IB Physics Topic C3 Wave Phenomena; SL & HL

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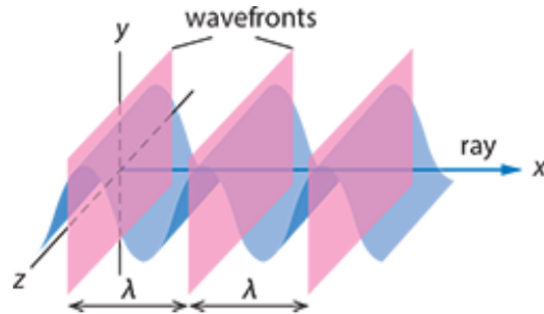
1   Wavefronts and Rays

Figure 1: Wavefronts and Rays

- Wavefronts are surfaces that move with the wave and are **perpendicular to the direction of the wave motion**. Consecutive wavefronts are imagined to be one wavelength apart

- Rays are lines that show the **direction of energy transfer** by the wave. They are locally perpendicular to the wavefront.

2   Refraction

Refraction refers to the change in speed and direction of a wave as it passes from one medium to another. The change in speed will depend on the optical density of the medium.

A general rule of thumb when considering the bending of the wave is that

- If the wave slows down, it will bend towards the normal

- If the wave speeds up, it will bend away from the normal

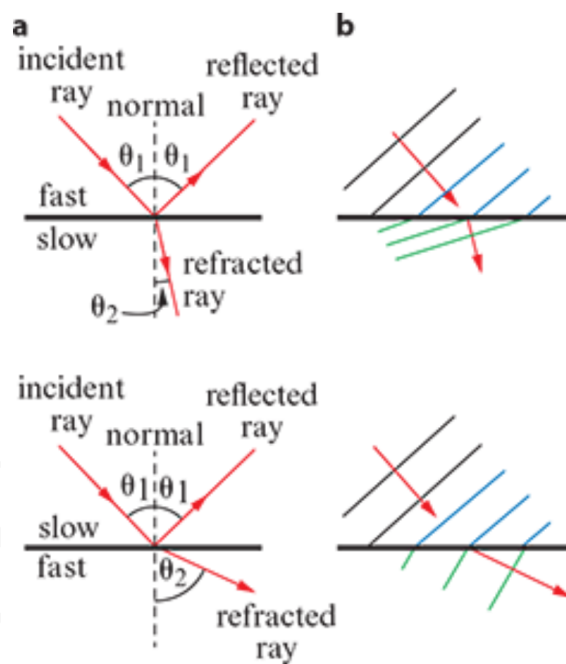The image below shows how refraction is drawn.

Figure 2: Refraction of Wavefronts

2.1 Snell's Law

In refraction, the frequency stays the same as the wave passes from one medium to another. However, the speed and wavelength of the wave will change. The full relationship is given as follows

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

where the quantities with subscript 1 are the quantities in the first medium and the quantities with subscript 2 are the quantities in the second medium. n is the refractive index of a medium.

The way to remember this is, a chained equality of ratios, all except the refractive index is in the same order (medium 1 over medium 2, or vice versa).

2.2 Critical Angle

The critical angle is defined as the angle of incidence that produces an angle of refraction of 90 degrees. This is the angle beyond which total internal reflection occurs. The critical angle is given as follows

$$\sin c = \frac{n_2}{n_1}$$

where n_1 is the refractive index of the medium the wave is coming from and n_2 is the refractive index of the medium the wave is entering. By this definition, this only works if the wave is going from a medium with a higher refractive index to a medium with a lower refractive index (i.e. $n_1 > n_2$), since the sine function cannot exceed 1 in any way.

3   Reflection

The law of reflection states that the angle of incidence is equal to the angle of reflection.

However, reflection can be partial or total.

- Partial reflection is when part of the wave is reflected while the rest is transmitted through the medium and refracted.

- A total internal reflection is when all of the waves are reflected and none are transmitted. This is often used in optical fibres, periscopes, binoculars, and endoscopy (medicine).

Total internal reflection occurs when the angle of incidence is greater than the critical angle.

