

IB Physics Topic E5 Fusion and Stars; SL & HL

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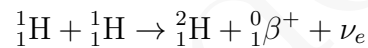
1 Fusion in Stars

Conditions for proton-proton cycle and thus hydrogen fusion to occur:

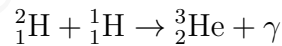
- High temperature: to overcome the electrostatic repulsion between the nuclei
- High pressure: to overcome the gravitational attraction between the nuclei

The hydrogen fusion has three stages

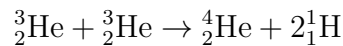
1. **Proton-proton reaction:** Two protons (hydrogen nuclei) collide at extremely high temperatures and pressures, fusing together to form a *deuterium nucleus* (one proton and one neutron), releasing a positron and a neutrino, while converting a proton into a neutron via beta-plus decay.



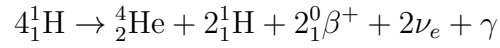
2. **Deuterium-proton reaction:** The newly formed deuterium nucleus fuses with another proton, resulting in a helium-3 nucleus (two protons and one neutron). This releases a gamma ray.



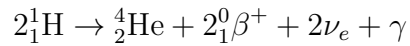
3. **Helium-3 fusion:** Two helium-3 nuclei from stage 2 fuse to form a helium-4 nucleus (two protons and two neutrons) while releasing two protons. This stage releases about 12.9 MeV of energy.



The overall reaction is written as



or equivalently



The difficulties of using fusion to produce energy include:

- Requires high pressure and temperature
- Reactants must overcome Coulomb/intermolecular repulsion
- Difficult to contain and control at high temperatures
- Difficult to capture energy from fusion reactions
- Difficult to maintain/sustain a constant reaction rate

Advantages of using fusion to produce energy include:

- Plentiful fuel supplies
- Larger specific energy
- Larger energy density
- Little or no major radioactive waste products

2 HR Diagrams

By the Stefan-Boltzmann law, large stars with high temperatures must be bright. The pattern of stars can be shown on a scatter plot of luminosity against surface temperature; this is known as the Hertzsprung-Russell (HR) diagram.

Notes about the diagram:

- The plot is logarithmic on both axes.
- The log. temperature (x) axis is plotted in decreasing order from left to right.
- The log. luminosity (y) axis is plotted in increasing order from bottom to top.
- An alternative x -axis is the spectral class, which is another measure of the temperature of the star.
- Each of the lines in Figure 1 represents a set of stars with the same radius: By the Stefan-Boltzmann law, $L = 4\pi\sigma R^2T^4$; for a constant radius, $L \propto T^4$ and on a log-log plot, this is $\ln L \propto \ln T$, a linear relationship.

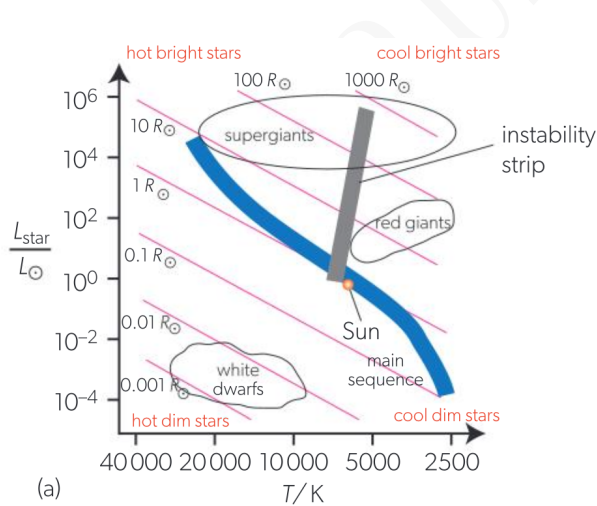


Figure 1: Principle regions and lines of equal radii

(b)

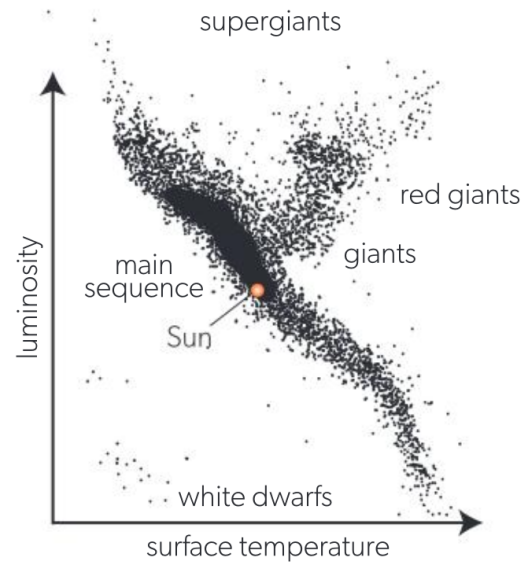


Figure 2: Scatter plot

Components of the graph:

