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Safety Barrier Certificates for Heterogeneous Multi-Robot Systems*

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Abstract— **This paper presents a formal framework for collision avoidance in multi-robot systems, wherein an existing controller is modified in a minimally invasive fashion to ensure safety. We build this framework through the use of control barrier functions (CBFs) which guarantee forward invariance of a safe set; these yield safety barrier certificates in the context of heterogeneous robot dynamics subject to acceleration bounds. Moreover, safety barrier certificates are extended to a distributed control framework, wherein neighboring agent dynamics are unknown, through local parameter identification. The end result is an optimization-based controller that formally guarantees collision free behavior in heterogeneous multi-agent systems by minimally modifying the desired controller via safety barrier constraints. This formal result is verified in simulation on a multi-robot system consisting of both "sluggish" and "agile" robots.**

I. INTRODUCTION

When designing coordinated controllers for teams of mobile robots, the primary control objective tends to drive the behavior of the team so as to realize tasks such as achieving and maintaining formations, covering areas, or collective transport [6], [8]. *Safety*, in terms of collision-avoidance, is oftentimes added as a secondary controller that overrides the existing controllers on individual robots if they are about to collide, e.g., following the behavior-based control paradigm [4]. As a result, what is actually deployed is not always what the design calls for, and as the robot density increases, the team spends more and more time avoiding collisions as opposed to progressing toward the primary design objective.

One remedy to this problem is to make collision-avoidance an explicit part of the design. This, however, means that many of the already established, coordinated multi-robot controllers in the literature [6], [8], [11] are no longer valid and must be revised. An alternative view, as is for example pursued in [12] for two aircrafts performing optimal evasive maneuvers, is to introduce a minimally invasive collision-avoidance controller, i.e., a controller that only changes the original control program when it is absolutely necessary. But the heavy computation associated with solving the Hamilton-Jacobian-Bellman Equations prohibits the applicability of [12] to large-scale mutli-robot systems. Similarly, the concept of "velocity obstacle" was developed in [13] to generate collision free trajectory in cluttered multi-agent workspace, while the constant velocity assumption severely limits available control options. This approach was further pursued in [5], where the main idea is to let the actual control input associated with Robot i, u_i, be as close to the designed control input $\hat{u}_i$ in a least-squares sense, subject to safety contstraints.

The way that safety constraints were encoded in [5] was using distributed *barrier functions* that prevented the robots from entering unsafe states. This line of inquiry is continued in this paper, but in the context of *heterogeneous* robot teams. In particular, the barrier functions in [5] were symmetric in the sense that the responsibility for avoiding collisions was shared in an equitable manner among the robots. But, in a heterogeneous multi-robot system, not all agents are equally nimble and can respond to potential collisions in the same way, due to such factors as different maximal accelerations. In this paper, we pursue this question and we show how barrier functions can be used also for teams of heterogeneous networks, even when the robots are unaware of which class neighboring robots belong.

The reason why heterogeneous multi-agent systems are of importance is that they, through the robots' diverse set of capabilities, can solve some tasks more effectively than their homogeneous counter parts, i.e., [1]. Moreover, heterogeneity already exists in many systems, such as transportation systems with automobiles and trucks [3], multirobot systems with ground and aerial robots [7], mobile sensor network with nodes with varying locomotion and sensing capabilities [10], just to name a few. As such, collision-avoidance algorithms must be extended also to heterogeneous systems. Yet, such an extension is not straightforward in that agents with "aggressive", "neutral" or even "timid" behaviors must be able to respond to possible collisions in dramatically different manners.

Motivated by these considerations, this paper extends previous work on safety barrier certificates in [5] in two important directions. First, we propose a provably safe way to decentralize the barrier certificates that explicitly takes the agents' heterogeneous dynamics into account. In this paper, the robotic swarm is heterogeneous in the sense that agents have different acceleration limits (agile or sluggish), and use different barrier certificate parameters (aggressive, neutral or conservative). Second, we formally ensure safety of the robotic swarm when no prior information about neighboring agents' dynamical properties is provided. To achieve this, the agents will have to estimate the dynamical properties of neighboring agents with local measurements, and update online their barrier certificate parameters to generate more reasonable evasive maneuvers. The enabling

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technique for this heterogeneous safety barrier certificates is Control Barrier Function [2], [14]. Control Barrier Function is similar to Control Lyapunov Function in that it provides a way to guarantee the forward invariance of the safety set without computing the system's reachable set. A Quadratic Program (QP) based controller with safety barrier constraints is developed to check the safety of the pre-designed control strategy, and generate minimally-invasive and collision-free control actions.

The remainder of this paper is organized as follows. Section II revisits the concepts of (zeroing) control barrier functions, which are incorporated into the optimization based controller as safety barrier constraints. Heterogeneous safety barrier certificates are then constructed in Section III to generate collision free behaviors for agents' with different dynamical capabilities. Incorporating unknown parameters into heterogeneous barrier certificates without losing safety guarantee is the topic of Section IV. Simulation results for heterogeneous barrier certificates are presented in Section V. At last, we conclude the paper with a summary and discussion of future work in Section VI.

