IB Physics Topic D4 Electromagnetic Induction; HL

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1 Fleming's Left and Right Hand Rules

The left-hand rule is for the motor effect; the right-hand equivalent is for the generator effect.

Induction and motor effects are intertwined and must co-occur. Consider a bar moving through a magnetic field. The motion would induce a current in the rod, which is explained by the left-hand rule that suggests that electrons feel a force and thus there is a current. The induced current would lead to a motor effect, creating a force that opposes the bar's motion. Thus there is a deceleration, if the system is isolated.

2 The Generator Effect

When there is a relative motion between a conductor and a magnetic field, a current/ε is induced in the conductor. Example scenarios include:

- Moving a magnet away from or into a coil.
- Moving a bar of conductor in a magnetic field
- Moving a charge in a magnetic field

2.1 Lenz's Law

The induced magnetic current in a coil by inserting a magnet into the coil must oppose the motion creating the current. Analogously, the insertion must be north-to-north or south-to-south; conversely, if we are pulling the magnet away from the coil, the facing poles should be opposite and attractive.

The induced current is continuous only if there is a complete circuit for charge to flow.

2.2 Right-hand Grip

This rule determines where the north-pole of a solenoid/coil is. The direction in which the fingers curl is the direction of current, and the thumb points to the north-pole.

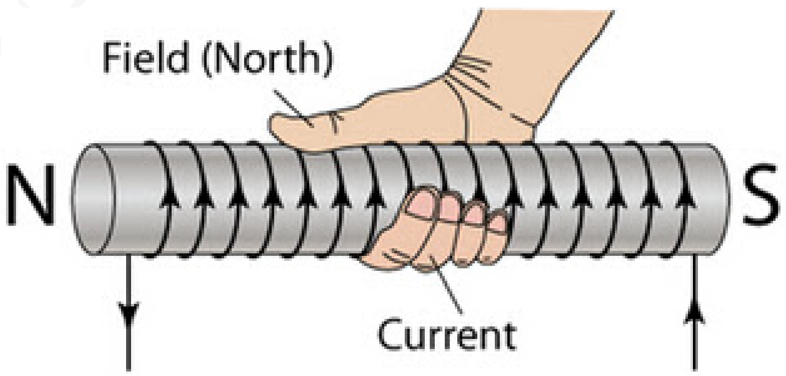

Figure 1: Right Hand Grip

2.3 Explaining the Generator Effect

Consider a conducting bar moving through a magnetic field.

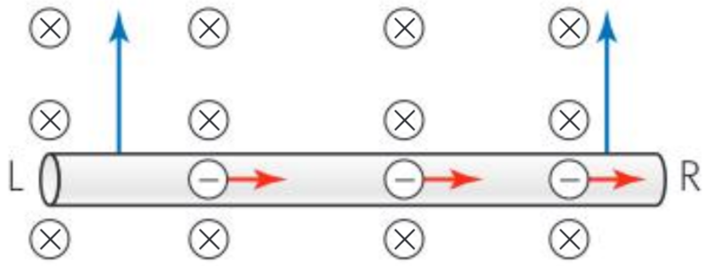

Figure 2: A conductor moving in a uniform magnetic directed into the page

Using Flemming's left-hand rule to analyze **the motion of the electrons**, we can see that there is a (conventional) current to the left, since the electrons are pushed to the right. There is now a p.d. between L and R, with L at a higher potential.

As the negative charges accumulate, another force, opposing the force pushing the electrons to R, arises — the electric force. Like charges repel, and hence, as the electrons accumulate, the electric force increases, opposing the motion of the electrons. Eventually, the two forces balance out.

- The force pushing the electrons to R is the magnetic force, given by $F_B = Bqv$
- The opposing electric force is given by $F_e = Eq$, where $E = \varepsilon$ the induced **emf**; by $E = \dfrac{V}{d}$, we have $F_e = \dfrac{\varepsilon q}{d}$, where d is, in fact, the length of the rod. Let's denote that as l and hence $F_e = \dfrac{\varepsilon q}{l}$
- Equating the two gives the following, which allows us to find the **induced emf**

$$Bv = \frac{\varepsilon}{l}$$

$$\varepsilon = Blv$$

2.4 External Force to Achieve Constant Velocity

Suppose, we have a conducting bar rolling (without friction) on a pair of parallel conducting rails.

