

# **IB Physics Topic B3 Gas Laws; SL & HL**

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# 1 Pressure

Pressure is defined as the force per unit area.

$$P = \frac{F}{A} \quad (1)$$

- **Solid:** The pressure due to the weight  $W$  over an area  $A$  is given by

$$P = \frac{W}{A} \quad (2)$$

- **Liquid:** The pressure in a liquid at a depth  $h$  is given by

$$P = \rho gh \quad (3)$$

where  $\rho$  is the density of the liquid,  $g$  is the acceleration due to gravity, and  $h$  is the depth.

- **Gas:** Will be discussed later.

## 1.1 Avogadro's Number and the Mole

The **mole** is the SI unit for the amount of substance. It has been historically defined as the number of atoms in approximately 12 grams of carbon-12. This quantity is known as the Avogadro number,  $N_A$ ,

$$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$$

“Every mol is  $N_A$  molecules”. For instance, 3 mol of electrons is simply the quantity of  $3N_A$  electrons.

### 1.1.1 Molar Mass

**Molar mass** is the mass of one mole of a substance, typically given in grams per mole (g/mol). The molar mass of a substance is numerically equal to the atomic mass of the substance in atomic mass units (u). E.g. water has a molar mass of 18.015 g/mol  $\equiv$  18.015 u, and the mass of a single water molecule is  $\frac{18.015}{N_A}$  g or  $\frac{0.018015}{N_A}$  kg.

## 2 Gas Laws

### 2.1 Boyle's Law

This states that the pressure of a gas is inversely proportional to its volume at constant temperature. Mathematically, this is

$$P \propto \frac{1}{V} \quad \text{or} \quad P_1 V_1 = P_2 V_2$$

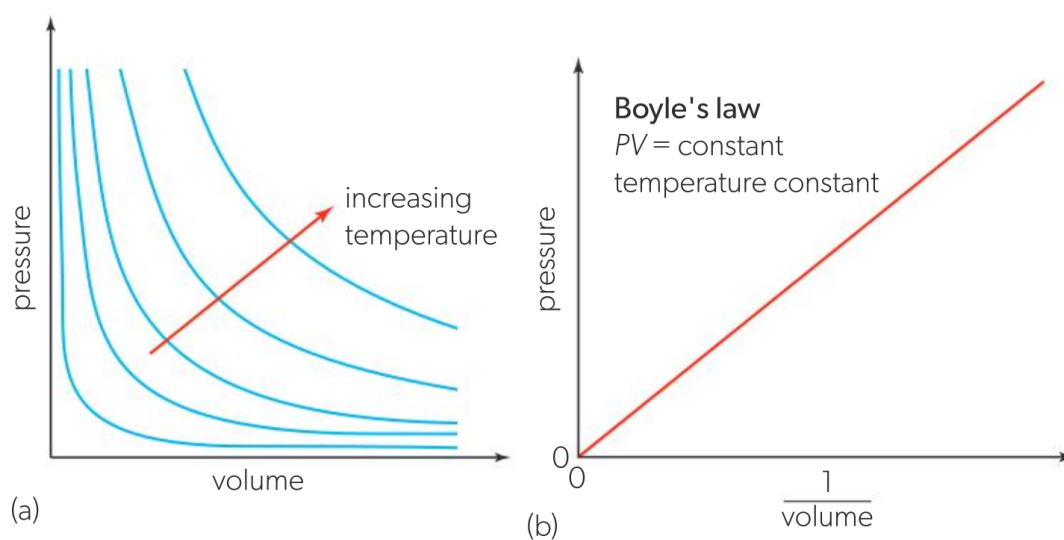


Figure 1: Two graphs that arise from Boyle's Law

- Every one of the curves in (a) is an isothermal curve.
- In the second graph
  1. For a higher temperature and the same mass of gas, the curve is steeper but still linear
  2. For a higher mass of gas and the same temperature, the curve would also be steeper.