4   Double Source Interference

Consider the waves coming from two identical sources S_1 and S_2; they have the same amplitude, frequency, and wavelength.
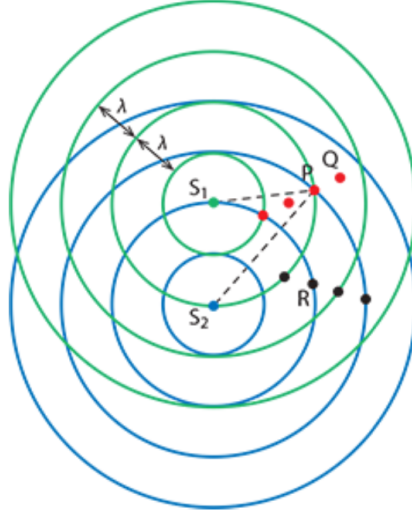

Figure 3: Double Source Interference

At any general point P, the **path difference** is the difference in the distance travelled by the two waves. Numerically, that is

$$\Delta r = |S_1 P - S_2 P|$$

In this particular example, for the specific point P,

$$\Delta r_P = |2\lambda - 3\lambda| = \lambda$$

The path difference will tell us whether the interference is constructive or destructive.

- If the path difference is an integer multiple of the wavelength, the interference is constructive. I.e., when $\Delta r = n\lambda$, where $n \in \mathbb{Z}$; the amplitude is doubled.

- If $\Delta r = (n + \frac{1}{2})\lambda$, where $n \in \mathbb{Z}$, then, it is a destructive interference; the superposed amplitude at this point is 0.

5   Diffraction

Diffraction is the spreading of waves as they pass through an aperture or around an obstacle. The amount of diffraction that occurs depends on the wavelength of the wave and the size of the gap or obstacle.
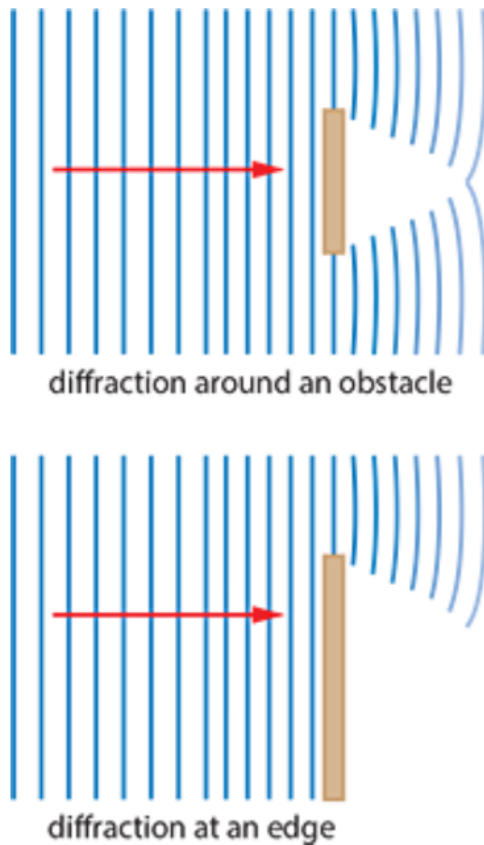