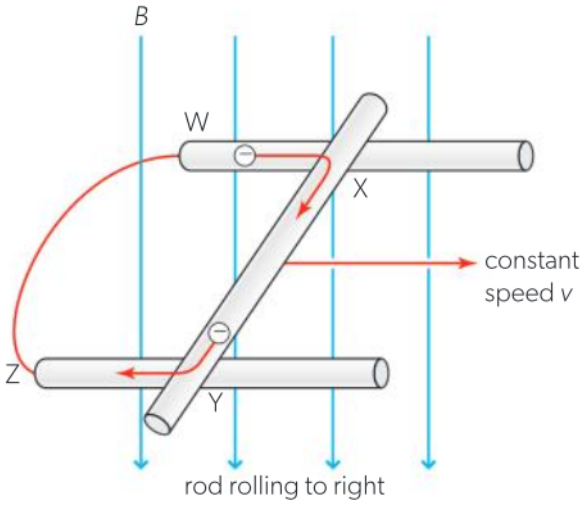

Figure 3: 3D View

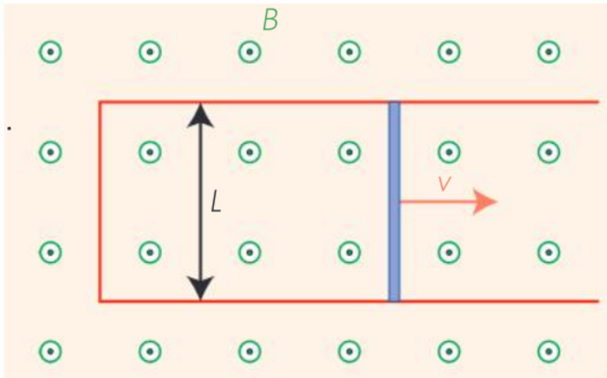

Figure 4: Top View

There must be an external force opposing the magnetic force causing the rod to roll to the right. This force is given by $F_{\text{ext}} = BIL$, where I is the current in the rod. This force is given by $F_B = BIL$ because it is the negative equivalent of F_B.

If we are considering the rate at which work is done to oppose the motion to the right then it's $F_B v$ using $P = Fv$ or equivalently $P = I\varepsilon$, using the circuit equation.

2.5 Deriving the Induced emf Using Energy

We know $\varepsilon = \frac{W}{Q}$, thus

$$\begin{aligned}\varepsilon &= \frac{\text{work done by the magnetic force}}{\text{total charge flowing}} \\ &= \frac{F_B \times vt}{It} \\ &= \frac{BILvt}{It} \\ \varepsilon &= Blv\end{aligned}$$

2.6 Area Swept Out by the Rod

Consider the area swept out by the rod, which is given by $A = Lvt$ and $\frac{\Delta A}{\Delta t} = Lv$

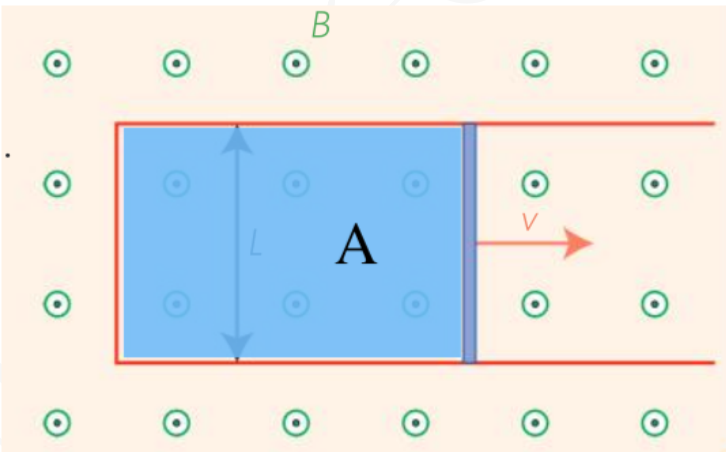

Figure 5: The area swept by the rod

we can incorporate this into our derivation of the induced emf and get

$$\varepsilon = B \times \frac{\Delta A}{\Delta t} = B \times \text{ rate of change of area} \tag{1}$$

3 Magnetic Flux

The **magnetic flux** is given by

$$\Phi = BA\cos\theta$$

where B is the magnetic field strength, A is the area of the coil, and θ is the angle between the magnetic field and the normal to the surface. The quantity Φ has unit webers (Wb).

