IB Physics Topic C3 Wave Phenomena; SL & HL

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1 Wavefronts and Rays

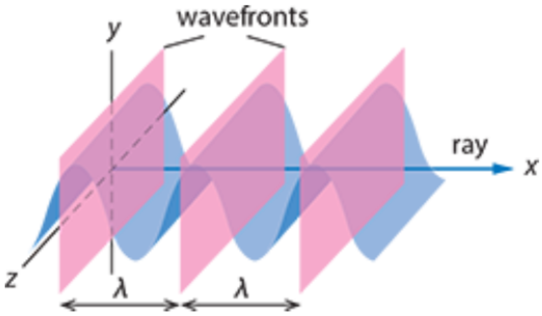

Figure 1: Wavefronts and Rays

- Wavefronts are surfaces that move with the wave and are **perpendicular to the direction of the wave motion**. Consecutive wavefronts are imagined to be one wavelength apart
- Rays are lines that show the **direction of energy transfer** by the wave. They are locally perpendicular to the wavefront.

2 Refraction

Refraction refers to the change in speed and direction of a wave as it passes from one medium to another. The change in speed will depend on the optical density of the medium. In refraction, the frequency of the wave remains constant, but the wavelength and speed of the wave will change.

A general rule of thumb when considering the bending of the wave is that

- If the wave slows down, it will bend towards the normal
- If the wave speeds up, it will bend away from the normal

The image below shows how refraction is drawn.

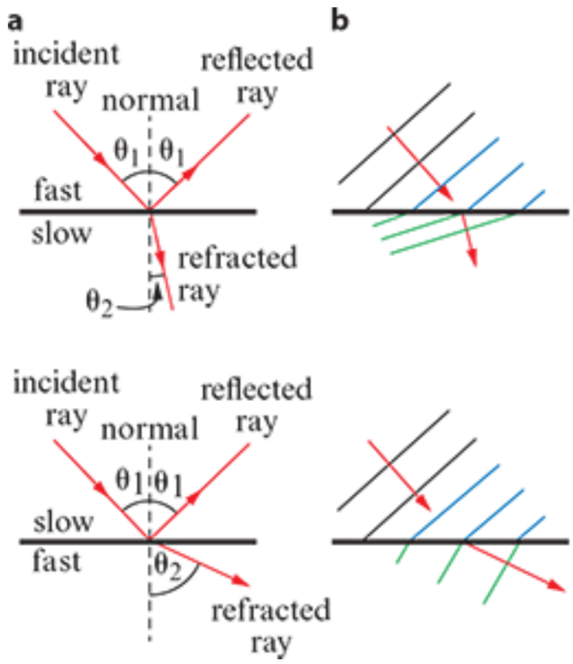

Figure 2: Refraction of Wavefronts

2.1 Snell's Law

In refraction, the frequency stays the same as the wave passes from one medium to another. However, the speed and wavelength of the wave will change. The full relationship is given as follows

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

where the quantities with subscript 1 are the quantities in the first medium and the quantities with subscript 2 are the quantities in the second medium. n is the refractive index of a medium.

The way to remember this is, a chained equality of ratios, all except the refractive index are in the same order (medium 1 over medium 2, or vice versa).

2.2 Critical Angle

The critical angle is defined as the angle of incidence that produces an angle of refraction of 90 degrees. This is the angle beyond which total internal reflection occurs. The critical angle is given as follows

$$\sin c = \frac{n_2}{n_1}$$

where n_1 is the refractive index of the medium the wave is coming from and n_2 is the refractive index of the medium the wave is entering. By this definition, this only works if the wave is going from a medium with a higher refractive index to a medium with a lower refractive index (i.e. $n_1 > n_2$), since the sine function cannot exceed 1 in any way.

2.3 Chaining Refractions

When a wave passes through multiple media, the wave will refract at each boundary. The angle of incidence at each boundary will be the angle of refraction from the previous boundary (consider alternate angles).

A refractive index can be absolute or relative:

- The absolute refractive index n_X of a material X is defined as

$$n_X = \frac{c}{v_X}$$

  where c is the speed of light in a vacuum (or in the air, if this approximation is allowed/inferred) and v_X is the speed of light in material X.

  Notice that this is substituting the speed of light and the absolute refractive index of the air into

$$\frac{n_X}{n_{\text{air}}} = \frac{v_{\text{air}}}{v_X}$$

- The relative refractive index of a material Y with respect to another material X defined as

$$_X n_Y = \frac{n_Y}{n_X} = \frac{v_X}{v_Y}$$

Consider a wave that travels through materials A, B, and C in that order, then, if one were to treat the transition of $A \to B \to C$ altogether as $A \to C$, then the refractive index of the combined medium $A \to C$ is given as

$$_A n_C = \frac{n_C}{n_A}$$

if we are instead given the relative refractive indices $_A n_B$ and $_B n_C$, then the refractive index of the combined medium is given as

$$_A n_C = \frac{n_C}{n_A} = \frac{n_B}{n_A} \times \frac{n_C}{n_B} = {_A n_B} \times {_B n_C}$$

In short, simply multiply the relative refractive indices to get the refractive index of the combined medium.

3 Reflection

The law of reflection states that the angle of incidence is equal to the angle of reflection.

However, reflection can be partial or total.

- Partial reflection is when part of the wave is reflected while the rest is transmitted through the medium and refracted.
- A total internal reflection is when all of the waves are reflected and none are transmitted. This is often used in optical fibres, periscopes, binoculars, and endoscopy (medicine).

Total internal reflection occurs when the angle of incidence is greater than the critical angle.

An important thing to note since it has come up in an exam question was that at the point of reflection from the surface of a medium with higher refractive index, there is a phase change of π (or 180 degrees) in the reflected wave.

4 Double Source Interference

Consider the waves coming from two identical sources S_1 and S_2; they have the same amplitude, frequency, and wavelength.

