IB Physics Topic E5 Fusion and Stars; SL & HL

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1 Fusion in Stars

Conditions for proton-proton cycle and thus hydrogen fusion to occur:

- High temperature: to overcome the electrostatic repulsion between the nuclei
- High pressure: to overcome the gravitational attraction between the nuclei

The hydrogen fusion has three stages

1. **Proton-proton reaction**: Two protons (hydrogen nuclei) collide at extremely high temperatures and pressures, fusing together to form a *deuterium nucleus* (one proton and one neutron), releasing a positron and a neutrino, while converting a proton into a neutron via beta-plus decay.

$$ {}^1_1\mathrm{H} + {}^1_1\mathrm{H} \rightarrow {}^2_1\mathrm{H} + {}^0_1\beta^+ + \nu_e $$

2. **Deuterium-proton reaction**: The newly formed deuterium nucleus fuses with another proton, resulting in a helium-3 nucleus (two protons and one neutron). This releases a gamma ray.

$$ {}^2_1\mathrm{H} + {}^1_1\mathrm{H} \rightarrow {}^3_2\mathrm{He} + \gamma $$

3. **Helium-3 fusion**: Two helium-3 nuclei from stage 2 fuse to form a helium-4 nucleus (two protons and two neutrons) while releasing two protons. This stage releases about 12.9 MeV of energy.

$$ {}^3_2\mathrm{He} + {}^3_2\mathrm{He} \rightarrow {}^4_2\mathrm{He} + 2{}^1_1\mathrm{H} $$

The overall reaction is written as

$$4{}_1^1\mathrm{H} \rightarrow {}_2^4\mathrm{He} + 2{}_1^1\mathrm{H} + 2{}_1^0\beta^+ + 2\nu_e + \gamma$$

or equivalently

$$2{}_1^1\mathrm{H} \rightarrow {}_2^4\mathrm{He} + 2{}_1^0\beta^+ + 2\nu_e + \gamma$$

2 HR Diagrams

By the Stefan-Boltzmann law, large stars with high temperatures must be bright. The pattern of stars can be shown on a scatter plot of luminosity against surface temperature; this is known as the Hertzsprung-Russell (HR) diagram.

Notes about the diagram:

- The plot is logarithmic on both axes.
- The log. temperature (x) axis is plotted in decreasing order from left to right.
- The log. luminosity (y) axis is plotted in increasing order from bottom to top.
- An alternative x-axis is the spectral class, which is another measure of the temperature of the star.
- Each of the lines in Figure 1 represents a set of stars with the same radius: By the Stefan-Boltzmann law, $L = 4\pi\sigma R^2 T^4$; for a constant radius, $L \propto T^4$ and on a log-log plot, this is $\ln L \propto \ln T$, a linear relationship.

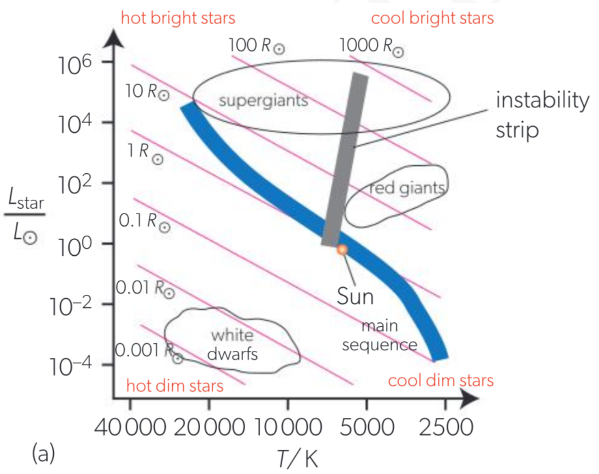
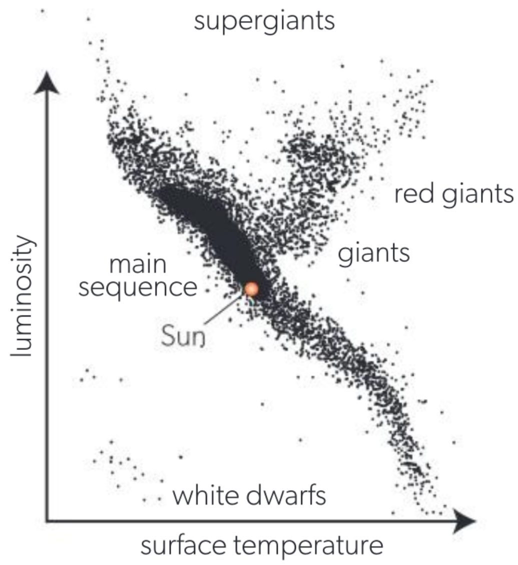


