IB Physics Topic D3 Motion in E.M. Fields; SL & HL

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1 Notes on Graphical Notation

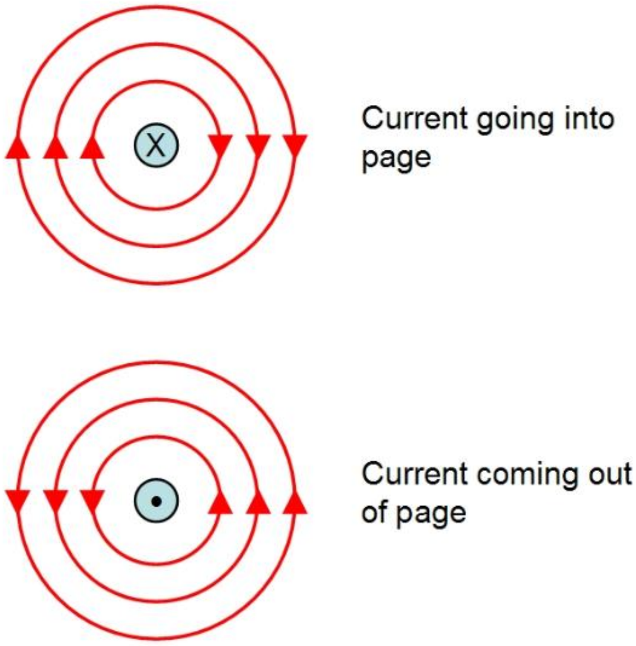


Figure 1: Into and out of the page

This notation uses the direction of **conventional current**.

Imagine an arrow going in the direction of the current: The tail looks like an X and so that would be into the page, and vice versa.

It is important to note that it can represent both currents and magnetic field lines; read the question carefully to determine which one it is referring to.

2 Field Strength around a Current-Carrying Wire

Consider a single straight wire carrying a current I; at a distance of r from the wire, the **magnetic field strength** is given by

$$B = \frac{\mu_0 I}{2\pi r} \tag{1}$$

where μ_0 is the permeability of free space (**not to confuse with permittivity!**), with a value of $4\pi \times 10^{-7}\mathrm{T\,m\,A^{-1}} \equiv 4\pi \times 10^{-7}\mathrm{kg\,m\,A^{-2}\,s^{-2}}$.

Also notice that, unlike past field strengths we have seen, it does not follow the inverse square law and is instead an inverse proportionality.

3 Motion — Straight Wire and Bar Magnet

3.1 Direction

Consider a current-carrying wire placed in a magnetic field between two bar magnets.

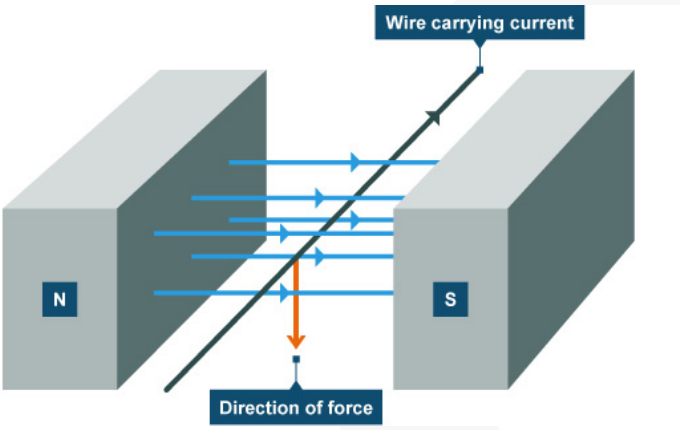


Figure 2: Wire in a magnetic field

There is a resultant force on the wire; in the scenario illustrated in the diagram, it is downwards. This is because, at the top of the wire, the field lines of the wire and the magnetic field respectively are in the same direction, creating a strong force; conversely, at the bottom, the field lines are in the opposite direction, creating an area of weak force.

