From Nash bargaining FOC's, we derived

$$w = \frac{r}{1+r} U + \phi\left(y - \frac{r}{1+r} U\right)$$

Nash bargaining wage solution

compensates for at least worker's annuitized value of being unemployed

Plus a share of firm's output in excess of worker's annuitized value of unemployment

We can rewrite this annuity value of U out further as a function of model primitives and equilibrium market tightness θ.

To do this, see Nash bargaining FOCs:

$$\left.\begin{aligned} E - U &= \phi S \\ J &= (1-\phi) S \end{aligned}\right\} \Rightarrow E - U = \left(\frac{\phi}{1-\phi}\right) J$$

From firm's Bellman equation with free-entry to posting ($V = 0$), we derived

$$J = \frac{c}{\beta q(\theta)}$$

So, we can also write

$$\boxed{E - U = \left(\frac{\phi}{1-\phi}\right) \cdot \left(\frac{c}{\beta q(\theta)}\right)} \quad \blacksquare$$

Recall the Bellman equation (evaluated at steady state $u_{t+1} = u_t = u$) for an unemployed person is:

$$U = z + \beta \left[p(\theta) E + (1 - p(\theta)) U \right]$$

Rewrite for $(E - U)$:

$$E - U = \frac{(1-\beta)U - z}{\beta p(\theta)} \qquad [2]$$

Plug [2] into LHS of [1]:

$$(1-\beta)U - z = \beta p(\theta) \left(\frac{\phi}{1-\phi} \right) \left(\frac{c}{\beta q(\theta)} \right)$$

Tidy up like Marie Kondo (noting that $p(\theta) = \theta q(\theta)$)

$$(1-\beta)U = z + \left(\frac{\phi}{1-\phi} \right) c\theta$$

$$\Rightarrow \left(1 - \frac{1}{1+r}\right)u = z + \left(\frac{\phi}{1-\phi}\right)c\theta$$

$$\Rightarrow \boxed{\left(\frac{r}{1+r}\right)u = z + \left(\frac{\phi}{1-\phi}\right)c\theta} \quad [3]$$

Annuity value of unemployment

outside option value

share of firm's average vacancy cost

If we plug [3] into [0], we have an alternative expression for the Nash bargaining wage solution as:

$$w = z + \left(\frac{\phi}{1-\phi}\right)c\theta + \phi\left[y - z - \left(\frac{\phi}{1-\phi}\right)c\theta\right]$$

$$\Rightarrow w = z + \beta\left[y - z + c\theta\right]$$

Nash bargaining wage solution compensates for ...

① pays at least worker's outside option

② wage premium:
- depends on worker's power β
- a share of firm's output net of z and average vacancy cost.

(Note: $c\theta$ is an average resource cost to everyone.)