1. The blue area highlights the region of **main sequence** stars; this contains 90% of all stars.
2. **Red giants** are cooler than the Sun but have a larger radius. Although they emit less energy per unit area, their combined area is larger, making them equally bright as some main-sequence stars.
3. **Supergiants** are brighter than most main sequence stars and very large. Some of them emit 10^5 times the energy of the Sun, and thus have a S.A. 10^5 times that of the Sun. This means the radius is $\sqrt{10^5}$ times that of the Sun.

Together with the red giants, these only constitute 1% of all stars; they have a much shorter lifetime in comparison to the main-sequence.
4. **White dwarfs**: The residue of old stars that are extremely hot and dense but have a small radius and a low luminosity. They make up about 9% of all stars. They take billions of years to cool.
5. **The instability strip**: Indicated by the almost vertical gray line in [Figure 1](#). Stars above the mass of the Sun enter this region when they reach the end of the main sequence lifetime, become unstable, and pulsate, changing their size cyclically (alternating between increasing and decreasing). Thus, their luminosity varies too.

2.1 Note on Notation

The subscript \odot denotes the value of this particular variable of the Sun. For example, L_{\odot} is the luminosity of the Sun, R_{\odot} is the radius of the Sun, T_{\odot} is the temperature of the Sun, and M_{\odot} is the mass of the Sun.

3 Stellar Evolution

3.1 Birth of Stars

The *stellar medium* is the region of space between stars that contains gas (mainly hydrogen and helium) and dust (mostly carbon and silicon). At a cool temperature, the density of this medium is as low as 10^{-4} ions/cm³. This is better than any achievable vacuum on Earth.

A gas cloud of the **stellar medium** is compressed by gravity and the cloud begins to increase in temperature and pressure. This is because the GPE of particles is released and transferred to their thermal energy. The cloud becomes a **protostar**.

Later, the star enters the HR-diagram from the right-most side, as a **main sequence star**.

3.2 Main Sequence

During the stability period as a main sequence star, the star is in **hydrostatic equilibrium**, where the inward gravitational force is balanced by the outward thermal & radiation pressure force. The star is stable and does not change in size. The **greater the mass** of the star, the **greater both forces** are. At this stage, the **core is composed mostly of hydrogen**, and the **outer layer is composed of helium**, which is produced by the fusion of hydrogen. The above suggests that the **luminosity** of the star has a positive correlation with the **mass** of the star.

1. For a more massive star, the core temperature must be higher to produce a higher pressure, balancing the higher gravitational force.
2. The higher core temperature results in a higher rate of fusion, producing more energy.

By the theory of nuclear physics, if T is the lifetime of the star and M is the mass of the star, then

$$T \propto M^{-2.5}$$

Thus, one can conclude the following

The larger the mass, the shorter the time the star remains on the main sequence.

Approaching the end of the main sequence, the star begins to **run out of hydrogen** fuel, as most have been turned into helium and no longer available for fusion. The outward pressure decreases — the forces are now unbalanced and the star **begins to shrink** due to the stronger gravitational force. The core temperature increases, and the outer layers expand, cooling down and turning red. The star becomes a **red giant** or a **supergiant**, depending on the mass of the star.

3.3 Stellar Evolution of Low-Mass Stars

This includes those with mass $M < 4M_{\odot}$.