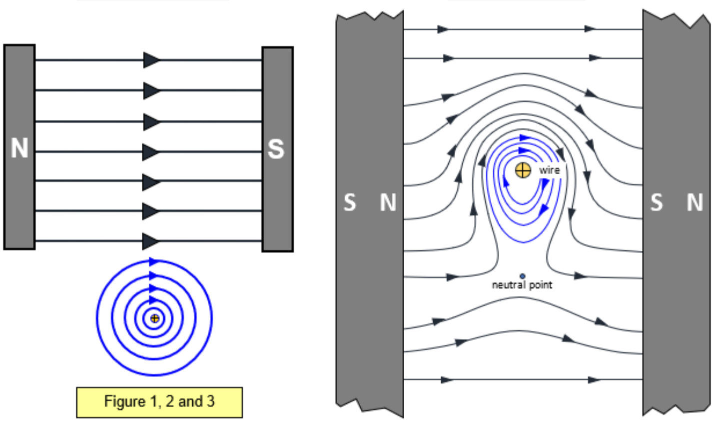


Figure 3: Force direction

3.2 Magnitude

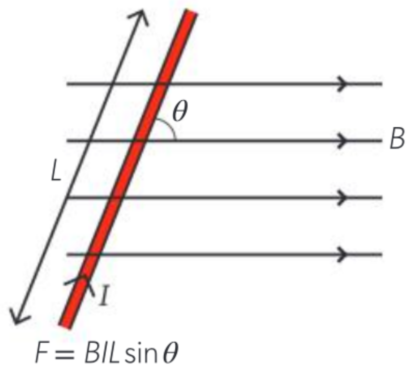


Figure 4: Cutting at an angle θ

The following relation quantifies this force:

$$F = BIL\sin\theta \tag{2}$$

where

- F is the force on the wire,
- B is the magnetic field strength, unit $\mathrm{T} \equiv \mathrm{N\,A^{-1}\,m^{-1}}$,
- I is the current in the wire, and
- L is the length of the wire **in the magnetic field**.

An alternative form is

$$F = qvB\sin\theta \tag{3}$$

where

- F is the force acting on a charge q,
- v is the velocity of the charge, and
- B is the magnetic field strength.

4 Motion — Two Current-Carrying Wire

Consider the case of two straight current-carrying wires placed parallel to each other side by side. We will study both the direction and magnitude of the resultant field.

4.1 Direction

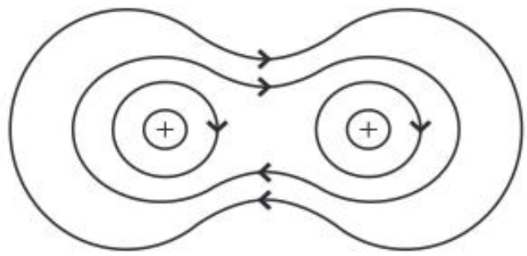


Figure 5: Two wires with current in the same direction

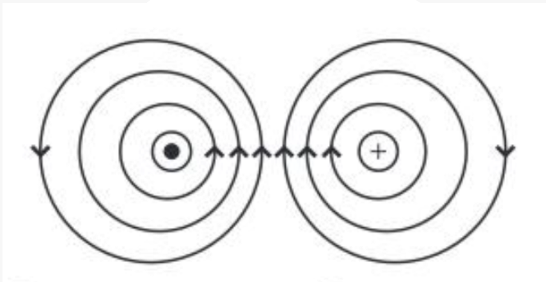


Figure 6: Two wires with current in the different directions

The field in between the wires is effectively ”canceled out” as the field lines in that region are opposing. The resulting field would produce an attractive force that brings the wires together.

The field in the central region is now reinforced as the field lines are in the same direction. The wires would repel each other as a result.

4.2 Magnitude

Consider the following configuration:

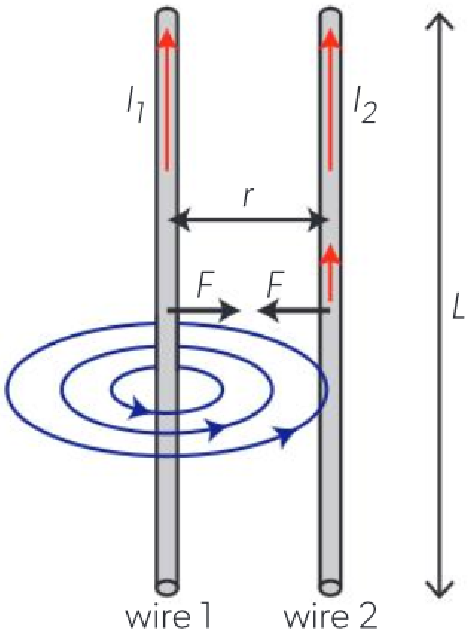


Figure 7: Force between two wires

Let r be the separation distance between the two wires, and I_1 and I_2 be the currents in the wires. Let L be the length over which the two wires are influencing each other.

