IB Physics Topic C4 Standing Waves and Resonance; SL & HL

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1 Formation of Standing Waves

A standing wave originates from the superposition of two waves of the **same frequency and amplitude** traveling in **opposite directions**.

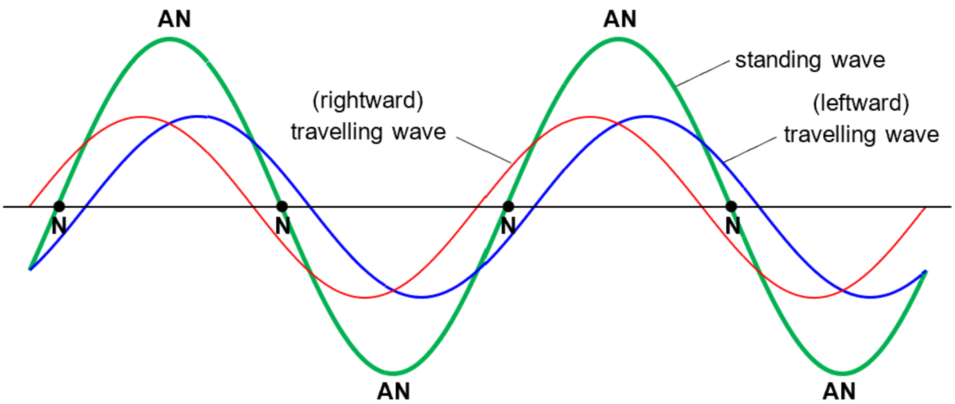

Figure 1: Formation of a standing wave

In the diagram above, the blue and red waves are two traveling waves of the same frequency and amplitude, moving in opposite directions. The result of their superposition (the summation of the amplitudes of the two waves at every point of overlap) is the standing wave shown in green. The standing wave has **nodes** (points of zero amplitude; destructive inteference) and **antinodes** (points of maximum amplitude; constructive interference).

Consider a few examples:

1. A guitar string is plucked, and the wave travels along the string. The wave reflects off the end of the string and interferes with the incoming wave. This interference creates a standing wave.

2. A sound wave is produced in a pipe. The wave reflects off the end of the pipe and interferes with the incoming wave. This interference creates a standing wave.

One important characteristic to distinguish between a traveling and a stationary wave is the **amplitude**. In a traveling wave, the amplitude is **the same at all points**. In a standing wave, at each point on the wave, the amplitude is different from that of its neighboring points.

Let a *nodal region* be the region of points between any two adjacent nodes.

- All points in a nodal region are in phase (0 phase difference).
- Points in nodal region R_1 are out of phase with points in one of the two neighboring nodal regions R_2 by π radians.

1.1 Harmonics

A harmonic is a standing wave pattern of a particular frequency. The first harmonic occurs at the fundamental frequency.

