IB Physics Topic B3 Gas Laws; SL & HL

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Table of Contents

1   Pressure

Pressure is defined as the force per unit area.

$$P = \frac{F}{A} \tag{1}$$

- **Solid**: The pressure due to the weight W over an area A is given by

$$P = \frac{W}{A} \tag{2}$$

- **Liquid**: The pressure in a liquid at a depth h is given by

$$P = \rho g h \tag{3}$$

  where ρ is the density of the liquid, g is the acceleration due to gravity, and h is the depth.

- **Gas**: Will be discussed later.

1.1   Avogadro's Number and the Mole

The **mole** is the SI unit for the amount of substance. It has been historically defined as the number of atoms in approximately 12 grams of carbon-12. This quantity is known as the Avogadro number, N_A,

$$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$$

"Every mol is N_A molecules". For instance, 3 mol of electrons is simply the quantity of $3N_A$ electrons.

1.1.1 Molar Mass

Molar mass is the mass of one mole of a substance, typically given in grams per mole (g/mol). The molar mass of a substance is numerically equal to the atomic mass of the substance in atomic mass units (u). E.g. water has a molar mass of 18.015 g/mol $\equiv$ 18.015 u, and the mass of a single water molecule is $\dfrac{18.015}{N_A}$ g or $\dfrac{0.018015}{N_A}$ kg.

2   Gas Laws

2.1   Boyle's Law

This states that the pressure of a gas is inversely proportional to its volume at constant temperature. Mathematically, this is

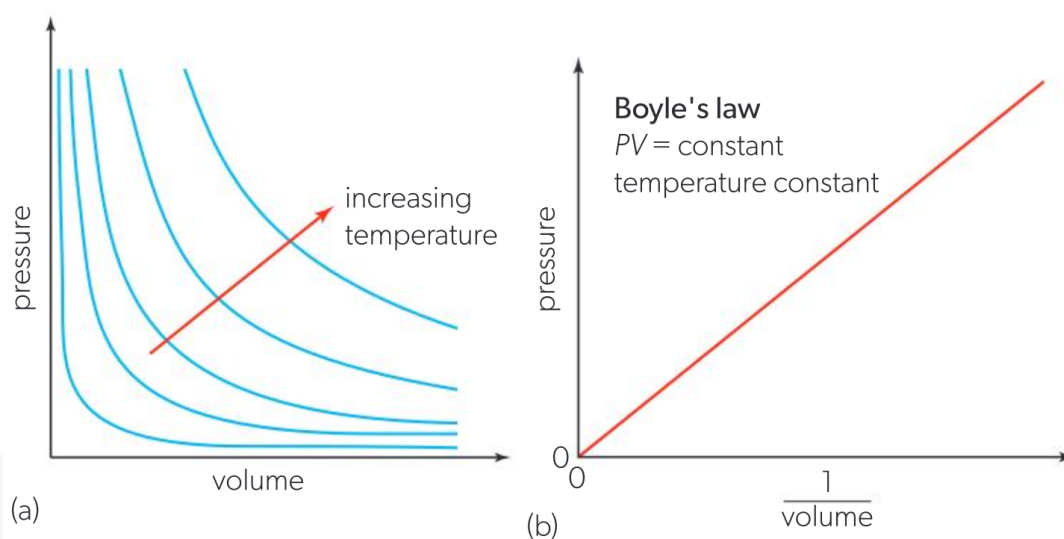$$P \propto \frac{1}{V} \quad \text{or} \quad P_1V_1 = P_2V_2$$

Figure 1: Two graphs that arise from Boyle's Law

- Every one of the curves in (a) is an isothermal curve.

