IB Physics Topic D2 Electric and Magnetic Fields; SL & HL

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1 Electrostatics — Interaction of Charges

Like charges attract; opposite charges repel. What else do you expect me to say here?

2 Conservation of Charge

The sum of the currents into a junction is equal to the sum of the currents away from the junction.

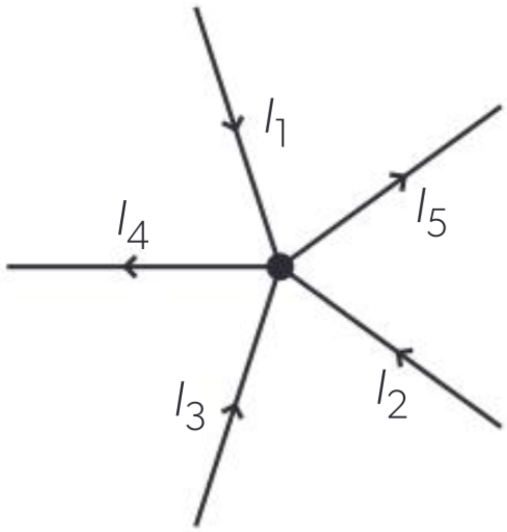


Figure 1: Conservation of Charge

In the diagram above, the currents flowing into the junction are I_1, I_2, and I_3. The currents flowing out of the junction are I_4 and I_5. By conservation of charge, $I_1 + I_2 + I_3 = I_4 + I_5$.

3 Mechanisms of Charge Transfer

3.1 Friction

- When two different materials are rubbed together, electrons can be transferred from one material to the other.
- The objects end up with opposite charges — one has a deficit of electrons and hence a positive overall charge, while the other has an excess of electrons and hence a negative overall charge, causing them to attract each other due to electrostatic forces.

3.2 Contact

- When a charged object touches a neutral object, electrons are transferred.
- If the charged object has an excess of electrons, some will move to the neutral object; if it has a deficit, it may draw electrons from the neutral object.
- After contact, both objects have similar charges, though the total charge is shared between them.

3.3 Induction

- Bringing a charged object close to but without touching a neutral one.
- This charged object causes electrons within the neutral object to move (but not transferred), either attracting them to the surface or repelling them to the other end of the object.
- The object temporarily develops opposite charges on opposite sides (polarization). If grounded, it can be left with a permanent charge opposite to the one on the inducing object.

4 Electric Fields

4.1 Coulomb's Law

This is the attractive force between two charged objects.

$$F = k\frac{q_1 q_2}{r^2} \tag{1}$$

where

- $k = 8.99 \times 10^9$ is Coulomb's constant.
- q_1 and q_2 are the charges; if they have the same sign, then the force is positive and thus repulsive; conversely, if they have opposite signs, the force is negative and attractive.

The important thing to note is that this force is a mutual property of a pair of charges, and so if a charge q_1 exerts a force of magnitude F on q_2, then q_2 exerts a force of the same magnitude F on q_1.

4.1.1 Coulomb's Constant

Coulumb's constant is given by

$$k = \frac{1}{4\pi\epsilon_0} \tag{2}$$

where

- $\epsilon_0 = 8.85 \times 10^{-12}$ is the permittivity of free space.

The above only applies to calculations in a vacuum. In a different medium of permitivity ϵ, k would be newly defined as $k = \dfrac{1}{4\pi\epsilon}$.

4.2 Electric Field Strength

The electric field strength E is given as

$$F = Eq \quad \text{or} \quad E = \frac{F}{q} \tag{3}$$

Formally, it is defined as the force per unit charge experienced by a small positive test charge place at that point. This is analogous to the idea of gravitational field strength.

Recall (if you don't, go read my notes) how we defined the gravitational field strength as

$$g = \frac{GM}{r^2}$$

Combining Equation (1) and Equation (3), we can derive an analogous expression for the electric field strength experienced by a small positive test charge at a distance r from a charge Q:

$$E = \frac{kQ}{r^2} \tag{4}$$

Observations of the similar forms:

- Both are instances of the inverse square law.
- Both do not require information about the test object specifically.
- Both involve a constant of proportionality, one is G and the other is k.
- Both are dependent on either the mass or charge of the source.

4.3 Field Line Patterns

A general rule of thumb is that field lines always point away from positive charges and towards negative charges. The field lines are always perpendicular to the surface of the charge. Also, the denser the field lines are in a region, the stronger the electric field in that region.

The following two diagrams show the field line patterns for two charges of opposite and similar signs respectively.