2 Pulse Reflection — Boundary Conditions

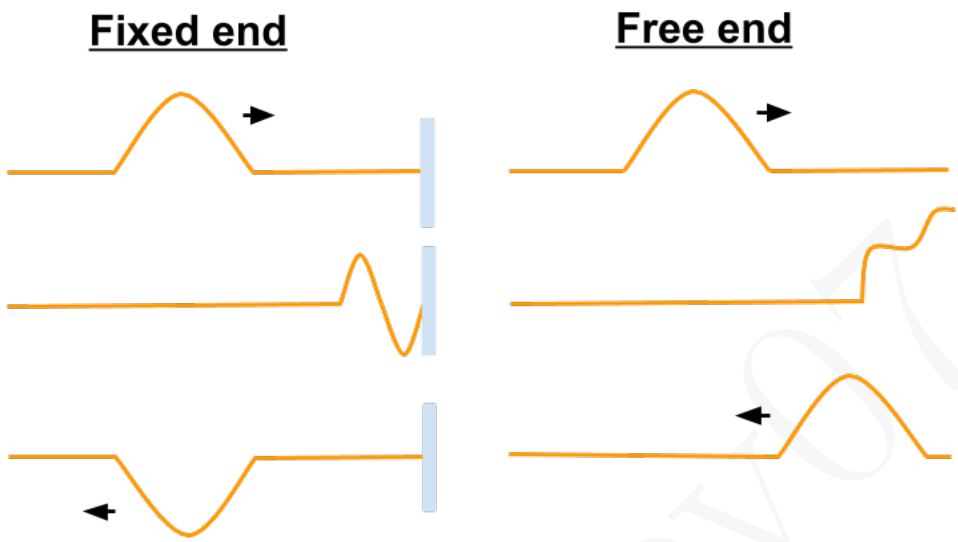

Figure 2: Pulse reflection

1. Fixed end; the reflected wave is inverted.
2. Free end; the reflected wave is not inverted.

3 Standing Waves on a String

3.1 Fixed-Fixed

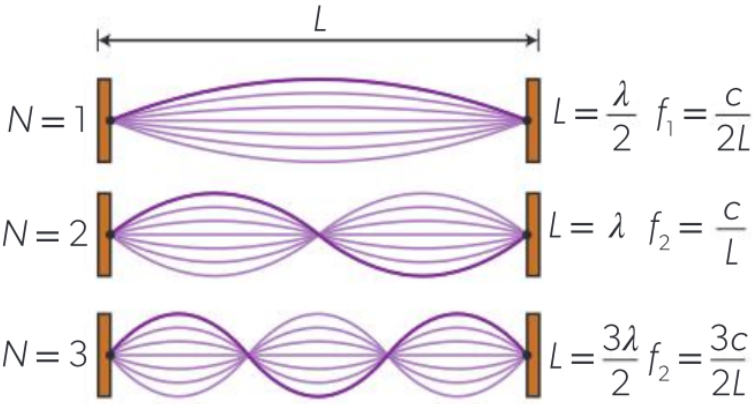

Figure 3: Fixed-Fixed

For the n-th harmonic:

- $\lambda_n = \dfrac{2L}{n}$
- The fundamental frequency is $f_1 = \dfrac{v}{2L}$
- Subsequent harmonics have frequencies $f_n = nf_1 = \dfrac{nv}{2L}$

3.2 Fixed-Free

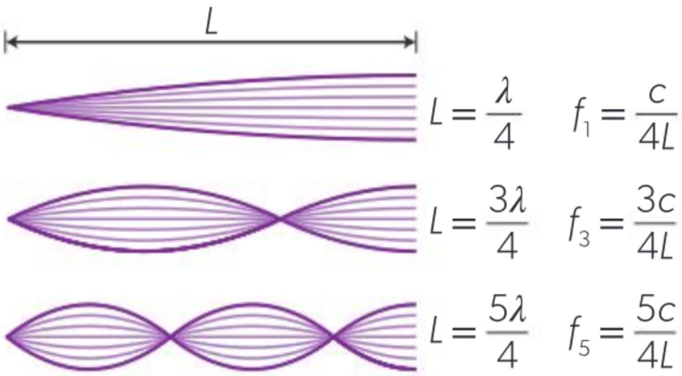

Figure 4: Fixed-Free

It must be noted that the second harmonic that exists actually has a harmonic number of 3. Similarly, the third harmonic has a harmonic number of 5, and so on.

For the harmonic with harmonic number n (not the nth harmonic that exists!), where n is odd:

- $\lambda_n = \dfrac{4L}{n}$
- The fundamental frequency is $f_1 = \dfrac{v}{4L}$
- Subsequent harmonics have frequencies $f_n = nf_1 = \dfrac{nv}{4L}$

3.3 Free-Free

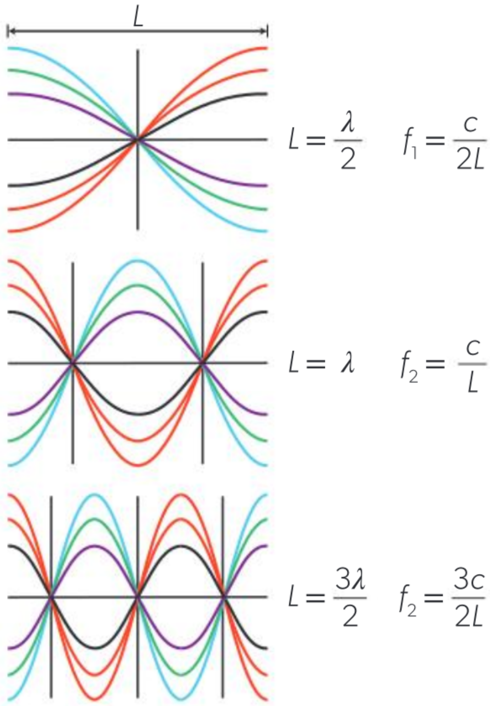

Figure 5: Free-Free

The formulae are identical to those for the fixed-free string. The only difference is the positions of the nodes and antinodes.

4 Standing Waves in Pipes

In a pipe, there can also be a standing wave pattern. It is slightly different from the case of strings, but similar analysis can be performed. Consider the following diagram, where the dots represent vibrating air molecules.

