Non-Walrasian Labor Markets: Search and Matching

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Outline

Motivation

- *Statistically*, level of unemployment measured as number of people
  1. not currently employed, and
  2. actively seeking work.
- Data: non-trivial and quite persistent rates of unemployment.
- Deficiency in some theories:
  1. limitations to Walrasian equilibrium theories of labor market.
  2. "level of employment" measured at the "intensive margin" – i.e. can vary the amount of hours work.
  3. Individuals do fall out of employment altogether – no variations on aggregate employment on the "extensive margin".
  4. earlier unemployment models based on efficiency wages or labor union theories similar.

An alternative theory:

- Heterogenous goods (labor) with no single marketplace to clear excess demand or supply; and
- Parties to a trade must search for someone to trade with.
- Double coincidence of wants problem.

i.e. involve transactions costs – it takes time for a person to look for a job and it takes time for a vacancy to find a worker.

Search frictions can potentially rationalize significant transit times between unemployment and employment in a equilibrium where markets do not necessarily need to clear.

We looked at the first of two basic models:

1. one-sided, partial-equilibrium models of job search: McCall (1970) – an optimal stopping time example.
2. Mortensen (1982), Pissarides (2000) and Diamond (1982) search and matching model.

Extensions: see recent survey of the state of the literature, see Rogerson, Shimer, and Wright (2004).

Diamond-Mortensen-Pissarides

- A matching function (like a production function) that takes **unemployment** and **vacancies** as "inputs" and maps into a **measure of matches**.
- A match pairs a worker and a firm. Bargain over **match surplus**.
- Here decentralized matching of worker-firm pairs is generally inefficient – *contra* markets that function according to the fictional Walrasian price mechanism.
- Large gross flows of job separation and job creation.
- Many workers looking for jobs. Many jobs looking for workers.

The model

Population size normalized to 1. So aggregate unemployment rate is the average unemployment rate.

Workers:

- exist on a continuum $[0, 1]$.
- infinitely lived, risk neutral, max. expected discounted value of wages, w, and leisure, z.
- Current utility of being employed, w.
- Discount factor $\beta = (1 + r)^{-1}$.
- each produces y units of output.

Firms:

- incurs vacancy cost c if it searches for a worker
- a match produces firm earnings per period of $y - w$
- Probability a match is destroyed each period is $s \in (0, 1)$.
- free entry $\Rightarrow$ firm's expected discounted profits is zero. Discount factor $\beta = (1 + r)^{-1}$.
- Wage determined by Nash bargaining with worker's bargaining strength $\phi \in (0, 1]$.

Matching function

- A reduced-form modelling device. Analogous to production function.
- $M : Input \rightarrow Output$
- Here **inputs** are:
  1. u aggregate unemployment (also $=$ unemployment rate here.)
  2. v aggregate vacancy (also $=$ average vacancy here.)
- **Output** is **aggregate number of job matches** per period.

Formally we write the number of succesful matches as $M = M(u, v)$. Some assumptions:

1. You can't tango alone.
2. The more parties involved the bigger the total number of matches. But more unemployed workers there are makes it harder for an unemployed person to find a match. Same intuition for $M_{vv} < 0$.
3. It is easier for unemployed workers to find a match if the number of vacancies rise.
4. You double the number of u and v, you double M. Rule out scale effect of population size on equilibrium unemployment rate.

Assumption

A1 $M(0, v) = M(u, 0) = 0.$

A2 $M_u > 0, M_v > 0, M_{uu} < 0, M_{vv} < 0.$

A3 $M_{uv} > 0.$

A4 $M(u, v)$ *is homogeneous of degree* 1.

Let $\theta := v/u$. Then **probability of filling a vacancy or a worker is matched** is

$$q(\theta) = \frac{M(u,v)}{v} = M\left(\frac{1}{\theta}, 1\right)$$

Probability of unemployed entering employment is

$$p(\theta) = \frac{M(u,v)}{u} = M(1, \theta)$$

by virtue of A4. Note relationship $p(\theta) = \theta q(\theta)$.

Example

Cobb-Douglas matching function.

$$M(u,v) = Au^{\alpha}v^{1-\alpha}$$

Let $\theta := v/u$. Then **probability of filling a vacancy or a worker is matched** is

$$q(\theta) = \frac{M(u,v)}{v} = A\theta^{-\alpha}$$

Probability of unemployed exiting unemployment is

$$p(\theta) = \frac{M(u,v)}{u} = A\theta^{1-\alpha} = \theta q(\theta).$$

Why this is not an equilibrium of the Walrasian kind.

1. Transition probabilities depend on market tightness (relative number of traders) – a **trading externality**.
2. Trading externality arises because price is not the only allocative mechanism.
3. **Stochastic rationing** exists and cannot be eliminated by price adjustments.
    - In one period, there is a positive probability $1 - q(\theta)$ that a vacancy will not find a worker.
    - In one period, there is a positive probability $1 - \theta q(\theta)$ that an unemployed worker will not be matched.
4. θ measures **labor market tightness**. If $\theta = v/u$ increases, the probability of rationing by the average firm rises – i.e. matches less likely to happen.
5. i.e. Searching firms and workers create trading externality (or **search** or **congestion externality**) for each other.
6. This implies the equilibrium will not be efficient (in the sense of the first fundamental welfare theorem).