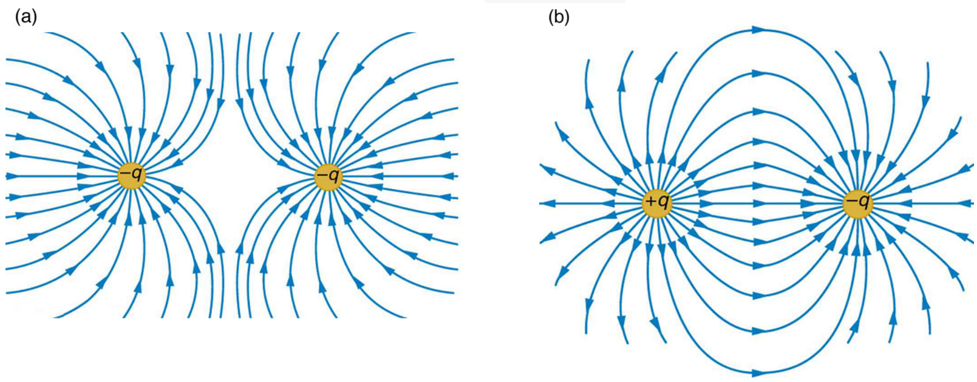


Figure 2: Field Line Patterns

The following is the field line pattern between two plates of opposite charges

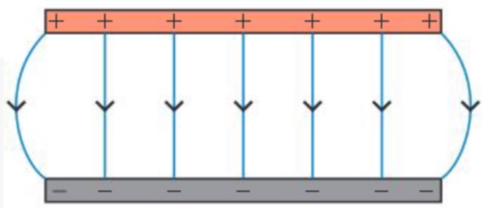


Figure 3: Field Line Patterns for Parallel Plates

- In the middle, the field lines are parallel and equidistant; this is a region of uniform electric field.
- On the edges, the field lines are curved and weaker; these are known as the edge effects.

4.4 Parallel Plates Equations

Recall that the definition of potential in general is the work done **per unit something** to move a test object from infinity to a point. Using the $F = Eq$ equation, we can derive the following equation for the **potential difference** between two plates of opposite charges:

$$\Delta V_e = Ed \tag{5}$$

where d is the separation distance between the plates and E is the field strength (**not the energy!**). This equation only works for uniform fields.

This equation is not merely used for parallel plates; it applies to the calculation of any change in potential in a uniform electric field through a distance d.

The rule of thumb for the movement of a charge in a uniform electric field is

- If $q > 0$, the charge moves from high potential to low potential.
- If $q < 0$, the charge moves from low potential to high potential.

4.5 Electron Volt Conversions

One electronvolt (eV) is the energy gained by one electron when it moves through a potential difference of one volt. This is equivalent to 1.6×10^{-19} Joules.

Conversely, one Joule is equivalent to 6.24×10^{18} electronvolts.

If we consider a particle of charge ne with $n \in \mathbb{N}$ that is accelerated through a potential difference of V, the work done on the particle is neV.

4.6 Field Close to a Conductor

Consider a plate with area A and total charge q, we define its surface charge density as $\sigma = \frac{q}{A}$. We desire to find the electric field close to the plate.

It is known that the total charge q is given as the following

$$q = \frac{VA}{4\pi kd} \tag{6}$$

We can then use $E = \frac{V}{d}$ to obtain

$$\begin{aligned} q &= \frac{V}{d} \times \frac{A}{4\pi k} \\ &= \frac{EA}{4\pi k} \\ E &= \frac{q}{A} \times 4\pi k \\ &= 4\pi k\sigma \end{aligned}$$

This expression gives the electric field between the two parallel plates, where each plate contributes half and so close to one plate, the field is $2\pi k\sigma$.

Recall that $E = \frac{V}{d}$ uses the assumption that the field is uniform; this gives away why we need the ”close to the conductor” condition, because at the surface, the surface is locally flat and so the field can be treated as a uniform field.

4.7 Millikan's Oil Drop Experiment

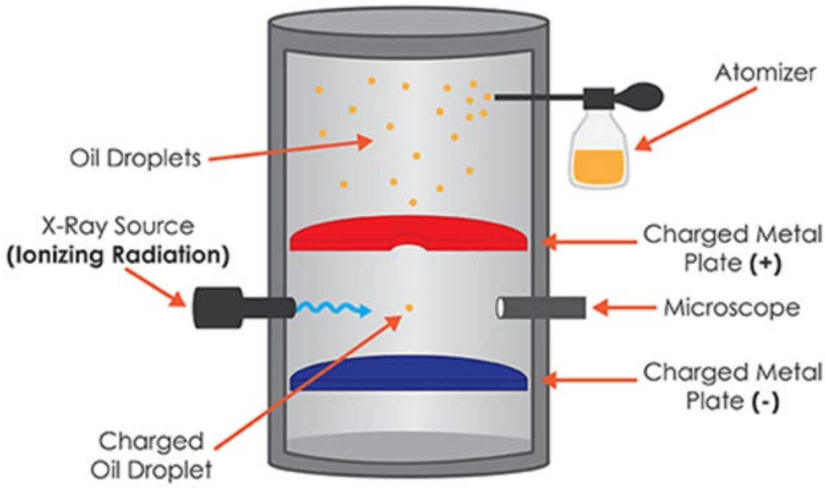


Figure 4: Millikan's Oil Drop Experiment

Set-up:

- Millikan sprayed a fine mist of oil droplets between two parallel, horizontal conducting plates.
- The droplets were tiny enough to remain suspended in the air for a while and were able to be observed through a microscope.
- An electric field was applied across the plates by connecting them to a power source, with one plate positively charged and the other negatively charged.
- X-rays were used to ionize the air, causing some oil droplets to pick up extra electrons, giving them a net negative charge.

