IB Physics Topic C1 S.H.M; SL & HL

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1 Isochronous Oscillations

This refers to periodic oscillations that maintain a constant frequency regardless of changes in amplitude. In reality, the amplitude of the motion will gradually drop because of energy losses; but isochronous oscillations will maintain a constant frequency.

2 Defining Periodic Motion

No matter which system we are looking at, be it the spring-mass or pendulum, the displacement, velocity, and acceleration of the system will be sin/cos functions of time. The following displacement-time diagram shows important properties of an s.h.m. system.

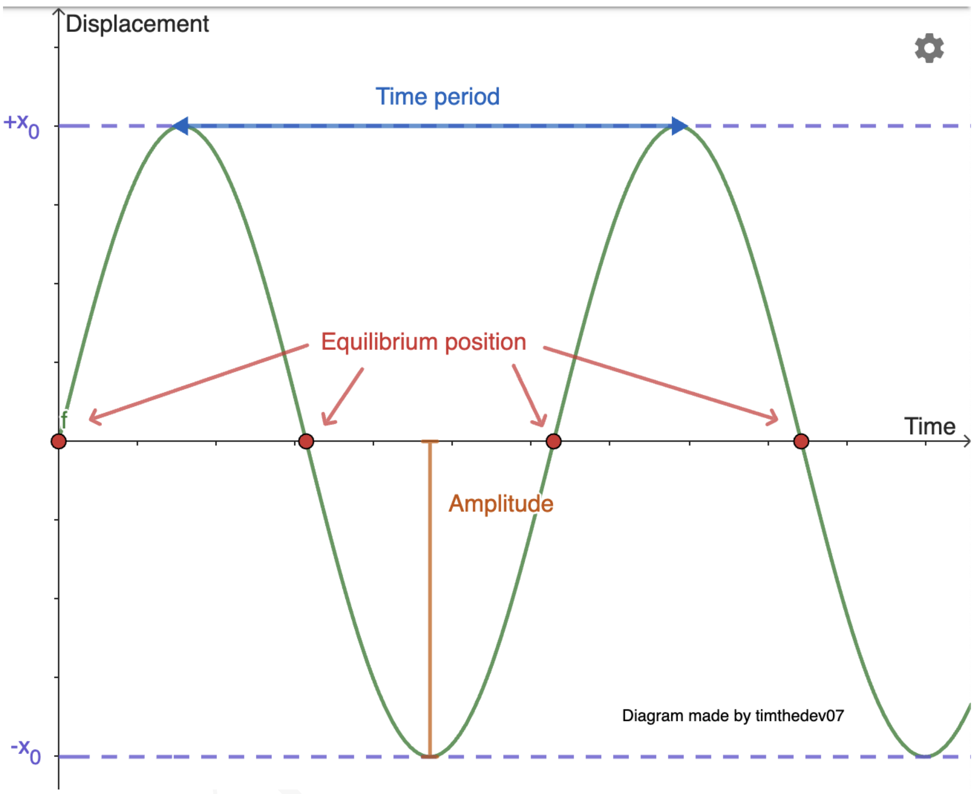

Figure 1: Labeled s.h.m. system

- The displacement is simply the distance from the equilibrium position, and the amplitude is the maximum displacement from the equilibrium position.
- The equilibrium position is the position where the system is naturally at rest (this is also where the acceleration is 0 and velocity is maximum when the system is oscillating).
- The period is the time taken for the system to complete one full cycle/oscillation.

- The frequency — number of cycles per second, is given by

$$f = \frac{1}{T}$$

where T is the period.

3 S.H.M. Basic Equations

S.h.m is a type of isochronous oscillatory motion where the force acting on the oscillator is directly proportional to its displacement from a central equilibrium position and is directed toward that position. The constant of proportionality is the square of the angular frequency, namely ω^2. The equation of motion for s.h.m. is given by

$$F = -\omega^2 x \tag{1}$$

This means that the force/displacement (or analogously acceleration/displacement) graph will be a negative-slope straight line through the origin.

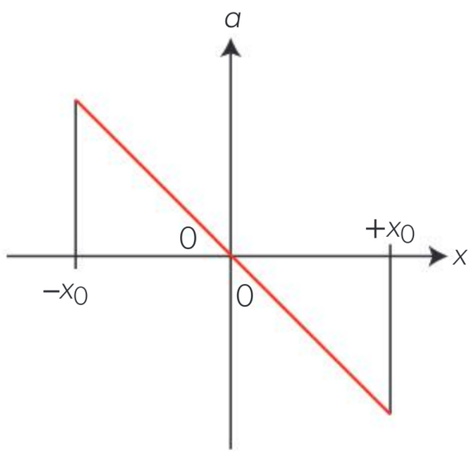

Figure 2: Force-displacement graph for s.h.m.

The angular frequency ω is given by

$$\omega = \frac{2\pi}{T} = 2\pi f$$

where f is the frequency of the oscillator. Although angular frequency can take on the unit $\mathrm{rad\,s^{-1}}$, the radians are often omitted since it is a unitless ratio — it is simply 2π times the frequency, which has unit $\mathrm{Hz} \equiv \mathrm{s}^{-1}$.

There are two systems we study at the IB level: the spring-mass system and the pendulum system. We will examine each one separately.