Figure 1: Principle regions and lines of equal radii



(b)

Figure 2: Scatter plot

Components of the graph:

1. The blue area highlights the region of **main sequence** stars; this contains 90% of all stars.

2. **Red giants** are cooler than the Sun but have a larger radius. Although they emit less energy per unit area, their combined area is larger, making them equally bright as some main-sequence stars.

3. **Supergiants** are brighter than most main sequence stars and very large. Some of them emit 10^5 times the energy of the Sun, and thus have a S.A. 10^5 times that of the Sun. This means the radius is $\sqrt{10^5}$ times that of the Sun.

   Together with the red giants, these only constitute 1% of all stars; they have a much shorter lifetime in comparison to the main-sequence.

4. **White dwarfs**: The residue of old stars that are extremely hot and dense but have a small radius and a low luminosity. They make up about 9% of all stars. They take billions of years to cool.

5. **The instability strip**: Indicated by the almost vertical gray line in Figure 1. Stars above the mass of the Sun enter this region when they reach the end of the main sequence lifetime, become unstable, and pulsate, changing their size cyclically (alternating between increasing and decreasing). Thus, their luminosity varies too.

2.1 Note on Notation

The subscript $\odot$ denotes the value of this particular variable of the Sun. For example, $L_\odot$ is the luminosity of the Sun, $R_\odot$ is the radius of the Sun, $T_\odot$ is the temperature of the Sun, and $M_\odot$ is the mass of the Sun.

3 Stellar Evolution

3.1 Birth of Stars

The *stellar medium* is the region of space between stars that contains gas (mainly hydrogen and helium) and dust (mostly carbon and silicon). At a cool temperature, the density of this medium is as low as 10^{-4} ions/cm^3. This is better than any achievable vacuum on Earth.

A gas cloud of the **stellar medium** is compressed by gravity and the cloud begin to increase in temperature and pressure. This is because the GPE of particles is released and transferred to their thermal energy. The cloud becomes a **protostar**.

Later, the star enters the HR-diagram from the right-most side, as a **main sequence star**.

3.2 Main Sequence

During the stability period as a main sequence star, the star is in **hydrostatic equilibrium**, where the inward gravitational force is balanced by the outward thermal & radiation pressure force. The star is stable and does not change in size. The **greater the mass** of the star, the **greater both forces** are.

The above suggests that the **luminosity** of the star has a positive correlation with the **mass** of the star.

1. For a more massive star, the core temperature must be higher to produce a higher pressure, balancing the higher gravitational force.

2. The higher core temperature results in a higher rate of fusion, producing more energy.

By the theory of nuclear physics, if T is the lifetime of the star and M is the mass of the star, then

$$T \propto M^{-2.5}$$

Thus, one can conclude the following

The larger the mass, the shorter the time the star remains on the main sequence.

Approaching the end of the main sequence, the star begins to **run out of hydrogen** fuel, as most have been turned into helium and no longer available for fusion. The outward pressure decreases — the forces are now unbalanced and the star **begins to shrink** due to the stronger gravitational force. The core temperature increases, and the outer layers expand, cooling down and turning red. The star becomes a **red giant** or a **supergiant**, depending on the mass of the star.