## 2.2 Charles' Law

This states that the **volume** of a gas is **directly proportional** to its **temperature** at **constant pressure**. Mathematically, this is

$$V \propto T \quad \text{or} \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

## 2.3 Gay-Lussac's Law

This states that the **pressure** of a gas is **directly proportional** to its **temperature** at **constant volume**. Mathematically, this is

$$P \propto T \quad \text{or} \quad \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

## 2.4 Avogadro's Law

This states that the **volume** of a gas is **directly proportional** to the **quantity (in mol)** of the gas at **constant temperature and pressure**. Mathematically, this is

$$V \propto n \quad \text{or} \quad \frac{V_1}{n_1} = \frac{V_2}{n_2}$$

## 2.5 The Ideal Gas Law

Derived from the previous laws; they combine to give

$$PV = nRT \quad \text{or} \quad R = \frac{PV}{nT}$$

where  $R$  is the ideal gas constant,  $8.31 \text{ J mol}^{-1} \text{ K}^{-1}$ .

The relation can also be defined in terms of the **number of molecules**  $N$  and the Boltzmann constant  $k$  as

$$\frac{PV}{NT} = k_B \quad \text{or} \quad pV = Nk_B T \quad (4)$$

where  $k_B = \frac{R}{N_A}$  has value  $1.38 \times 10^{-23} \text{ J K}^{-1}$ .

### 2.5.1 Proportionality Form

In calculations, the following form is also often useful

$$\frac{P_1 V_1}{N_1 T_1} = \frac{P_2 V_2}{N_2 T_2}$$

### 3 Brownian Motion

- **Definition:** Brownian motion refers to the unpredictable and irregular movement of microscopic particles suspended in a fluid (liquid or gas) caused by collisions with molecules of the fluid.
- **Collisions:** Brownian motion occurs because particles are constantly bombarded by surrounding molecules, whose motion is due to thermal energy. These molecular collisions are random in both direction and magnitude.
- **Temperature dependence:** The motion increases with temperature, as higher temperatures lead to more energetic collisions between fluid molecules and the suspended particles.
- **Continuous motion:** Any fluid above the temperature of absolute zero will exhibit Brownian motion, as the fluid molecules are always in motion because of their KE.



## 4 Kinetic Model of Ideal Gases

### 4.1 Assumptions of the Model

1. All gas molecules are identical.
2. Brownian motion constantly occurs
3. The total volume of the gas is negligible compared to the volume of the container.
4. The molecules collide elastically with each other and the walls of the container.
5. The internal energy of the gas is entirely kinetic and does not include potential energy; intramolecular forces between the particles and the walls are negligible except during collisions.
6. The time of collision is negligible compared to the time between collisions.
7. External forces such as gravity are ignored.

### 4.2 The Model

Consider a cubic container with length  $L$  and volume  $L^3$ .

Also consider a single particle with mass  $m$  and  $x$ -velocity  $v_x$  that hits the wall of the container at a right angle. The **impulse** is  $-2mv_x$ , and the *average* force exerted on the wall is  $F_x = \frac{\Delta p}{\Delta t} = -\frac{2mv_x}{T}$ , where  $T$  is the time taken for the collision and the particle to bounce back to the same contact point. This uses [assumption 4](#). Throughout  $T$ , the cube has traveled twice the length of the cube, so  $T = \frac{2L}{v_x}$ .

The average force along the  $x$ -axis is then

$$F_x = -\frac{2mv_x}{2L/v_x} = -\frac{mv_x^2}{L}$$

This analysis applies to any of the three components ( $x$ ,  $y$ , or  $z$ ), and if there are  $N$  particles in the container, then, this analysis also applies to any of the  $N$  particles.

Now, we return to considering the  $x$ -dimension, without loss of generality. Let  $\{v_{x_1}, v_{x_2}, \dots, v_{x_N}\}$  be the  $x$ -velocities of the particles, the total  $x$ -force is given by

$$F_x = F_{x_1} + F_{x_2} + \dots + F_{x_N} = \frac{m(v_{x_1}^2 + v_{x_2}^2 + \dots + v_{x_N}^2)}{L}$$

We then take the **mean square speed** of the  $x$ -components across the  $N$  particles

$$\overline{v_x^2} = \frac{v_{x_1}^2 + v_{x_2}^2 + \dots + v_{x_N}^2}{N}$$

Hence, we obtain that the average force is given by

$$\overline{F_x} = \frac{Nm}{L} \times \overline{v_x^2}$$

Finally, we now combine the three dimensions:

1. The magnitude of a 3D vector  $\vec{v}$  is given by  $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ .
2. We can apply this to the mean square speed to get  $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$ ; this represents the **mean square speed of the molecules**.