3.1 Spring Mass

Consider a mass m attached to a spring with spring constant k, moving on a frictionless surface along the horizontal axis. Initially, it is at rest on the surface and the spring is relaxed. We pull the mass x_0 away from the initial position — this will be the amplitude of the subsequent s.h.m.

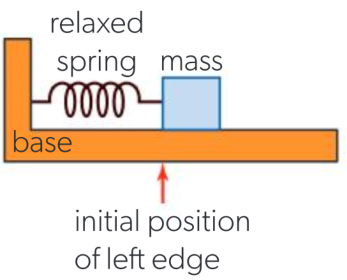

Figure 3: Spring-mass system at equilibrium

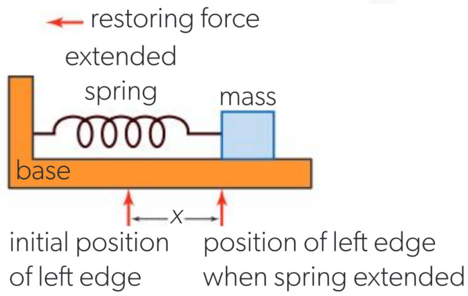

Figure 4: Spring-mass system displaced

Using Hooke's Law on the oscillator, we obtain that the restoring force is

$$F = ma = -kx$$

where k is the spring constant. Comparing coefficients with the s.h.m. equation, we get that the angular frequency is given by

$$\omega = \sqrt{\frac{k}{m}}$$

Similarly, the period and frequency are given by

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

My advice is forget about these two, as they look similar to each other and to ω, and so it's easy to confuse them.

3.2 Pendulum

Consider a pendulum of length l and mass m swinging in a gravitational field of strength g. In this case, the displacement will be the arc length that the mass makes with the origin at any point during oscillation.

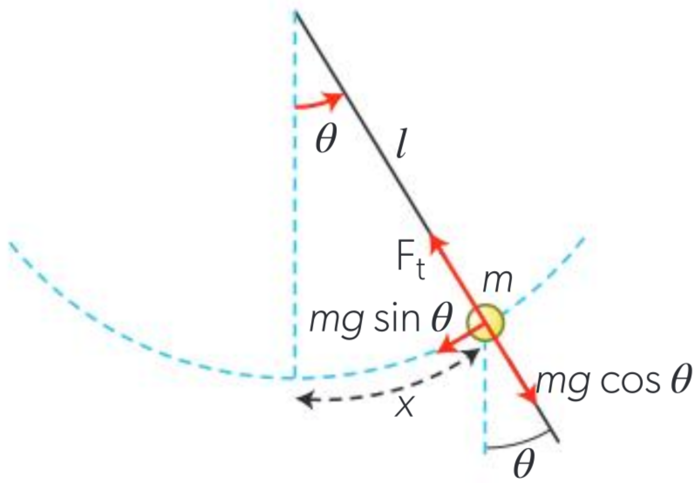

Figure 5: Labeled pendulum system

It must be noted that the pendulum obeys the s.h.m only for small angles of θ (typically less than 10 degrees).

In this case, the restoring acceleration is $mg\sin\theta$; using small angle approximation and the fact that l is the radius of the arc x subtended by the angle θ, we get that the angular frequency is given by

$$\omega = \sqrt{\frac{g}{l}} \tag{2}$$

4 S.H.M. Equations — Circular Motion Form

As mentioned earlier, the displacement, velocity, and acceleration of the system will be sin/cos functions of time. Let us now transform this oscillatory system into a circular motion system to obtain information about these quantities.

Firstly, we must know that the trajectory of a circle with radius r centered at the origin on the Cartesian plane with axes y/x can be expressed in a parametric form as follows

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

where r is the radius of the circle and $\theta = \omega t$ is the so called "phase angle". We can use this to transform the s.h.m. system into a circular motion system. The following diagram shows the transformation.

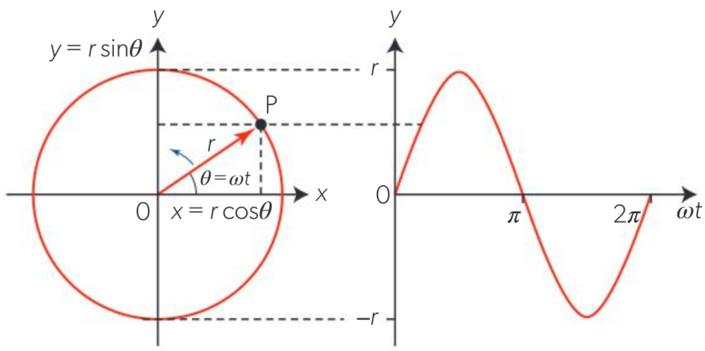

Figure 6: Transformation of s.h.m. to circular motion

From Figure 6, notice that the peak height of the trig. function, which is the amplitude, maps to the radius of the circular motion. This means that $r = x_0$. The $r\cos\theta$ is for s.h.m. beginning at an extreme position, and inversely, $r\sin\theta$ is for s.h.m. beginning at the equilibrium position.

5 S.H.M. Energy Equations

6 Phase Difference

7 Energy Revisit — KE, GPE, and Total Energy