II. Background: Control Barrier Functions

In this section, we will review the fundamentals of Control Barrier Functions (CBFs), which is employed as a means to ensure that the robots execute collision-free trajectories. CBFs are conceptually similar to Control Lyapunov Functions (CLFs) in that they can be used to guarantee desired system properties without explicitly having to compute the forward reachable set. Analogously to CLFs, by constraining the time derivative of the CBFs within prescribed bounds, CBFs can formally guarantee the forward invariance of a desired set, e.g., safe set.

The fundamental idea behind CBFs is thus to design them in such a way that the agents always remain in the safe set. We are particularly interested in control affine dynamic systems because they result in affine safety barrier constraints, which can be incorporated into simple QP based controllers. Even though the main focus of this paper is on double integrator dynamics, we start the exposition with the general control affine case. In particular, consider a nonlinear control system in affine form

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

where $x \in \mathbb{R}^n$ and $u \in U \subset \mathbb{R}^m$, with f and g locally Lipschitz continuous. Note that, for the sake of simplicity, we will assume that (1) is forward complete, i.e. solutions $x(t)$ are defined for all $t \geq 0$.

Suppose now that we have a set $\mathscr{C} \subset \mathbb{R}^n$ where we wish the state of all robots to remain. The goal is thus to design a controller u that guarantees the forward invariance of $\mathscr{C}$, i.e., solutions to (1) that start in $\mathscr{C}$, stay in $\mathscr{C}$ for all time. We will assume that the set $\mathscr{C}$ can be defined as the level set to a particular function $h(x)$,

$$\mathscr{C} = \{x \in \mathbb{R}^n \mid h(x) \geq 0\}, \tag{2}$$

and we have the following definition that allows us to be precise about what safety entails, as was done in [14],

Definition 1: Given a dynamical system (1) and the set $\mathscr{C}$ defined by (2) for a continuously differentiable function $h : \mathbb{R}^n \to \mathbb{R}$, if there exist a locally Lipschitz extended class $\mathscr{K}$ function α (strictly increasing and $\alpha(0) = 0$) and a set $\mathscr{C} \subseteq \mathscr{D} \subset \mathbb{R}^n$ such that, for all $x \in \mathscr{D}$,

$$\sup_{u \in U} \left[L_f h(x) + L_g h(x) u + \alpha(h(x)) \right] \geq 0, \tag{3}$$

then the function h is called a Zeroing Control Barrier Function (ZCBF) defined on $\mathscr{D}$.

Note that the Lie derivative formulation comes from

$$\dot{h}(x) = \frac{\partial h}{\partial x}(f(x) + g(x)u) = L_f h(x) + L_g h(x) u.$$

Now, given a ZCBF h, the set of feasible control inputs is

$$K(x) = \left\{ u \in U \mid L_f h(x) + L_g h(x) u + \alpha(h(x)) \geq 0 \right\},$$

and in [14], the following key result was obtained;

Theorem [14]: *Given a set $\mathscr{C} \subset \mathbb{R}^n$ defined by (2) and a ZCBF h defined on $\mathscr{D}$ with $\mathscr{C} \subseteq \mathscr{D} \subset \mathbb{R}^n$, any Lipschitz continuous controller $u \colon \mathscr{D} \to \mathbb{R}$ such that $u \in K(x)$ for the system (1) renders the set $\mathscr{C}$ forward invariant.*

ZCBFs also imply asymptotic stability of the set $\mathscr{C}$, which provides robustness to different perturbations [14].

III. Heterogeneous Safety Barrier Certificates

This section focuses on constructing the decentralized heterogeneous safety barrier certificates that take into account the heterogeneity in agents' dynamical properties. Importantly, in an effort to reduce the amount of information required when executing barrier certificates, we will explore safety guarantees subject to unknown parameters of neigboring agents in Section IV.

A. Problem Formulation

Consider a heterogeneous robotic swarm containing N mobile agents with index set $\mathscr{M} = \{1, 2, \ldots, N\}$, the robot agent $i \in \mathscr{M}$ is modelled with double integrator dynamics. Agents in the robotic swarm are heterogeneous in the sense that each of them has different dynamical capability, which is modelled with different speed and acceleration limits,

$$\begin{bmatrix} \dot{\mathbf{p}}_i \\ \dot{\mathbf{v}}_i \end{bmatrix} = \begin{bmatrix} 0 & I_{2\times 2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_i \\ \mathbf{v}_i \end{bmatrix} + \begin{bmatrix} 0 \\ I_{2\times 2} \end{bmatrix} \mathbf{u}_i, \tag{4}$$

where $\mathbf{p}_i \in \mathbb{R}^2$, $\mathbf{v}_i \in \mathbb{R}^2$, and $\mathbf{u}_i \in \mathbb{R}^2$ are the position, velocity, and acceleration of agent i respectively. The ensemble form is $\mathbf{p} \in \mathbb{R}^{2N}$, $\mathbf{v} \in \mathbb{R}^{2N}$, and $\mathbf{u} \in \mathbb{R}^{2N}$. The speed and acceleration limits of agent i are $\|\mathbf{v}_i\|_\infty \leq \beta_i$ and $\|\mathbf{u}_i\|_\infty \leq \alpha_i$. The relative position and relative velocity between agent i and j are denoted as $\Delta\mathbf{p}_{ij} = \mathbf{p}_i - \mathbf{p}_j$ and $\Delta\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$.