3.3 Stellar Evolution of Low-Mass Stars

This includes those with mass $M < 4M_{\odot}$.

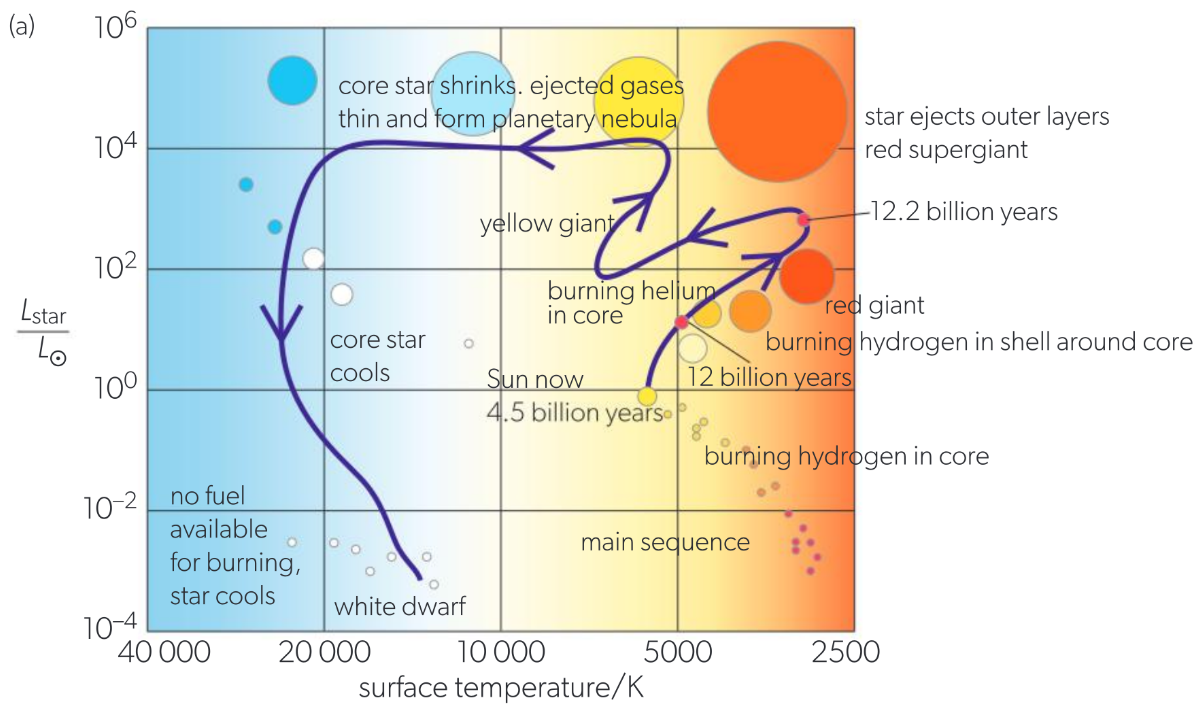


Figure 3: Stellar evolution of low-mass stars

1. Due to having lower masses, the core of these stars do not suffice for fusion of any product beyond carbon. When the helium in the core is exhausted, the core contracts as it continues to emit radiation.

2. A double shell forms around the core: An inner shell of helium and an outer shell of hydrogen. The outer shell of hydrogen begins to fuse, causing the star to expand and cool down, becoming a **red giant**.

3. As the outer shell expands away from the core, a **planetary nebula** is formed. The core then further contracts; it will shrink to the size of the Earth, containing ions together with a degenerate electron gas.

4. At this stage, the core cannot shrink further. The electron gas exerts a pressure called the **electron degeneracy pressure**, which prevents the star from further collapsing. The star is now a **white dwarf**.

