IB Physics Topic C5 The Doppler Effect; SL & HL

By timthedev07, M25 Cohort

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1 The Doppler Effect for Sound

The Doppler effect is a change in observed the frequency and wavelength of a wave that arises from relative motion between the source and the observer. In the following parts we will look at how one can compute the shifted frequencies; it must be noted, however, that these only hold for low speeds, below $0.2c$. Otherwise, one must use the relativistic Doppler effect, which is not covered.

1.1 Case I: Stationary Observer, Moving Source

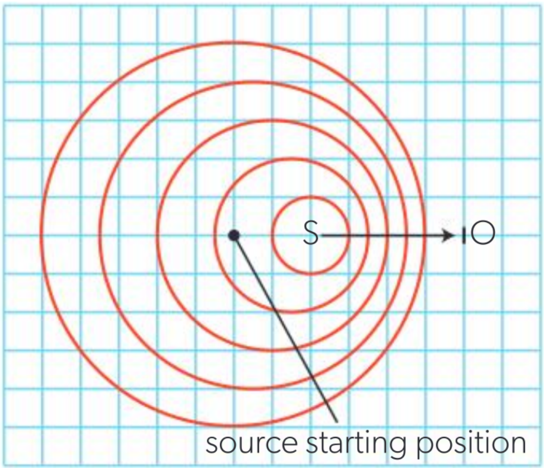

Figure 1: Moving towards the observer

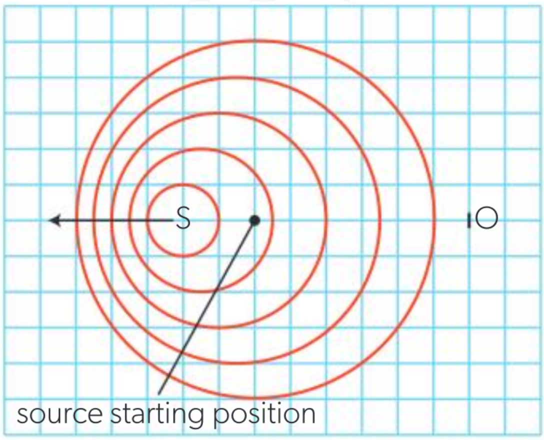

Figure 2: Moving away from the observer

- In the first sub-case, where the relative velocity between the two is u_s, the the observed frequency is higher, as the wavefronts are compressed. The formula for the shifted frequency is given by the following, **where v is the wave speed**

$$f' = f\left(\frac{v}{v - |u_s|}\right) > f \tag{1}$$

- In the second sub-case, where the source is moving away from the observer, the observed frequency is lower, as the wavefronts are stretched.

$$f' = f\left(\frac{v}{v + |u_s|}\right) < f \tag{2}$$

1.2 Case II: Moving Observer, Stationary Source

When the source is stationary and the observer is moving, there are also shifts in the observed frequency.

- If the observer is moving towards the source, the observed frequency is higher, as it will cross more wavefronts in a given time than if it were stationary.

$$f' = f\left(\frac{v + |u_o|}{v}\right) > f \tag{3}$$

- Inversely, if the observer is moving away from the source, the observed frequency is lower, as it will cross fewer wavefronts in a given time.

$$f' = f\left(\frac{v - |u_o|}{v}\right) < f \tag{4}$$

1.3 The Combined Effect

The complete equation that encapsulates both cases is given by

$$f' = f\left(\frac{v \pm |u_O|}{v \pm |u_S|}\right) \tag{5}$$

where

- f is the frequency of the source
- f' is the observed frequency
- v is the wave speed
- u_O is the velocity of the observer
- u_S is the velocity of the source

The ones discussed before are just special cases of this general relation.

2 The Doppler Effect for E.M. Waves

The previous analysis does not apply to light for the following reasons:

- E.M. waves do not require a medium
- Light travels at the same speed in all frames of reference, but this is not the case with sound.
- In special relativity, there is no such concept as a source and an observer, as all motion is relative. Thus, only the relative velocity between the source and the observer matters.

When Δu, **the relative velocity between the two is much less than the speed of light**, we may use a relation that is modified by

- substituting the speed of light for the wavespeed
- using the relative velocity between the source and the observer, denote it as Δu

Eventually we arrive with the relation

$$f' = f\left(1 - \frac{\Delta u}{c}\right) \tag{6}$$

If we denote the change in frequency as $\Delta f = f - f'$, we can rewrite the equation as

$$\Delta f = f\frac{\Delta u}{c} \iff \frac{\Delta \lambda}{\lambda} = \frac{\Delta f}{f} = \frac{\Delta u}{c} \tag{7}$$

2.1 Stellar and Galatic Motion

The Doppler effect is used to determine the motion of stars and galaxies.

Observe the light spectra of light emitted by a star at two distinct timestamps.

- If the most recent one's spectra lines are shifted toward the blue terminal of the spectrum, which represents a blue shift (increase in frequency and hence decrease in wavelength), the star is moving towards the observer.
- If the most recent one's spectra lines are shifted toward the red terminal of the spectrum, which represents a red shift (decrease in frequency and hence increase in wavelength), the star is moving away from the observer.