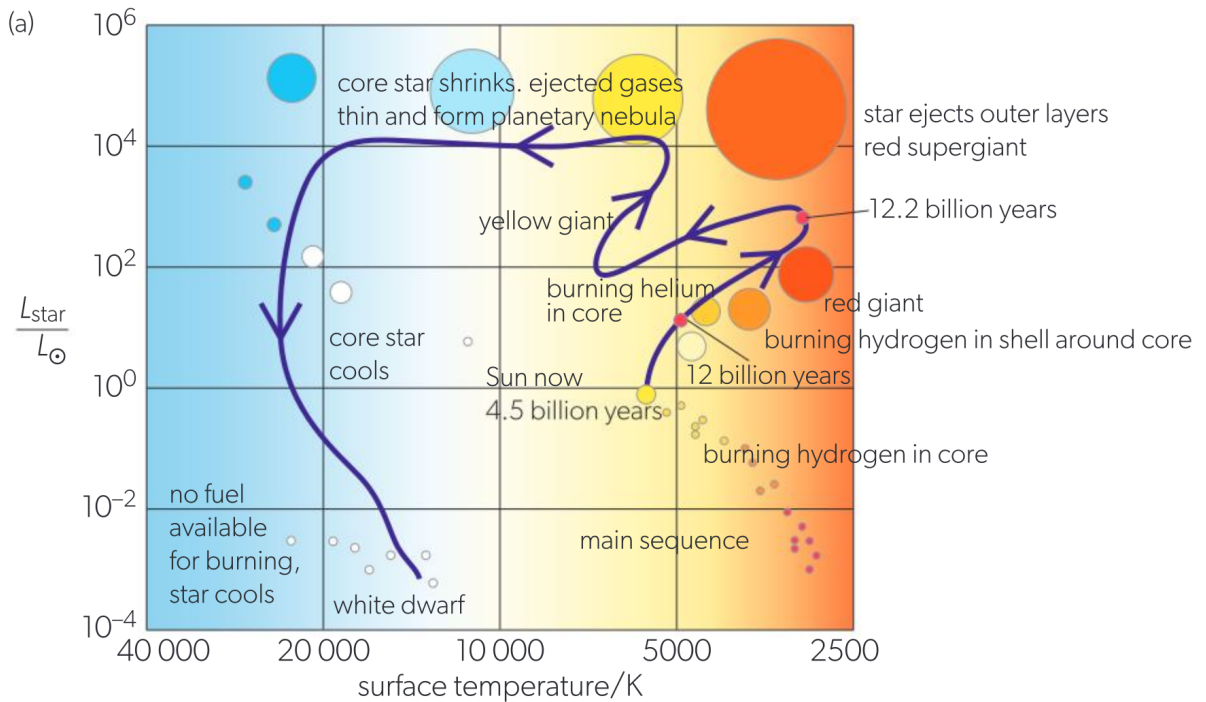


Figure 3: Stellar evolution of low-mass stars

1. Due to having lower masses, the core of these stars do not suffice for fusion of any product beyond carbon. When the helium in the core is exhausted, the core contracts as it continues to emit radiation.
2. A double shell forms around the core: An inner shell of helium and an outer shell of hydrogen. The outer shell of hydrogen begins to fuse, causing the star to expand and cool down, becoming a **red giant**. In this form, the primary energy source is the fusion of helium in the core.
3. As the outer shell expands away from the core, a **planetary nebula** is formed. The core then further contracts; it will shrink to the size of the Earth, containing ions together with a degenerate electron gas.
4. At this stage, the core cannot shrink further. The electron gas exerts a pressure called the **electron degeneracy pressure**, which prevents the star from further collapsing. The star is now a **white dwarf**.

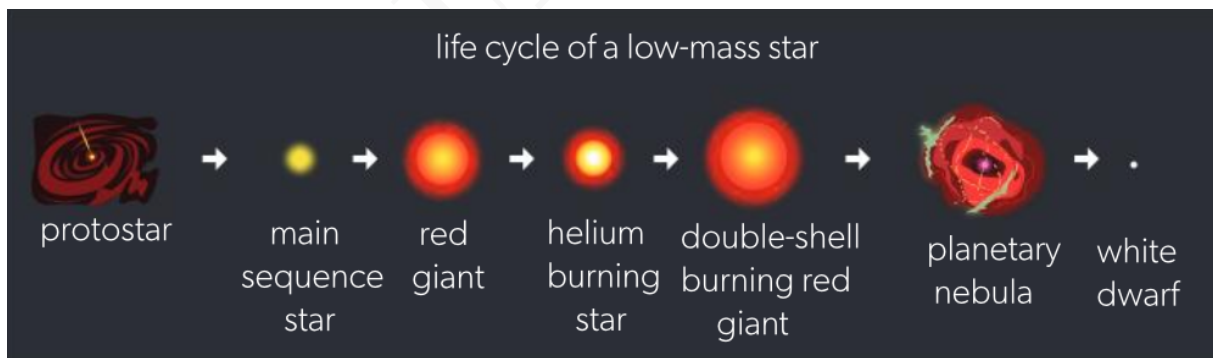


Figure 4: Stellar evolution of low-mass stars

3.4 Stellar Evolution of Large-Mass Stars

This includes those with mass $M > 4M_{\odot}$.