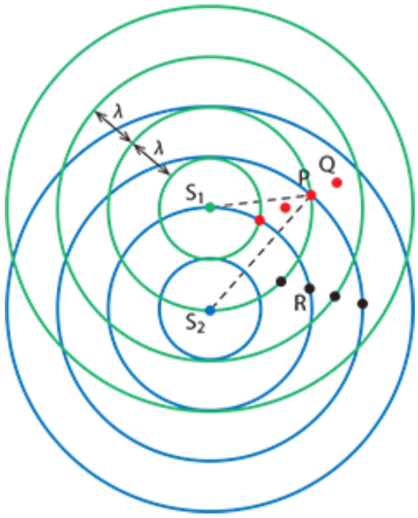

Figure 3: Double Source Interference

At any general point P, the **path difference** is the difference in the distance travelled by the two waves. Numerically, that is

$$\Delta r = |S_1P - S_2P|$$

In this particular example, for the specific point P,

$$\Delta r_P = |2\lambda - 3\lambda| = \lambda$$

The path difference will tell us whether the interference is constructive or destructive.

- If the path difference is an integer multiple of the wavelength, the interference is constructive. I.e., when $\Delta r = n\lambda$, where $n \in \mathbb{Z}$; the amplitude is doubled.
- If $\Delta r = (n + \frac{1}{2})\lambda$, where $n \in \mathbb{Z}$, then, it is a destructive interference; the superposed amplitude at this point is 0.

5 Diffraction

Diffraction is the spreading of waves as they pass through an aperture or around an obstacle. The amount of diffraction that occurs depends on the wavelength of the wave and the size of the gap or obstacle.

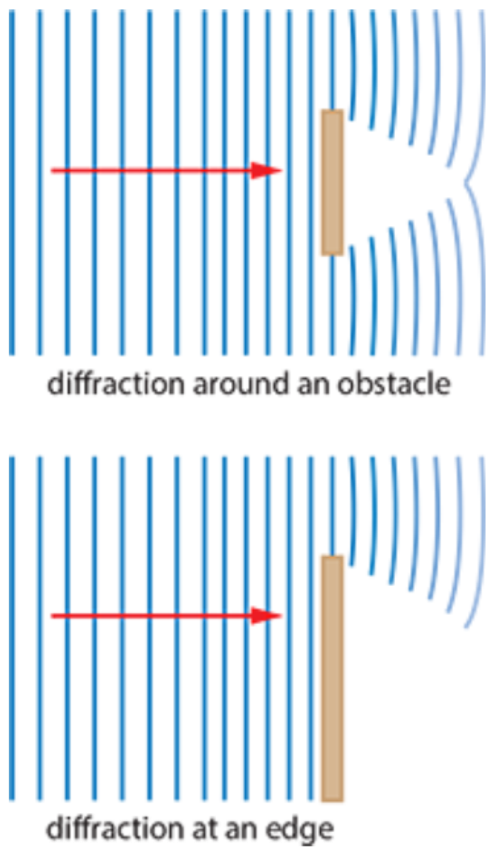

Figure 4: Diffraction around an edge

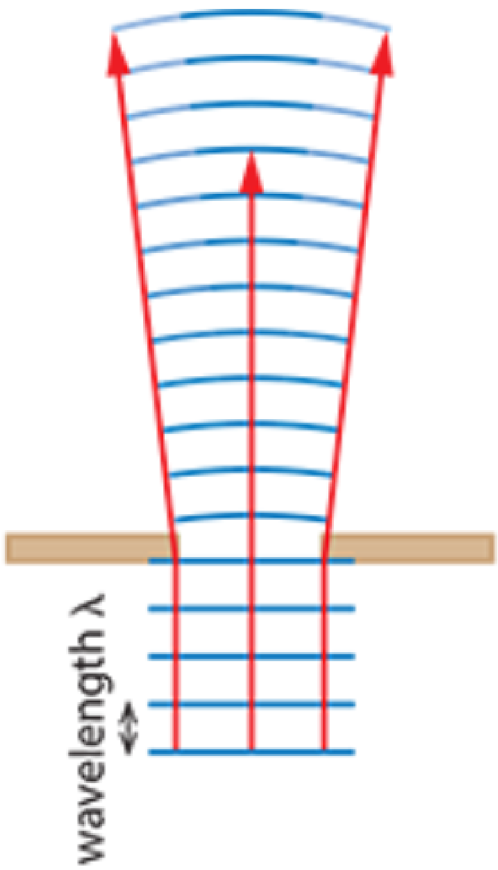

Figure 5: Diffraction through an aperture

6 Double Slit Diffraction

Consider the following scenario: a wave is incident on a surface that has two slits separated by a distance d. The wavefronts from the two slits will interfere with each other, constructively at some points and destructively at others. A requirement of this is that the waves coming out of the two slits must be **coherent**, i.e. they sustain a constant phase difference over time.

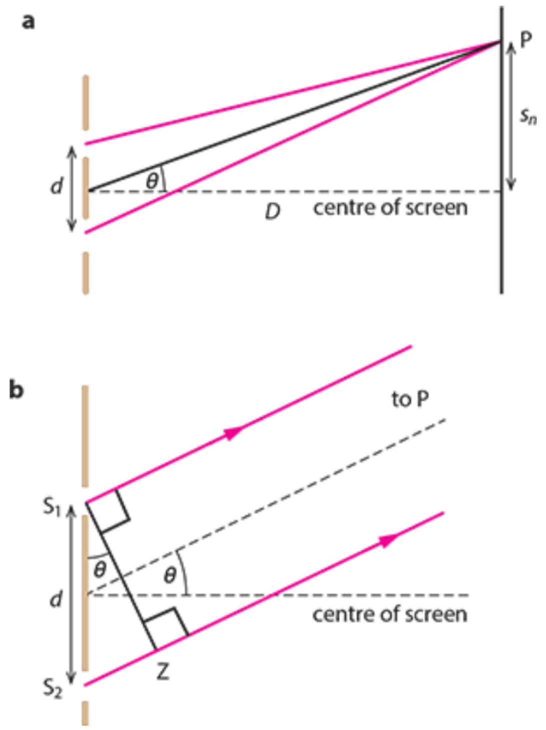

Figure 6: Double Slit Diffraction

1. The slit separation d is almost negligible, thus we can consider the two rays to be parallel.

2. Consider P to be a point on the screen, we now analyze the interference at this point.

3. Define θ as shown in the diagram. With some geometric sense, the path difference

is given as

$$\Delta r = |S_2 Z| = d \sin\theta$$

4. If we want to find points of constructive interference, we require $\Delta r = n\lambda$, where $n \in \mathbb{Z}$ is represents the nth maximum. Thus, we have

$$d \sin\theta = n\lambda \tag{1}$$

5. Conversely, if we want to find points of destructive interference, we require $\Delta r = (n + \frac{1}{2})\lambda$, where $n \in \mathbb{Z}$ represents the $(n-1)$th dark spot. Thus, we have

$$d \sin\theta = (n + \frac{1}{2})\lambda \tag{2}$$

6. An important concept here is that θ is also very small, so we can use the small angle approximation $\sin\theta \approx \tan\theta$ (just take limits...)