- In the second graph

  1. For a higher temperature and the same mass of gas, the curve is steeper but still linear

  2. For a higher mass of gas and the same temperature, the curve would also be steeper.

2.2   Charles' Law

This states that the **volume** of a gas is **directly proportional** to its **temperature** at **constant pressure**. Mathematically, this is

$$V \propto T \quad \text{or} \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

2.3   Gay-Lussac's Law

This states that the **pressure** of a gas is **directly proportional** to its **temperature** at **constant volume**. Mathematically, this is

$$P \propto T \quad \text{or} \quad \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

2.4   Avogadro's Law

This states that the **volume** of a gas is **directly proportional** to the **quantity (in mol)** of the gas at **constant temperature and pressure**. Mathematically, this is

$$V \propto n \quad \text{or} \quad \frac{V_1}{n_1} = \frac{V_2}{n_2}$$

2.5   The Ideal Gas Law

Derived from the previous laws; they combine to give the following <mark>empirical</mark> relation

$$PV = nRT \quad \text{or} \quad R = \frac{PV}{nT}$$

where R is the ideal gas constant, 8.31 J mol^{-1} K^{-1}.

The relation can also be defined in terms of the **number of molecules** N and the Boltzmann constant k as

$$\frac{PV}{NT} = k_B \quad \text{or} \quad pV = Nk_BT \tag{4}$$

where $k_B = \dfrac{R}{N_A}$ has value 1.38×10^{-23} J K^{-1}.

2.5.1   Proportionality Form

In calculations, the following form is also often useful

$$\frac{P_1V_1}{N_1T_1} = \frac{P_2V_2}{N_2T_2}$$

2.5.2   Link to Internal Energy

$$U = \frac{3}{2}nRT = \frac{3}{2}PV$$

3   Brownian Motion

- **Definition**: Brownian motion refers to the unpredictable and irregular movement of microscopic particles suspended in a fluid (liquid or gas) caused by collisions with molecules of the fluid.

- **Collisions**: Brownian motion occurs because particles are constantly bombarded by surrounding molecules, whose motion is due to thermal energy. These molecular collisions are random in both direction and magnitude.

- **Temperature dependence**: The motion increases with temperature, as higher temperatures lead to more energetic collisions between fluid molecules and the suspended particles.

- **Continuous motion**: Any fluid above the temperature of absolute zero will exhibit Brownian motion, as the fluid molecules are always in motion because of their KE.

4   Kinetic Model of Ideal Gases

4.1   Assumptions of the Model

1. All gas molecules are identical.

2. Brownian motion constantly occurs

3. The total volume of the gas is negligible compared to the volume of the container.

4. The molecules collide <mark>elastically</mark> with each other and the walls of the container.

5. The internal energy of the gas is entirely kinetic and does not include potential energy; intramolecular forces between the particles and the walls are negligible <mark>except during collisions</mark>.

6. The <mark>time of collision is negligible</mark> compared to the time between collisions.

7. External forces such as gravity are ignored.

4.2   The Model

Consider a cubic container with length L and volume L^3.

Also consider a single particle with mass m and x-velocity v_x that hits the wall of the container at a right angle. The **impulse** is $-2mv_x$, and the *average* force exerted on the wall is $F_x = \dfrac{\Delta p}{\Delta t} = -\dfrac{2mv_x}{T}$, where T is the time taken for the collision and the particle to bounce back to the same contact point. This uses assumption 4. Throughout T, the cube has traveled twice the length of the cube, so $T = \dfrac{2L}{v_x}$.

The average force along the x-axis is then

$$F_x = -\frac{2mv_x}{2L/v_x} = -\frac{mv_x^2}{L}$$

This analysis applies to any of the three components (x, y, or z), and if there are N particles in the container, then, this analysis also applies to any of the N particles.

Now, we return to considering the x-dimension, without loss of generality. Let $\{v_{x_1}, v_{x_2}, \ldots, v_{x_N}\}$ be the x-velocities of the particles, the total x-force is given by

$$F_x = F_{x_1} + F_{x_2} + \cdots + F_{x_N} = \frac{m(v_{x_1}^2 + v_{x_2}^2 + \cdots + v_{x_N}^2)}{L}$$

We then take the **mean square speed** of the x-components across the N particles

$$\overline{v_x^2} = \frac{v_{x_1}^2 + v_{x_2}^2 + \cdots + v_{x_N}^2}{N}$$

Hence, we obtain that the average force is given by

$$\overline{F_x} = \frac{Nm}{L} \times \overline{v_x^2}$$

Finally, we now combine the three dimensions:

1. The magnitude of a 3D vector $\vec{v}$ is given by $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$.

2. We can apply this to the mean square speed to get $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$; this represents the **mean square speed of the molecules**.