Measurements:

- Before turning on the electric field, the droplets were allowed to fall and eventually reach a terminal velocity.
- At this constant velocity, weight = drag force + upthrust.

  - The upthrust the buoyancy force acting on the droplet due to the air and can be found by $\rho g V$.
  - The drag force can be approximated as $6\pi\eta r v$.

- Thus, he was able to work out the mass of the droplet.
- With the electric field present, the droplets were observed to be eventually suspended in air, at which stage, the forces of the electric field ($F_e = Eq$, upwards) and the weight of the droplet ($w = mg$, downwards) were balanced.
- Rearranging gives that the charge on the droplet is $q = mg/E$.

Evidence for quantization of charge:

- Millikan measured the charge on many droplets and found that these charges were always whole-number multiples of a smallest, consistent value, e.
- This observation led to the conclusion that charge is quantized — it comes in discrete packets, rather than being continuous.
- Experiments failed to find any charge less than 1.6×10^{-19} Coulombs; this is then the value of the elementary charge, e.

Note: it is helpful to remember the derivation of this experiment — it may appear in exams. Quite simply it is the highlighted line, from there you can form equations and rearrange to find desired expressions.

4.8 Electric Potential

Recall the definition of electric potential

"The work done per unit charge to move a positive test charge from infinity to a point."

Which means that the electric potential different between two points is given by

$$\Delta V_e = \frac{W}{q} \tag{7}$$

where q is the test charge and not the charge creating the field. The unit for electric potential is the volt, which is equivalent to a Joule per Coulomb (V $\equiv \mathrm{J\,C^{-1}}$).

The electric potential at a distance r from the center of a field created by a charge $\pm Q$ is given by

$$V_e = \pm\frac{kQ}{r} \tag{8}$$

The $\pm$ signs just indicate that Q and V have the same sign.

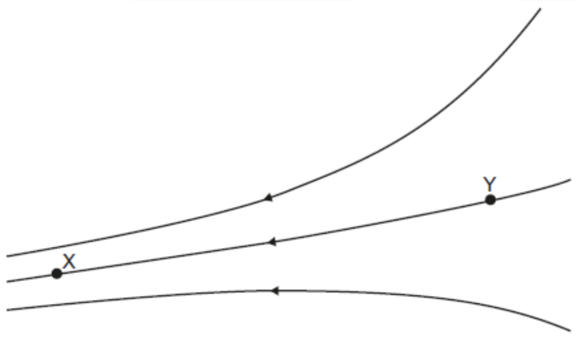


Figure 5: Potential: Points on field lines

In the image above, the potential at point Y is greater than the potential at point X because the potential increases in the direction opposite to field strength.

4.9 Accelerating an Electron

Consider an electron accelerated through a potential difference V. The work done on the electron is given by

$$W = eV \tag{9}$$

where e is the elementary charge. This work done is then converted into kinetic energy, so the maximum speed of the electron is given by

$$\begin{aligned} \frac{1}{2}m_e v^2 &= eV \\ v &= \sqrt{\frac{2eV}{m_e}} \end{aligned}$$

4.9.1 Equipotentials

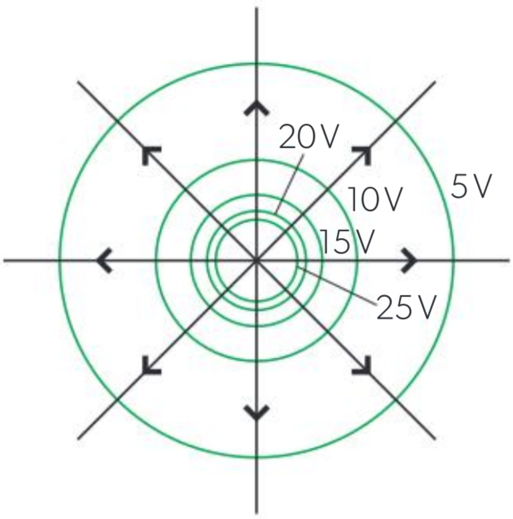


Figure 6: Equipotentials

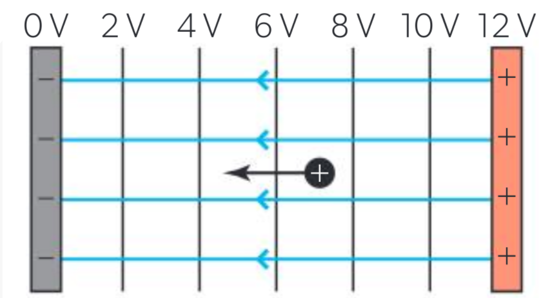


Figure 7: Equipotentials for Parallel Plates

We have met this idea in D1. Around a charge Q, the equipotentials are *concentric spheres* not equally spaced. The closer the spheres are to the charge, the closer they are to each other. As before, the electric field lines are always perpendicular to the equipotentials.