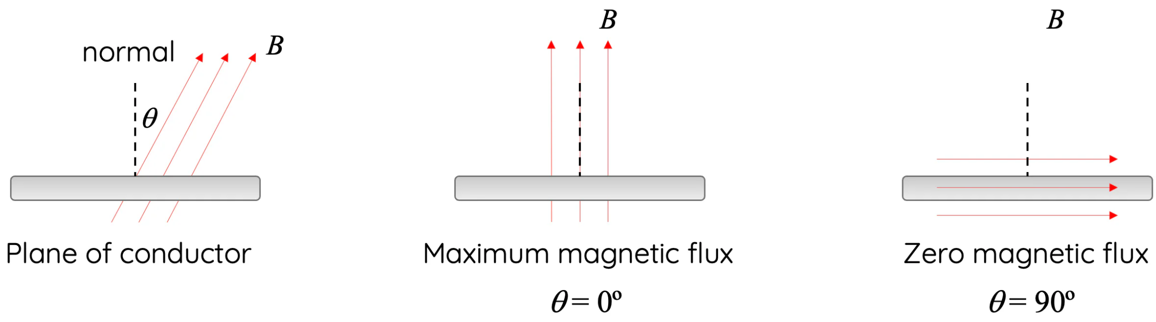

Figure 6: Magnetic Flux Density

The **magnetic flux density** is numerically analogous to the electric field strength; both quantities are denoted by B. It represents the number of field lines per unit area. Thus, the magnetic flux is the number of field lines passing through a surface of area A.

This also allows us to rewrite Equation (1) as

$$\varepsilon = \frac{\Delta\Phi}{\Delta t}$$

and we can then define the unit of flux, a weber, as **the flux that produces an emf of 1 volt per second**. Also, one tesla is defined as 1 weber per square meter.

$$1\text{T} \equiv 1\text{Wb}\,\text{m}^{-2}$$

3.1 Magnetic Flux Linkage

Previously, we considered a single rod rolling along two rails. Now, let's consider a coil of N turns, each of area A, in a magnetic field. The magnetic flux linkage is given by

$$\Lambda = N\Phi = BAN$$

the unit of this quantity is also the weber, or weber-turns.

In this case, the induced emf is given by

$$\varepsilon = \frac{\Delta\Lambda}{\Delta t} = N\frac{\Delta\Phi}{\Delta t} = N\frac{B\Delta A}{\Delta t}$$

3.2 Faraday's Law

Faraday's law states that the induced emf is directly proportional to the rate of change of magnetic flux linkage. This is given by

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$$

which is the Neumann's equation that includes both the ideas of Lenz and Faraday.

It suggests that the magnetic flux can be changed by changing at least one of the following quantities:

- $\dfrac{\Delta A}{\Delta t}$
- $\dfrac{\Delta \cos\theta}{\Delta t}$
- $\dfrac{\Delta B}{\Delta t}$

4 Relative Motion Between Coil & Field

4.1 Coil Remains Within a Uniform Field

This is when a coil moves at a **constant speed** from one position to another **completely within a uniform magnetic field**. In this case, there is **no change in flux linkage**, because the same number of field lines are being cut on opposite sides of the coil, where the current is in opposite directions.

4.2 Coil Moves Out of A Uniform Field

When a coil of N turns moves from a position where the flux is Φ to a position where the flux is 0, the change in flux linkage is $-N\Phi$, and thus, the induced emf is given by

$$\varepsilon = -\frac{N\Phi}{\Delta t}$$

4.3 Rotation of a Coil in a Uniform Field

When the coil is rotated by $180°$ in a uniform field, the change in flux linkage is $2N\Phi$, and thus, the induced emf is given by

$$\varepsilon = -\frac{2N\Phi}{\Delta t}$$

because the field lines reverse their direction.

4.4 Changing Magnetic Field

The coil remains stationary, but the magnetic field cutting through it changes. In this case, the induced emf is given by

$$\varepsilon = -NA\frac{\Delta B}{\Delta t}$$

We do not need to know the exact field strength, only the **rate of change of the field strength** is sufficient.

There are two graphs that can be drawn.

