

# Math AA HL at KCA - Chapters 1 & 2 Notes

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## 1 Discriminants

The discriminants of a quadratic function

$$ax^2 + bx + c$$

is given by

$$D = b^2 - 4ac$$

1. When  $D = 0$ , there is **only one** distinct root
2. When  $D > 0$ , there are **two** distinct roots
3. When  $D < 0$ , there are **two complex** roots, pairwise conjugate

### Typical Exam Question

Let  $f(x)$  be a quadratic and  $g(x)$  be another quadratic or a linear function. Now the question is, "Find some sort of constant, e.g.  $k$ , in the coefficient of  $f(x)$ ", when

1. Either  $f(x)$  and  $g(x)$  has no intersections,
2. Or  $f(x)$  and  $g(x)$  has two intersections
3. Or  $f(x)$  and  $g(x)$  has exactly one intersection

**Key idea:** set  $f(x) - g(x) = 0$  and use the discriminant of the resulting function  $f(x) - g(x)$  to tackle the target case.

## 2 Linear Functions and Gaussian Elimination

The straight line distance between two points is given by

$$\sqrt{\Delta x^2 + \Delta y^2}$$

Properties of linear functions

- Two parallel lines have gradients  $m_1 = m_2$
- Two perpendicular lines have gradients  $m_1 = -\frac{1}{m_2} \iff m_2 = -\frac{1}{m_1}$
- The point of intersection between  $f(x)$  and  $g(x)$  is found by solving  $f(x) = g(x)$

Gaussian elimination is a way of solving a system of linear equations. The goal is to transform all numbers below the main diagonal to zero.

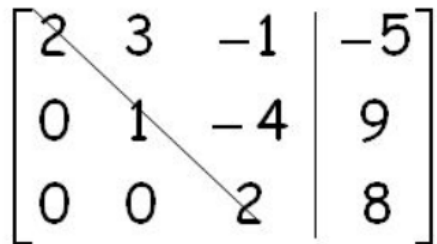

$$\left[ \begin{array}{ccc|c} 2 & 3 & -1 & -5 \\ 0 & 1 & -4 & 9 \\ 0 & 0 & 2 & 8 \end{array} \right]$$

Figure 1: Gaussian Elimination

Let the third column represent the variable  $z$  and the final row after elimination be  $az = b$ . The following summarizes the implications of the final row

1.  $0z = b, b \neq 0 \implies z$  has no solution; **inconsistent**
2.  $0z = 0 \implies z$  has infinitely many solutions, and the values of the other variables will be written in forms such as parametric equations.  
E.g.  $z = t, x = 3t - 2, y = \frac{t}{2}$ ; **consistent**
3.  $az = b$  with  $a \neq 0 \implies$  the system has a unique solution; **consistent**.