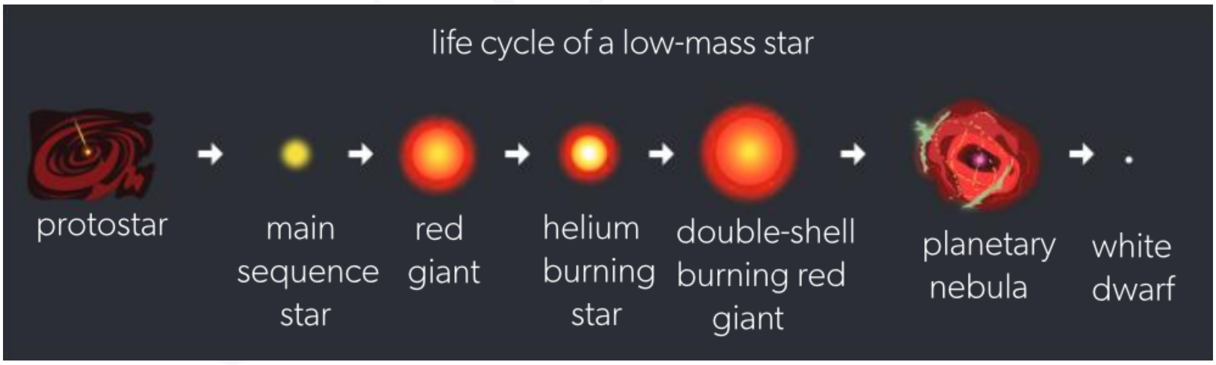


Figure 4: Stellar evolution of low-mass stars

3.4 Stellar Evolution of Large-Mass Stars

This includes those with mass $M > 4M_{\odot}$.

1. In the giant stage, the higher temperature and pressure create elements heavier than carbon.
2. The giant phase ends when the star has a layered structure (like an onion), with iron ash in the center. The inner layers have elements with higher proton numbers — i.e. the proton number decreases as we move outwards.
3. Similar to low-mass stars, the core contracts; but it does not reach stability due to the higher mass.
4. The **Chandrasekhar limit** predicts that stars with a remnant mass above 1.4 $M_{\odot}$ cannot turn into white dwarfs.
5. As the core continue to contract, electrons and protons combine to form neutrons, releasing neutrinos.
6. Both the cores and outer layers of the star rapidly collapse, causing a huge explosion known as a **supernova**.
7. After the explosion, the outer layers are blown away, leaving only the core, small but extremely dense. Two outcomes are possible from this point
   (a) The core remains as a **neutron star**. By the Oppenheimer-Volkoff limit, the mass of a neutron star is limited to 2-3 $M_{\odot}$. Beyond this limit, the core collapses further into a black hole.
   (b) The core collapses further, forming a **black hole**.