1. In the giant stage, the higher temperature and pressure create elements heavier than carbon.
2. The giant phase ends when the star has a layered structure (like an onion), with iron ash in the center. The inner layers have elements with higher proton numbers — i.e. the proton number decreases as we move outwards.
3. Similar to low-mass stars, the core contracts; but it does not reach stability due to the higher mass.
4. The **Chandrasekhar limit** predicts that stars with a remnant mass above $1.4 M_{\odot}$ cannot turn into white dwarfs.
5. As the core continue to contract, electrons and protons combine to form neutrons, releasing neutrinos.
6. Both the cores and outer layers of the star rapidly collapse, causing a huge explosion known as a **supernova**.
7. After the explosion, the outer layers are blown away, leaving only the core, small but extremely dense. Two outcomes are possible from this point
 - (a) The core remains as a **neutron star**. By the Oppenheimer-Volkoff limit, the mass of a neutron star is limited to $2\text{--}3 M_{\odot}$. Beyond this limit, the core collapses further into a black hole.
 - (b) The core collapses further, forming a **black hole**.

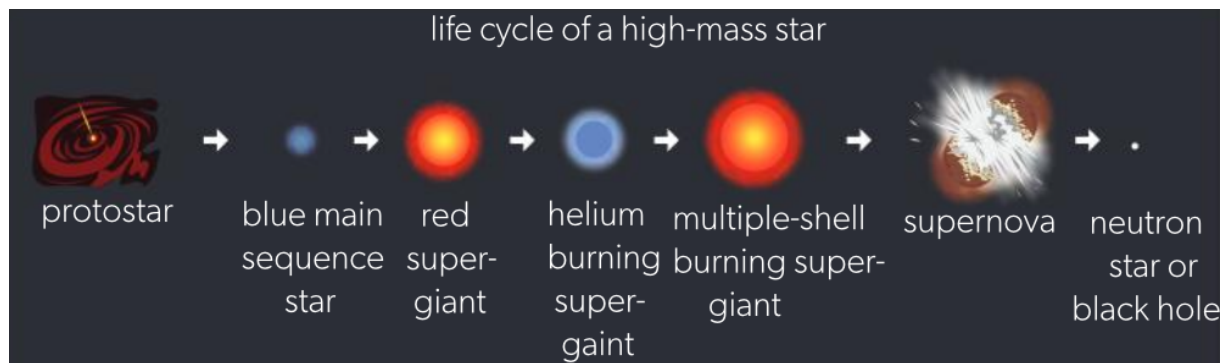


Figure 5: Stellar evolution of high-mass stars

3.4.1 Detecting Black Holes

1. X-rays are emitted as mass spirals towards the edge of the black hole and heats up at it travels. Satellites with X-ray detectors can detect these emissions.
2. Conjecture: Only rotating black holes can emit jets of matter from the core of galaxies.
3. Some stars spiral for no apparent reason; this is likely due to the strong gravitational pull of a black hole nearby.

4 Astronomical Distances and Measurement

- *Light years*: The distance light travels in a year, approximately 9.47×10^{15} m.
- *Astronomical units*: The **average distance** between the Earth and the Sun, approximately 1.5×10^{11} m or equivalently 8 light minutes.
- *Parsecs*: 3.26 light years, or approximately 3.1×10^{16} m. The typical order of nearby stars (to the Earth) is parsec, that of distant stars in the same galaxy is kiloparsec, and that of distant galaxies is megaparsec/gigaparsec.

5 Stellar Parallax

- *Parallax* is the visual effect where the position of an object relative to a background appears to change as the viewer's position changes.
- *Stellar parallax* is the use of the geometry of parallax to find the distance of nearby stars.

We use this method when the stars we are observing are “fixed” in the background.