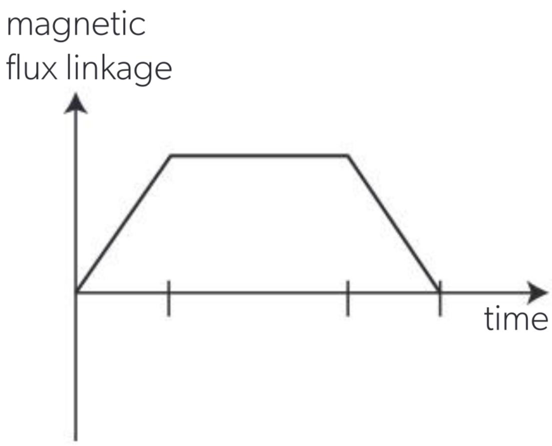

Figure 7: Flux Linkage vs. Time

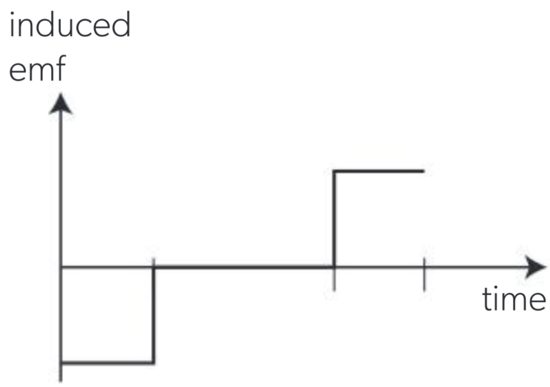

Figure 8: emf vs. Time

- The gradient of the first graph is the emf
- The area under the second graph is the total change in flux linkage.

5 Generators

Key components of an AC generator include:

- A coil of wire rotating in a magnetic field
- A magnetic field (e.g. from a bar magnet)
- Relative movement between the coil and the field
- A suitable connection to the static circuit outside the generator

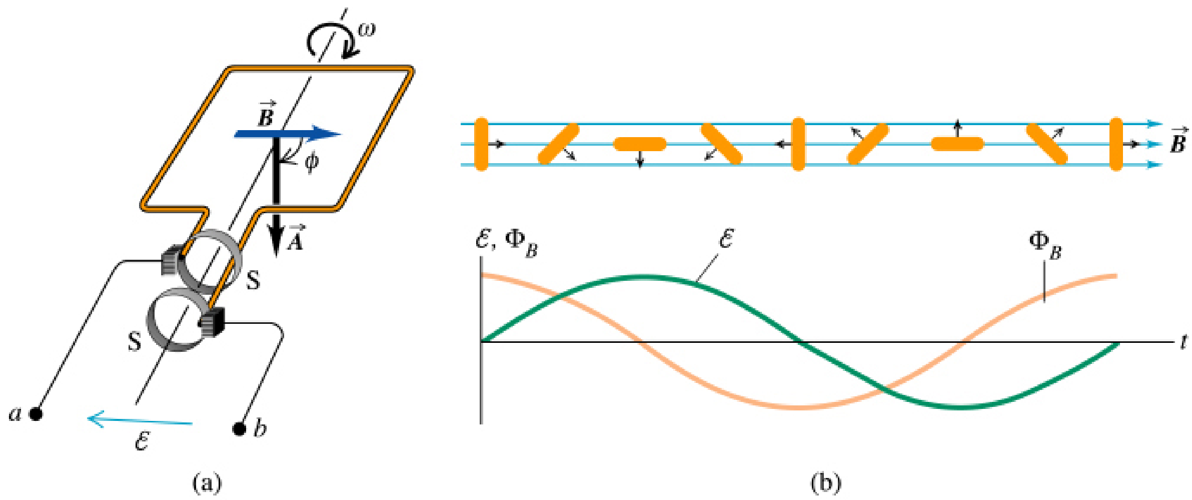

Figure 9: A simple generator

Relationship between emf and flux linkage:

- The induced emf is the rate of change of magnetic flux linkage.
- Since $\Phi = BA\cos\theta$, the magnetic flux linkage is maximum when $\theta = 0$, e.g. when the coil is perpendicular to the field. However, this is the moment at which the induced emf is 0; consider the taking the partial derivative of Φ with respect to θ — ε will involve $\sin\theta$.

- Conversely, the magnetic flux linkage is minimum when $\theta = 90°$, e.g. when the coil is parallel to the field. This is the moment at which the induced emf is maximum.

Effect of increasing the angular speed of the coil:

- Frequency (cycles per second) increases
- This squashes the graph, increasing the rate of change of linkage with respect to time, thus increasing the peak emf.