Previously, we have seen the magnetic field strength at a point around a single wire. In our case, we have two wires, we can combine them as follows:

1. The field strength at the position of wire 1 due to wire 2 is

$$B_1 = \frac{\mu_0 I_2}{2\pi r}$$

2. Using $F = BIL$ with $I = I_1$ to make a substitution

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi r}$$

3. We may rewrite it as follows

$$\frac{F}{L} = -\frac{\mu_0 I_1 I_2}{2\pi r} \tag{4}$$

The form in Equation (4) gives the force per unit length between two wires carrying currents I_1 and I_2 respectively.

This force is mutual: wire 1 exerts this force on wire 2 and so does wire 2 on wire 1.

For the sake of consistency, following the convention that attractive forces are negative, we have added in a negative sign in the equation:

- When the two wires are in the same direction, $\frac{\mu_0 I_1 I_2}{2\pi r}$ is positive, and so to reflect the attractive nature of the force, we add a negative sign.
- In contrast, when the wires are in opposite directions, $\frac{\mu_0 I_1 I_2}{2\pi r}$ is negative, and so to reflect the repulsive nature of the force, we add a negative sign to obtain a positive value.

It is important to always remember that this force is mutual to both wires. For example, consider

> Two long parallel wires X and Y carry equal currents I. The magnetic force exerted per unit length of each wire is F. The current in X is halved and the current in Y is doubled. What is the force per unit length of each wire after the change?

	Force per unit length of X	**Force per unit length of Y**
A.	F	F
B.	$\frac{F}{2}$	$2F$
C.	$2F$	$\frac{F}{2}$
D.	$2F$	$2F$

Figure 8: Options

Using this knowledge, we can immediately eliminate B and C where the two magnitudes are different. Then, the formula gives option A.

4.3 Net Force on a Loop

Consider the following scenario: A long straight wire carries a current of 2.0 A. A square conducting loop ABCD of side length 0.20 m is placed near the straight wire, with side AB at a distance of 0.30 m from the wire. There is a current of 1.0 A in the loop. The directions of the currents are shown.

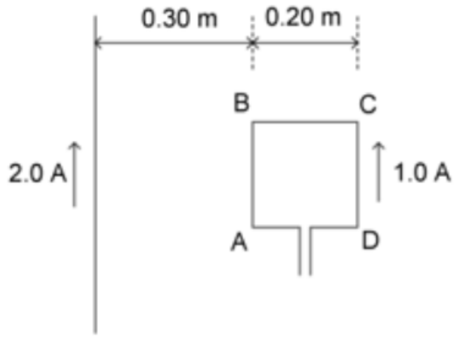


Figure 9: Diagram

In this case, the current through DC is in the same direction as the 2.0A current, but the current through BA is in the opposite direction. The net force acting on this loop will be the force between CD and 2.0A + the force between BA and 2.0A

$$
\begin{aligned}
F_{\text{DC}} &= -(0.2)(\frac{\mu_0(1)(2)}{2\pi(0.5)}) \\
F_{\text{BA}} &= (0.2)(\frac{\mu_0(1)(2)}{2\pi(0.3)}) \\
\Sigma F &= F_{\text{DC}} + F_{\text{BA}} \\
&= 1.1 \times 10^{-7}\,\text{N}
\end{aligned}
$$

This is telling us that the overall force is repulsive (since positive), and this is intuitive even without calculation. The repelled segment is closer to the straight wire than the attracted segment, and so the repulsive force is stronger.

5 Moving Charge

5.1 Uniform Electric Field

Consider a charge q moving across a uniform electric field perpendicular to the field lines. The ”tunnel” has a length of L. We will **ignore gravity, air resistance, and the edge effects** at the boundaries where the charge enters and exits the field. The path in blue is a parabolic trajectory.

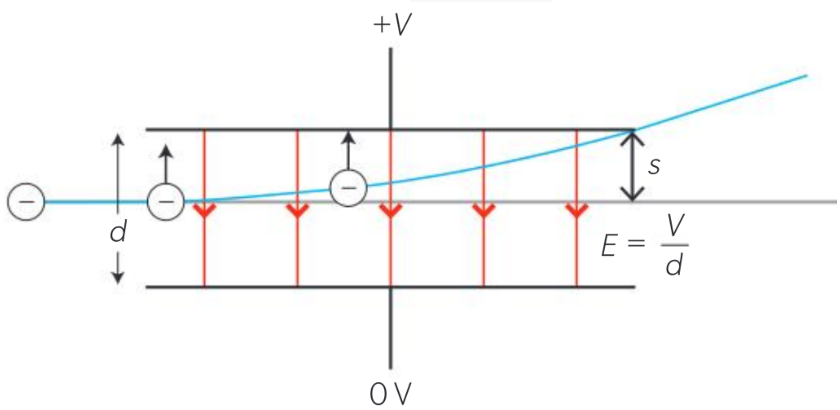


Figure 10: Charge moving in a uniform electric field

Consider the acceleration of the charge. The force acting on the charge is given by $F = \frac{qV}{d}$