Next, we need to formulate an appropriate safe set $\mathscr{C}$ that characterizes the safety of the robotic swarm, which ensures that *all* pairwise collisions are excluded from the safe set. To meet this requirement, a pairwise safety constraint is formulated to ensure that all agents will always keep a safety distance D_s away from each other when the maximum braking force is applied. As illustrated in Fig 1, the normal component of the relative velocity ($\Delta\bar{v} = \frac{\Delta \mathbf{p}_{ij}^T}{\|\Delta \mathbf{p}_{ij}\|}\Delta \mathbf{v}_{ij}$) between agent i and j is the component that might lead to collision, while the tangent component only leads to rotation around each other. Therefore, the pairwise safety constraint can be derived by regulating the normal component of the relative velocity $\Delta\bar{v}$ such that the maximum braking acceleration $(\alpha_i + \alpha_j)$ can prevent imminent collisions,

$$\|\Delta \mathbf{p}_{ij}\| - \frac{(\Delta\bar{v})^2}{2(\alpha_i+\alpha_j)} \geq D_s, \quad \forall i \neq j. \tag{5}$$

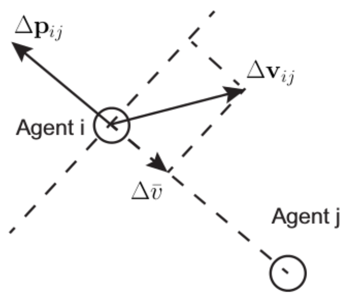


Fig. 1: Relative position and velocity between agent i, j

Note that when two agents are moving closer to each other ($\Delta\bar{v} \leq 0$), (5) regulates how fast the approaching speed could be; when they are moving away from each other ($\Delta\bar{v} > 0$), no constraint is enforced because safety is not endangered. Combining these observations, we can derive the safety constraints and formally define the safe set $\mathscr{C}$ as

$$\begin{aligned}\mathscr{C}_{ij} \quad &= \quad \{(\mathbf{p}_i,\mathbf{v}_i) | h_{ij} = \sqrt{2(\alpha_i+\alpha_j)(\|\Delta \mathbf{p}_{ij}\| - D_s)} \\ &\qquad + \frac{\Delta \mathbf{p}_{ij}^T}{\|\Delta \mathbf{p}_{ij}\|}\Delta \mathbf{v}_{ij} \geq 0\}, j \neq i, \end{aligned} \tag{6}$$

$$\mathscr{C} \quad = \quad \prod_{i\in\mathscr{M}} \left\{ \bigcap_{\substack{j\in\mathscr{M} \\ j\neq i}} \mathscr{C}_{ij} \right\}, \tag{7}$$

where h_{ij}, short for $h_{ij}(\Delta \mathbf{p}_{ij}, \Delta \mathbf{v}_{ij})$, is also a ZCBF candidate for $\mathscr{C}_{ij}$. $\prod_{i\in\mathscr{M}}$ is the Cartesian product over the states of all agents in the set of robots.

Definition 2: The robotic swarm with index set $\mathscr{M}$ with dynamics given in (4) is defined to be *safe* if the state $(\mathbf{p}, \mathbf{v})$ of the system stays in $\mathscr{C}$ for all time.

According to *Definition 2*, the robotic swarm needs to simultaneously satisfy all the pairwise safety constraints to ensure safety. ZCBF constraints are constructed to guarantee the forward invariance of the safe set $\mathscr{C}$, i.e. there are the following pairwise CBF constraints

$$L_f h_{ij} + L_g h_{ij}\mathbf{u} + \gamma h_{ij}^3 \geq 0, \forall i \neq j. \tag{8}$$

Theorem 3.1: The robotic swarm represented with $\mathscr{M}$ is *safe*, if the control variable $\mathbf{u}$ satisfies all the pairwise ZCBF constraints in (8).

Proof: If the control variable $\mathbf{u}$ satisfies the pairwise ZCBF constraints in (8), then h_{ij} is a valid ZCBF for $\mathscr{C}_{ij}$ with $\alpha(x) = \gamma x^3$ according to *Definition 1*. Following *Theorem [14]*, the forward invariance of $\mathscr{C}$ is guaranteed, which means the robotic swarm with index set $\mathscr{M}$ is *safe*. ■

Combining (6) with (8) gives the ZCBF constraint,

$$\begin{aligned}-\Delta \mathbf{p}_{ij}^T \Delta \mathbf{u}_{ij} \leq \gamma h_{ij}^3 \|\Delta \mathbf{p}_{ij}\| - \frac{(\Delta \mathbf{v}_{ij}^T \Delta \mathbf{p}_{ij})^2}{\|\Delta \mathbf{p}_{ij}\|^2} \\ + \|\Delta \mathbf{v}_{ij}\|^2 + \frac{(\alpha_i+\alpha_j)\Delta \mathbf{v}_{ij}^T \Delta \mathbf{p}_{ij}}{\sqrt{2(\alpha_i+\alpha_j)(\|\Delta \mathbf{p}_{ij}\| - D_s)}}, \quad \forall i \neq j. \end{aligned} \tag{9}$$

This safety barrier constraint can be represented as linear constraints on the control variable $\mathbf{u}$ as $A_{ij}\mathbf{u} \leq b_{ij}$, where

$$A_{ij} = [0, ..., \underbrace{-\Delta \mathbf{p}_{ij}^T}_{\text{agent } i}, ..., \underbrace{\Delta \mathbf{p}_{ij}^T}_{\text{agent } j}, ..., 0],$$

and b_{ij} is the right side of (9).