Other ways of increasing the induced emf

- Increasing flux density
- Increasing the number of turns on the coil
- Increasing the coil area

5.1 Power

The voltage (induced emf) and current graphs are both sinusoidal, with different vertical scaling factors but the same zeroes. Since power is calculated by $P = IV$, the power graph will be a **sine squared graph**.

- The mean power is given by $\frac{1}{2}V_{\text{rms}}I_{\text{rms}}$
- $I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}}$ (root mean square)
- $V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}}$ (root mean square)

6 Mutual and Self-Induction

6.1 Mutual Induction

This is the process by which a changing current in one coil induces an emf in another coil nearby. The emf induced is given by

$$\varepsilon = -M\frac{\Delta I_1}{\Delta t}$$

where M is the mutual inductance of the two coils. The unit of mutual inductance is the henry (H).

- When I_1 is increasing or decreasing, there is a change in the magnetic field and thus an induced emf.
- When I_1 reaches a constant level and stops changing, the induced emf is 0.
- This is because the presence of an induced emf requires a **non-zero rate of change in the magnetic flux linkage**.
- By Lenz's Law, the induced emf/current **opposes the change in** I_1 (not I_1). I.e., if the current is increasing, the induced current is in the opposite direction; if the current is decreasing, the induced current is in the same direction.

This is the effect behind transformers.

6.2 Self-Induction

Occurrence of the phenomenon:

- Current starts flowing through a wire or coil. As the current increases, a magnetic field grows around the wire.
- But when the current changes, the magnetic field changes too. According to Faraday's Law, a changing magnetic field induces a voltage.

- This newly induced voltage, however, opposes the very change that created it (by Lenz's Law). It is as if the coil resists the current's attempt to speed up or slow down. This resistance to change is what we call self-induction.

7 Exam Questions

7.1 Terminal Velocity

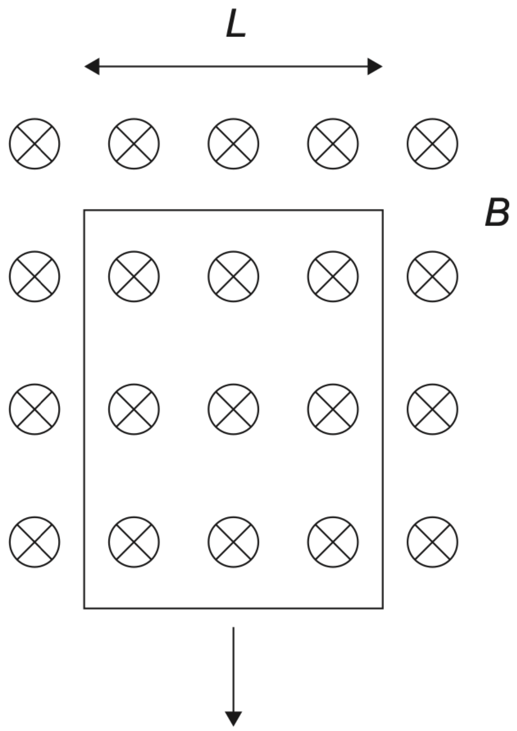

Figure 10: Dropped loop

A vertical rectangular loop of conducting wire is dropped in a region of horizontal magnetic field. The diagram shows the loop as it leaves the region of the magnetic field. Just before the loop is about to completely exit the region of magnetic field, the loop moves with constant terminal speed v. The following data is available; find v.

Mass of loop	$m = 4.0\text{g}$
Resistance of loop	$R = 25\,\text{m}\Omega$
Width of loop	$L = 15\,\text{cm}$
Magnetic flux density	$B = 0.80\,\text{T}$

1. The terminal velocity is reached when forces are balanced. Let's identify the two opposing force that must be equal in magnitude in this case:

   - The downward force is due to gravity, $w = mg$.
   - The upward force is due to the magnetic field, $F = BIL$.

2. Let us now list the known quantities

   - $w = mg = 4 \times 10^{-3} \times 9.8$
   - $BL = 0.8 \times 0.15$

3. Now we must bring the velocity into this equation somehow, which hints at the use of the formula $\varepsilon = Blv$. However, the emf can be expressed as $\varepsilon = IR$, and thus $Blv = IR$.

4. In the equation $mg = BIL$, I is the unknown quantity that we do not desire to find, thus we proceed by making the substitution $I = \frac{Blv}{R}$.

5. Combining everything together, we have

$$mg = BL(\frac{Blv}{R})$$
$$v = \frac{mgR}{B^2L^2}$$

6. This will allow us to compute the desired value of v.