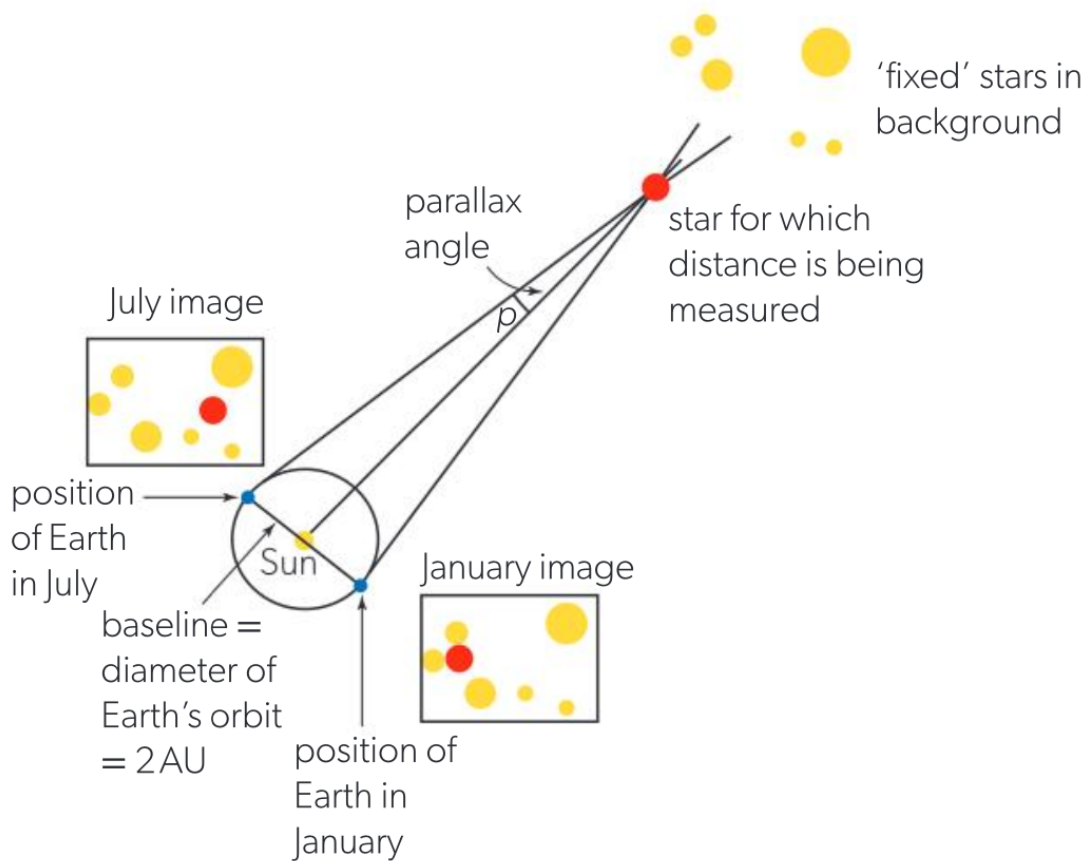
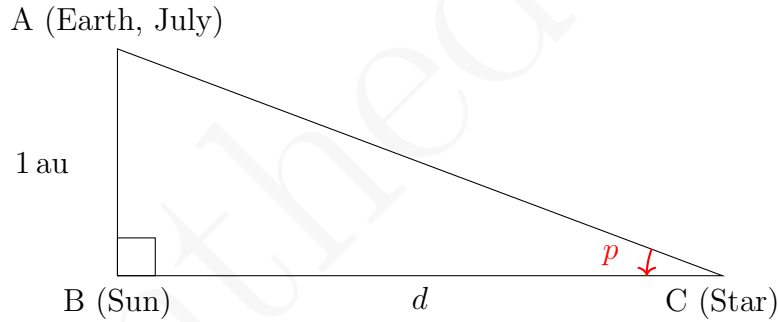


Figure 6: Stellar parallax

Outline of the procedure:

1. Observe the star at two different times, six months apart. The Earth's position is now on the opposite side of the Sun.
2. The angle between the two lines of sight is the displacement angle, $\Delta\theta$.
3. The parallax angle is found by halving the displacement angle, i.e. $p = \frac{1}{2}\Delta\theta$
4. We know that the distance between the two points of observation is 2 au, where 1 au is the average distance between the Earth and the Sun (and thus the orbital radius).
5. We can then obtain the following right-triangle, where BC is the desired distance from the distant star to the Sun.



6. By small angle approximation, $p \approx \tan p = \frac{1 \text{ au}}{d}$.
7. In parsecs, this is simply $d = \frac{1}{p}$.

5.1 Convenient Units

Because we are dealing with “microscopic angles” in stellar parallax, scientists have introduced the unit of *arcseconds* ($''$) to measure these angles. This is defined as $\frac{1}{3600}$ of a degree. Similarly, the unit of *arcminutes* is defined as $\frac{1}{60}$ of a degree.

6 Stellar Radius

Determining the stellar radius:

1. By the Stefan-Boltzmann law, $L = 4\pi\sigma R^2 T^4 = \sigma AT^4$, where L is the luminosity, R is the radius, and T is the temperature of the star.
2. The apparent brightness of a star is given by $b = \frac{L}{4\pi d^2}$, where d is the distance from the star to the observer. Note that this equation only applies to the light emitted and not reflected by the star.
3. These quantities can be compared with those of the sun; this is helpful in questions where we are just asked to determine the quantity of a star in terms of that of the sun

(a) The luminosity ratio (star:sun) is given by

$$\frac{L}{L_{\odot}} = \frac{R^2 T^4}{R_{\odot}^2 T_{\odot}^2}$$

(b) The stellar radius ratio (star:sun) is given by

$$\frac{R}{R_{\odot}} = \sqrt{\frac{L}{L_{\odot}} \frac{T_{\odot}^4}{T^4}} = \frac{T_{\odot}^2}{T^2} \sqrt{\frac{L}{L_{\odot}}}$$

(c) The apparent brightness ratio (star:sun) is given by

$$\frac{b}{b_{\odot}} = \frac{L}{L_{\odot}} \frac{d_{\odot}^2}{d^2}$$

7 Exam Questions

[Maximum mark: 1]

EXE.1A.SL.TZ0.26

A star is on the main sequence.