7. By construction, we have that

$$\tan\theta = \frac{s_n}{D}$$

where D is the distance to the screen and s_n is the distance from the central maximum to the nth maximum.

8. If we just focus on the points of constructive interference, we can rewrite Equation (1) as

$$\frac{s_n d}{D} = n\lambda$$

which can be rearranged to give

$$s_n = \frac{n\lambda D}{d} \tag{3}$$

9. Now, this allows us to work out the **separation between any two maxima or**

minima on the screen, s:

$$s = s_{n+1} - s_n = \frac{\lambda D}{d}$$

$$s = \frac{\lambda D}{d} \tag{4}$$

The displacement or angular displacement vs. intensity graph **only concerning the effect of double slit diffraction** is shown below.

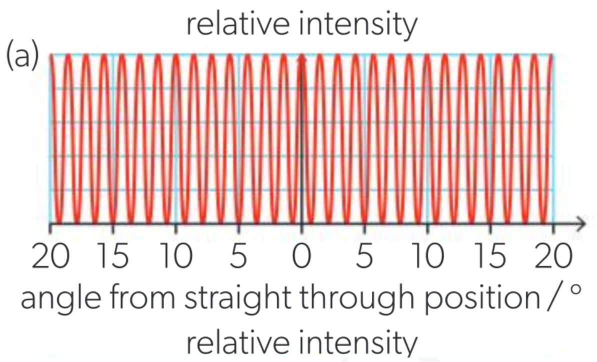

Figure 7: Graph of intensity vs. Angular displacement

An important feature is that, the **intensity is proportional to the square of the amplitude**; this means that, two waves of amplitude A interfering constructively at point P will have an amplitude of $2A$ and an intensity of four times the original.

Assumption. 1

It must be noted that this graph, with peaks at roughly the same intensities, is only true when the slit widths are negligible so that the single slit effect will not be taken into account.

7 Single Slit Diffraction

Consider a single slit of width b and a wave incident on it. The wavefronts will spread out as they pass through the slit.

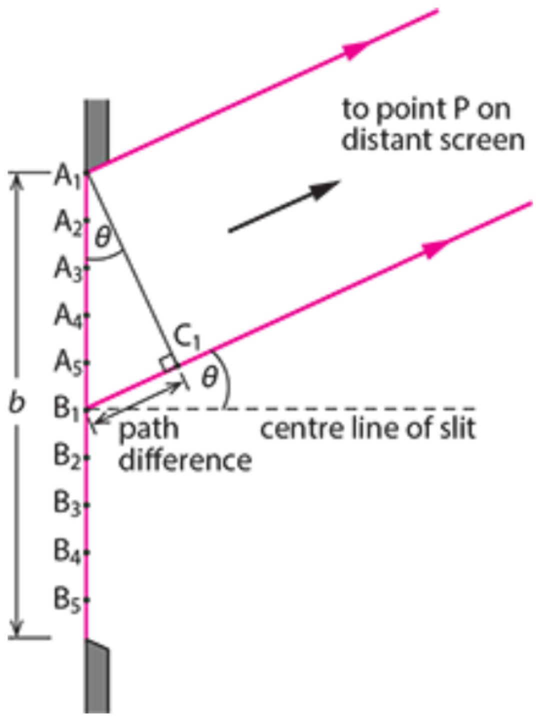

Figure 8: Single Slit Diffraction

Our aim is now to identify the **angular displacement of the first minimum** (first point of destructive inteference). Before we proceed, we need to accept the assumptions of Huygen's principle:

Every point on a wavefront acts as a source of secondary spherical wavelets, and the wavefront at any later time is the envelope of these wavelets.

If we consider the wave incident on the slit, we can split the slit into a number of point sources, each of which will emit a wavefront.

1. Consider the pair of wavelets emitted from A_1 and B_1; the path difference between the two wavelets is given as

$$\Delta r = |A_1P - B_1P| = \frac{b}{2}\sin\theta$$

2. Again, this relies on the approximation of the two rays being parallel.

3. To achieve destructive interference, we require $\Delta r = (n + \frac{1}{2})\lambda$, where $n \in \mathbb{Z}$. Thus, we have

$$\frac{b}{2}\sin\theta = (n + \frac{1}{2})\lambda \tag{5}$$

4. Since we are only concerned with the first minimum, we take $n = 0$.

5. We can now rearrange Equation (5) to give

$$b\sin\theta = \lambda \tag{6}$$

6. With small angle approximation, we obtain that

$$\theta = \frac{\lambda}{b} \tag{7}$$

The intensity graph is as follows:

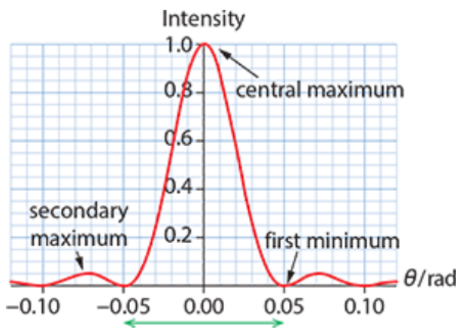

Figure 9: Graph of intensity vs. angular displacement

Effects of decreasing the slit width (assuming a fixed wavelength):

- The angular displacement of the first minimum (and in fact, other minima) will increase. This means that the graph will be more spread out.
- Suppose the slit width is dropped to $\frac{1}{k}$ of the original, then, the amplitude will also drop to $\frac{1}{k}$ of the original. Hence, the intensity will drop to $\frac{1}{k^2}$ of the original.

Also note that the angular position of the first minimum is a measure of the width of the central maximum.

The key thing to remember is that the angular width of the central maximum is twice the angular width of any other maximum.