We now make a further approximation based on the idea that the motion is random:

1. The motion is said to be **isotropic**; there is no preferred direction

2. We can then take $v_x^2 = v_y^2 = v_z^2$, this gives that $\overline{v^2} = 3\overline{v_x^2}$ and equivalently $\overline{v_x^2} = \frac{1}{3}\overline{v^2}$

Substitution of this gives:

$$\overline{F} = \frac{Nm}{L} \times \frac{1}{3}\overline{v^2}$$

Since pressure is defined as the force per unit area, we divide by L^2

$$P = \frac{Nm}{3L^3} \times \overline{v^2} = \frac{Nm}{3V} \times \overline{v^2}$$

Also notice that $N \times m$ is the number of molecules $\times$ mass of each molecule; this is exactly the total mass of the gas, M. Then, $\dfrac{M}{V}$ is simply the density of the gas. Hence, we obtain

$$P = \frac{1}{3}\rho\overline{v^2} \tag{5}$$

4.3   Temperature

We can combine Equation (4) and Equation (5) (the algebra is omitted) to get

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T \tag{6}$$

Notice that the LHS of this equation measures the kinetic energy of the gas molecules. By previous argument, the internal energy of gas is almost entirely made of kinetic energy, as the intermolecular force is negligible. Thus, we can claim that

$$\textbf{total internal energy of an idea gas} = \frac{3}{2}Nk_B T = \frac{3}{2}PV$$

This gives us the following properties of an ideal gas

- It's total energy is directly proportional to the temperature in Kelvin.

- At constant temperature, the internal energy of the gas is proportional to the number of molecules.

A useful form for calculating the speed is

$$\overline{v^2} = \frac{3k_B T}{m} = \frac{3PV}{M}$$

4.4   Ideal vs. Real Gases

The previously developed model only applies to *monatomic* (single atom) gas molecules.

An example of a real gas behavior is **liquefaction**, this is impossible for an idea gas.

- The process of turning a gas into a liquid.

- This occurs when the gas is cooled and compressed. The gas molecules are then close enough to each other that the intermolecular forces become significant. This is why the ideal gas law fails at high pressures and low temperatures.

Let us revisit the following assumptions made for the ideal gas model:

- Particles themselves **do not have volume**.

- There are **no intermolecular forces** between the particles except during collisions.

- All collisions are perfectly **elastic**.

However, for a real gas,

- Real gas particles **have volume**.

- **There are intermolecular forces** between gas particles, especially at lower temperatures and higher pressures.

- **Collisions are not perfectly elastic** because of the intermolecular forces.

Under these conditions, the behavior of real gases deviate from the ideal gas law

- At *low temperatures*: Real gas particles move **slower**, so **intermolecular forces** become more significant, leading to **liquefaction**.

- At *high pressures*: The volume of the particles becomes significant compared to the total volume of the gas, violating the ideal gas assumption of negligible particle volume.

This means that, for real gases, Equation (4) no longer holds, this means that

$$\frac{PV}{RT}$$

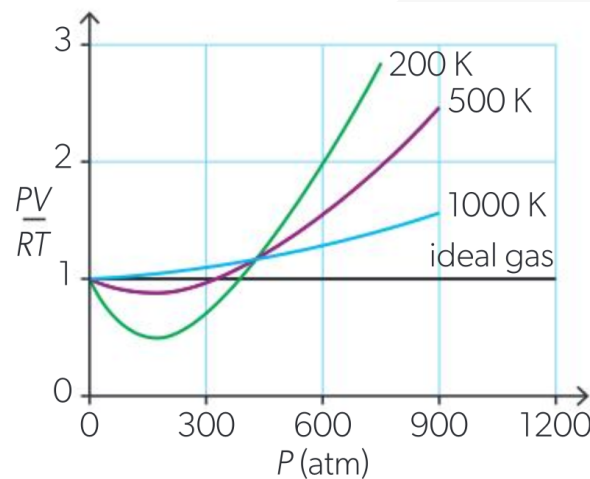is no longer a constant. To visualize this graphically:

Figure 2: A graph showing the deviation of real gases from the ideal gas law, for a **single mole of gas**

It must be noted that the liquefaction of a gas it only possible under a certain temperature threshold.