What are the most abundant element(s) in the core of the star and in the outer layer of the star?

	Most abundant element(s) in the core	Most abundant element(s) in the outer layer
A.	Helium and lithium	Hydrogen
B.	Hydrogen	Helium
C.	Hydrogen and helium	Hydrogen and lithium
D.	Hydrogen	Helium and beryllium

[1]

Markscheme

B

Figure 7: Question 1

[Maximum mark: 1]

EXE.1A.SL.TZ0.27

A star has a radius 13 times that of the Sun and a luminosity that is 400 000 times that of the Sun.

The surface temperature of the Sun is 5700 K.

What is the surface temperature of the star?

- A. 4.0×10^4 K
- B. 7.6×10^4 K
- C. 1.0×10^5 K
- D. 1.4×10^7 K

[1]

Markscheme

A

Figure 8: Question 2

[Maximum mark: 1]

EXE.1A.SL.TZ0.25

Star X has the same surface temperature as the Sun and a luminosity of $10^4 L_{\odot}$

What is $\frac{\text{radius of X}}{\text{radius of the Sun}}$?

- A. 10
- B. 10^2
- C. 10^3
- D. 10^4

[1]

Markscheme
B

Figure 9: Question 3

[Maximum mark: 1]

EXE.1A.SL.TZ0.23

Which process is the primary energy source in a red giant star?

- A. gravitational contraction
- B. nuclear fusion of hydrogen in the core
- C. nuclear fusion of helium in the core
- D. nuclear fusion of helium in the shell surrounding the core

[1]

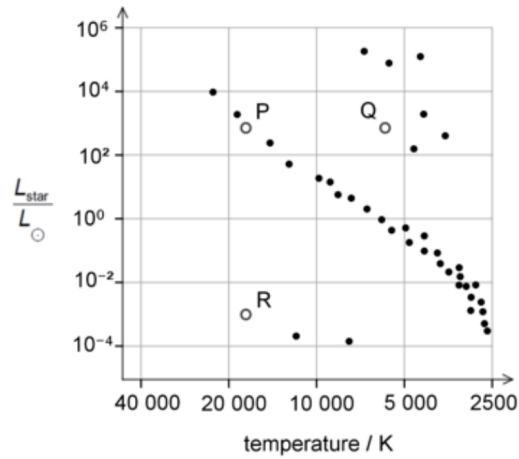
Markscheme
C

Figure 10: Question 4

[Maximum mark: 1]

EXE.1A.SL.TZ0.24

Three stars P, Q and R are plotted in the Hertzsprung-Russell diagram with empty circles.



Which of the following lists the stars in the order of increasing radius?

- A. Q, R, P
- B. P, R, Q
- C. R, Q, P
- D. R, P, Q

[1]

Markscheme

D

Figure 11: Question 5

[Maximum mark: 1]

EXE.1A.SL.TZ0.28

What is the likely evolutionary outcome for a star with the same mass as the Sun?

- A. main sequence star → red giant → supernova → white dwarf
- B. main sequence star → red giant → planetary nebula → white dwarf
- C. main sequence star → super giant → supernova → neutron star
- D. main sequence star → super giant → planetary nebula → black hole

[1]

Markscheme

B

Figure 12: Question 6

[Maximum mark: 2]

23M.2.HL.TZ1.5ci

- (i) **two** difficulties of energy production by nuclear fusion.

[2]

Markscheme

Requires high temp/pressure ✓
Must overcome Coulomb/intermolecular repulsion ✓
Difficult to contain / control «at high temp/pressure» ✓
Difficult to produce excess energy/often energy input greater than output / *OWTTE* ✓
Difficult to capture energy from fusion reactions ✓
Difficult to maintain/sustain a constant reaction rate ✓

Figure 13: Question 7

[Maximum mark: 1]

23M.2.HL.TZ1.5cii

- (ii) **one** advantage of energy production by nuclear fusion compared to nuclear fission.

