Math AA HL at KCA - Chapter 10 to 11 Notes

Tim Bao 2023-2025

February 26, 2024

Contents

1 Radians, Arc Length, and Area of a Circle

Radian to degree conversion

$$\theta^\circ = \frac{180}{\pi}\theta^c$$

Degree to radian conversion

$$\theta^c = \frac{\theta^\circ}{180}\pi$$

Arc length l enclosed by two radii r at an angle θ

$$l = r\theta$$

Area of a sector

$$A = \frac{r^2\theta}{2}$$

2 Sine Rule and Area of a Triangle

Full sine rule

$$2R = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where

- R is the circumradius of the triangle

Given two sides a, b and the angle in between C, the area of the triangle is given by

$$A = \frac{1}{2}ab\sin C$$

3 Cosine Rule

Cosine rule for a side

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

For an angle

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

4 Cones

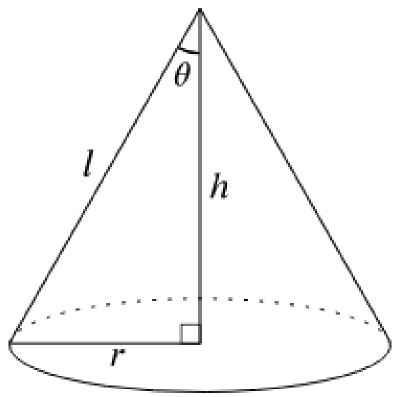

Figure 1: Cone

- $l^2 = h^2 + r^2$
- $A = \pi r^2 + \pi rl$
- $V = \frac{1}{3}\pi r^2 h$

5 Radian-Degree Special Values

Degrees	*Radians*	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0°	0	0	1	0	–	1	–
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	–	1	–	0

Figure 2: Conversion Table

6 Trigonometric Graphs

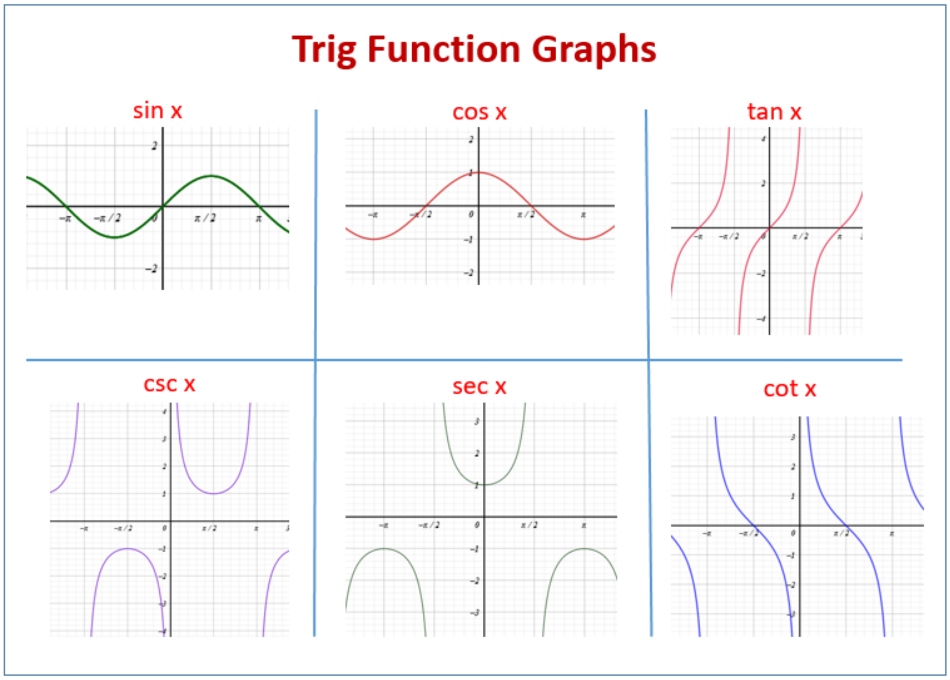

Figure 3: Graphs

Periodicity:

