau=1}^{N} \sum_{(\tau,\varrho)\in\hat{E}} \dfrac{1}{\alpha_{\tau\varrho}} \\ \quad \times \displaystyle\int_\Omega \left(\hat{\omega}^* - \sigma_{\tau\varrho}(x,\vartheta_k)\right)^2 dx,\ t \in (\vartheta_k, \zeta_{k+1}) \end{cases}$$

in which $k \in Z^+$, $\hat{\omega}^*$ is a positive number.

For $\zeta_k \le t \le \vartheta_k$, from Lemma 4

$$\begin{aligned} &D^+ V_2(t) \\ &\le \left(F + \frac{\mu}{2} - \xi\right) \sum_{\tau=1}^{N} \sum_{\varrho=1,\varrho\ne\tau}^{N} \int_\Omega e_{\tau\varrho}^T(x,t) e_{\tau\varrho}(x,t) dx \\ &\quad - N \sum_{\tau=1}^{N} \sum_{(\tau,\varrho)\in E\setminus\hat{E}} c_{\tau\varrho}\chi_{\tau\varrho} \int_\Omega e_{\tau\varrho}^T(x,t) H e_{\tau\varrho}(x,t) dx \\ &\quad - N\hat{\omega}^* e^{-\mu T} \sum_{\tau=1}^{N} \sum_{(\tau,\varrho)\in\hat{E}} c_{\tau\varrho} \int_\Omega e_{\tau\varrho}^T(x,t) H e_{\tau\varrho}(x,t) dx \\ &\quad - \mu V_2(t). \end{aligned} \tag{19}$$

Similar to (12)

$$\begin{aligned} &\sum_{\tau=1}^{N} \sum_{(\tau,\varrho)\in E\setminus\hat{E}} c_{\tau\varrho}\chi_{\tau\varrho} e_{\tau\varrho}^T(x,t) H e_{\tau\varrho}(x,t) \\ &= 2\hat{e}^T(x,t)(\hat{\Omega} \otimes H)\hat{e}(x,t) \end{aligned} \tag{20}$$

$$\begin{aligned} &\sum_{\tau=1}^{N} \sum_{(\tau,\varrho)\in\hat{E}} c_{\tau\varrho} e_{\tau\varrho}^T(x,t) H e_{\tau\varrho}(x,t) \\ &\ge 2\lambda_2(\hat{L})\hat{e}^T(x,t)(I_N \otimes H)\hat{e}(x,t). \end{aligned} \tag{21}$$

Since $\hat{\mathcal{G}} = (V, \hat{E})$ is connected, $\lambda_2(\hat{L}) > 0$, which implies that a suitable $\hat{\omega}^*$ can be selected such that

$$\begin{aligned} &\left(F + \frac{\mu}{2} - \xi\right)(M \otimes I_n) - \hat{\Omega} \otimes H - \hat{\omega}^* N e^{-\mu T} \lambda_2(\hat{L})(I_N \otimes H) \\ &\quad \le 0 \end{aligned}$$

which combines with (19)–(21)

$$D^+ V_2(t) \le -\mu V_2(t) \quad \zeta_k \le t \le \vartheta_k.$$

The rest of the proof is the same as that of Theorem 1. ■

If the weights of edges are independent of the space, the coupled model (1) is degenerated as

$$\begin{aligned} \frac{\partial y_\tau(x,t)}{\partial t} &= D\Delta y_\tau(x,t) + f(y_\tau(x,t)) \\ &\quad + \sum_{k=0}^{+\infty} \sum_{\varrho=1}^{N} c_{\tau\varrho}\sigma_{\tau\varrho}(t) H y_\varrho(x,t)\delta_k(t),\ \tau \in \mathscr{N}. \end{aligned} \tag{22}$$