8 Combined Effect

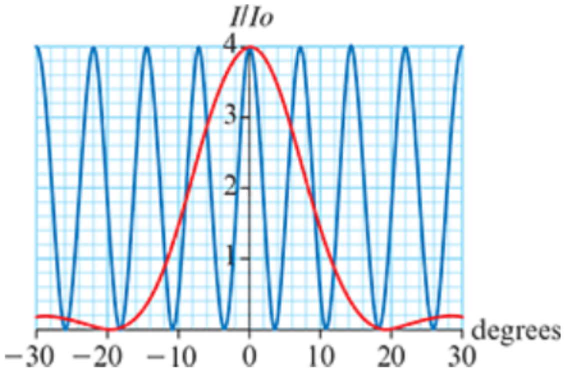

Figure 10: Overlay of the effects

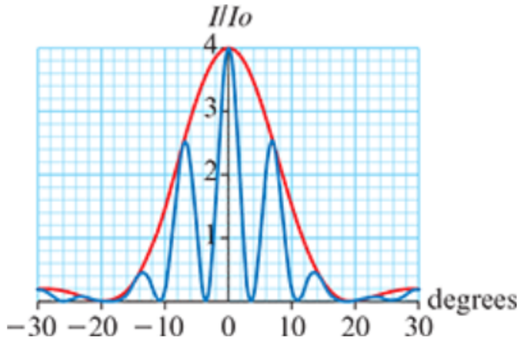

Figure 11: Resulting graph

In Figure 11, the blue curve shows the resulting intensity graph. It is essentially the original double slit graph suppressed by the single red curve.

With the single slit diffraction effect, we had

$$\theta = \frac{\lambda}{b}$$

With the double slit diffraction effect, we had

$$s = \frac{\lambda D}{d}$$

We can link these two using their common term λ to give

$$b\theta_{\text{1st min}} = \frac{sd}{D}$$

This may be of use in questions where both effects are present.

9 Multi-Slit Diffraction

Consider adding more and more slits. Below shows the graphs for four and six slits respectively.

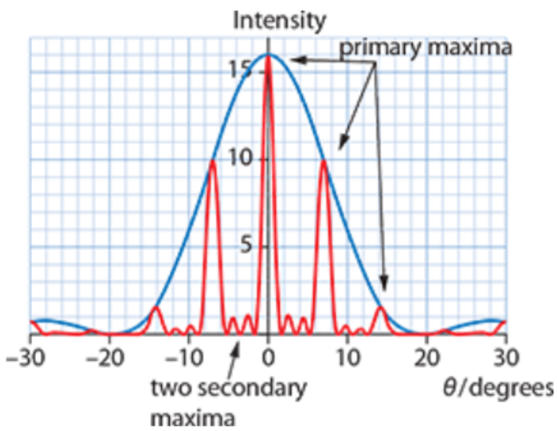

Figure 12: Four Slit Diffraction

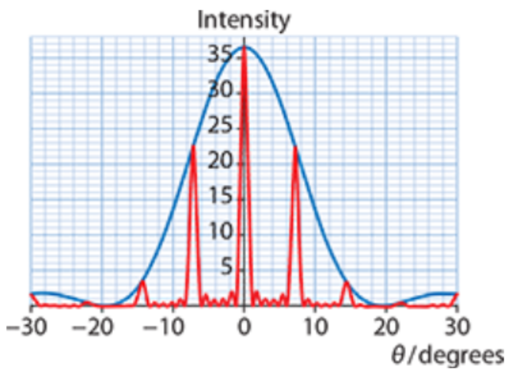

Figure 13: Six Slit Diffraction

Previously, in Figure 11, there is only one *secondary minimum* between every pair of maxima. However, as observed, in the case of four slits, there are 2 secondary minima between every pair of maxima. The general rule of thumb for N slits is that:

- There are $N - 2$ secondary minima between every pair of maxima.;
- The intensity of the central maximum is N^2 times the intensity of just one slit by itself.

With increasing N:

- The primary maxima remain at the same angles
- The primary maxima get narrower and brighter
- The secondary maxima become unimportant.

9.1 Diffraction Grating

A diffraction grating is a configuration that consists of a large number of slits of negligible width.

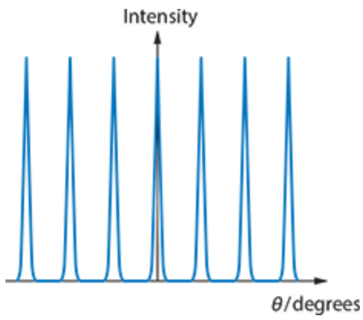

Figure 14: Diffraction Grating

The equations for destructive and constructive interference are the same as the ones developed previously, namely Equation (1) and Equation (2).

One thing to note about colors is that, in a non-monochromatic light, the different wavelengths will diffract at different angles. In particular, the longer the wave length (e.g. red), the further away the maxima will be from the central maximum.

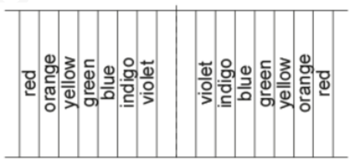

Figure 15: Diffraction Grating

9.2 Grating Spacing

Since the slit width is negligible, we can treat each slit as a single line. The grating spacing is the distance between two adjacent lines. If we are told that the grating is N lines per unit length, then the grating spacing is given as $d = \frac{1}{N}$ in that unit.

10 Exam Questions

10.1 May 2023 Paper 2 HL TZ1 Question 3

(a) Monochromatic light is incident on two very narrow slits. The light that passes through the slits is observed on a screen. M is directly opposite the midpoint of the slits. x represents the displacement from M in the direction shown.

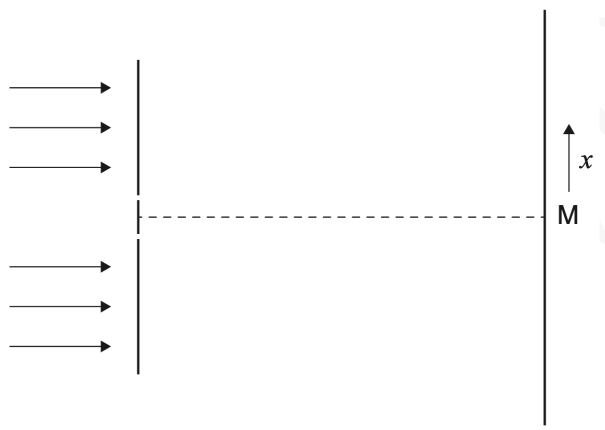

Figure 16: Question 1

A student argues that what will be observed on the screen will be a total of two bright spots opposite the slits. Explain why the student's argument is incorrect.