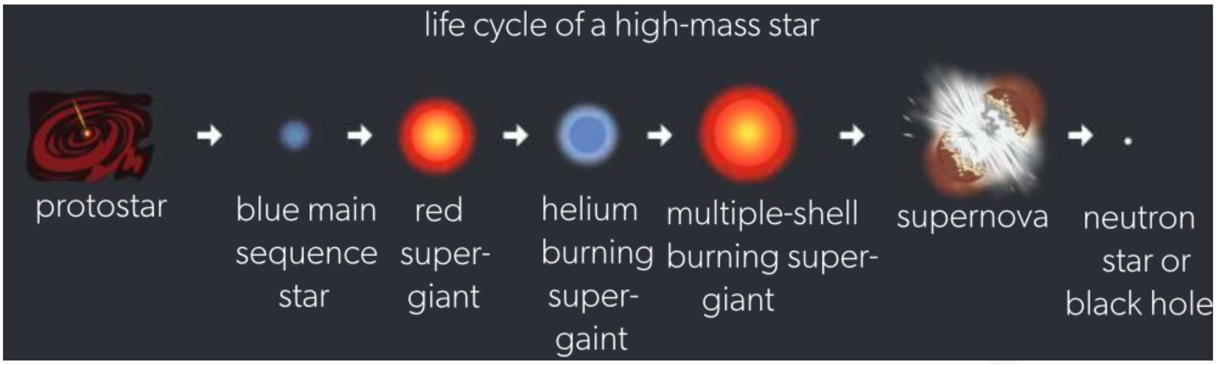


Figure 5: Stellar evolution of high-mass stars

3.4.1 Detecting Black Holes

1. X-rays are emitted as mass spirals towards the edge of the black hole and heats up at it travels. Satellites with X-ray detectors can detect these emissions.

2. Conjecture: Only rotating black holes can emit jets of matter from the core of galaxies.

3. Some stars spiral for no apparent reason; this is likely due to the strong gravitational pull of a black hole nearby.

4 Astronomical Distances and Measurement

- *Light years*: The distance light travels in a year, approximately 9.47×10^{15} m.
- *Astronomical units*: The **average distance** between the Earth and the Sun, approximately 1.5×10^{11} m or equivalently 8 light minutes.
- *Parsecs*: 3.26 light years, or approximately 3.1×10^{16} m. The typical order of nearby stars (to the Earth) is parsec, that of distant stars in the same galaxy is kiloparsec, and that of distant galaxies is megaparsec/gigaparsec.

5 Stellar Parallax

- *Parallax* is the visual effect where the position of an object relative to a background appears to change as the viewer's position changes.
- *Stellar parallax* is the use of the geometry of parallax to find the distance of nearby stars.

We use this method when the stars we are observing are "fixed" in the background.

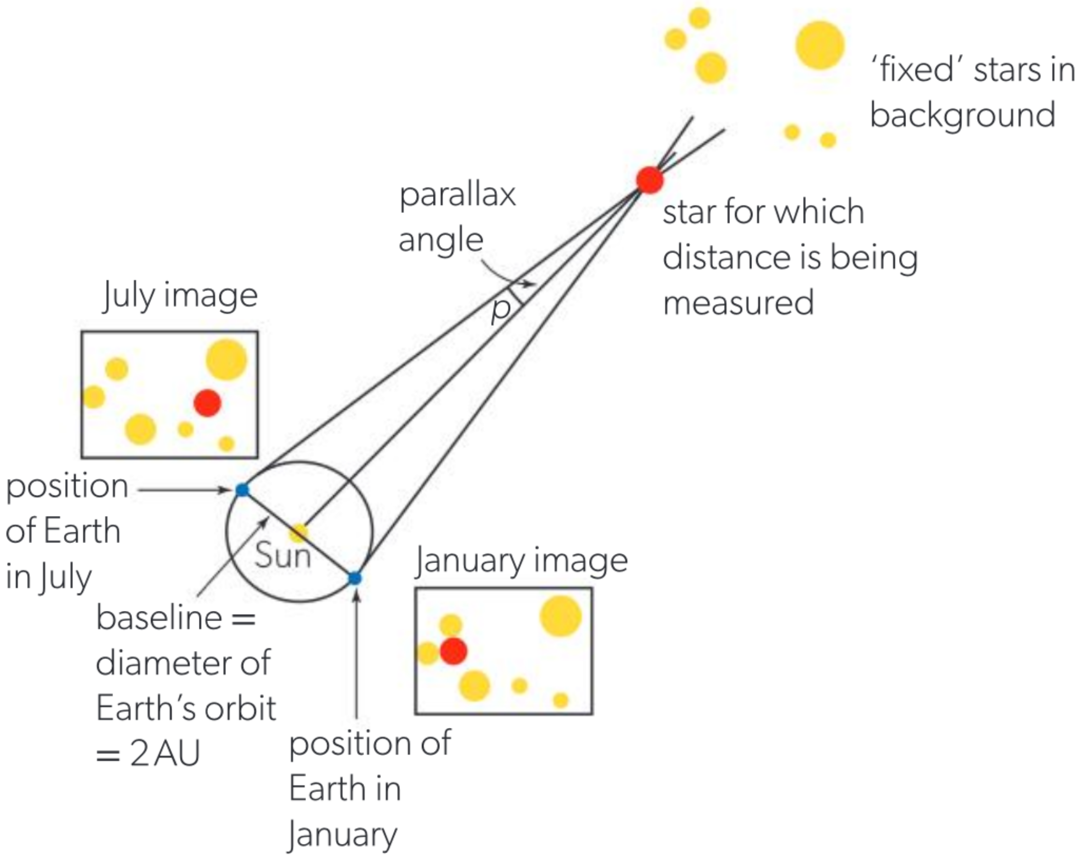
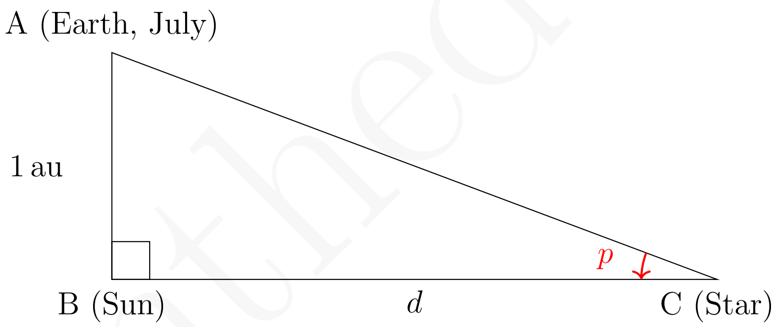