Corollary 1: Based on Assumptions 1 and 2, the synchronization of spatiotemporal model (22) is reached under the following intermittent adaptive pinning protocol:

$$\begin{cases} \dot{\sigma}_{\tau\varrho}(t) = \alpha_{\tau\varrho} c_{\tau\varrho} \displaystyle\int_\Omega \left(y_\varrho(x,t) - y_\tau(x,t)\right)^T \\ \qquad \times H\left(y_\varrho(x,t) - y_\tau(x,t)\right) dx,\ \zeta_k \le t \le \vartheta_k \\ \sigma_{\tau\varrho}(\zeta_{k+1}) = \sigma_{\tau\varrho}(\vartheta_k) \\ \sigma_{\tau\varrho}(t) = 0,\ \vartheta_k < t < \zeta_{k+1}, (\tau,\varrho) \in \hat{E} \end{cases} \tag{23}$$

where $\sigma_{\tau\varrho}(0) \ge 0$ for $(\tau,\varrho) \in E$, and $\alpha_{\tau\varrho} = \alpha_{\varrho\tau} > 0$ with $(\tau,\varrho) \in \hat{E}$.

Proof: Define an auxiliary function

$$V_3(t) = U(t) + W_3(t)$$

in which

$$W_3(t) = \begin{cases} \dfrac{N}{2} e^{-\mu(t-\zeta_k)} \sum_{\tau=1}^{N} \sum_{(\tau,\varrho)\in\hat{E}} \dfrac{1}{\alpha_{\tau\varrho}} \left(\hat{\omega}^* - \sigma_{\tau\varrho}(t)\right)^2, \\ \quad \zeta_k \le t \le \vartheta_k \\ \dfrac{N}{2} e^{2F(t-\vartheta_k) - \mu(\vartheta_k - \zeta_k)} \sum_{\tau=1}^{N} \\ \sum_{(\tau,\varrho)\in\hat{E}} \frac{1}{\alpha_{\tau\varrho}} \left(\hat{\omega}^* - \sigma_{\tau\varrho}(\vartheta_k)\right)^2, \vartheta_k < t < \zeta_{k+1}. \end{cases}$$

The rest proof can follow from that of Theorem 2. ■

If the intermittent coupling is reduced to the continuous case, the pinning synchronization of the network (2) can be directly obtained from Theorem 2.

Corollary 2: Under Assumption 1, the synchronization of continuously coupled system (2) is achieved by the following adaptive pinning protocol:

$$\begin{aligned} \frac{\partial \sigma_{\tau\varrho}(x,t)}{\partial t} &= \alpha_{\tau\varrho} c_{\tau\varrho} \left(y_\varrho(x,t) - y_\tau(x,t)\right)^T \\ &\quad \times H\left(y_\varrho(x,t) - y_\tau(x,t)\right),\ t \ge 0,\ (\tau,\varrho) \in \hat{E} \end{aligned}$$

in which $\sigma_{\tau\varrho}(x,0)$ are non-negative and continuous for $x \in \overline{\Omega}$, $\alpha_{\tau\varrho} = \alpha_{\varrho\tau} > 0$ with $(\tau,\varrho) \in \hat{E}$.

If the weights of edges are independent of the space, the continuously coupled network (2) can be rewritten as

$$\begin{aligned} \frac{\partial y_\tau(x,t)}{\partial t} &= D\Delta y_\tau(x,t) + f(y_\tau(x,t)) \\ &\quad + \sum_{\varrho=1}^{N} c_{\tau\varrho}\sigma_{\tau\varrho}(t) H y_\varrho(x,t),\ \tau \in \mathscr{N}. \end{aligned} \tag{24}$$

The following result is a consequence of Corollary 1.