- First, we must understand the student's claim — it is the assertion that the light will shine direct through the slits and travel in a straight line until hitting the screen.
- The reason why this is not correct is that
  - Light acts as a wave and not a particle in this situation
  - Light at slits will diffract
  - Light passing through slits will interfere, creating bright and dark spots

(b) The graph shows the actual variation with displacement x from M of the intensity of the light on the screen. I_0 is the intensity of light at the screen from one slit only.

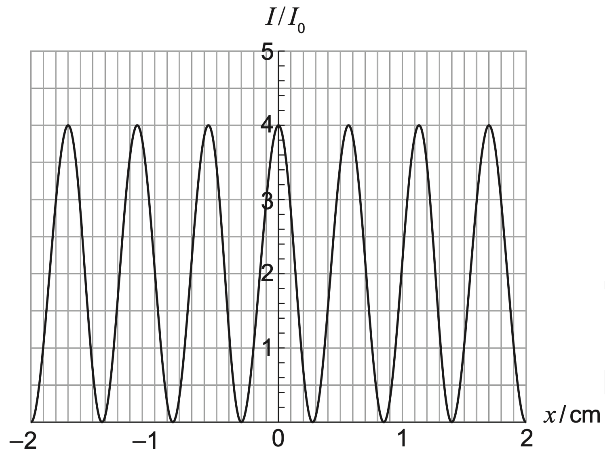

Figure 17: Question 2

(i) Explain why the intensity of light at $x = 0$ is $4I_0$.

- At $x = 0$, the light waves from the two slits are in phase, and so the waves will constructively interfere. Thus, the amplitude doubles.
- Since the intensity is proportional to the square of the amplitude, the intensity will be four times the original.

(ii) The slits are separated by a distance of 0.18 mm and the distance to the screen is 2.2 m. Determine, in m, the wavelength of light.

- Let's list out the known quantities
  - $d = 1.8 \times 10^{-4}$ m
  - $D = 2.2$ m
  - we can spot from the graph that the separation between any two maxima is $s = \dfrac{1.7}{300}$m

- From the double slit formula, we have

$$\begin{aligned}
\lambda &= \frac{sd}{D} \\
&= \frac{1.7 \times 1.8 \times 10^{-4}}{300 \times 2.2} \\
&\approx 4.6 \times 10^{-7} \text{ m} \\
&= 460 \text{ nm}
\end{aligned}$$

- The two slits are replaced by many slits of the same separation. State one feature of the intensity pattern that will remain the same and one that will change.
  - Unchanged: The peak separation s remains unchanged, since all of its dependencies remain the same.
  - Changed: The peak intensity

10.2 May 2023 Paper 2 SL TZ1 Question 3

Blue light of wavelength λ is incident on a double slit. Light from the double slit falls on a screen. A student measures the distance between nine successive fringes on the screen to be 15cm. The separation of the double slit is 60µm; the double slit is 2.5m from the screen.

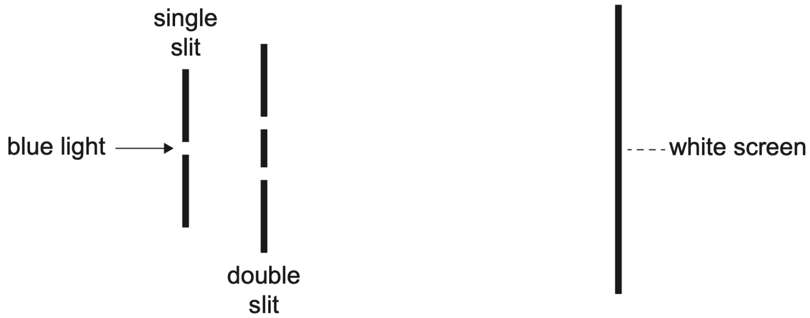

Figure 18: Diagram

(a) Explain the pattern seen on the screen.

- There will be interference.
- Bright fringe occurs when light from the slits arrives in phase.
- Dark fringe occurs when light from the slits arrives π out of phase.

(b) Calculate, in nm, λ.

- First, we must pay close attention to the given information: The distance between nine successive fringes is 15cm. This means that 8 times the separation between the fringes is 15cm. Hence, we find that $s = \frac{15}{800}$m.

$$\begin{aligned}\lambda &= \frac{sd}{D} \\ &= \frac{15 \times 6 \times 10^{-5}}{2.5 \times 800} = 4.5 \times 10^{-7} \text{ m} \\ &= 450 \text{ nm}\end{aligned}$$

(c) The student changes the light source to one that emits two colors:

- blue light of wavelength λ, and
- red light of wavelength 1.5λ.

Predict the pattern that the student will see on the screen.

- We should consider the two colors separately:
  - For the blue light, the pattern will be as calculated in part (b). It will not change.
  - For the red light, since $s = \dfrac{\lambda D}{d}$, the separation between the fringes will be $1.5s$. This means that the red light will have a larger separation between the fringes, with a scale factor of 1.5.
- Also, at some points, the two colors coincide to make a purple fringe.

10.3 May 2019 Paper 2 SL TZ1 Question 3

A beam of microwaves is incident normally on a pair of identical narrow slits S1 and S2.

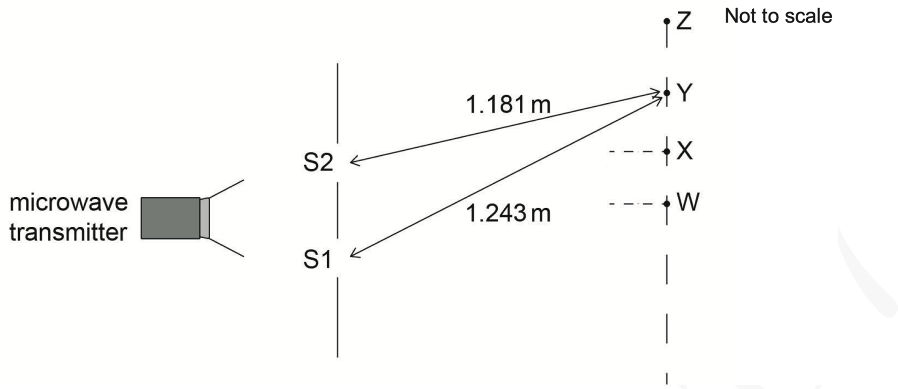

Figure 19: Diagram

When a microwave receiver is initially placed at W which is equidistant from the slits, a maximum in intensity is observed. The receiver is then moved towards Z along a line parallel to the slits. Intensity maxima are observed at X and Y with one minimum between them. W, X and Y are consecutive maxima.