For parallel plates, the equipotentials are *parallel lines* equally spaced. The electric field lines are always perpendicular to the equipotentials.

It is worth noting that in Figure 7, the most work is done when one moves the positive charge towards the positive plate against the arrows, and conversely, the least work is done when one moves the positive charge towards the negative plate with the arrows.

Also, the motion of the charge in Figure 7 is uniformly accelerated, since the field is uniform.

Lastly, it must be noted that in Figure 7, any motion perpendicular to the field lines does not require work to be done; there is no change in potential.

4.10 Electric Potential and Field Strength

Recall, from D1, that gravitational field strength can be given as

$$g = -\frac{\mathrm{d}V_g}{\mathrm{d}r}$$

Analogously, we can define the electric field strength as

$$E = -\frac{\mathrm{d}V_e}{\mathrm{d}r} \tag{10}$$

The graphical interpretation is that the electric field strength is the tangential gradient of the electric potential graph at some point. We can rewrite by integrating both sides:

$$\int E \, \mathrm{d}r = -V_e$$

The graphical interpretation of this is that on a graph of electric field strength against distance, the area under the curve between two points r_1 and r_2 is the potential difference between the two points.

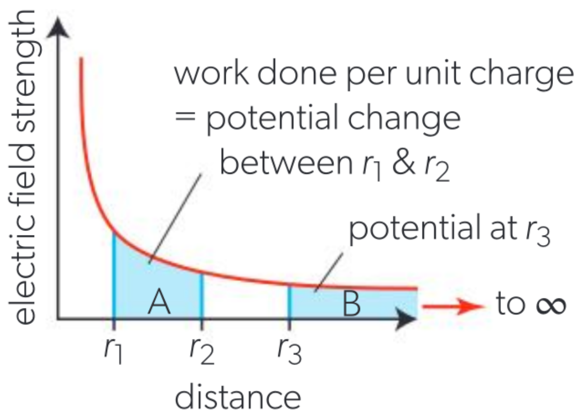


Figure 8: Electric Field Strength/Distance Graph

4.11 Potential and Fields Inside a Hollow Sphere

Outside the sphere, the calculations are the same as when we treat the sphere as a point charge.

Inside the sphere, however, the field is zero. This is because the field lines are canceled out by the opposite charges on the inner surface of the sphere.

Recall that $E = -\frac{\mathrm{d}V_e}{\mathrm{d}r}$; if the field is zero, then

$$E = -\frac{\mathrm{d}V_e}{\mathrm{d}r} = 0$$

This means that the potential inside the sphere is constant (since its rate of change is 0) and equal to the potential at the surface of the sphere.

4.12 Electric Potential Energy

The electric potential energy is the work done to bring a charge from infinity to a point in an electric field. Similar to GPE, it is a property of a two-charge system. It is given by

$$E_p = F_e \times r = k\frac{q_1 q_2}{r} \tag{11}$$

If we know the potential of charge q_1 at position r in the field of q_2, we can find the potential energy of charge q_1 at that point by multiplying the potential by the charge:

$$E_p = q_1 V \tag{12}$$

4.13 Combined Potentials

First consider two identical point charges Q_1 and Q_2 at a distance R from each other.

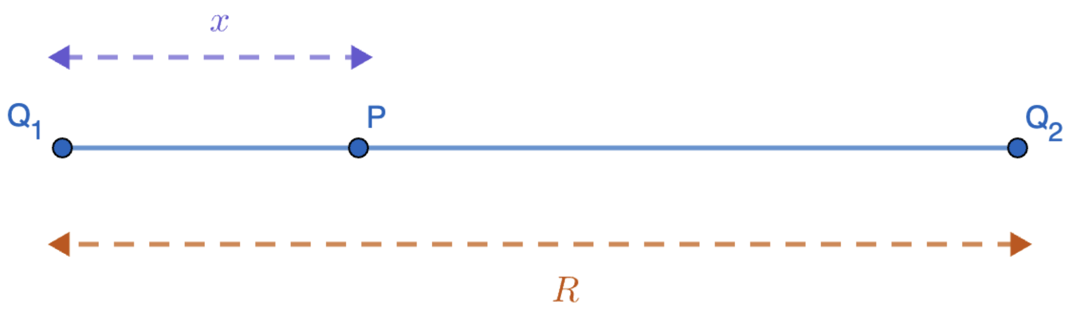


Figure 9: Combined Potentials

The resulting potential at a point P that is x away from Q_1 can be derived as follows:

$$\sum V = V_1 + V_2 = \frac{kQ_1}{x} + \frac{kQ_2}{R-x}$$

It is worth noting that this graph has a minimum at $x = \frac{1}{2}R$ Now consider the case when the two charges are replaced by two identical conducting spheres.