[1]

Markscheme

Plentiful fuel supplies **OR** larger specific energy **OR** larger energy density **OR** little or no «major radioactive» waste products ✓

Allow descriptions such as "more energy per unit mass" or "more energy per unit volume"

Figure 14: Question 8

[Maximum mark: 1]

22N.1A.SL.TZ0.25

A fusion reaction of one nucleus of hydrogen-2 and one nucleus of hydrogen-3 converts 0.019 u to energy. A fission reaction of one nucleus of uranium-235 converts a mass of 0.190 u to energy.

What is the ratio $\frac{\text{specific energy of this fusion of hydrogen}}{\text{specific energy of this fission of uranium}}$?

A. 0.1

B. 0.2

C. 5

D. 10

[1]

Markscheme

C

Examiners report

The most frequently chosen alternative was (incorrect) A. This alternative represents the ratio of energies released in both reactions, but since the masses of the nuclides are so much different in fission and fusion, the ratio of specific energies *cannot* be equal to 0.1. Just remembering that fusion generally releases more energy than fission per unit mass of fuel should be sufficient to eliminate alternatives A and B.

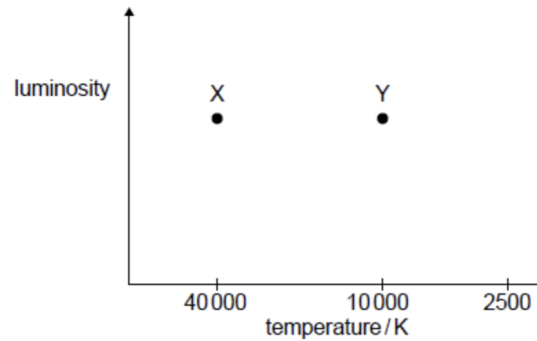
The fusion reaction involves two isotopes of hydrogen of different masses. In this case, specific energy refers to the energy available from unit mass of a fuel containing both isotopes in the same mass proportion as in the fusion reaction. The simplest way to think about it is that the specific energy is the energy released in the reaction divided by the combined mass of the reactants, regardless of whether they are the same or different nuclides.

Figure 15: Question 9

[Maximum mark: 1]

SPM.1A.HL.TZ0.37

The Hertzsprung–Russell diagram shows two stars, X and Y.



What is $\frac{\text{radius of X}}{\text{radius of Y}}$?

- A. $\frac{1}{16}$
- B. $\frac{1}{4}$
- C. 4
- D. 16

[1]

Markscheme
A

Figure 16: Question 10

[Maximum mark: 1]

SPM.1A.HL.TZ0.33

What is the sequence for the evolution of a main sequence star of about 2 solar masses?

- A. Red super giant → supernova → neutron star
- B. Red giant → planetary nebula → white dwarf
- C. Red giant → supernova → white dwarf
- D. Red super giant → planetary nebula → neutron star

[1]

Markscheme
B

Figure 17: Question 11

[Maximum mark: 1]

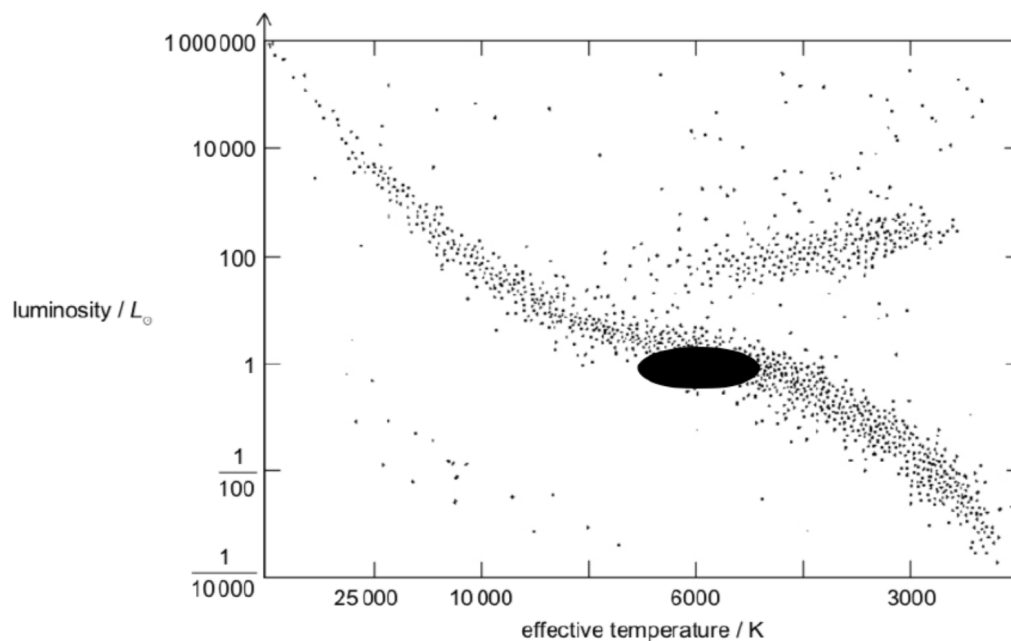
19M.2.SL.TZ2.15a

- (a) Identify, on the HR diagram, the position of the Sun. Label the position S.