(a) Explain why intensity maxima are observed at X and Y.

- At the points X and Y, there is constructive interference between the two waves from the two slits.
- This is because the **path difference** is an integer multiple of the wavelength.

(b) The distance from S1 to Y is 1.243m and the distance from S2 to Y is 1.181m. Determine the frequency of the microwaves.

- Since Y is the second (0-indexed counting) maximum, the path difference is 2λ.

$$\lambda = \frac{1.243 - 1.181}{2} = 0.031 \text{ m}$$

- Then, the frequency is given by

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{0.031} = 9.7 \times 10^9 \text{ Hz}$$

(c) Outline one reason why the maxima observed at W, X and Y will have different intensities from each other.

- The intensity graph is modulated by a single slit diffraction envelope (recall the section on the combined effect)
- Also, the intensity varies with distance, and the maxima are further and further away from the slits.

10.4 Diffraction Grating and Particle Scattering

In an experiment to determine the radius of a carbon-12 nucleus, a beam of neutrons is scattered by a thin film of carbon-12. The graph shows the variation of intensity of the scattered neutrons with scattering angle. The de Broglie wavelength of the neutrons is 1.6×10^{-15} m

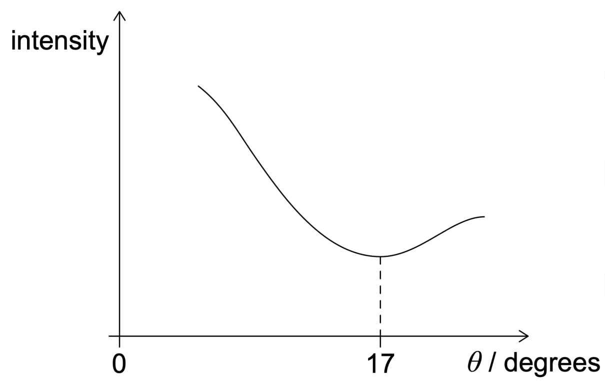

Figure 20: Diagram

Estimate, using the graph, the radius of a carbon-12 nucleus.

- This question may initially be very daunting, but let us look at it from the perspective of diffraction grating.
- The phrase "thin film" gives away the that you can consider the material as a single layer of carbon-12 nuclei. This then becomes the layer of slits in the diffraction grating.

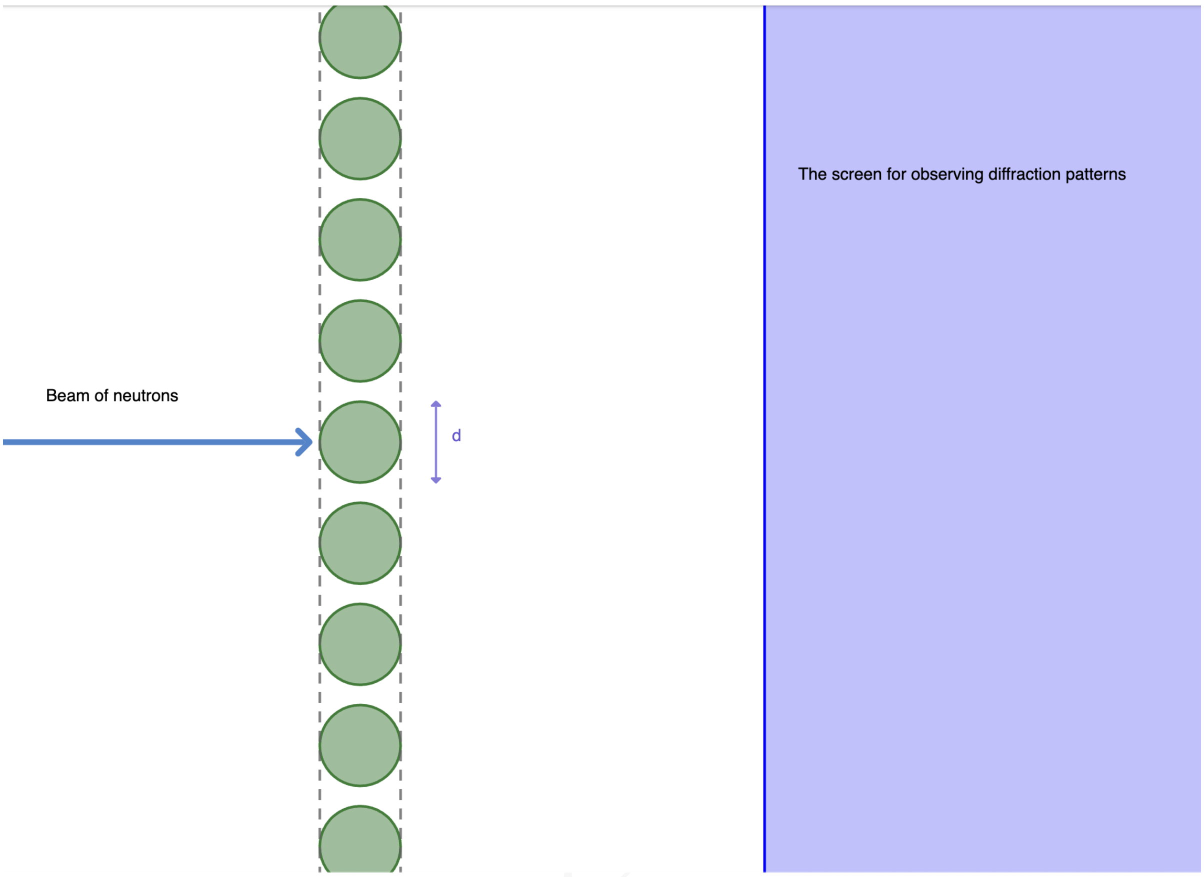

Figure 21: The grating setup

- The slit spacing in this case is the diameter of the carbon-12 nucleus.
- The wavelength of the neutron beam is given as 1.6×10^{-15} m.
- The angle of the first minimum is given in the diagram as 17°.

