

Math AA HL at KCA - Chapter 10 to 11 Notes

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February 26, 2024

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1 Radians, Arc Length, and Area of a Circle

Radian to degree conversion

$$\theta^\circ = \frac{180}{\pi} \theta^c$$

Degree to radian conversion

$$\theta^c = \frac{\theta^\circ}{180} \pi$$

Arc length l enclosed by two radii r at an angle θ

$$l = r\theta$$

Area of a sector

$$A = \frac{r^2 \theta}{2}$$

2 Sine Rule and Area of a Triangle

Full sine rule

$$2R = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where

- R is the circumradius of the triangle

Given two sides a, b and the angle in between C , the area of the triangle is given by

$$A = \frac{1}{2} ab \sin C$$

3 Cosine Rule

Cosine rule for a side

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

For an angle

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

4 Cones

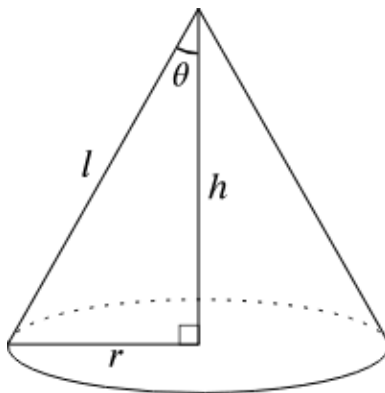


Figure 1: Cone

- $l^2 = h^2 + r^2$
- $A = \pi r^2 + \pi r l$
- $V = \frac{1}{3}\pi r^2 h$

5 Radian-Degree Special Values

<i>Degrees</i>	<i>Radians</i>	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
0°	0	0	1	0	—	1	—
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	—	1	—	0

Figure 2: Conversion Table

6 Trigonometric Graphs

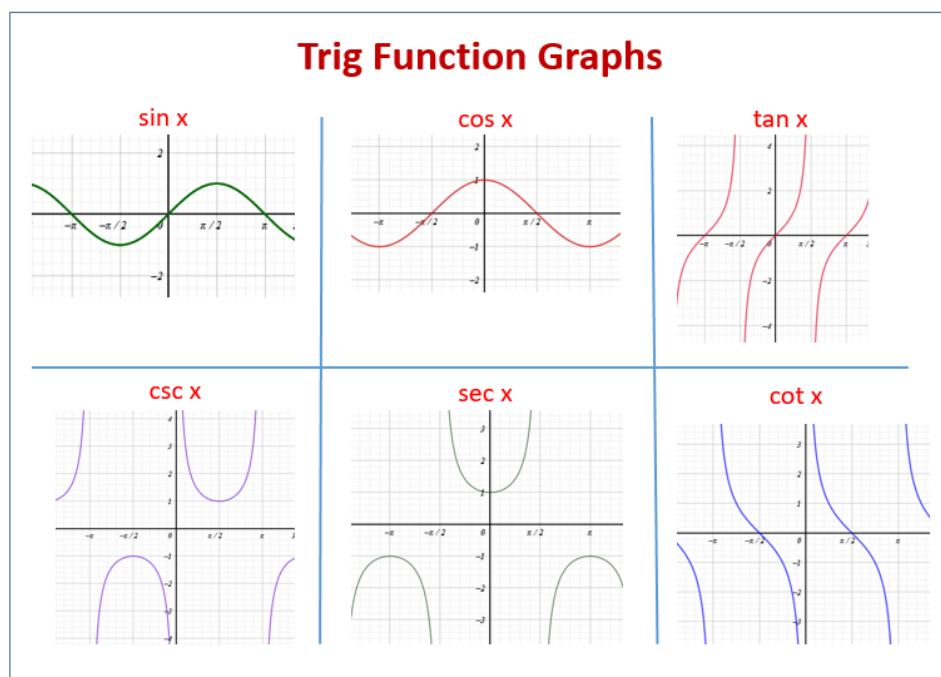


Figure 3: Graphs

Periodicity:

- $\tan(\theta + k\pi) = \tan(\theta), k \in \mathbb{Z}$
- $\sin(\theta + 2k\pi) = \sin(\theta), k \in \mathbb{Z}$
- $\cos(\theta + 2k\pi) = \cos(\theta), k \in \mathbb{Z}$

6.1 Transformations of the Sine/Cosine Graphs

$$y = a \sin(b(x - c)) + d, \quad a, b > 0$$

- Amplitude = $|a|$
- Period = $\frac{2\pi}{b}$
- Principle axis $y = d$
- Maximum $a + d$, minimum $-a + d$

It is obtained from the transformations

- if $a < 0$ then a reflection in the x -axis
- vertical stretch by factor $|a|$
- horizontal stretch by factor $\frac{1}{b}$
- a translation through $\begin{pmatrix} c \\ d \end{pmatrix}$

7 The Unit Circle

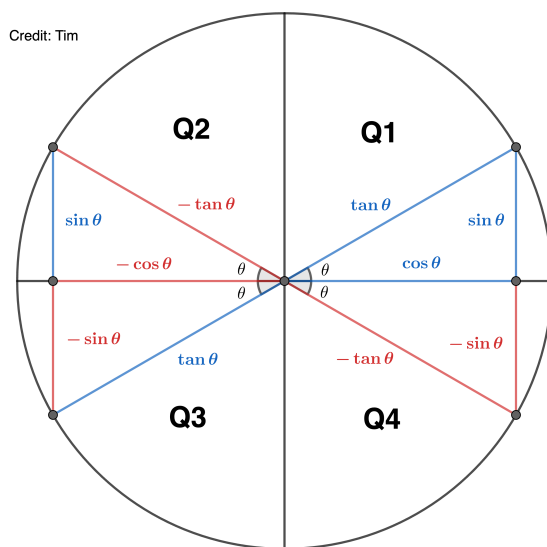


Figure 4: The unit circle

Angles measured counter-clockwise from the positive x -axis are positive, and angles measured clockwise from the x -axis are negative.

A "reference angle" is the angle θ in the first quadrant.

- The corresponding angle in the *second quadrant* is $\pi - \theta$
- The corresponding angle in the *third quadrant* is $\pi + \theta$
- The corresponding angle in the *fourth quadrant* is $-\theta$

Each angle's trigonometric ratios have the same *magnitude* as its reference angle; using information about the quadrant will help to determine the signs.

8 Trigonometric Identities

9 Compound Angle and Half Angle Formulae