Corollary 3: Under Assumption 1, the synchronization for coupled model (24) is realized via the following adaptive pinning strategy:

$$\begin{aligned} \dot{\sigma}_{\tau\varrho}(t) &= \alpha_{\tau\varrho} c_{\tau\varrho} \int_\Omega \left(y_\varrho(x,t) - y_\tau(x,t)\right)^T \\ &\quad \times H\left(y_\varrho(x,t) - y_\tau(x,t)\right) dx,\ t \ge 0,\ (\tau,\varrho) \in \hat{E} \end{aligned}$$

where $\sigma_{\tau\varrho}(0) \ge 0$, $\alpha_{\tau\varrho} = \alpha_{\varrho\tau} > 0$ with $(\tau,\varrho) \in \hat{E}$.

Remark 2: Actually, adaptive distributed control has been studied to explore synchronization or consensus of continuously coupled systems [21], [23]–[25]. In comparison with these results, spatial factor is considered in the modeling of networks and the distributed adaptive design in this article. Moreover, the designed adaptive laws for edge weights are intermittently updated, which is an essential difference from the traditional continuous adaptive distributed scheme.

Remark 3: In [27]–[30], some kinds of continuously coupled networks with reaction–diffusion were studied by designing adaptive adjustment strategies for edge weights, where the time-varying weights of edges are independent of the space. Unlike these results, a class of intermittent coupled spatiotemporal networks is addressed in this article and the adaptive weights of edges depend on both the time and the space which are consistent with the spatiotemporal characteristics of the considered networks. Furthermore, by comparison, some previous excellent work such as the results in [28] and [29] can be regarded as some natural consequences of Corollary 3 given in this article.

Remark 4: In [21], [23]–[25], and [28]–[31], distributed adaptive method was utilized to analyze the synchronization of coupled networks with continuous coupling, in which the central state $\bar{y} = \frac{1}{N}\sum_{\varrho=1}^{N} y_\varrho$ was introduced and the central errors $e_\tau = y_\tau - \bar{y}$ were computed. Different from the central error method, a direct error method is utilized in this article and a new equality about the direct synchronization errors $e_{\tau\varrho} = y_\varrho - y_\tau$ is established in Lemma 4 to discuss the synchronization of ICSNs with edge-based weights.

IV. Numerical Simulations

Recently, coupled neural networks with reaction–diffusion terms have been extensively investigated in view of their potential applications in the shortest path solution, pattern recognition, image processing [14], [27]–[31], where the coupling is supposed to be continuous. Here, we consider a class of intermittently coupled neural networks described by

$$\begin{aligned} \frac{\partial y_\tau(x,t)}{\partial t} &= D\Delta y_\tau(x,t) - y_\tau(x,t) + Qg(y_\tau(x,t)) \\ &\quad + \sum_{k=0}^{+\infty}\sum_{\varrho=1}^{9} c_{\tau\varrho}\sigma_{\tau\varrho}(x,t) y_\varrho(x,t)\delta_k(t) \end{aligned} \tag{25}$$

where $y_\tau = (y_\tau^1, y_\tau^2, y_\tau^3)^T$, $D = \mathrm{diag}\{0.01, 0.01, 0.01\}$, $x \in \Omega = \{x \in R : |x| < 1\}$, $g(y_\tau) = (g_1(y_\tau^1), g_2(y_\tau^2), g_3(y_\tau^3))^T$, $g_\varrho(y_\tau^\varrho) = 0.5(|y_\tau^\varrho + 1| - |y_\tau^\varrho - 1|)$ with $\varrho = 1, 2, 3$ and

$$Q = \begin{pmatrix} 1.25 & -3.2 & -3.2 \\ -3.2 & 1.1 & -4.4 \\ -3.2 & 4.4 & 1.0 \end{pmatrix}$$

the time sequences $\{\zeta_k\}$ and ϑ_k are given by

$$\begin{aligned} \{\zeta_k\} = \ &\{0, 0.8, 1.5, 3.0, 4.0, 6.0, 7.6, 8.8 \\ &10, 11.2, 12.4, 13.4, 14.6, 16, \cdots\} \end{aligned}$$

$$\begin{aligned} \{\vartheta_k\} = \ &\{0.5, 1.2, 2.2, 3.6, 5.4, 7.0, 8.4, 9.6 \\ &10.5, 12, 13.0, 14.0, 15.4, \ldots\} \end{aligned}$$

and the topology of the network (25) is given in Fig. 1.