$$a = \frac{F}{m_e} = \frac{qV}{m_e d}$$

All values on the RHS are constant, which means that the charge has a constant acceleration, and thus we can use kinematic equations to analyze the situation.

Also note that there is no horizontal force acting on the charge and so the horizontal velocity will remain constant. This means that we can find out the duration of the entire journey through the field:

$$T = \frac{L}{v_x}$$

where v_x is the horizontal velocity of the charge.

5.2 Uniform Magnetic Field

Consider the following scneario: A charge q is moving in a uniform magnetic field perpendicular to the field lines (the currents are directed into/out of the page and so the charge will be traveling on the plane of the page).

In short, the charge will move in a circle. Let's see why.

Consider an electron (e^-) moving in the magnetic field shown. As per Flemming's left-hand rule (it uses convention current so we take the opposite direction in which the electron travels), the force acting on the electron is perpendicular to the direction of motion and directed inwards. This force is constant in magnitude and is the centripetal force that keeps the electron in a circular path. The above explanation is adapted from exam mark schemes, feel safe to use it in your exams.

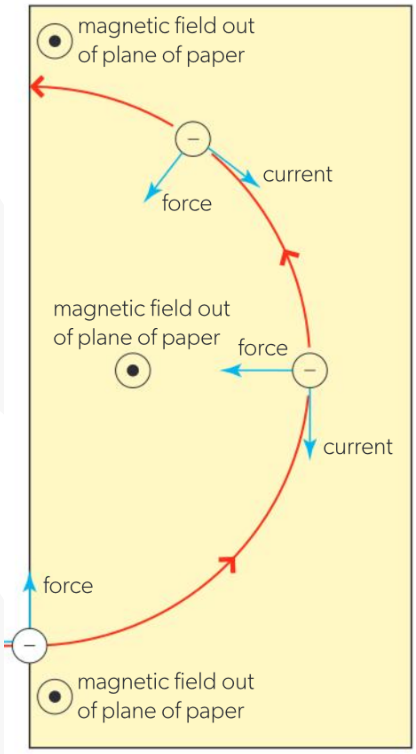


Figure 11: Charge moving in a uniform magnetic field

The equations about the trajectory outlined below are VERY IMPORTANT and worth memorising and understanding. There are loads of questions on them.

The centripetal force is provided by $F = evB$, we equate the two quantities to obtain

$$evB = \frac{m_e v^2}{r}$$
$$v = \frac{Ber}{m_e}$$

In the scenario under study, the tangential velocity v is constant throughout. Let us now rearrange to find an expression for the radius of the circular trajectory:

$$r = \frac{m_e v}{Be}$$

Notice that the numerator is the momentum of the charge, so just bear that in mind if you are given only the momentum instead of the mass and the velocity. Moreover, $m_e v = \sqrt{2m_e E_k}$, where E_k is the kinetic energy of the charge; this form is useful when you are not given the velocity but the kinetic energy instead.

Now consider tilting the plane of the page so that it makes an angle θ with the magnetic field. The radius of the circular path is now given by

$$r = \frac{m_e v \sin\theta}{Be}$$

However, the tilted plane will mean that the charge will move in a helical path, "escalating" along the direction of the magnetic field.

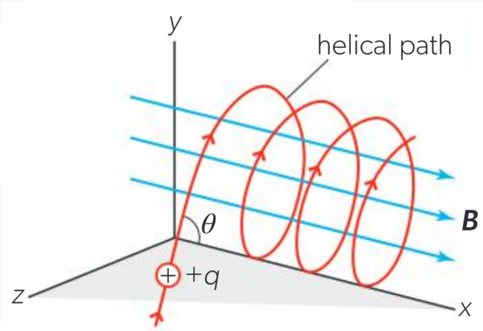


Figure 12: Helical path

5.3 Perpendicular Magnetic and Electric Fields

In the below configuration, an electric field is combined with a magnetic field at a right angle.