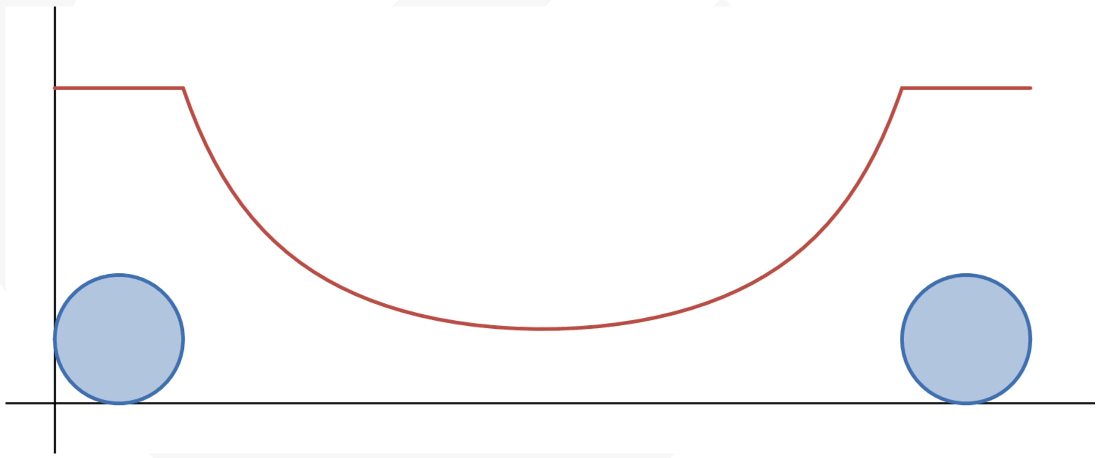


Figure 10: Combined Potentials for Conducting Spheres

The above is the shape of the graph of $\sum V$.

In the region between the two spheres, the graph has the same shape as if the two charges were point charges. However, once reaching into one of the spheres, the potential becomes a horizontal segment. This is because the field inside a conductor is zero, and so the potential is constant, since

$$E = -\frac{\mathrm{d}V}{\mathrm{d}r} = 0$$

In this case, the potential due to the sphere on the other side no longer matters, because any contribution to the electric field from external charges (e.g., right sphere) is canceled out by the redistribution of charge on the surface of the left sphere. This is a property of conductors, where external fields cannot penetrate inside.

5 Magnetic Fields

Magnetic field lines have similar properties to those of electric field lines:

- North to south
- Higher field line density $\implies$ stronger field
- Field lines never cross
- Opposite poles attract, similar poles repel

5.1 Field Line Patterns

5.1.1 Straight Wire

Consider a straight current-carrying wire — the field lines are concentric circles around the wire. The direction of the circular field lines can be determined using the right-hand grip rule. **This rule assumes conventional current flow, from positive to negative.**

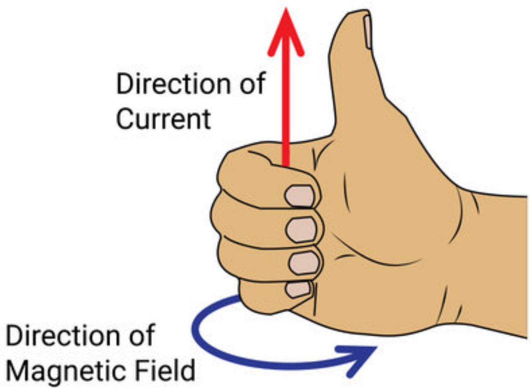


Figure 11: Field Lines for a Straight Wire

5.1.2 Single Loop of Circular Coil

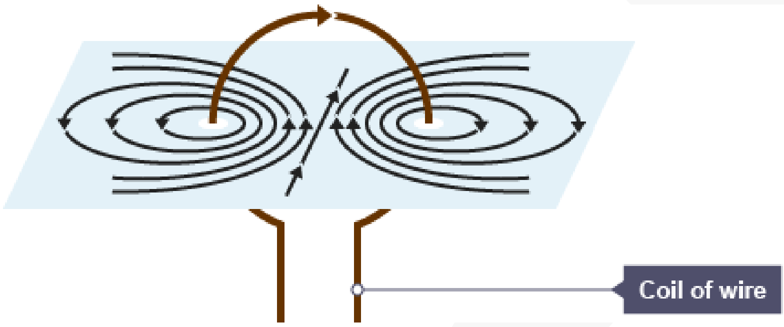


Figure 12: Field Lines for a Single Loop

The direction of the field lines can also be determined using the right-hand grip rule. This time, the fingers curl in the direction of the current and the thumb points in the direction of the field lines.

On either sides of the coil, the field lines are in the opposite direction to the field lines running through the center.

