

Math AA HL at KCA - Chapter 3 Notes

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1 Laws of Exponents

1. $a^m \times a^n = a^{m+n}$

2. $a^m \div a^n = a^{m-n}$

3. $(a^m)^n = a^{mn}$

4. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

5. $a^{-m} = \frac{1}{a^m}$

6. $a^0 = 1, a \neq 0$

2 Exponential Functions

They take the form of $y = ab^x + c$ and can grow very rapidly.

The graph of exponential functions will be shown later.

3 Logarithms

1. $\ln x = \log_e(x)$

2. $\log x = \log_{10}(x)$

3. $\log_a(x) = \frac{\log_b(x)}{\log_b(a)} = \frac{\ln x}{\ln a}$, where b can be any real number.

4. $\ln(xy) = \ln x + \ln y$

5. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

6. $\ln(x^k) = k \ln x$

4 Exponential Growth and Decay

The exponential function in the form

$$y = p \times a^{x-h} + k$$

- will **grow** if $a > 1$; the function is increasing
- will **decay** if $a < 1$; the function is decreasing

The following shows an exponentially growing (red) and a decaying (purple) function.

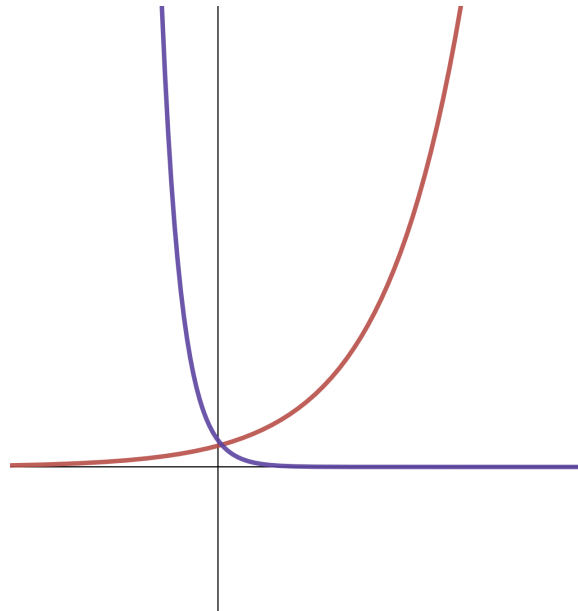


Figure 1: Exponential functions

5 Modeling Exponential and Power functions

1. An exponential function in the form of $y = ab^x$ can be modeled using the linear function $\log y = x \log b + \log a$
2. Power functions take the form $y = kx^n$ and $\log y = n \log x + \log a$ will be a straight line.