3 Applications of the Doppler Effect

3.1 Blood Flow Measurement

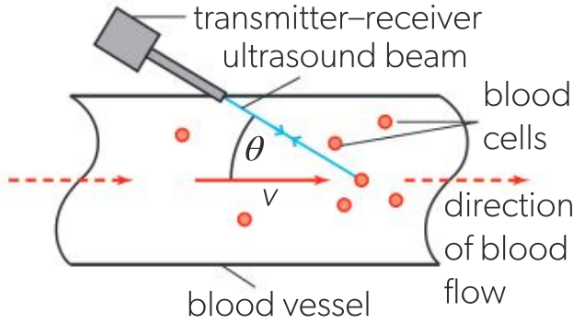

Figure 3: Blood flow measurement

$$\frac{\Delta f}{f} = \frac{2u\cos\theta}{v} \tag{8}$$

where

- u is the speed of the blood (wrong in the diagram)
- θ is the angle between the direction of the blood flow and the direction of the sound wave
- v is the speed of sound in the blood

3.2 RADAR

This includes the following applications:

- flow measurements, e.g. medical, rain cloud speed measurements, weather forecasting
- vehicle speed determinations (police speed traps)
- remote sensing of ocean currents
- measurement of turbulence in river and ocean flow

4 Exam Questions

4.1 Intensity and Frequency Changes

A train approaches a station and sounds a horn of constant frequency and constant intensity. An observer waiting at the station detects a frequency f_{obs} and an intensity I_{obs}. What are the changes, if any, in f_{obs} and I_{obs} as the train slows down?

I_{obs}	f_{obs}
no change	decreases
increases	increases
no change	increases
increases	decreases

- It is tempted to say that f_{obs} increases because the source is approaching the observer. However, the question is not asking for f_{obs} relative to f_{source}, but rather the change in f_{obs} itself as the train slows down. In this case, the source is approaching the observer at a slower rate, so while f_{obs} will be higher than f_{source}, it will decrease as the train slows down.

- Recall that the **observed intensity** I_{obs} obeys the inverse square law

$$I_{\text{obs}} = \frac{P_{\text{source}}}{4\pi r^2}$$

we are given that the intensity of the source is constant and thus the power of the source is constant. As the train slows down, the distance between the source and the observer decreases, thus increasing I_{obs}.

4.2 Applying the Doppler Effect

Sea waves move towards a beach at a constant speed of 2.0 m s^{-1}. They arrive at the beach with a frequency of 0.10 Hz . A girl on a surfboard is moving in the sea at right angles to the wave fronts. She observes that the surfboard crosses the wave fronts with a frequency of 0.40 Hz .

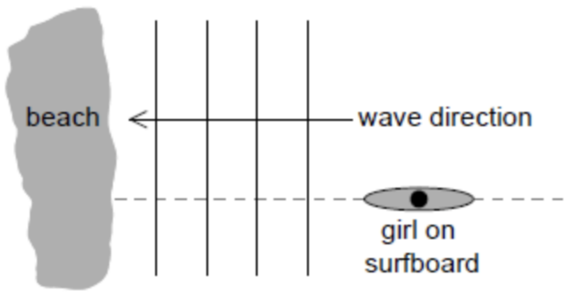

Figure 4: Sea waves

What is the speed of the surfboard and the direction of motion of the surfboard relative to the beach? To tackle Doppler effect questions,

1. Identify the source and observer — in this case, the source is somewhere in the sea and stationary (no explicit references to the source moving); the observer is the girl.

2. Identify the wave speed — this is given as $v = 2.0$ m s^{-1}.

3. Identify the direction of motion of the observer relative to the source — this is not explicitly given, but if we think about the clue in the question that the observer detects a higher frequency, we can infer that the observer is moving towards the source, so we know that the desired relation is

$$f' = f\left(\frac{v + |u_o|}{v}\right)$$

4. Plug in the values and solve for u_o.

4.3 Reflected Doppler Effect

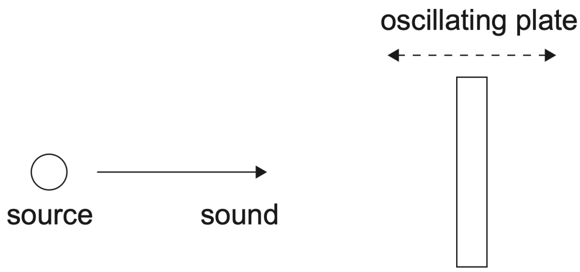

Figure 5: Plate

A plate performs simple harmonic oscillations with a frequency of 39 Hz and an amplitude of 8.0 cm. Sound of frequency 2400 Hz is emitted from a stationary source towards the oscillating plate. The speed of sound is $340\,\mathrm{m\,s^{-1}}$. Determine the maximum frequency of the sound that is received back at the source after reflection at the plate.

1. The reflected sound wave will have the highest frequency when the plate is at its maximum speed towards the source. Consider kicking a soccer ball approaching you, if you kick in the opposite motion of the approaching ball with the greatest strength and speed, the ball will have the highest speed when it goes back.

2. By s.h.m. equations in C1, we conclude that the highest speed of the plate is $19.6\,\mathrm{m\,s^{-1}}$.

3. Putting all this together, let's first calculate the frequency of the sound that arrives at the plate when it is at $19.6\,\mathrm{m\,s^{-1}}$ towards the source:

$$f' = f\left(\frac{v+|u_o|}{v}\right) = 2400\left(\frac{340+19.6}{340}\right) = 2400\left(\frac{359.6}{340}\right) = 2538\text{ Hz}$$

4. Now, we must consider the reflected sound. In this half of the journey, the plate acts as the "source" and it is moving towards the actual source. Thus, the frequency of

the reflected sound is

$$f'' = f'\left(\frac{v}{v-|u_s|}\right) = 2538\left(\frac{340}{340-19.6}\right) = 2694 \approx 2700 \text{ Hz}$$