To strengthen the field:

- Increase current
- Add more turns to turn into a solenoid

5.1.3 Solenoid

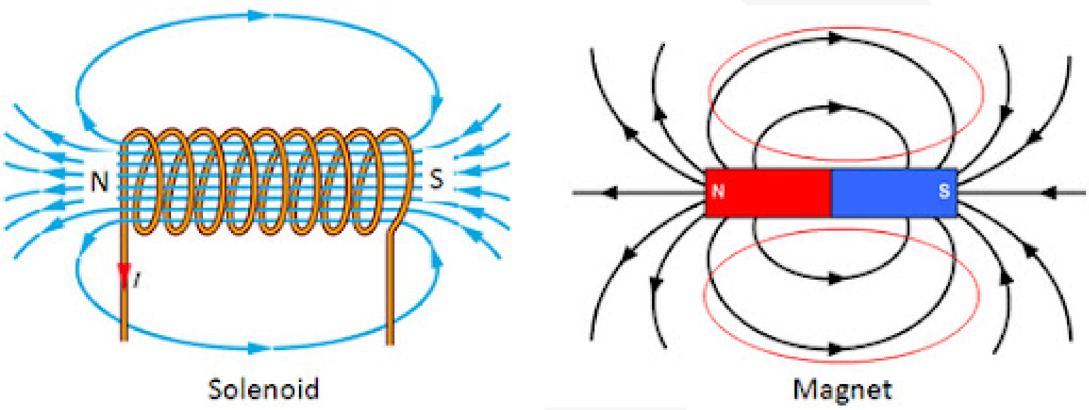


Figure 13: Field Lines for a Solenoid

Again, the right-hand rule determines the direction of the field. The current in the diagram is conventional current.

To strengthen the field:

- Increase the current
- Increase the number of turns
- Add an iron core

The shape of the field lines look similar to those of a bar magnet; the end of the solenoid from which the field lines emerge is the north pole, and vice versa.

6 Exam Questions

6.1 Force and Field Strength Directions

P and Q are two opposite point charges. The force F acting on P due to Q and the electric field strength E at P are shown.

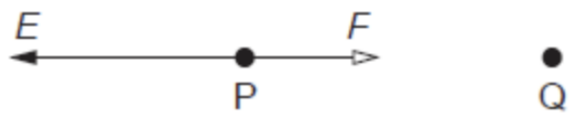


Figure 14: Force and Field Strength Directions

Which diagram shows the force on Q due to P and the electric field strength at Q?

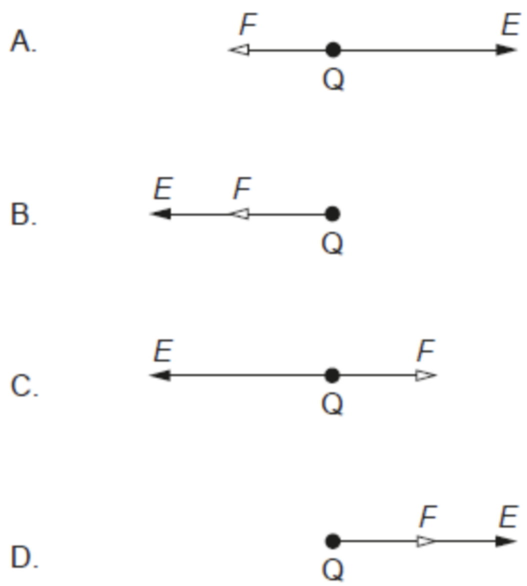


Figure 15: Options

1. Firstly, since P and Q have opposite charges, there must be an attractive force between them, this means that while P points toward Q, Q also points toward P. This eliminates options C and D.

2. Recall that the electric field strength is the force per unit charge experienced by a small positive test charge at that point. The fact that E_P and F_P are in opposite directions suggests that P has a negative charge. This means that Q must have a positive charge, and so the field lines must point toward P (since field lines go from positive to negative). This gives option B.

6.2 Parallel Plates — Work Done in Moving a Charge

Two parallel plates are a distance apart with a potential difference between them. A point charge moves from the negatively charged plate to the positively charged plate. The charge gains kinetic energy W. The distance between the plates is doubled and the potential difference between them is halved. What is the kinetic energy gained by an identical charge moving between these plates?

1. Here, the doubled distance is simply a distraction; the only thing that matters is the potential difference between the plates, since

$$W = q\Delta V$$

6.3 Combining Potentials

Four positive charges are fixed at the corners of a square that has a diagonal length d.Three of the charges have a charge of $+Q$. The total electric potential at the center of the square is $\frac{10kQ}{d}$. What is the magnitude of the fourth charge?