We now make a further approximation based on the idea that the motion is random:

1. The motion is said to be **isotropic**; there is no preferred direction
2. We can then take  $v_x^2 = v_y^2 = v_z^2$ , this gives that  $\overline{v^2} = 3\overline{v_x^2}$  and equivalently  $\overline{v_x^2} = \frac{1}{3}\overline{v^2}$

Substitution of this gives:

$$\overline{F} = \frac{Nm}{L} \times \frac{1}{3}\overline{v^2}$$

Since pressure is defined as the force per unit area, we divide by  $L^2$

$$P = \frac{Nm}{3L^3} \times \overline{v^2} = \frac{Nm}{3V} \times \overline{v^2}$$

Also notice that  $N \times m$  is the number of molecules  $\times$  mass of each molecule; this is exactly the total mass of the gas,  $M$ . Then,  $\frac{M}{V}$  is simply the density of the gas. Hence, we obtain

$$P = \frac{1}{3} \rho \overline{v^2} \quad (5)$$

### 4.3 Temperature

We can combine Equation (4) and Equation (5) (the algebra is omitted) to get

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T \quad (6)$$

Notice that the LHS of this equation measures the kinetic energy of the gas molecules. By previous argument, the internal energy of gas is almost entirely made of kinetic energy, as the intermolecular force is negligible. Thus, we can claim that

$$\text{total internal energy of an idea gas} = \frac{3}{2} N k_B T$$

This gives us the following properties of an ideal gas

- It's total energy is directly proportional to the temperature in Kelvin.
- At constant temperature, the internal energy of the gas is proportional to the number of molecules.

## 4.4 Ideal vs. Real Gases

The previously developed model only applies to *monatomic* (single atom) gas molecules.

An example of a real gas behavior is **liquefaction**, this is impossible for an idea gas.

- The process of turning a gas into a liquid.
- This occurs when the gas is cooled and compressed. The gas molecules are then close enough to each other that the intermolecular forces become significant. This is why the ideal gas law fails at high pressures and low temperatures.

Let us revisit the following assumptions made for the ideal gas model:

- Particles themselves **do not have volume**.
- There are **no intermolecular forces** between the particles except during collisions.
- All collisions are perfectly **elastic**.

However, for a real gas,

- Real gas particles **have volume**.
- **There are intermolecular forces** between gas particles, especially at lower temperatures and higher pressures.

Under these conditions, the behavior of real gases deviate from the ideal gas law

- At *low temperatures*: Real gas particles move **slower**, so **intermolecular forces** become more significant, leading to **liquefaction**.
- At *high pressures*: The volume of the particles becomes significant compared to the total volume of the gas, violating the ideal gas assumption of negligible particle volume.

This means that, for real gases, Equation (4) no longer holds, this means that

$$\frac{PV}{RT}$$

is no longer a constant. To visualize this graphically:

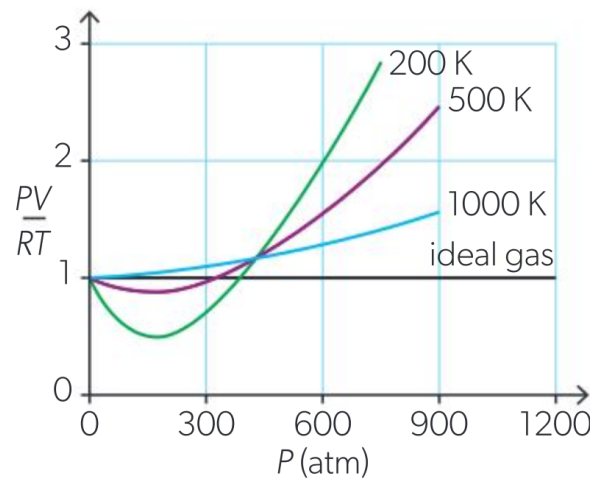


Figure 2: A graph showing the deviation of real gases from the ideal gas law, for a **single mole of gas**

It must be noted that the liquefaction of a gas is only possible under a certain temperature threshold.