- The calculation becomes trivial once we have recognized and understood the scenario.

$$
\begin{aligned}
r &= \frac{d}{2} \\
&= \frac{1}{2}\left(\frac{n\lambda}{\sin\theta}\right) \\
&= \frac{1}{2}\left(\frac{1 \times 1.6 \times 10^{-15}}{\sin 17^\circ}\right) \\
&\approx 2.8 \times 10^{-15} \text{ m}
\end{aligned}
$$

10.5 May 2021 Paper 2 HL TZ2 Question 8

1. Monochromatic light of wavelength λ is normally incident on a diffraction grating. The diagram shows adjacent slits of the diffraction grating labelled V, W and X. Light waves are diffracted through an angle θ to form a second-order diffraction maximum. Points Z and Y are labelled.

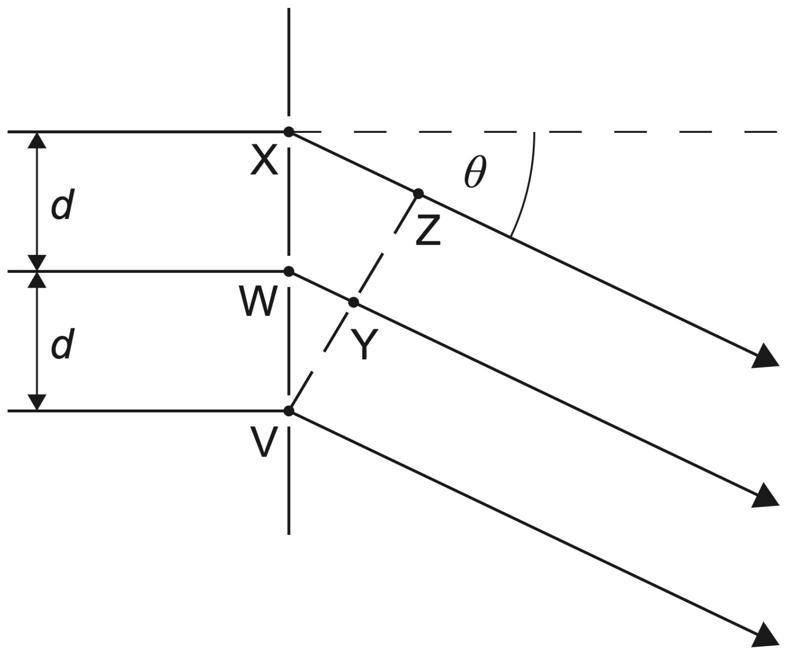

Figure 22: Diagram

(i) State the phase difference between the waves at V and Y.

- First, we must recognize that the shown rays are those that meet at a point of constructive interference
- This is where the path difference is 2λ and the phase difference is 0 mod 2π.

(ii) State, in terms of λ, the path length between points X and Z.

- You should know that in this kind of multi-slit configuration, $WY = 2\lambda$, $XZ = 4\lambda$, and so on for further slits.

(iii) The separation of adjacent slits is d. Show that for the second-order diffraction maximum $2\lambda = d \sin\theta$.

$$\begin{aligned}\sin\theta &= \frac{XZ}{VX}\\ &= \frac{4\lambda}{2d}\\ &= \frac{2\lambda}{d}\end{aligned}$$

2. Monochromatic light of wavelength 633nm is normally incident on a diffraction grating. The diffraction maxima incident on a screen are detected and their angle θ to the central beam is determined. The graph shows the variation of $\sin\theta$ with the order n of the maximum. The central order corresponds to $n = 0$.

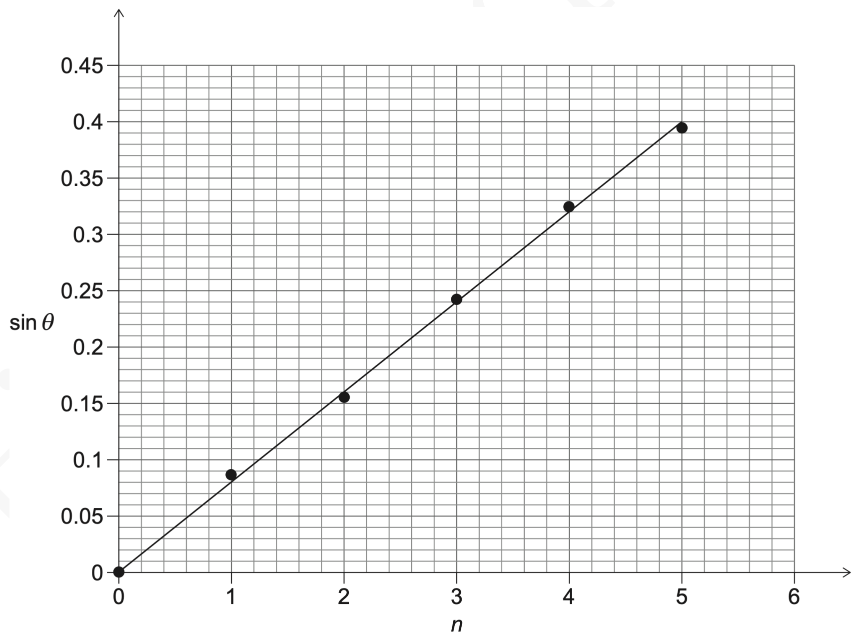

Figure 23: Graph

Determine a mean value for the number of slits per millimeter of the grating.

- The number of slits N contained in a unit length is given by $N = \dfrac{1}{d}$, where d is the slit spacing.

- Recall the multi-slit diffraction formula: $n\lambda = d\sin\theta$ rearranging gives

$$\frac{\sin\theta}{n} = \frac{\lambda}{d}$$

- First, let us convert the quantity λ into millimeters so that it's consistent with the unit of d asked in the question. This is $633 \times 10^{-9}\text{m} \equiv 633 \times 10^{-6}\text{mm}$.
- Graphically, this is the gradient of the given straight line, so

$$\begin{aligned} \frac{\lambda}{d} &= \frac{0.4}{5} \\ N = \frac{1}{d} &= \frac{0.4}{5\lambda} \approx 126 \end{aligned}$$

3. State the effect on the graph of the variation of $\sin\theta$ with n of:

   (i) using a light source with a smaller wavelength.

   - Gradient decreases

   (ii) increasing the distance between the diffraction grating and the screen.

   - No change. If we were talking about the actual position of the maximum (in length units), then, this would make a difference, however angles don't scale and stay constant.

10.6 Applying the Snell Equality Chain

A ray of monochromatic light is incident on the parallel interfaces between three media. The speeds of light in the media are v_1, v_2, and v_3 respectively.