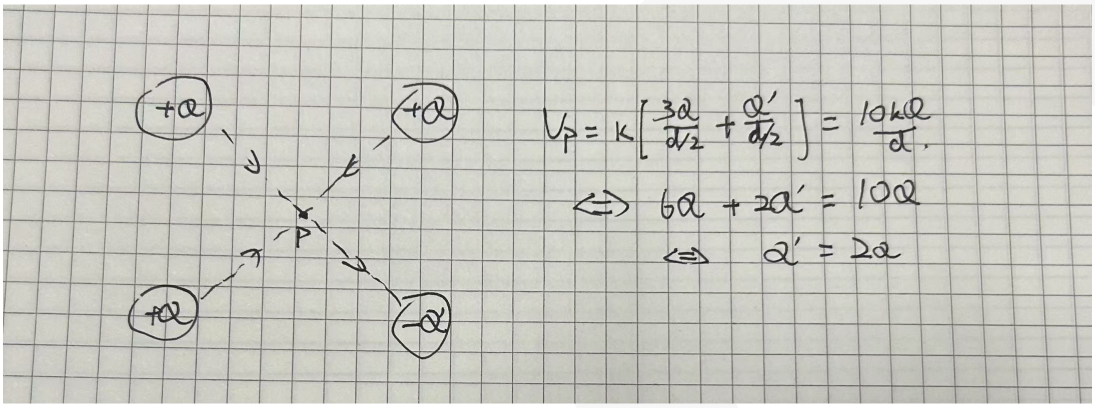


Figure 16: Solution

The key is not to mistakenly assume that the opposite-positioned positive charges are cancelling each other out.

6.4 Tilted Equipotentials

A positively charged particle is positioned in an electric field. Three equipotential lines are shown. The particle is released.

What is the initial direction of the velocity of the particle?

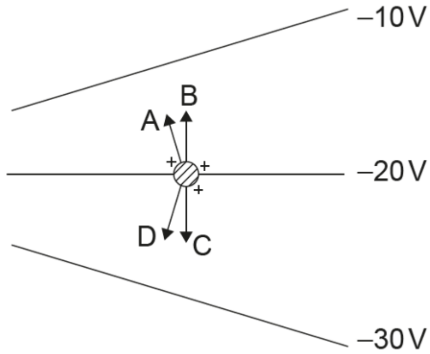
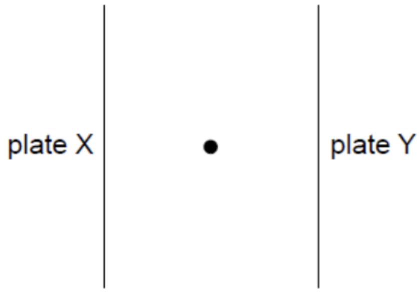


Figure 17: Diagram

- Firstly, we know that the positive test charge moves from high potential to low potential, which means from -20V to -30V. This eliminates A and B
- Consider this as some sort of circular motion when it moves from -20V to -30V. The initial velocity is tangent to the circular path at the point of the charge and so must be C.

6.5 Released Test Charge in Plates

Two very long parallel plates, X and Y, have equal and opposite charges. The potential on X is V_X and that on Y is V_Y where $V_X > V_Y$. A point particle of positive charge q and mass m is held at rest midway between the plates.



The particle is then released. Which plate will the particle move toward and what kinetic energy does it have when it reaches the plate?

Figure 18: Diagram

	Plate	Kinetic energy
A.	X	$q(V_X - V_Y)$
B.	X	$\frac{q(V_X - V_Y)}{2}$
C.	Y	$q(V_X - V_Y)$
D.	Y	$\frac{q(V_X - V_Y)}{2}$

Figure 19: Options

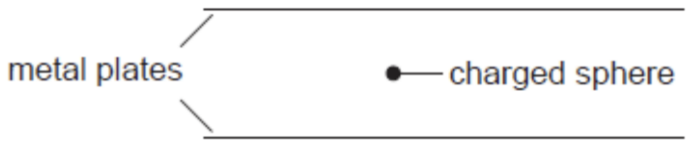
- Positive charges move from high to low potential, this eliminates A and B
- Recall that

$$W = \Delta V \times q$$

the charge moves through half of the separation between the plates and so the potential difference is halved. This gives D.

6.6 Accelerating a Charge Between Plates

A charged sphere in a gravitational field is initially stationary between two parallel metal plates. There is a potential difference *V* between the plates.



Three changes can be made:

I. Increase the separation of the metal plates
II. Increase *V*
III. Apply a magnetic field into the plane of the paper

What changes made separately will cause the charged sphere to accelerate?

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

Figure 20: Question

The magnitude of the resultant force is given by

$$\Sigma F = W - F_e = mg - Eq = mg - \frac{Vq}{d}$$

Both of the first two options changes this resultant force so that it is no longer 0 and so there will be an acceleration.

However, the third option, applying a magnetic field into the paper won't. This is because a magnetic force only acts on a moving charge (because of the cutting of field lines and Faraday's law). This eliminates III. Hence, the correct answer is A.

6.7 $F = Eq$ Directions

P and Q are two opposite point charges. The force F acting on P due to Q and the electric field strength E at P are shown.

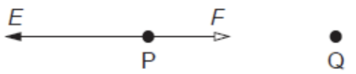


Figure 21: Diagram

Which diagram shows the force on Q due to P and the electric field strength at Q?