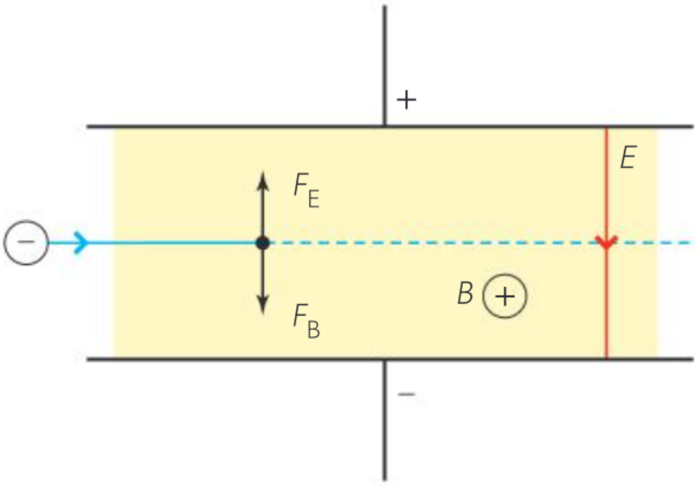


Figure 13: Charge moving in a magnetic and electric field

Let's recall the behavior of the charge in the two fields separately:

- In a magnetic field alone it will move in a circular path, as the magnetic force constantly changes its direction but not its speed.
- In an electric field alone, it will follow a parabolic trajectory due to the constant force exerted in one direction.

In the combined field, we can get a straight-line resultant pass under certain conditions. That is, when the upward and downward forces F_e and F_B respectively are equal in magnitude. This gives

$$F_e = F_B$$

$$qE = qvB$$

$$v = \frac{E}{B}$$

For every combination of (E, B), the velocity of the charged particle must be exactly $v = \dfrac{E}{B}$ to get a straight-line path without deviations.

This property allows the setup to be used as a *velocity selector* to select particles of a certain velocity.

- This setup acts as a velocity selector: only particles with the precise speed $v = \dfrac{E}{B}$ will experience balanced forces and continue in a straight line.
- Particles moving slower than this speed will be deflected upwards (since the magnetic force will be weaker than the electric force).
- Particles moving faster will be deflected downwards (since the magnetic force will be stronger).

It must be noted that, since the charge q was canceled out in the calculation, the velocity selector can be used for any charged particle, not just electrons, the only factor that determines deflection is the velocity of the particle, whether it matches $v = \dfrac{E}{B}$.

6 The Charge-Mass Ratio Derivation

The scenario described in Section 5.2 allows for the computation of the charge-mass ratio of the charge. The ratio is given by the following and is derived by equating KE and $W = eV_e$ and using circular motion equations:

$$\frac{e}{m_e} = \frac{2V}{B^2 r^2} \tag{5}$$

However, we can also apply this to any charge q and not just an electron:

$$\frac{q}{m_q} = \frac{2V}{B^2 r^2} \tag{6}$$

7 Exam Questions

7.1 A Wire and The Motor Effect

A long straight vertical conductor carries a current I upwards. An electron moves with horizontal speed v to the right.

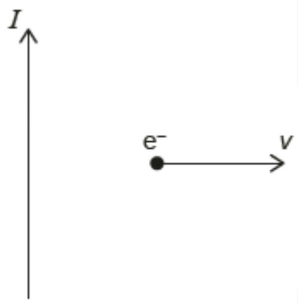


Figure 14: Wire and motor effect

What is the direction of the magnetic force on the electron?

1. Clearly we have to use the left-hand rule for motor effect.

   - The current is to the left, since it is the conventional current and opposite to the direction of electron flow.
   - The magnetic field in this case is into the page **at the point of the electron**: First, with the right-hand grip rule, we establish the direction of the circular field lines around the wire; its tangential direction is into the page at the electron.

2. It should then follow that the force on the electron is downwards.

7.2 Framework – Explaining the Force on a Wire

Two parallel wires A and B both carry an electrical current into the page. State and explain, using a diagram, why a force acts on B due to A in the plane of the paper.

1. B lies in the magnetic field of A
2. Using the right-hand grip rule (with a sketch)
3. It can be shown that the force on B is to the left and in the plane of the paper.
4. The sketch should also show a null point in the middle; refer to Figure 5
5. The wires will move to reduce stored energy and this is achieved by moving together so the force on B is to the left.