Figure 6: Stellar parallax

Outline of the procedure:

1. Observe the star at two different times, six months apart. The Earth's position is now on the opposite side of the Sun.
2. The angle between the two lines of sight is the displacement angle, $\Delta\theta$.
3. The parallax angle is found by halving the displacement angle, i.e. $p = \frac{1}{2}\Delta\theta$
4. We know that the distance between the two points of observation is $2\,\text{au}$, where $1\,\text{au}$ is the average distance between the Earth and the Sun (and thus the orbital radius).
5. We can then obtain the following right-triangle, where BC is the desired distance from the distant star to the Sun.



6. By small angle approximation, $p \approx \tan p = \dfrac{d}{1\,\text{au}}$.
7. In parsecs, this is simply $d = \dfrac{1}{p}$.

5.1 Convenient Units

Because we are dealing with "microscopic angles" in stellar parallax, scientists have introduced the unit of *arcseconds* $(^\circ)$ to measure these angles. This is defined as $\dfrac{1}{3600}$ of a degree. Similarly, the unit of *arcminutes* is defined as $\dfrac{1}{60}$ of a degree.

6 Stellar Radius

Determining the stellar radius:

1. By the Stefan-Boltzmann law, $L = 4\pi\sigma R^2 T^4$, where L is the luminosity, R is the radius, and T is the temperature of the star.
2. The apparent brightness of a star is given by $b = \frac{L}{4\pi d^2}$, where d is the distance from the star to the observer.
3. These quantities can be compared with those of the sun; this is helpful in questions where we are just asked to determine the quantity of a star in terms of that of the sun

   (a) The luminosity ratio (star:sun) is given by

   $$\frac{L}{L_\odot} = \frac{R^2 T^4}{R_\odot^2 T_\odot^2}$$

   (b) The stellar radius ratio (star:sun) is given by

   $$\frac{R}{R_\odot} = \sqrt{\frac{L}{L_\odot}\frac{T_\odot^4}{T^4}} = \frac{T_\odot^2}{T^2}\sqrt{\frac{L}{L_\odot}}$$

   (c) The apparent brightness ratio (star:sun) is given by

   $$\frac{b}{b_\odot} = \frac{L}{L_\odot}\frac{d_\odot^2}{d^2}$$