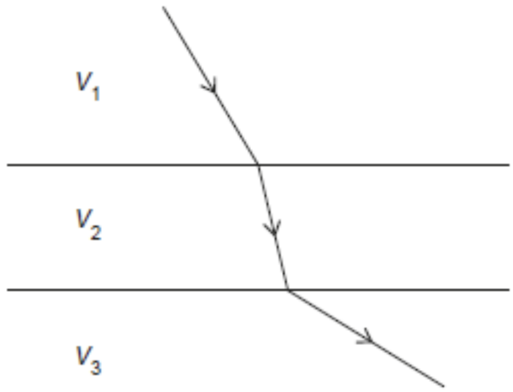

Figure 24: Diagram

- By the relation
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$
we know that the speed has a positive correlation with the angle in that medium. This means that, in a medium whose speed is greater, the angle of refraction will be greater.

- In medium 2, the angle is the smallest, so the speed is the smallest.

- By similar arguments, $v_2 < v_1 < v_3$.

10.7 Colors

In two different experiments, white light is passed through a single slit and then is either refracted through a prism or diffracted with a diffraction grating. The prism produces a band of colors from M to N. The diffraction grating produces a first order spectrum P to Q.

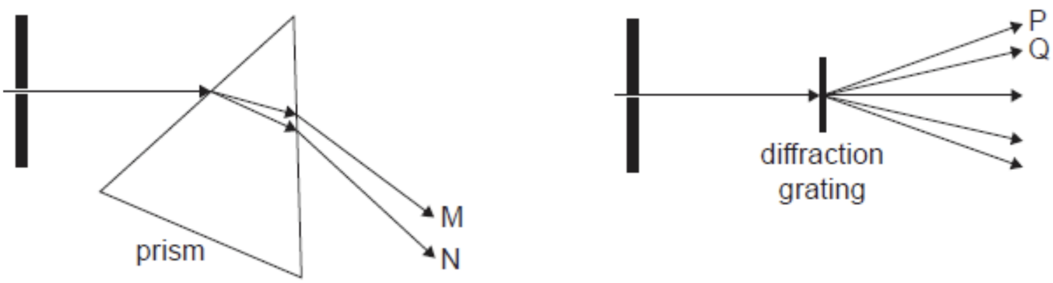

Figure 25: Diagram

	M	P
A	red	red
B	red	violet
C	violet	red
D	violet	violet

- Let's consider the first setup — this is clearly a refraction problem.
  - Looking at the two rays, the difference is the angles of refraction.
  - Using our knowledge about the Snell equality chain, the longer the wavelength, the larger the angle of refraction

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2}$$

  - Thus, we can see that, since M has a larger angle of refraction, it must have a longer wavelength and thus should be red.

- The second setup is a diffraction problem.
  - We know that
    $$\theta = \frac{\lambda}{b}$$
    is how spread out the first minimum in the diffraction grating pattern is. Thus, the longer the wavelength, the larger the angle.
  - Clearly, P is at a larger angular position and so it must possess a longer wavelength, thus it should also be red.

10.8 Observable Maxima

Light of wavelength 400 nm is incident normally on a diffraction grating. The slit separation of the diffraction grating is 1.0 µm What is the greatest number of maxima that can be observed?

- We invoke the equation for diffraction grating

$$d\sin\theta = n\lambda \iff n = \frac{d\sin\theta}{\lambda}$$

- The nth maximum is observed at the angle θ.
- For a maximum to be observed, we require $\theta < 90°$.

$$\sin\theta < 1 \implies n < \frac{d}{\lambda} \implies n < \frac{1.0\times 10^{-6}}{4.0\times 10^{-7}} = 2.5$$

- This indicates that on one side of the central maximum, there are two other maxima.
- In total, this gives $2 + 1 + 2 = 5$ maxima.

10.9 Path and Phase Diff.

Light of wavelength λ diffracts at a single rectangular slit of opening b. The diagram shows two rays of light leaving the top and middle of the slit. The rays come from the same wavefront. The angle of diffraction is θ. For small angles the approximation $\sin\theta \approx \theta$ may be used.

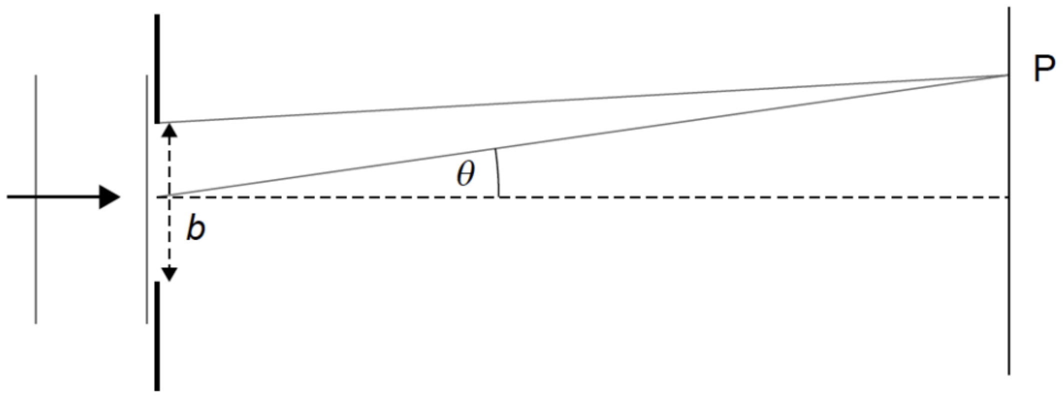

Figure 26: Diagram

(a) Show that the phase difference between the two rays at P is $\frac{\pi b\theta}{\lambda}$

- The path difference is given by

$$\frac{b}{2}\sin\theta \approx \frac{b\theta}{2}$$

- We invoke the ratios link

$$\begin{aligned}\frac{\Delta r}{\lambda} &= \frac{\Delta\phi}{2\pi} \\ \Delta\phi &= \frac{2\pi}{\lambda}\Delta r \\ &= \frac{2\pi}{\lambda}\times\frac{b\theta}{2} \\ &= \frac{\pi b\theta}{\lambda}\end{aligned}$$

(b) The two rays interfere destructively at P to form the first minimum of the single slit diffraction pattern. Explain why $\theta = \frac{\lambda}{b}$

- For the first minimum, the phase difference is $\Delta\phi = \pi$. We substitute this back to obtain

$$1 = \frac{b\theta}{\lambda} \implies \theta = \frac{\lambda}{b}$$