- $\tan(\theta + k\pi) = \tan\theta, k \in \mathbb{Z}$
- $\sin(\theta + 2k\pi) = \sin\theta, k \in \mathbb{Z}$
- $\cos(\theta + 2k\pi) = \cos\theta, k \in \mathbb{Z}$

6.1 Transformations of the Sine/Cosine Graphs

$$y = a\sin(b(x-c)) + d, \quad a, b > 0$$

- Amplitude $= |a|$
- Period $= \dfrac{2\pi}{b}$
- Principle axis $y = d$
- Maximum $a + d$, minimum $-a + d$

It is obtained from the transformations

- if $a < 0$ then a reflection in the x-axis
- vertical stretch by factor $|a|$
- horizontal stretch by factor $\dfrac{1}{b}$
- a translation through $\begin{pmatrix} c \\ d \end{pmatrix}$

7 The Unit Circle

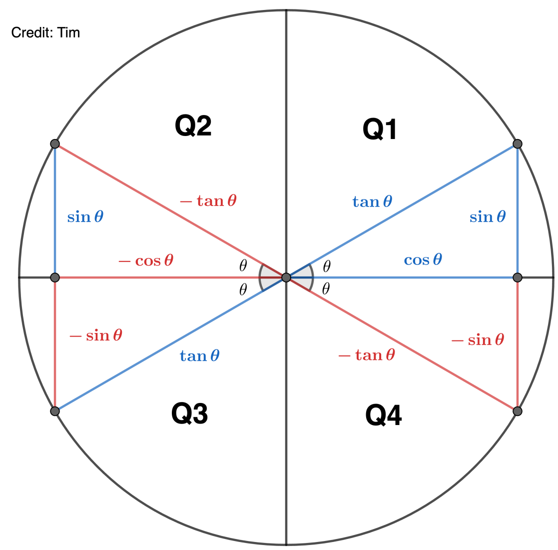

Figure 4: The unit circle

Angles measured counter-clockwise from the positive x-axis are positive, and angles measured clockwise from the x-axis are negative.
A "reference angle" is the angle θ in the first quadrant.

- The corresponding angle in the *second quadrant* is $\pi - \theta$
- The corresponding angle in the *third quadrant* is $\pi + \theta$
- The corresponding angle in the *fourth quadrant* is $-\theta$

Each angle's trigonometric ratios have the same *magnitude* as its reference angle; using information about the quadrant will help to determine the signs.

8 Inverse Trigonometric Functions

- $\csc\theta = \dfrac{1}{\sin\theta}$
- $\sec\theta = \dfrac{1}{\cos\theta}$
- $\cot\theta = \dfrac{1}{\tan\theta}$

9 Trigonometric Identities

9.1 Pythagorean and the tangent identities

- $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \csc^2\theta$

9.2 Double angle

These are just special cases of compound angle identities

- $\sin 2\theta = 2\cos\theta\sin\theta$
- $\cos 2\theta =$

$$\begin{aligned} &\cos^2\theta - \sin^2\theta \\ &= 1 - 2\sin^2\theta \\ &= 2\cos^2\theta - 1 \end{aligned}$$

- $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

9.3 Compound Angle

- $\sin A \pm B = \sin A \cos B \pm \cos A \sin B$
- $\cos A \pm B = \cos A \cos B \mp \sin A \sin B$
- $\tan A \pm B = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

9.4 Half Angle

- $\sin \dfrac{\theta}{2} = \pm\sqrt{\dfrac{1 - \cos\theta}{2}}$
- $\cos \dfrac{\theta}{2} = \pm\sqrt{\dfrac{1 + \cos\theta}{2}}$
- $\tan \dfrac{\theta}{2} = \dfrac{\sin\theta}{1 + \cos\theta} = \dfrac{1 - \cos\theta}{\sin\theta}$