The safety barrier constraints assembled together, termed the *safety barrier certificates*, defines the space of permissible controls. The objective of the safety barrier certificates is to validate the safety of pre-designed control strategy $\hat{\mathbf{u}}$, and interfere with minimal impact to the desired strategy when collision is truly imminent. The goals of collision avoidance and minimal interference are combined together using QP,

$$\begin{aligned}\mathbf{u}^* = \underset{\mathbf{u}}{\operatorname{argmin}} \quad & J(\mathbf{u}) = \sum_{i=1}^{N} \|\mathbf{u}_i - \hat{\mathbf{u}}_i\|^2 \\ \text{s.t.} \quad & A_{ij}\mathbf{u} \leq b_{ij}, \qquad \forall i \neq j, \\ & \|\mathbf{u}_i\|_\infty \leq \alpha_i, \qquad \forall i \in \mathscr{M}. \end{aligned} \tag{10}$$

This QP based controller follows pre-designed control strategy $\hat{\mathbf{u}}$ when the system is safe; takes over and computes the closest permissible control in a least-squares sense when collision is about to happen. Note that this QP-based controller is a centralized controller, demanding centralized computation, which provides a starting point for decentralized heterogeneous barrier certificates.

B. Decentralized Heterogeneous Barrier Certificates

Centralized safety barrier certificates face significantly increased communication and computation burden when the size of the robotic swarm grows. It is desirable to have decentralized barrier certificates that act only based on local information, while safety is still guaranteed. Thus we propose two different strategies to distribute the safety barrier certificates to each agent based on their acceleration limits. Motivated by the fact that agents with higher acceleration limits are more agile, these agile agents are assigned with larger portion of the admissible control space.

1) Strategy A distributes b_{ij} to two robot agents.

$$\begin{aligned}-\Delta \mathbf{p}_{ij}^T \mathbf{u}_i &\leq \frac{\alpha_i}{\alpha_i+\alpha_j} b_{ij}, \\ \Delta \mathbf{p}_{ij}^T \mathbf{u}_j &\leq \frac{\alpha_j}{\alpha_i+\alpha_j} b_{ij}.\end{aligned}$$

2) Strategy B partitions the terms containing acceleration limits in (9) and distributes other terms appropriately.

$$-\Delta\mathbf{p}_{ij}^T\mathbf{u}_i + \frac{\Delta\mathbf{p}_{ij}^T\Delta\mathbf{v}_{ij}}{\|\Delta\mathbf{p}_{ij}\|^2}\Delta\mathbf{p}_{ij}^T\mathbf{v}_i - \Delta\mathbf{v}_{ij}^T\mathbf{v}_i \le \frac{\alpha_i}{\alpha_i+\alpha_j}(\gamma h_{ij}^3\|\Delta\mathbf{p}_{ij}\| + \frac{\sqrt{\alpha_i+\alpha_j}\Delta\mathbf{p}_{ij}^T\Delta\mathbf{v}_{ij}}{\sqrt{2(\|\Delta\mathbf{p}_{ij}\|-D_s)}}), \quad (11)$$

$$\Delta\mathbf{p}_{ij}^T\mathbf{u}_j - \frac{\Delta\mathbf{p}_{ij}^T\Delta\mathbf{v}_{ij}}{\|\Delta\mathbf{p}_{ij}\|^2}\Delta\mathbf{p}_{ij}^T\mathbf{v}_j + \Delta\mathbf{v}_{ij}^T\mathbf{v}_j \le \frac{\alpha_j}{\alpha_i+\alpha_j}(\gamma h_{ij}^3\|\Delta\mathbf{p}_{ij}\| + \frac{\sqrt{\alpha_i+\alpha_j}\Delta\mathbf{p}_{ij}^T\Delta\mathbf{v}_{ij}}{\sqrt{2(\|\Delta\mathbf{p}_{ij}\|-D_s)}}). \quad (12)$$

These two decentralization strategies differ in the required amount of information to implement the safety barrier certificates as shown in TABLE I. The self known parameters and sensing data can be easily attained by the controller. Meanwhile, obtaining neighboring agents' parameters, e.g., acceleration limit α_j, requires identity recognition or communication. In terms of required information, strategy B surpasses A by not requiring neighbors' parameters. Handling unknown neighboring agents' safety barrier parameters using strategy B is the topic of Section IV.