Select $\hat{E} = \{(2,1), (3,1), (5,3), (5,4), (6,2)\}$. From Theorem 2, system (25) can realize synchronization based on the intermittent adaptive scheme (18). In order to show the theoretical result, in numerical simulations, $\alpha_{\tau\varrho} = 5$, $\psi_{\tau\varrho}(x) = |0.1\sin x|$, $\varphi_\tau(x)$ are some arbitrary constants in the interval $[-0.3, 0.3]$ for $\tau, \varrho \in \mathscr{N}$ and $x \in \Omega$, the synchronized error $E(x,t) = \sum_{\tau=1}^{N}\sum_{\varrho=1, \varrho\neq\tau}^{N} e_{\tau\varrho}^T(x,t)e_{\tau\varrho}(x,t)$ is shown in Fig. 2 and the evolutions of adaptive edge weights $\sigma_{\tau\varrho}(x,t)$ with $(\tau,\varrho) \in \hat{E}$ are demonstrated in Figs. 3– 7.

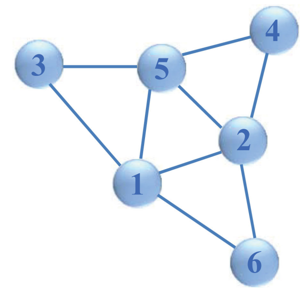

Fig. 1. Topology of the coupled system (25).

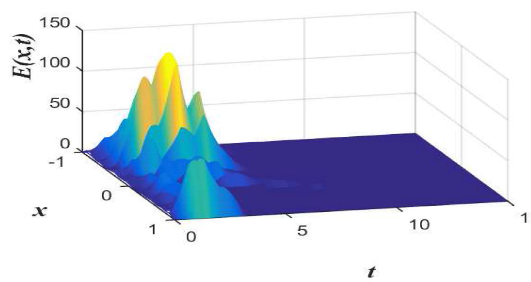

Fig. 2. Synchronization error of the network (25).

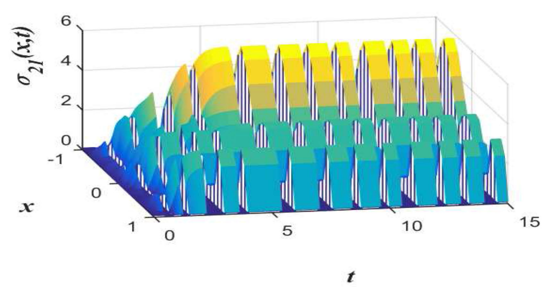

Fig. 3. Evolution of time-space dependent weight $\sigma_{21}(x,t)$.

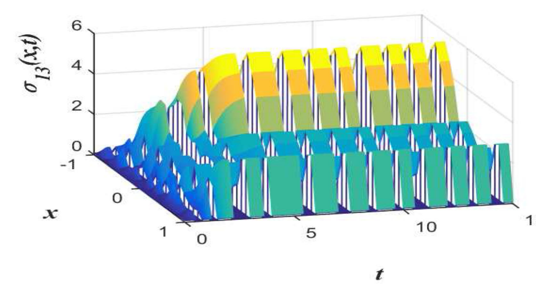

Fig. 4. Evolution of time-space dependent weight $\sigma_{13}(x,t)$.

