Math AA HL at KCA - Chapters 6 to 9 Notes

Tim Bao 2023-2025

February 25, 2024

1 Quadratics

A quadratic function can be in any of the following three forms

Form	General expression	Features
Standard	$ax^2 + bx + c$	$c = y$-intercept
Vertex	$a(x-h)^2 + k$	(h, k) vertex; $x = h$ line of symmetry
Factored	$a(x-\alpha)(x-\beta)$	α, β roots

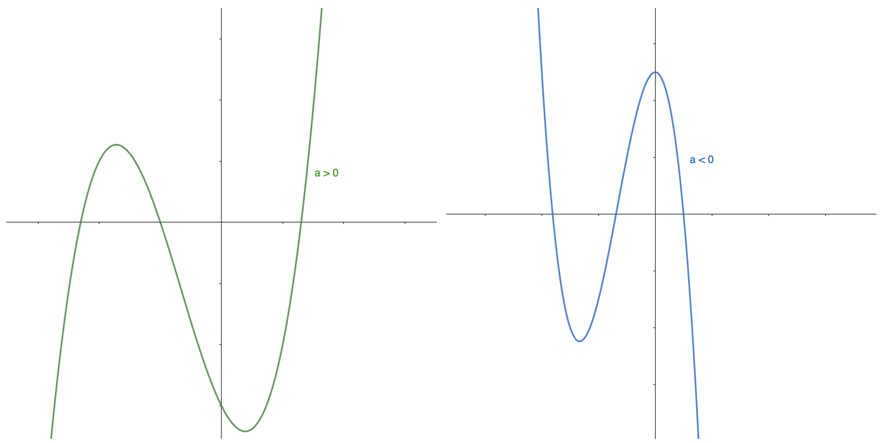
The leading coefficient a determines the concavity of the function

- $a > 0 \implies$ the function is convex (tends to ∞)
- $a < 0 \implies$ the function is concave (tends to $-\infty$)

2 Sketching Cubics and Higher Degree Graphs

A polynomial of degree n can have at most n distinct roots.

The following features are to be considered when sketching a polynomial $f(x)$ with leading coefficient a



(a) $a > 0$ (b) $a < 0$

1. Behaviors as $x \to \pm\infty$; for a cubic:
   (a) $a > 0 \implies$ as $x \to \pm\infty$, $y \to \pm\infty$
   (b) $a < 0 \implies$ as $x \to \pm\infty$, $y \to \mp\infty$
2. If the roots are α, β, γ, then the x-intercepts are $(\alpha, 0)$, $(\beta, 0)$, $(\gamma, 0)$.
3. The y-intercept, this can be found by $f(0)$
4. The turning point(s), if applicable and necessary.

3 Mappings

A **relation** describes how values are mapped from one set onto another.

In the case of functions, values from the **domain** are mapped onto values in the **range**.

There are four types of mapping

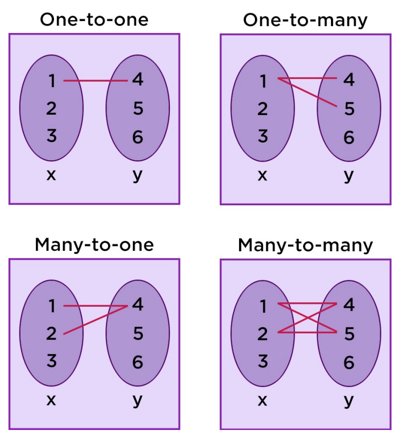


Figure 2: Mappings

For a relation to be a function, it has to be either one-to-one or many-to-one. This is because each x value inputted into the function must be associated with a single y value.

3.1 Interval Notations

An opening/closing square bracket represents an inclusive end.

An opening/closing parenthesis represents an exclusive end.

E.g.

- $x \in [a, b) \iff a \leq x < b$
- $x \in (-\infty, h] \iff x \leq h$

3.2 Domain and Range

The **range** is the set of x values that can go into a function, and the **range** is the set of y values that can come out of a function.

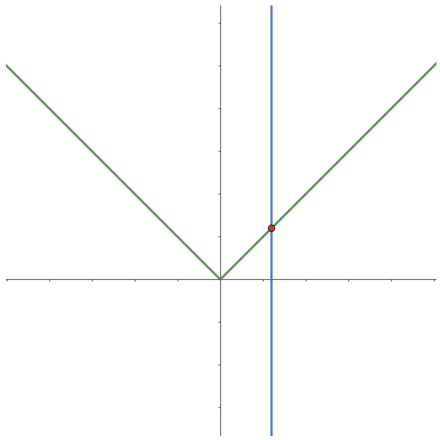
Notations for expressing a domain

1. $x \in [a, b]$
2. $a \leq x \leq b$
3. $R = \{x \mid a \leq x \leq b\}$

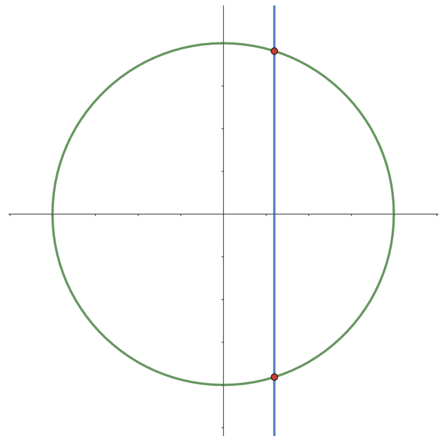
Notations for expressing a range

1. $y \in [a, b]$
2. $f(x) \in [a, b]$
3. $a \leq f(x) \leq b$
4. $D = \{y \mid a \leq y \leq b\}$

3.3 Restricting the Domain



(a) Function



(b) Not a function

A relation is not a function if it does not pass the **vertical line test**, which states that: If *at any point* in the domain of a relation, a vertical line intersects the graph at two or more points, then it is not a function.

For such a relation to be a function, the domain needs to be restricted such that the new domain passes the vertical line test.

4 Composite Functions

$$(f \circ g)(x) = f(g(x))$$

Properties

- Associativity $f \circ (g \circ h) = (f \circ g) \circ h$
- $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

5 Inverse Functions

The inverse function of a function $f(x)$ is denoted $f^{-1}(x)$, it is a reflection in the line $y = x$.

A **self-inverse** function is its own inverse, i.e. $f(x) \equiv f^{-1}(x)$. E.g. $y = \dfrac{1}{x}$ is a self-inverse function.

Implications:

- $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$
- The *domain* of f is the *range* of f^{-1}, and vice versa
- The intersections of $f(x)$ and $f^{-1}(x)$ lie on the line $y = x$ and can be found by $f(x) = x$ or $f^{-1}(x) = x$

To find an expression for the inverse function, exchange the x's and y's, then make the new y the subject.

6 Even and Odd Functions

1. Even functions: $f(-x) = f(x)$; symmetrical about the y-axis
   (a) $f(x)$ has only even powers of x
2. Odd functions: $f(-x) = -f(x)$; rotational symmetry of 180° about the origin
   (a) $f(x)$ has only odd powers of x