4.4.1   Conditions for Approximating Ideal Gas

- Low pressure: At low pressures, the intermolecular forces between gas particles become negligible, and the volume occupied by the gas molecules themselves is insignificant compared to the total volume of the gas.

- Low density: Minimal influence of intermolecular forces.

5   Exam Questions

5.1   Quick-Fire MCQ #1

Two samples of a gas are kept in separate containers. The molecules of each sample have the same average translational speed, but the samples have a different density. What is correct about the pressure and the temperature of the samples, as compared to each other?

	Pressure	Temperature
A.	same	same
B.	same	different
C.	different	same
D.	different	different

Figure 3

- We know that the pressure is given by

$$P = \frac{1}{3}\rho\overline{v^2}$$

the average speed is the same while the density is different, so the pressure must be different. This helps us to eliminate A and B.

- We also know that the temperature is given by

$$T = \frac{1}{2}m\overline{v^2}$$

$\overline{v^2}$ is the same, and so is the mass of the molecules. Hence, the temperature must be the same. This leads us to the answer, C.

5.2   Air Mixture and Different Molecular Speeds

A sample of air is a mixture of nitrogen, oxygen and other gases. Explain why the component gases of air in the container have different average translational speeds.

1. Average kinetic energy of the molecules is determined by the temperature only

2. The mass of a molecule is different for each type of gas

3. From $E_K = \frac{1}{2}mv^2$, the same E_K and different m leads to different v

The important takeaway is that, no matter what kind of gas mixture we have, as long as they are in the same sealed container, the average kinetic energy and hence the temperature of the molecules is the same for all substances.

5.3   Absolute Zero

Outline how the concept of absolute zero of temperature is interpreted in terms of:

(a) the ideal gas law ($PV = nRT$),

- it is the temperature at which the volume or pressure extrapolates to zero.

(b) the kinetic energy of particles in an ideal gas

- it is the temperature at which all the Brownian motion of particles stops

5.4   Explaining Pressure Increase due to Temperature

The temperature of the gas in the container is increased. Explain, using the kinetic theory, how this change leads to a change in pressure in the container.

1. Increased temperature means increased average KE and hence increased average **translational** speed.

2. This increases the momentum transfer ($m\Delta v$) at the walls during each collision.

3. It also increases the frequency of collisions with the walls.

4. Since $F_{\text{wall}} = \dfrac{\Delta p}{\Delta t}$ and $P = \dfrac{F}{A}$ the average force on the walls increases, therefore the pressure increases.

5.5 Miscellaneous #1

A fixed quantity of $4.5 \times 10^{-3} \, \mathrm{mol}$ of air is compressed at a constant temperature. The graph shows the variation of pressure P with volume V of the air.
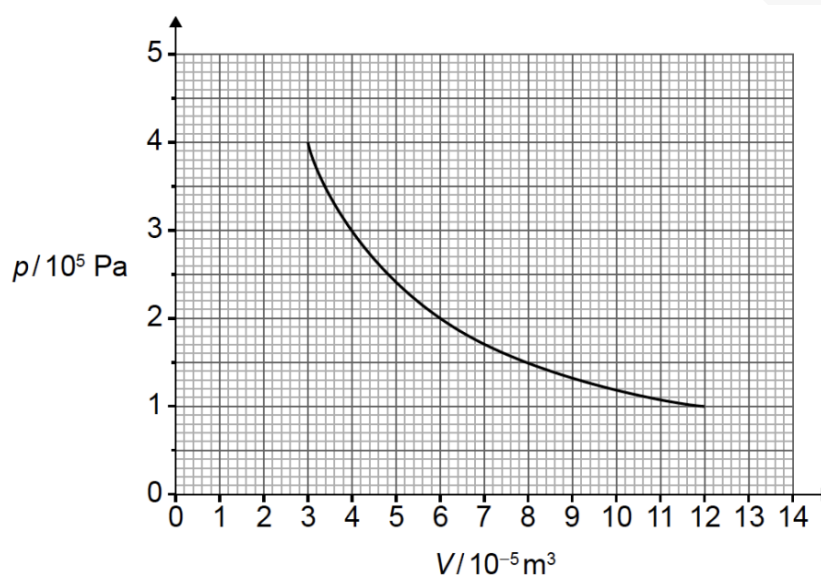