From the evolutions of the coupled weights shown in Figs. 3–7, the following facts can be observed. First, each $\sigma_{\tau\varrho}(x,t)$ is a piecewise function with regard to time t but is continuous with respect to the space x. Additionally, for any given $x \in \overline{\Omega}$, $\sigma_{\tau\varrho}(x,t) = 0$ in decoupling time and $\sigma_{\tau\varrho}(x,t)$ are nondecreasing in coupling time. Besides, when the synchronization is realized, each $\sigma_{\tau\varrho}(x,t)$ tends to a certain constant in coupling time for any fixed $x \in \overline{\Omega}$, but $\sigma_{\tau\varrho}(x,t)$ is still a function with respect to the variable of space x at the fixed time. Obviously,

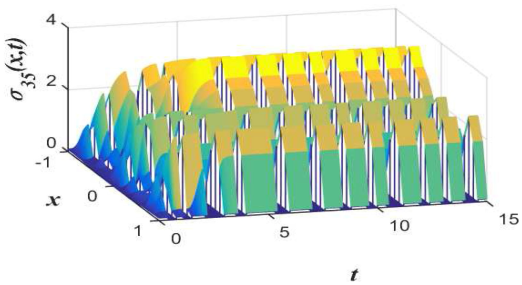

Fig. 5. Evolution of time-space dependent weight $\sigma_{35}(x,t)$.

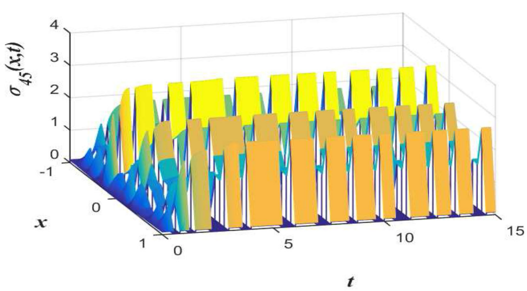

Fig. 6. Evolution of time-space dependent weight $\sigma_{45}(x,t)$.

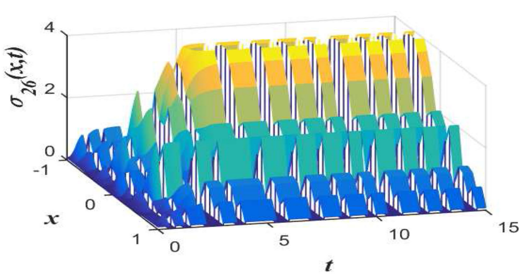

Fig. 7. Evolution of time-space dependent weight $\sigma_{26}(x,t)$.

these characteristics of the adaptive weights are consistent with the theoretical designs.

V. Conclusion

This article introduced a kind of spatiotemporal models with discontinuous coupling, in which the couplings among nodes are intermittent. Under the edge-based weights framework, a distributed intermittent adaptive scheme and its pinning form were proposed to determine the space-time dependent weights to achieve synchronization. It is revealed that the synchronization of the considered networks can be reached by intermittently tuning the weights of edges within a spanning tree. Besides, a direct error approach is established and several piecewise auxiliary functions are constructed in theoretical discussion, which are distinguishable entirely from the traditional central error means and continuous Lyapunov construction.

In this article, the network is considered to be connected and identical during each coupling period. If the topology is switching, the following coupled system can be obtained:

$$\frac{\partial y_\tau(x,t)}{\partial t} = D\Delta y_\tau(x,t) + f(y_\tau(x,t)) + \sum_{k=0}^{+\infty}\sum_{\varrho=1}^{N} c_{\tau\varrho}(k)\sigma_{\tau\varrho}(x,t)Hy_\varrho(x,t)\delta_k(t),\ \tau\in\mathscr{N}$$

where $C_k = (c_{\tau\varrho}(k))_{N\times N}$ is the connection matrix associated with the communication graph $\mathcal{G}_k$ within the kth coupling time. It is believed that it would be more interesting to explore the cooperative control of this coupled model under the joint or sequential connectivity. In addition, the index function of intermittent coupling in this article is assumed to be identical to all nodes, the case of inhomogeneity for the index function is not addressed. Moreover, communication delay is inevitable [41] and it is important to discuss the synchronization of ICSNs. These challenging problems will be deeply concerned in recent research.

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