7.3 Path Curvature in Magnetic Field

A particle with charge Q is accelerated from rest through a potential difference V over a distance s. The particle then enters a magnetic field of strength P at right angles to the magnetic field direction. What single change will make the radius of curvature of the path of the particle smaller.

- We simply invoke
$$r = \frac{mv}{PQ}$$
- Quite simply, we see that increasing the strength P will decrease the radius of curvature.

7.4 Combining Two Rules

Electrons in a conductor are moving down the page. A proton outside the wire is moving to the right.

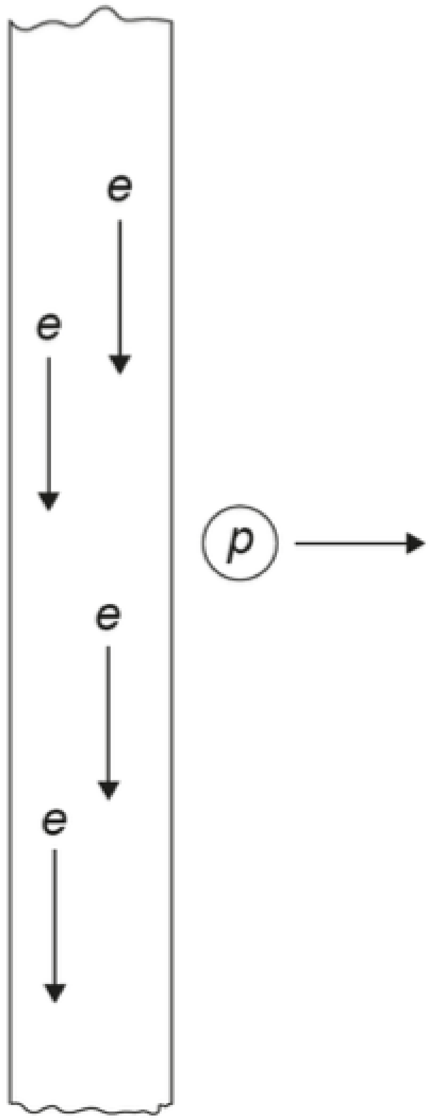


Figure 15: Diagram

What is the direction of the magnetic force acting on the proton?

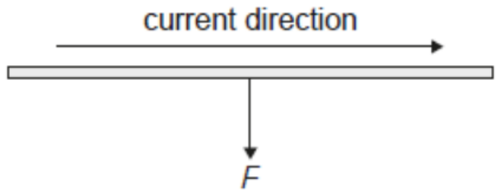
- This is about using the left-hand motor effect rule. Let's identify the elements involved in this rule:

  - The current is in fact the moving proton rather than the electrons in the wire! This is because we are predicting the motion of the proton. The direction of the proton is the equivalent of the conventional current, hence I is to the right.
  - The field is what is created by the electrons in the wire. Again, the flow of electrons is opposite to the direction of the conventional current, so the field at P is into the page.

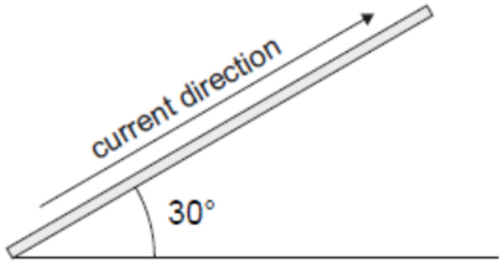
- Finally, using the left-hand rule, we can see that the proton will be deflected upwards.

7.5 Trick MCQ

A conductor is placed in a uniform magnetic field perpendicular to the plane of the paper. A force F acts on the conductor when there is a current in the conductor as shown.



The conductor is rotated 30° about the axis of the magnetic field.



What is the direction of the magnetic field and what is the magnitude of the force on the conductor after the rotation?

	Direction of magnetic field	Magnitude of force
A.	into the plane of the paper	F
B.	into the plane of the paper	$\frac{F}{2}$
C.	out of the plane of the paper	F
D.	out of the plane of the paper	$\frac{F}{2}$

Figure 16: Diagram

- Many would be tempted to say that the force is halved because of the factor of $\sin 30°$. However, that would be true if the field was in the same plane as the paper! Hence, this rotation does not affect the force acting on the conductor. This allows us to eliminate B and D.

- In the initial configuration, we can use the left-hand rule to predict that the field is out of the page. This remains unchanged in the new configuration. Hence, the answer is C.