[1]

Markscheme

the letter S should be in the region of the shaded area ✓



Examiners report

Locating the Sun's position on the HR diagram was correctly done by most candidates, although a few were unsure of the surface temperature of the Sun.

Figure 18: Question 12

[Maximum mark: 2]

19M.2.SL.TZ2.d

During its evolution, the Sun is likely to be a red giant of surface temperature 3000 K and luminosity $10^4 L_{\odot}$. Later it is likely to be a white dwarf of surface temperature 10 000 K and luminosity $10^{-4} L_{\odot}$. Calculate the radius of the Sun as a white dwarf
radius of the Sun as a red giant.

[2]

Markscheme

«use of $L = \sigma AT^4$ »

$$\frac{10^{-4}}{10^4} = \left(\frac{R_D}{R_G}\right)^2 \times \left(\frac{10000}{3000}\right)^4 \quad \checkmark$$

$$\frac{R_D}{R_G} = 9 \times 10^{-6} \quad \checkmark$$

Examiners report

Calculating the ratio of the radius of a white dwarf to a red giant star was done quite well by most candidates. However quite a few candidates made POT errors or forgot to take the final square root.

Figure 19: Question 13

[Maximum mark: 2]

20N.2.SL.TZ0.14a

- (a) The astronomical unit (AU) and light year (ly) are convenient measures of distance in astrophysics. Define each unit.

AU:

ly:

[2]

Markscheme

AU: «average» distance from the Earth to the Sun ✓

ly: distance light travels in one year ✓

Figure 20: Question 14

[Maximum mark: 1]

20N.2.SL.TZ0.15a

- (a) Show that the apparent brightness $b \propto \frac{AT^4}{d^2}$, where d is the distance of the object from Earth, T is the surface temperature of the object and A is the surface area of the object.

[1]

Markscheme

substitution of $L = \sigma AT^4$ into $b = \frac{L}{4\pi d^2}$ giving $b = \frac{\sigma AT^4}{4\pi d^2}$

Removal of constants σ and 4π is optional

Figure 21: Question 15

[Maximum mark: 2]

20N.2.SL.TZ0.15b

- (b) Two of the brightest objects in the night sky seen from Earth are the planet Venus and the star Sirius.
Explain why the equation $b \propto \frac{AT^4}{d^2}$ is applicable to Sirius but not to Venus.

[2]

Markscheme

equation applies to Sirius/stars that are luminous/emit light «from fusion» ✓

but Venus reflects the Sun's light/does not emit light «from fusion» ✓

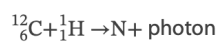
OWTTE

Figure 22: Question 16

[Maximum mark: 1]

22M.1A.SL.TZ1.27

Carbon (C-12) and hydrogen (H-1) undergo nuclear fusion to form nitrogen.



What is the number of neutrons and number of nucleons in the nitrogen nuclide?

	Number of neutrons	Number of nucleons
A.	7	13
B.	6	13
C.	6	7
D.	7	6

[1]

Markscheme

B

Examiners report

This question was well answered by HL candidates, although a significant number of candidates incorrectly identified the number of neutrons present in the nitrogen nuclide.

Figure 23: Question 17

[Maximum mark: 1]

23M.2.SL.TZ2.a

State the main element that is undergoing nuclear fusion in star C.

[1]

Markscheme

Hydrogen ✓

Figure 24: Question 18

[Maximum mark: 2]

23M.2.SL.TZ2.17c

- (c) White dwarfs with similar volumes to each other are shown on the HR diagram.

Sketch, on the HR diagram, to show the possible positions of other white dwarf stars with similar volumes to those marked on the HR diagram.

[2]

Markscheme

Any evidence of correct identification that three dots bottom left represent white dwarfs ✓

line passing through all 3 white dwarfs *OR* line continuing from 3 white dwarfs with approximately same gradient, in either direction ✓

Award MP2 if no line drawn through the three dots but just beyond them in either direction

Figure 25: Question 19

[Maximum mark: 2]

23M.2.SL.TZ2.11bi

- (i) Determine which star will appear to move more.

[2]

Markscheme

Star Y ✓

because parallax angle is greater *OR* star Y is closer «and that means movement relative to distant stars is greater» ✓

Allow reverse argument for star X

Figure 26: Question 20