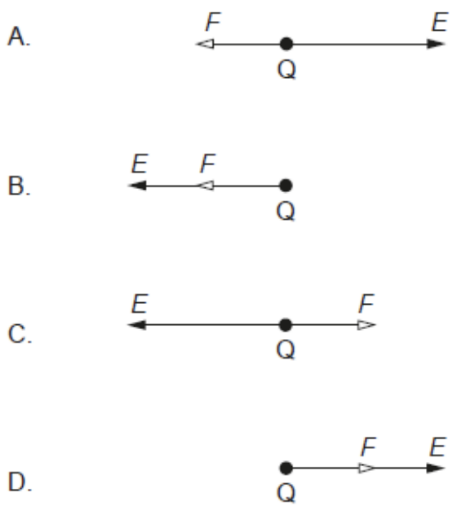


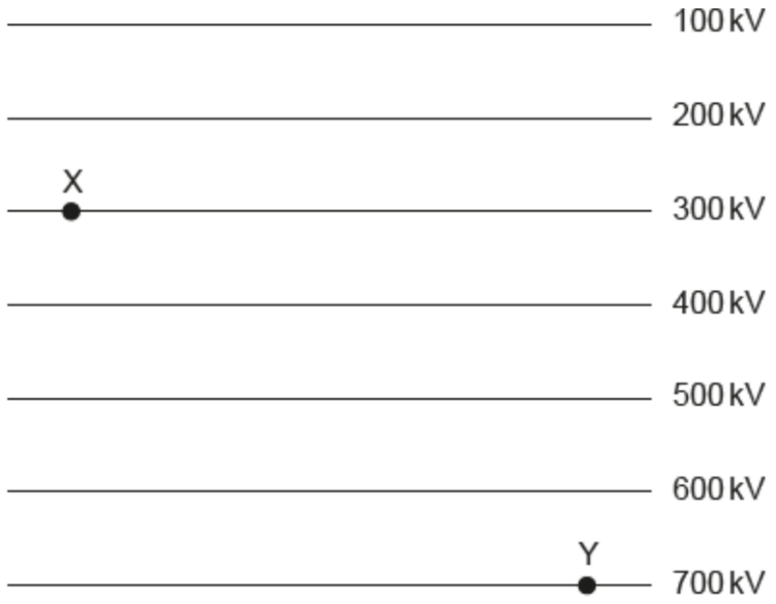
Figure 22: Options

Right children, let's do this the easy way.

- Invoke $F = Eq$ here. Let's first consider the point P, E and F are in opposite directions. This means that P has a negative charge. Hence, Q must be positive.
- Then, the force between P and Q must be attractive, which means that the force arrow at Q must be pointing from Q to P. This eliminates options C and D.
- Since Q is positive, its F and E must be in the same direction, leading us to the correct answer of B.

6.8 Quick-Fire MCQ #1

A particle with charge $-2.5 \times 10^{6}\,\mathrm{C}$ moves from point X to point Y due to a uniform electrostatic field. The diagram shows some equipotential lines of the field.



What is correct about the motion of the particle from X to Y and the magnitude of the work done by the field on the particle?

	Motion of the particle from X to Y	Magnitude of the work done by the field on the particle
A.	uniform linear	0 J
B.	uniform linear	1 J
C.	uniformly accelerated	0 J
D.	uniformly accelerated	1 J

Figure 23: Diagram

- Just know that this movement involves uniform acceleration... This eliminates A and B.
- By common sense, there is a change in potential and so the work done cannot be 0. Hence C is the correct answer.

6.9 Quick-Fire MCQ #2

The points X and Y are in a uniform electric field of strength E. The distance OX is x and the distance OY is y.

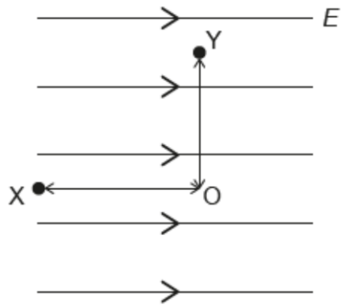


Figure 24: Diagram

What is the magnitude of the change in electric potential between X and Y?

A. Ex

B. Ey

C. $E(x+y)$

D. $E\sqrt{x^2+y^2}$

- Mate, this is a no brainer!
- From O to Y, there is no change in potential! It is moving along an equipotential!
- This means that anything with a y in it should be eliminated.
- This leaves us with A!

6.10 Electron Final Speed

Estimate the speed of an alpha particle that has accelerated from rest in a uniform electric field $E = 50\,\mathrm{N\,C^{-1}}$ over a distance of $2.0\,\mathrm{m}$.

A. $10^3\,\mathrm{m\,s^{-1}}$

B. $10^4\,\mathrm{m\,s^{-1}}$

C. $10^5\,\mathrm{m\,s^{-1}}$

D. $10^9\,\mathrm{m\,s^{-1}}$

We can use both suvat and energy. Here, we will use energy.

- The work done on the particle is given by $W = Fd = Eqd$.
- We equate this to the kinetic energy of the particle, $W = \dfrac{1}{2}(2m_p + 2m_n)v^2$.

$$(m_p + m_n)v^2 = Eqd \iff v = \sqrt{\frac{Eqd}{m_p + m_n}}$$

- A quick substitution gives $v = 97764.4\,\mathrm{m\,s^{-1}}$, which is roughly C.