TABLE I: Comparison of required information

Strategy	Self params	Sensing data	Neighbors' params
A	α_i, γ	$\Delta\mathbf{p}_{ij}, \Delta\mathbf{v}_{ij}$	α_j
B	α_i, γ	$\Delta\mathbf{p}_{ij}, \Delta\mathbf{v}_{ij}, \mathbf{v}_i,$	

Both decentralization strategies guarantees safety, because the safety barrier constraint (9) still holds with the partitions. With strategy B, we can come up with a decentralized QP-based controller that is minimally invasive to the predesigned controller and provably safe.

$$\begin{aligned}\mathbf{u}_i^* = \underset{\mathbf{u}_i}{\text{argmin}} \quad & J(\mathbf{u}_i) = \|\mathbf{u}_i - \hat{\mathbf{u}}_i\| \\ \text{s.t.} \quad & \bar{A}_{ij}\mathbf{u}_i \le \bar{b}_{ij}, \qquad \forall j \ne i, \\ & \|\mathbf{u}_i\|_\infty \le \alpha_i,\end{aligned} \quad (13)$$

where $\bar{A}_{ij} = -\Delta\mathbf{p}_{ij}^T$, $\bar{b}_{ij} = -\frac{\Delta\mathbf{p}_{ij}^T\Delta\mathbf{v}_{ij}}{\|\Delta\mathbf{p}_{ij}\|^2}\Delta\mathbf{p}_{ij}^T\mathbf{v}_i + \Delta\mathbf{v}_{ij}^T\mathbf{v}_i + \frac{\alpha_i}{\alpha_i+\alpha_j}(\gamma h_{ij}^3\|\Delta\mathbf{p}_{ij}\| + \frac{\sqrt{\alpha_i+\alpha_j}\Delta\mathbf{p}_{ij}^T\Delta\mathbf{v}_{ij}}{\sqrt{2(\|\Delta\mathbf{p}_{ij}\|-D_s)}})$.

IV. Barrier Certificates with Unknown Parameters

Heterogeneity in agents' dynamical capabilities brings extra challenge to collision avoidance. Agents need to first assess how effective other agents can respond to safety threats before making decisions for collision avoidance. Meanwhile, swarm robots are often designed to be simple and therefore lack the ability to obtain other agents' parameters. This section addresses scenarios that agents need to ensure safety when some dynamical parameters of other agents are unknown.

A. Barrier Certificates with Different γ

The safety barrier parameter γ determines how fast the agents' states can approach the boundary of the safe set $\mathscr{C}$. Agents with different γ are still safe when running the decentralized barrier certificates.

Lemma 4.1: Two heterogeneous agents $i, j \in \mathscr{M}$ regulated by safety barrier certificates (13) with different parameters γ_i, γ_j are guaranteed to be safe.

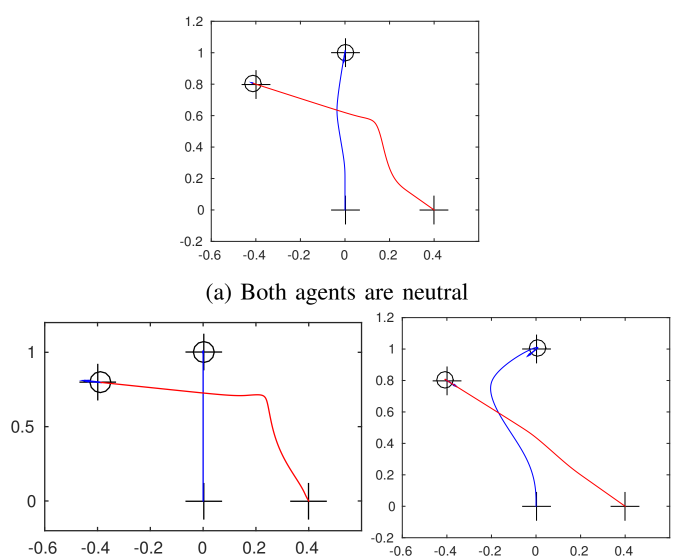
Proof: Agent i and j follow the safety barrier constriant given in (11) and (12) with different parameters γ_i, γ_j. Adding these two safety barrier constraints together gives

$$-\Delta\mathbf{p}_{ij}^T\Delta\mathbf{u}_{ij} \le \frac{\gamma'}{B_{ij}}h_{ij}^2\|\Delta\mathbf{p}_{ij}\| - \frac{(\Delta\mathbf{v}_{ij}^T\Delta\mathbf{p}_{ij})^2}{\|\Delta\mathbf{p}_{ij}\|^2} + \|\Delta\mathbf{v}_{ij}\|^2 + \frac{(\alpha_i+\alpha_j)\Delta\mathbf{v}_{ij}^T\Delta\mathbf{p}_{ij}}{\sqrt{2(\alpha_i+\alpha_j)(\|\Delta\mathbf{p}_{ij}\|-D_s)}}, \quad (14)$$

where $\gamma' = \frac{\alpha_i\gamma_i+\alpha_j\gamma_j}{\alpha_i+\alpha_j}$. This inequality can be rewritten as $-\dot{h}_{ij} \le \gamma' h_{ij}^3$, which guarantees safety as if a weighted version of γ is used in the safety barrier certificates. ■

This lemma provides the freedom for heterogeneous agents to choose their own γ without endangering safety. γ can be selected appropriately to prioritize certain agents over others, which resembles the real life case of the ambulance granted higher priority to go through the traffic flow.