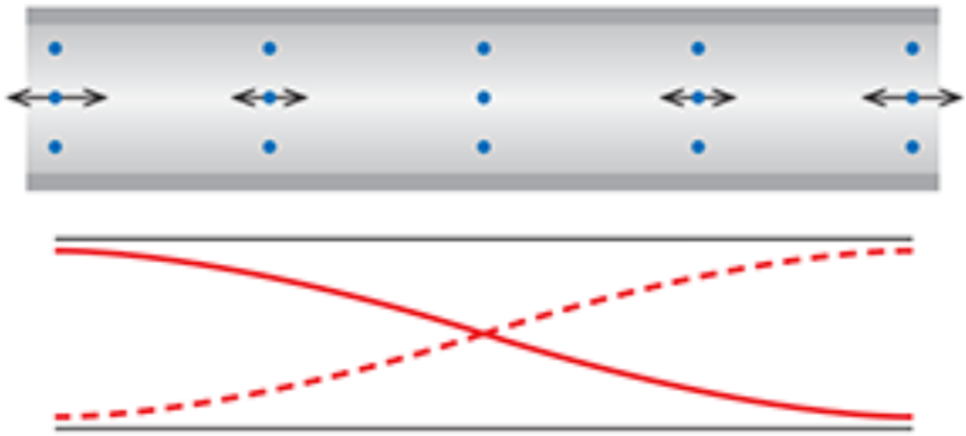

Figure 6: Standing wave in a pipe

- The particles oscillate left and right in a fixed position. The amplitude, previously the vertical displacement of a point on the string, is now the range of horizontal movement of the particles.
- A free end is an antinode
- A fixed end is a node

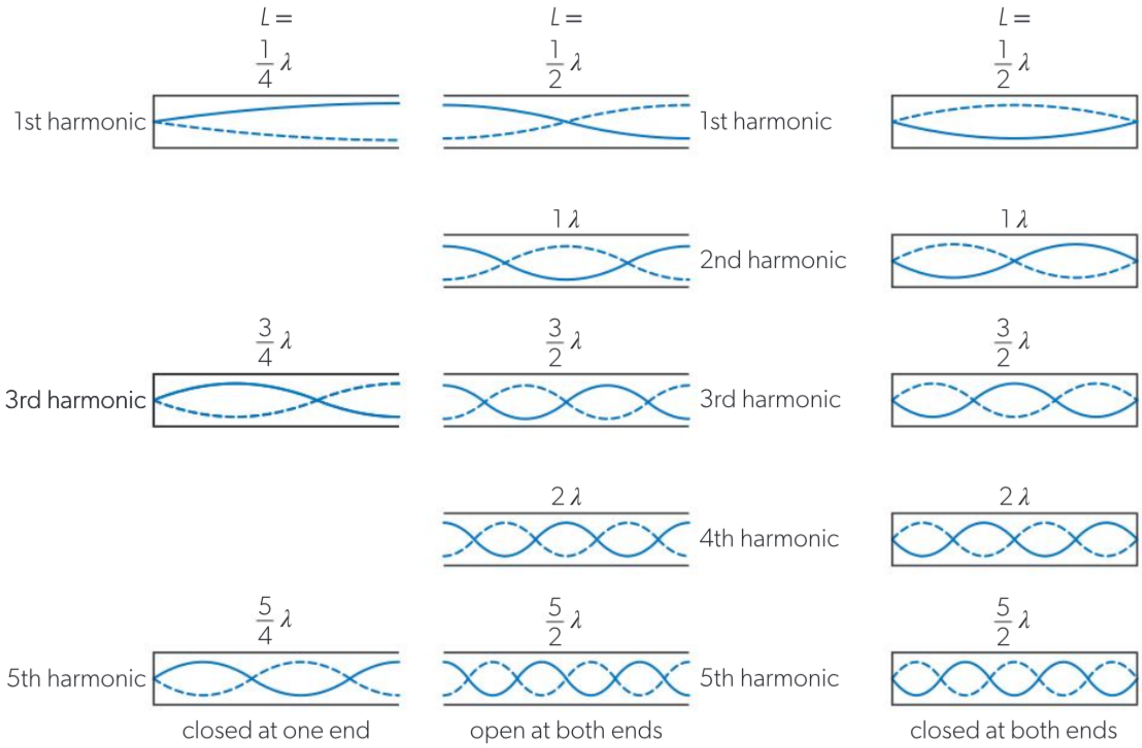

Figure 7: Summary of standing wave patterns in pipes

4.1 Closed-Closed

Notice that this is completely analogous to the fixed-fixed string case.

- For the nth harmonic, $\lambda_n = \dfrac{2L}{n}$
- The fundamental frequency is $f_1 = \dfrac{v}{2L}$
- Subsequent harmonics have frequencies $f_n = nf_1 = \dfrac{nv}{2L}$

4.2 Closed-Open

Notice that this is completely analogous to the fixed-free string case.

In this case, the even harmonics are omitted.

- For the harmonic numbered n, $\lambda_n = \dfrac{4L}{n}$

- The fundamental frequency is $f_1 = \dfrac{v}{4L}$
- Subsequent harmonics have frequencies $f_n = nf_1 = \dfrac{nv}{4L}$

4.3 Open-Open

This is completely analogous to the free-free string case as well as the closed-closed pipe case.