Figure 4: Graph

(a) Suggest whether the air behaves as an ideal gas during this change.

- The approach to this question is to find any two pairs of coordinates to show that $pV = 12$

- Because pV remains constant, the air behaves as an ideal gas.

(b) Outline how the kinetic theory of gases relates observable properties of a gas to the motion of the molecules.

- Absolute temperature is proportional to the KE of the molecules.

- Pressure is the result of molecular force on the container walls during collisions.

- Higher pressure is the result of higher KE of molecules in constant random motion or vice versa.

5.6   Isotope and Mass

What is the ratio $\dfrac{\text{number of atoms in 20g of Neon-20}}{\text{number of atoms in 40g of Krypton-80}}$?

Simple trick:

$$\frac{20 \div 20}{40 \div 80} = 2$$

Explanation:

- Each atom of Neon-20 has a mass of 20 u, and each atom of Krypton-80 has a mass of 80 u.

- Units are not important here, because we are working with ratios.

- Then, by dividing the total mass by the mass of each atom (regardless of discrepancies in units), we can find a "scaled version" of the number of atoms.

5.7   Ideal Gas Law Ratios

A balloon of volume V contains 10 mg of an ideal gas at a pressure P. An additional mass of the gas is added without changing the temperature of the balloon. This change causes the volume to increase to $2V$ and the pressure to increase to $3P$.

What is the mass of gas **added** to the balloon?

$$PV \propto N \propto M \text{(mass)}$$

$$PV \to (3P)(2V) = 6PV$$

$$\implies M \to 6M$$

$$6M - M = 5M = 50 \, \text{mg}$$

5.8   Common Pitfall – Increase in Pressure

An ideal gas of constant mass is heated in a container of constant volume. What is the reason for the increase in pressure of the gas?

  A. The average number of molecules per unit volume increases.

  B. The average force per impact at the container wall increases.

  C. Molecules collide with each other more frequently.

  D. Molecules occupy a greater fractional volume of the container.

If you think C is the answer, shame on you, wee fella! It's never about the intermolecular collisions but rather the collisions with the walls of the container. The answer is B.

5.9   Common Sense (no offense)

A substance in the gas state has a density about 1000 times less than when it is in the liquid state. The diameter of a molecule is d. What is the best estimate of the average distance between molecules in the gas state?

- If the density $\rho = \dfrac{M}{V}$ is 1000 times less as a gas, then the volume must be 1000 times larger.

- An estimate is found by taking the cube root of the volume, giving $\sqrt[3]{1000} = 10$.

- Cmon mate, think about a cube whose volume is 1000 times larger than another, the side length must be 10 times larger. Baby maths.

5.10   Simultaneous Changes

An ideal gas is in a closed container. Which changes to its volume and temperature when taken together must cause a decrease in the gas pressure?

	Volume	Temperature
A.	Decrease	Increase
B.	Decrease	Decrease
C.	Increase	Increase
D.	Increase	Decrease

$$P = \frac{nRT}{V}$$

To guarantee a decrease in pressure, we need to decrease something from the numerator AND increase something from the denominator; otherwise, there is no guarantee because the opposite undesired effect may override the desired effect.

Quite clearly, the only option is D.