Fig. 2 demonstrates how heterogeneous γ in safety barrier certificates can be used to coordinate conflicting agents. Two agents executing goal-to-goal controllers regulated by heterogeneous barrier certificates are simulated in three different scenarios. The case that both agents adopt the same γ is used as a benchmark in Fig. 2a. When the left agent uses larger γ, it moves straightly to its goal, while the other agent moves around it (Fig. 2b). When the left agent is assigned with smaller γ, it gives way to the other agent (Fig. 2c).



(a) Both agents are neutral

(b) Left agent aggressive

(c) Left agent conservative

Fig. 2: Trajectories of two agents regulated by safety barrier certificates with different parameter γ

With heterogeneous safety barrier certificates, we can define the notion of neighborhood to reduce the pairs of necessary safety barrier constraints,

$$\mathscr{N}_i = \{ j \in \mathscr{M} \mid \|\Delta \mathbf{p}_{ij}\| \leq D^i_{\mathscr{N}}, j \neq i \}, \tag{15}$$

where $D^i_{\mathscr{N}} = D_s + \frac{1}{2(\alpha_i+\alpha_{\min})}(\sqrt[3]{\frac{2(\alpha_i+\alpha_{\max})}{\gamma_i}} + \beta_i + \beta_{\max})^2$, $\alpha_{\max} = \max_{j\in\mathscr{M}}\{\alpha_j\}$ and $\alpha_{\min} = \min_{j\in\mathscr{M}}\{\alpha_j\}$ are the upper and lower bounds of acceleration limits of all agents, $\beta_{\max} = \max_{j\in\mathscr{M}}\{\beta_j\}$ is the upper bound of speed limit of all agents. The neighborhood notion is helpful in reducing computation intensity and sensing requirement. This notion is valid because there is no threat of collision when agents are sufficiently far away from each other.

Theorem 4.2: Any agent $i \in \mathscr{M}$ is guaranteed to be safe if it only forms ZCBFs with its heterogeneous neighbors defined by (15).

Proof: Heterogeneous agents each possesses a safety neighbor disk with different radius. Thus there are generally three scenarios considering $\forall j \in \mathscr{M}, j \neq i$, i.e. $\|\Delta\mathbf{p}_{ij}\| > \max\{D^i_{\mathscr{N}}, D^j_{\mathscr{N}}\}$, $\max\{D^i_{\mathscr{N}}, D^j_{\mathscr{N}}\} \geq \|\Delta\mathbf{p}_{ij}\| \geq \min\{D^i_{\mathscr{N}}, D^j_{\mathscr{N}}\}$ or $\|\Delta\mathbf{p}_{ij}\| < \min\{D^i_{\mathscr{N}}, D^j_{\mathscr{N}}\}$.

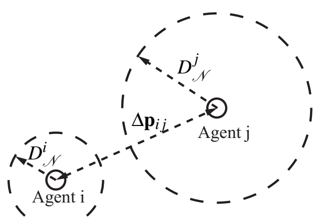


Fig. 3: Two agents with different safety neighborhood disks

In all of the three cases, it can be proved that $-\dot{h}_{ij} \leq \max\{\gamma_i, \gamma_j\} h^3_{ij}$ following similar reasoning of *Theorem* [5] and *Lemma* 4.1 by considering the worst-case scenario. Therefore, safety is guaranteed in all three cases. Heterogeneous agents only needs to form ZCBFs with their neighbors to guarantee safety. ■

B. Barrier Certificates with Unknown Acceleration Limits

The acceleration limits of neighboring agents might not be known prior. It can be proved that safety is still guaranteed when conservative estimates of neighbors' acceleration limits are used. With the estimated parameters, the safe set definition will be slightly different for different agents. Let α_i and α_{ij} be agent i's acceleration limit and estimate of agent j's acceleration limit. The pairwise safe set $\bar{\mathscr{C}}_{ij}$ is

$$\bar{\mathscr{C}}_{ij} = \{ (\mathbf{p}_i, \mathbf{v}_i) \mid h_{ij}(\alpha_i + \alpha_{ij}) = \frac{\Delta\mathbf{p}^T_{ij}}{\|\Delta\mathbf{p}_{ij}\|}\Delta\mathbf{v}_{ij} + \sqrt{2(\alpha_i + \alpha_{ij})(\|\Delta\mathbf{p}_{ij}\| - D_s)} \geq 0 \}, j \neq i.$$

The corresponding safety barrier constraint of agent i is

$$-\Delta\mathbf{p}^T_{ij}\mathbf{u}_i + \frac{\Delta\mathbf{p}^T_{ij}\Delta\mathbf{v}_{ij}}{\|\Delta\mathbf{p}_{ij}\|^2}\Delta\mathbf{p}^T_{ij}\mathbf{v}_i - \Delta\mathbf{v}^T_{ij}\mathbf{v}_i \leq \frac{\alpha_i}{\alpha_i + \alpha_{ij}}(\gamma_i h^3_{ij}(\alpha_i + \alpha_{ij})\|\Delta\mathbf{p}_{ij}\| + \frac{\sqrt{\alpha_i + \alpha_{ij}}\Delta\mathbf{p}^T_{ij}\Delta\mathbf{v}_{ij}}{\sqrt{2(\|\Delta\mathbf{p}_{ij}\| - D_s)}}). \tag{16}$$