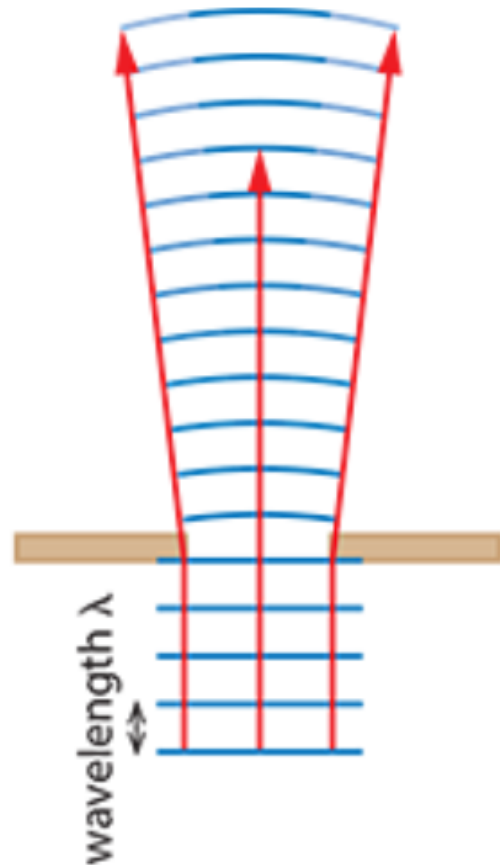Figure 4: Diffraction around an edge

Figure 5: Diffraction through an aperture

6   Double Slit Diffraction

Consider the following scenario: a wave is incident on a surface that has two slits separated by a distance d. The wavefronts from the two slits will interfere with each other, constructively at some points and destructively at others. A requirement of this is that the waves coming out of the two slits must be **coherent**, i.e. they sustain a constant phase difference over time.
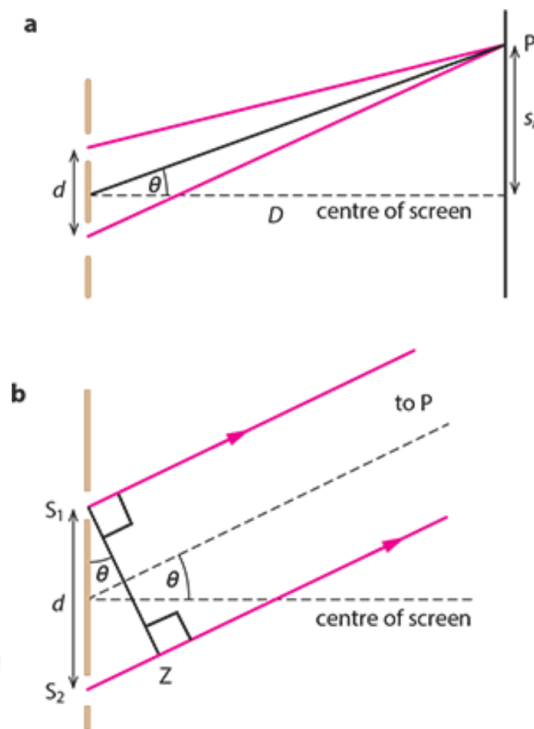

Figure 6: Double Slit Diffraction

1. The slit separation d is almost negligible, thus we can consider the two rays to be parallel.

2. Consider P to be a point on the screen, we now analyze the interference at this point.

3. Define θ as shown in the diagram. With some geometric sense, the path difference

is given as

$$\Delta r = |S_2 Z| = d \sin \theta$$

4. If we want to find points of constructive interference, we require $\Delta r = n\lambda$, where $n \in \mathbb{Z}$. Thus, we have

$$d \sin \theta = n\lambda \tag{1}$$

5. Conversely, if we want to find points of destructive interference, we require $\Delta r = (n + \frac{1}{2})\lambda$, where $n \in \mathbb{Z}$. Thus, we have

$$d \sin \theta = (n + \frac{1}{2})\lambda \tag{2}$$

6. An important concept here is that θ is also very small, so we can use the small angle approximation $\sin \theta \approx \tan \theta$ (just take limits...)

7. By construction, we have that

$$\tan \theta = \frac{s_n}{D}$$

where D is the distance to the screen and s_n is the distance from the central maximum to the nth maximum.

8. If we just focus on the points of constructive interference, we can rewrite Equation (1) as

$$\frac{s_n d}{D} = n\lambda$$

which can be rearranged to give

$$s_n = \frac{n\lambda D}{d} \tag{3}$$

9. Now, this allows us to work out the **separation between any two maxima or**

minima on the screen, s:

$$s = s_{n+1} - s_n = \frac{\lambda D}{d}$$

$$s = \frac{\lambda D}{d} \tag{4}$$

The displacement or angular displacement vs. intensity graph **only concerning the effect of double slit diffraction** is shown below.
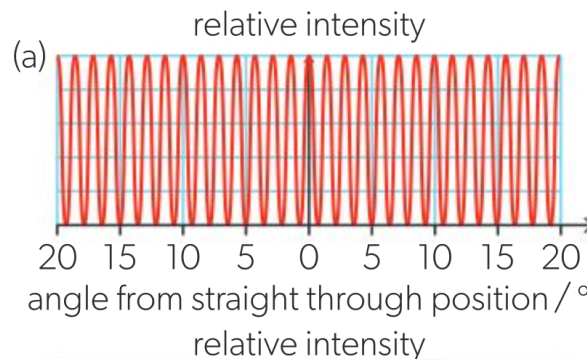

Figure 7: Graph of intensity vs. Angular displacement

An important feature is that, the **intensity is proportional to the square of the amplitude**; this means that, two waves of amplitude A interfering constructively at point P will have an amplitude of $2A$ and an intensity of four times the original.

Assumption. 1

It must be noted that this graph, with peaks at roughly the same intensities, is only true when the slit widths are negligible so that the single slit effect will not be taken into account.