5 Summary — Standing Wave Equations

The following table works for both the string and pipe cases, since both are analogous.

Case	Wavelength	Frequency
Both ends same condition; $n \in \mathbb{Z}^+$	$\lambda_n = \frac{2L}{n}$	$f_n = nf_1 = \frac{nv}{2L}$
Two ends different conditions; n is odd	$\lambda_n = \frac{4L}{n}$	$f_n = nf_1 = \frac{nv}{4L}$

6 Resonance

Resonance occurs when the driving frequency f_D is equal to the natural frequency f_0 of the system. The amplitude of the system increases significantly; without damping, the amplitude would tend towards infinity, which is impossible.

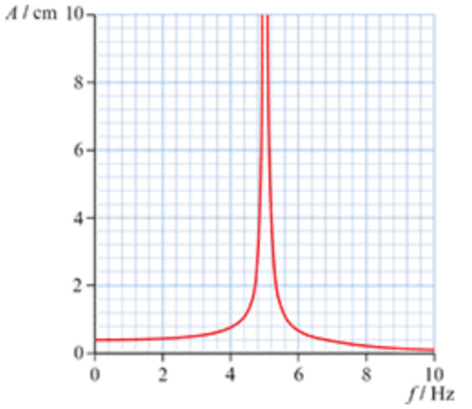

Figure 8: Resonance

The driven oscillator may initially oscillate at a different frequency than the driving frequency, but it will eventually reach the driving frequency.

- *Natural frequency*: The frequency at which a system oscillates on its own.
- *Driving frequency*: The frequency of an external force driving the system.

6.1 Resonance with Damping

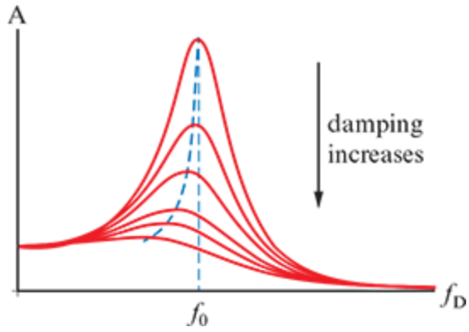

Figure 9: Resonance with damping

Let's now investigate this refined graph with damping considered.

- Each red curve of the family represents an oscillatory system with a different level of damping.
- The lower the peak of a red curve, the higher the damping associated with it.
- With low damping, the peak occurs closest to the natural frequency. As damping increases, the peak shifts away from the natural frequency and becomes flatter.

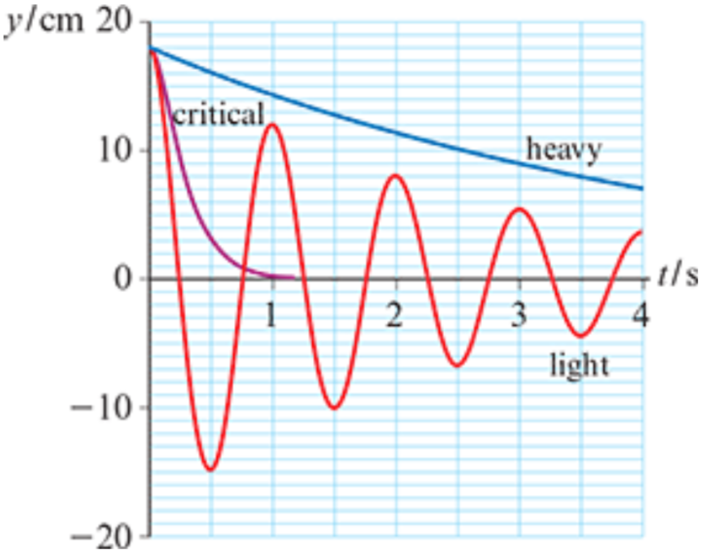

Figure 10: Damping

- *Critical damping*: The system returns to equilibrium as quickly as possible without oscillating.
- *Heavy damping*: The system returns to equilibrium slowly without oscillating.
- *Light damping*: The system returns to equilibrium slowly with oscillations; the amplitude decreases exponentially (decay) with time.

6.2 Pros and Cons of Resonance

Pros	Cons
Microwave Ovens: Resonance excites water molecules to heat food efficiently.	Structural Damage: Bridges like Tacoma Narrows and Millennium Footbridge suffered instability due to resonance; fixed with dampers.
Ozone Layer: Resonance absorbs harmful UV radiation, protecting living tissue.	Mechanical Vibrations: Unwanted resonance in vehicle mirrors, engines, and washing machines causes noise and wear.
Nuclear Magnetic Resonance (NMR): Used in MRI for medical diagnostics.	
Lasers: Produced by setting up standing waves at specific light frequencies.	