In order to guarantee safety with inaccurate parameters, it is desirable to ensure that $\bar{\mathscr{C}}_{ij}$ is always subset of $\mathscr{C}_{ij}$. Notice that when $\alpha_{ji} \leq \alpha_i, \alpha_{ij} \leq \alpha_j$, we have $\bar{\mathscr{C}}_{ij} \subseteq \mathscr{C}_{ij}$. It can be shown that agents are safe if conservative estimates of neighboring agents' acceleration limits are used for decentralized heterogeneous barrier certificates.

Lemma 4.3: If $\alpha_{ji} \leq \alpha_i, \alpha_{ij} \leq \alpha_j$ and the safety barrier constraints (16) is satisfied, safety is still guaranteed.

Proof: When agents i and j use their own estimates of acceleration limits based on (16), we can get

$$\begin{aligned} &-\Delta\mathbf{p}^T_{ij}\Delta\mathbf{u}_{ij} + \frac{(\Delta\mathbf{p}^T_{ij}\Delta\mathbf{v}_{ij})^2}{\|\Delta\mathbf{p}_{ij}\|^2} - \Delta\mathbf{v}^T_{ij}\Delta\mathbf{v}_{ij} \\ &\leq \frac{\alpha_i\gamma_i h^3_{ij}(\alpha_i + \alpha_{ij})}{\alpha_i + \alpha_{ij}}\|\Delta\mathbf{p}_{ij}\| + \frac{\alpha_j\gamma_j h^3_{ji}(\alpha_j + \alpha_{ji})}{\alpha_j + \alpha_{ji}}\|\Delta\mathbf{p}_{ij}\| \\ &+ (\frac{\alpha_i}{\sqrt{\alpha_i + \alpha_{ij}}} + \frac{\alpha_j}{\sqrt{\alpha_j + \alpha_{ji}}})\frac{\Delta\mathbf{p}^T_{ij}\Delta\mathbf{v}_{ij}}{\sqrt{2(\|\Delta\mathbf{p}_{ij}\| - D_s)}}. \end{aligned} \tag{17}$$

Next, we will discuss about two scenarios where two agents are moving closer or further away from each other.

1) when $\Delta\mathbf{p}^T_{ij}\Delta\mathbf{v}_{ij} \leq 0$, agents i and j are moving closer to each other. With $\alpha_{ji} \leq \alpha_i, \alpha_{ij} \leq \alpha_j$, we have $\frac{\alpha_i}{\sqrt{\alpha_i+\alpha_{ij}}} + \frac{\alpha_j}{\sqrt{\alpha_j+\alpha_{ji}}} \geq \sqrt{\alpha_i + \alpha_j}$. Thus
$$\begin{aligned} &-\Delta\mathbf{p}^T_{ij}\Delta\mathbf{u}_{ij} + \frac{(\Delta\mathbf{p}^T_{ij}\Delta\mathbf{v}_{ij})^2}{\|\Delta\mathbf{p}_{ij}\|^2} - \Delta\mathbf{v}^T_{ij}\Delta\mathbf{v}_{ij} \\ &\leq \quad \bar{\gamma}h^3_{ij}(\alpha_i + \alpha_j)\|\Delta\mathbf{p}_{ij}\| + \frac{\sqrt{\alpha_i+\alpha_j}\Delta\mathbf{p}^T_{ij}\Delta\mathbf{v}_{ij}}{\sqrt{2(\|\Delta\mathbf{p}_{ij}\| - D_s)}}, \end{aligned} \tag{18}$$
where $\bar{\gamma} = \frac{\alpha_i\gamma_i}{\alpha_i+\alpha_{ij}} + \frac{\alpha_j\gamma_j}{\alpha_j+\alpha_{ji}}$. Compared with (9), this inequality can be rewritten as $-\dot{h}_{ij}(\alpha_i + \alpha_j) \leq \bar{\gamma}h_{ij}(\alpha_i + \alpha_j)^3$, which guarantees safety as if a weighted version of γ is adopted. This means that, if $\Delta\mathbf{p}^T_{ij}\Delta\mathbf{v}_{ij} \leq 0$, the forward invariance of the nominal safe set $\mathscr{C}$ is guaranteed.
2) when $\Delta\mathbf{p}^T_{ij}\Delta\mathbf{v}_{ij} > 0$ (agents are moving away from each other), it is guaranteed to have $h_{ij}(\alpha_i + \alpha_j) \geq 0$. Thus agents always stay in the nominal safe set $\mathscr{C}$ in this scenario.

It can be shown that safety is still guaranteed if agents switch back and forth between these two cases. In case (1), the forward set invariance requires agent i to always start in $\mathscr{C}$ after each switching. Due to the second order dynamical model used for barrier certificates, $\Delta\mathbf{p}^T_{ij}\Delta\mathbf{v}_{ij}$ is continuous with respect to time. Thus the switching between two cases always occurs at $\Delta\mathbf{p}^T_{ij}\Delta\mathbf{v}_{ij} = 0$, where $h_{ij}(\alpha_i + \alpha_j) \geq 0$.