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7 Single Slit Diffraction

Consider a single slit of width b and a wave incident on it. The wavefronts will spread out as they pass through the slit.
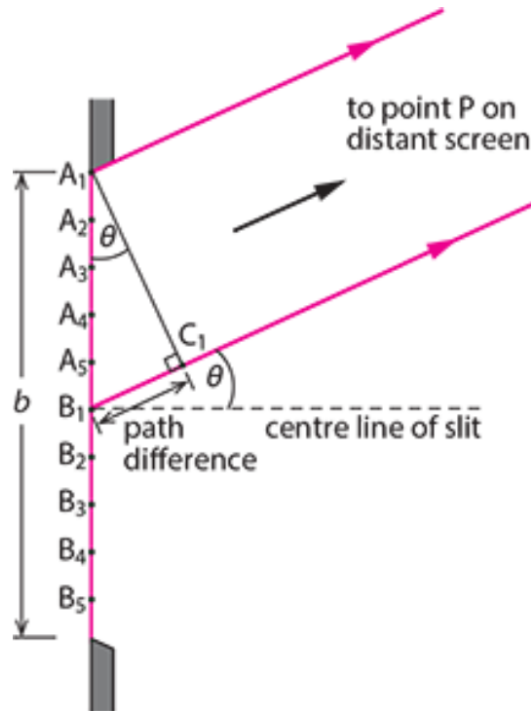

Figure 8: Single Slit Diffraction

Our aim is now to identify the **angular displacement of the first minimum** (first point of destructive inteference). Before we proceed, we need to accept the assumptions of Huygen's principle:

Every point on a wavefront acts as a source of secondary spherical wavelets, and the wavefront at any later time is the envelope of these wavelets.

If we consider the wave incident on the slit, we can split the slit into a number of point sources, each of which will emit a wavefront.

1. Consider the pair of wavelets emitted from A_1 and B_1; the path difference between the two wavelets is given as

$$\Delta r = |A_1 P - B_1 P| = \frac{b}{2} \sin \theta$$

2. Again, this relies on the approximation of the two rays being parallel.

3. To achieve destructive interference, we require $\Delta r = (n + \frac{1}{2})\lambda$, where $n \in \mathbb{Z}$. Thus, we have

$$\frac{b}{2} \sin \theta = (n + \frac{1}{2})\lambda \tag{5}$$

4. Since we are only concerned with the first minimum, we take $n = 0$.

5. We can now rearrange Equation (5) to give

$$b \sin \theta = \lambda \tag{6}$$

6. With small angle approximation, we obtain that

$$\theta = \frac{\lambda}{b} \tag{7}$$
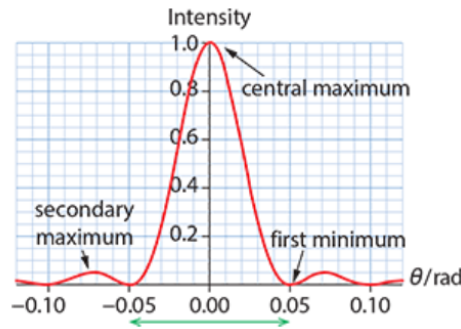
The intensity graph is as follows:

Figure 9: Graph of intensity vs. angular displacement

Effects of <mark>decreasing</mark> the slit width (assuming a fixed wavelength):

- The angular displacement of the first minimum (and in fact, other minima) will increase. This means that the graph will be more spread out.

- Suppose the slit width is dropped to $\frac{1}{k}$ of the original, then, the amplitude will also drop to $\frac{1}{k}$ of the original. Hence, the intensity will drop to $\frac{1}{k^2}$ of the original.
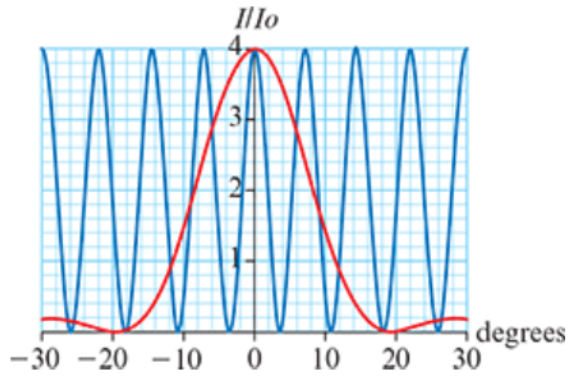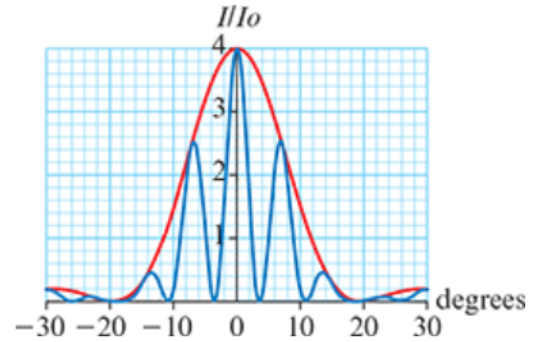
8   Combined Effect

