The Beveridge Curve (BC) is derived, in the model, from the stock-flow accounting relation for u_t:

$$u_{t+1} = s(1-u_t) + [1 - p(\theta)]u_t$$

by setting $u_{t+1} = u_t = u$ (steady state).

$$\Rightarrow \quad u = \frac{s}{s + p(\theta)} \qquad (BC)$$

<u>Ex. 1</u> $M(u,v)$ is Cobb-Douglas.

$$m = Au^{\alpha}v^{1-\alpha}, \quad A > 0, \quad \alpha \in (0,1)$$

So,

$$p(\theta) := \frac{m}{v} = A\left(\frac{u}{v}\right)^{\alpha} \equiv A\theta^{-\alpha}$$

The BC is given by

$$u = \frac{s}{s + A\theta^{-\alpha}}$$

Note that for all t:

$$\left.\frac{dv}{du}\right|_{u_t = u} = \frac{-(s + M_u)}{M_v}$$

$$= \frac{\overset{\ominus}{-}(\overset{\oplus}{\overbrace{s}} + \overset{\oplus}{\overbrace{\alpha A \theta^{1-\alpha}}})}{\underbrace{(1-\alpha) A \theta^{-\alpha}}_{\oplus}} < 0$$

and,

$$\left.\frac{d^2 v}{du^2}\right|_{u_t = u} = -\frac{M_v M_{uu} + (s + M_u) M_{vu}}{(M_v)^2} > 0$$

since $M_v, M_u > 0$, $M_{uu} = \alpha(1-\alpha) A \theta^{-\alpha}\left(-\frac{v}{u^2}\right) < 0$,
$M_{vu} = -\alpha(1-\alpha) A \theta^{-\alpha-1}\left(\frac{v}{u^2}\right)(-1) > 0$.

Ex. 2 CES

When $M(u,v) = A\left[\alpha u^{1-\varepsilon} + (1-\alpha) v^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$

Note that $\varepsilon \to 1$ gives the Cobb-Douglas case as a special limit of CES.

Let $\rho \equiv -(1-\varepsilon)$

We're interested in the behavior of this function as σ varies, for each fixed (u,v).

So let

$$M(\rho) = A\left[\alpha u^{-\rho} + (1-\alpha) v^{-\rho}\right]^{-\frac{1}{\rho}}$$

$$\Rightarrow \frac{M(\rho)}{A} = \left[\alpha u^{-\rho} + (1-\alpha) v^{-\rho}\right]^{-\frac{1}{\rho}}$$

Take a monotone transform, via log:

$$m(\rho) := \ln\left[\frac{\mu(\rho)}{A}\right] = \frac{-\ln\left[\alpha u^{-\rho} + (1-\alpha) v^{-\rho}\right]}{\rho} \equiv \frac{f(\rho)}{g(\rho)}$$

Note we cannot evaluate directly what happens to $\mu(\rho)$ as $\rho \to 0$ (or $\varepsilon \to 1$) since $1/\rho \to \infty$!

But we can use L'Hôpital's rule which says:

$$\lim_{\rho \to 0} m(\rho) = \lim_{\rho \to 0} \ln\left[\frac{\mu(\rho)}{A}\right]$$

$$= \lim_{\rho \to 0} \frac{f'(\rho)}{g'(\rho)}$$

Note:

$$f'(\rho) = \frac{-\left[-\alpha u^{-\rho}\ln(u) - (1-\alpha) v^{-\rho}\ln(v)\right]}{\alpha u^{-\rho} + (1-\alpha) v^{-\rho}}$$

$$\Rightarrow \lim_{\rho \to 0} f'(\rho) = \alpha \ln(u) + (1-\alpha)\ln(v)$$

$$\lim_{\rho \to 0} g'(\rho) = 1.$$

So,

$$\lim_{\rho \to 0} \left\{ \ln\left[\mu(\rho)/A\right] \right\} = \alpha \ln(u) + (1-\alpha)\ln(v)$$

$$\Rightarrow \lim_{\rho \to 0} \left\{ \ln\left[\mu(\rho)\right] \right\} = \ln(A) + \alpha \ln(u) + (1-\alpha)\ln(v)$$

$$\Rightarrow \lim_{\varepsilon \to 1} \left\{ \mu(\varepsilon) \right\} = A u^{\alpha} v^{1-\alpha}$$

take $\exp\{\cdot\}$ on both sides.

<u>Note</u>: Can show that $\sigma \equiv 1/\varepsilon$ is the elasticity of substitution between the inputs.