Math AA HL at KCA - Chapter 1 & 2 Notes

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1 Discriminants

The discriminants of a quadratic function

$$ax^2 + bx + c$$

is given by

$$D = b^2 - 4ac$$

1. When $D = 0$, there is **only one** distinct root
2. When $D > 0$, there are **two** distinct roots
3. When $D < 0$, there are **two complex** roots, pairwise conjugate

Typical Exam Question

Let $f(x)$ be a quadratic and $g(x)$ be another quadratic or a linear function. Now the question is, "Find some sort of constant, e.g. k, in the coefficient of $f(x)$", when

1. Either $f(x)$ and $g(x)$ has no intersections,
2. Or $f(x)$ and $g(x)$ has two intersections
3. Or $f(x)$ and $g(x)$ has exactly one intersection

Key idea: set $f(x) - g(x) = 0$ and use the discriminant of the resulting function $f(x) - g(x)$ to tackle the target case.

2 Linear Functions and Gaussian Elimination

The straight line distance between two points is given by

$$\sqrt{\Delta x^2 + \Delta y^2}$$

Properties of linear functions

- Two parallel lines have gradients $m_1 = m_2$
- Two perpendicular lines have gradients $m_1 = -\frac{1}{m_2} \iff m_2 = -\frac{1}{m_1}$
- The point of intersection between $f(x)$ and $g(x)$ is found by solving $f(x) = g(x)$

Gaussian elimination is a way of solving a system of linear equations. The goal is to transform all numbers below the main diagonal to zero.

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & -5 \\ 0 & 1 & -4 & 9 \\ 0 & 0 & 2 & 8 \end{array}\right]$$

Figure 1: Gaussian Elimination

Let the third column represent the variable z and the final row after elimination be $az = b$. The following summarizes the implications of the final row

1. $0z = b\,, b \neq 0 \implies z$ has no solution; **inconsistent**
2. $0z = 0 \implies z$ has infinitely many solutions, and the values of the other variables will be written in forms such as parametric equations. E.g. $z = t, x = 3t - 2, y = \frac{t}{2}$; **consistent**
3. $az = b$ with $a \neq 0 \implies$ the system has a unique solution; **consistent**.