Figure 10: Overlay of the effects

Figure 11: Resulting graph

In Figure 11, the blue curve shows the resulting intensity graph. It is essentially the original double slit graph suppressed by the single red curve.

9   Multi-Slit Diffraction

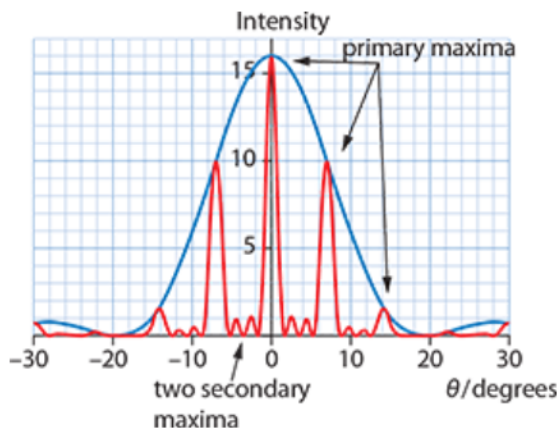Consider adding more and more slits. Below shows the graphs for four and six slits respectively.
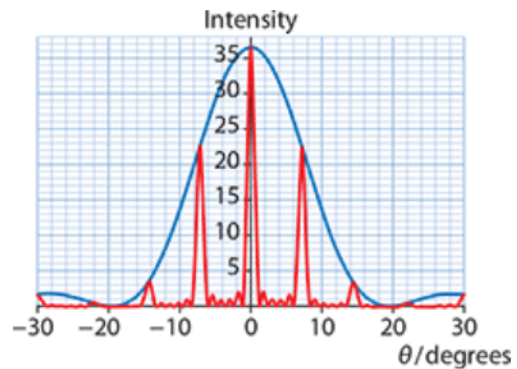

Figure 12: Four Slit Diffraction

Figure 13: Six Slit Diffraction

Previously, in Figure 11, there is only one *secondary minimum* between every pair of maxima. However, as observed, in the case of four slits, there are 2 secondary minima between every pair of maxima. The general rule of thumb for N slits is that:

- There are $N - 2$ secondary minima between every pair of maxima.;

- The intensity of the central maximum is N^2 times the intensity of just one slit by itself.

With increasing N:

- The primary maxima remain at the same angles

- The primary maxima get narrower and brighter

- The secondary maxima become unimportant.

9.1 Diffraction Grating

A diffraction grating is a configuration that consists of a large number of slits of negligible width.
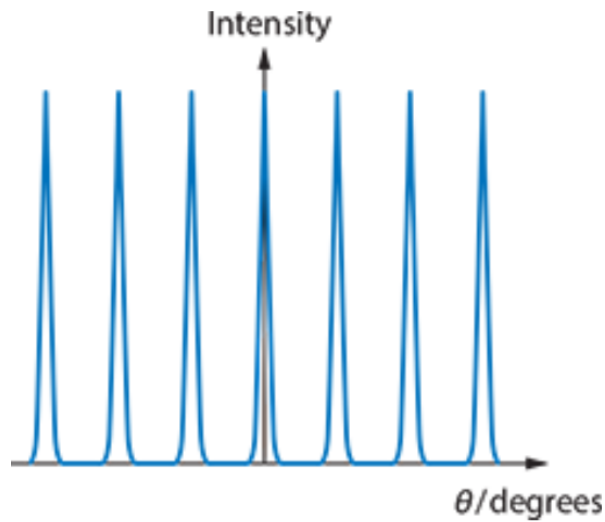

Figure 14: Diffraction Grating

The equations for destructive and constructive interference are the same as the ones developed previously, namely Equation (1) and Equation (2).

9.2 Grating Spacing

Since the slit width is negligible, we can treat each slit as a single line. The grating spacing is the distance between two adjacent lines. If we are told that the grating is N lines per unit length, then the grating spacing is given as

$$d = \frac{1}{N}$$

in that unit.