Combining these two scenarios with the safe switching condition, agent i is guaranteed to be safe with respect to the nominal safe set $\mathscr{C}$. ■

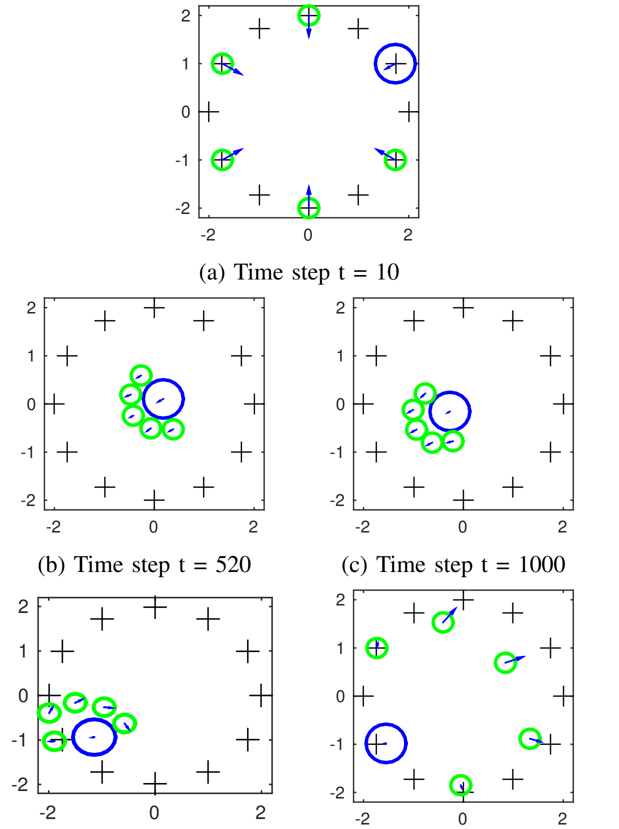
With the local sensor measurements of neighboring agents, we can construct a distributed least squares estimator or Kalman filter [8] to estimate the current acceleration $\|\bar{\mathbf{u}}_j\|$ of agent j. Agent i's estimate of agent j's acceleration limit $\bar{\alpha}_{ij}$ can be updated with $\dot{\bar{\alpha}}_{ij} = \max\{\bar{\alpha}_{ij}, \|\bar{\mathbf{u}}_j\|\} - \bar{\alpha}_{ij}$.

This strategy will ensure that parameter estimation satisfies $\bar{\alpha}_{ji} \leq \alpha_i, \bar{\alpha}_{ij} \leq \alpha_j$. Thus safety is still guaranteed using the estimated parameters due to *lemma* 4.3. With this estimation strategy, agents do not need to know the acceleration limits of neighboring agents. They can start with conservative initial guesses, and gradually improve their knowledge with local observations without endangering safety.

V. Simulation Results

A multi-robot system with six heterogeneous agents is simulated with MATLAB. Each agent is modelled with double integrator dynamics and executes a goal-to-goal controller without considering collision avoidance. This system contains two types of agents: small agile agents ($\alpha_s = 1.2\ m/s^2$, safety radius is $0.2\ m$); large sluggish agent ($\alpha_l = 0.6\ m/s^2$, safety radius is $0.4\ m$). As illustrated in Fig.4, the objective of the pre-designed controller is to make all agents swap position with the agents on the opposite side. Without collision avoidance strategy, the goal-to-goal controller will lead to collision of all agents in the middle.

The heterogeneous safety barrier certificates were wrapped around the pre-designed control strategy. All agents started heading towards the center following the goal-to-goal controller (Fig. 4a). As they moved closer to each other, the safety barrier certificates were activated and kept all agents with enough safety distance away from each other (Fig. 4b). The small agents are more agile and deviated from their original paths to avoid collisions, while the sluggish agent continued its own path because of inertia (Fig. 4c). After the large agent reached its destination, other small agents were safe to pursue their own goals without worrying about colliding with the large agent (Fig. 4d). At last, all agents successfully navigated out of the "crowded" scenarios and achieved their objectives.



(a) Time step t = 10

(b) Time step t = 520

(c) Time step t = 1000

(d) Time step t = 2400

(e) Time step t = 2830

Fig. 4: Six robot agents regulated by heterogeneous safety barrier certificates. The small and large circles represent the safety radius of different agents. Units for X and Y coordinates in the plots are in meters.

VI. Conclusions

The heterogeneous safety barrier certificates proposed in this paper provides a provable way to address the challenges in collision avoidance brought by heterogeneity in robots' dynamical capabilities. The simulation results validate the effectiveness of the proposed approach. While studying those results, several interesting future research directions also arise. When the objectives of several agents conflict with each other, the agents sometimes get into a deadlock. It is important to design a strategy that breaks deadlock to ensure task completion. In some high density situations, the optimization-based controller might become infeasible. Safety barrier certificates with guaranteed feasibility need to be synthesized for those safety critical systems.

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