Steady-state solution

What is the equilibrium θ, w and u at steady state?

To find solution:

1. Impose zero-profit condition for vacancies: Job Creation (JC) curve.
2. Solve Nash bargaining determination of wage rate: Wage-setting (WC) curve.
3. Solution must be on Beveridge Curve (BC).

The Beveridge Curve

Now, aggregate unemployment (i.e. unemployment rate with labor force population $= 1$) evolves according to:

$$u_{t+1} = s\left(1 - u_t\right) + \left[1 - \theta q\left(\theta\right)\right] u_t$$

We'll focus on the steady state equilibrium outcomes. In a steady state, $u_{t+1} = u_t = u$. So then we have

$$s\left(1 - u\right) = \theta q\left(\theta\right) u = M\left(u, v\right).$$

That is, **the flow of job destruction equals the flow of job creation**

Or, $s\left(1-u\right)=p\left(\theta\right)u$ means

$$\begin{gathered}\#\text{ of people exiting employment per period}\\=\\\#\text{ of people exiting unemployment per period.}\end{gathered}$$

Rearranging we have **unemployment rate** as

$$u=\frac{s}{s+\theta q\left(\theta\right)}=\frac{s}{s+p\left(\theta\right)} \tag{BC}$$

Beveridge Curve

A long run aggregate relationship: locus of (u, v) points where **the flow of job destruction equals the flow of job creation**.

$$u = \frac{s}{s + \theta q(\theta)} = \frac{s}{s + p(\theta)} \tag{BC}$$

Beveridge (1944) found a negative relationship between aggregate unemployment and vacancies.

We can derive our Beveridge curve from the model's steady state. Recall:

$$s\,(1-u) = \theta q\,(\theta)\,u = M\,(u, v)\,.$$

Totally differentiate this expression we get the slope of the BC as:

$$\left.\frac{dv}{du}\right|_{u_{t+1}=u_t=u} = \frac{-\,(s+M_u)}{M_v} < 0$$

and the BC is convex to the origin in (u, v) space:

$$\left.\frac{d^2v}{du^2}\right|_{u_{t+1}=u_t=u} = \frac{-M_v M_{uu} + (s+M_u)\,M_{vu}}{(M_v)^2} > 0.$$

by virtue of A2 and A3.

Exercise

Derive the Beveridge curve when the matching function is a Cobb-Douglas function:

$$M(u, v) = Au^{\alpha}v^{1-\alpha}$$

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Derive the Beveridge curve when the matching function is a constant-elasticity-of-substitution (CES) function:

$$M(u,v) = A\left[\alpha u^{1-\varepsilon} + (1-\alpha)\, v^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

where $\alpha \in (0,1)$ *and* $\varepsilon \leq 1$.
Which part of Assumption 1 must be relaxed for this matching function? Prove that your answer in this case is the same as the one previously, when $\varepsilon \to 1$. *What happens when* $\varepsilon \to -\infty$?

Firms and Job Creation

Note we are evaluating the firm's Bellman equation at steady state. Let J be the optimal value of a filled job, and V the optimal value of a vacancy.
If a firm finds a match, the value of that match for the firm that behaves optimally is

$$J = y - w + \beta \left[sV + (1 - s) J \right]$$

Its value of a vacancy will be

$$V = -c + \beta \left[q(\theta) J + (1 - q(\theta)) V \right].$$

With free entry of firms, vacancies earn zero expected PV of profits, $V = 0$. This implies

$$J = \frac{c}{\beta q(\theta)},$$

and we can derive:

Job Creation Condition

$$w = y - \frac{c}{q(\theta)}(r+s). \tag{JC}$$

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Note this is downward sloping in (θ, w) space – a "labor demand". $r = \beta^{-1}(1 - \beta)$ is the firm's rate of time preference.

Note:

- If $c = 0$, no cost of posting a vacancy (hiring cost), then $w = MPL = y$.
- $1/q(\theta)$ is expected duration of vacancy. So $c(r+s)/q(\theta)$ is **expected capitalized value of firms hiring cost.**
- If $c > 0$, optimizing firm is willing to pay a wage equal MPL y, net of, wedge: $c(r+s)/q(\theta)$.
- Labor demand measure θ negatively related to w.
- w will be determined by bargaining between firm and worker.

Wage-setting Condition

Realistically workers affect w through

- their job search (more intense effort, can raise w) and
- their bargaining power.

Here all workers have same MPL, y, and fixed search intensity. The only influence workers have on w is their bargaining strength, $\phi \in (0, 1)$.

Let E be value function of wages, for an employed worker and U be value of wages unemployed worker, *at the steady state*. Unemployed worker has outside option z. What can z represent?

Worker's steady-state Bellman equation is given by

$$E = w + \beta \left[sU + (1 - s) E \right]$$

$$U = z + \beta \left[p(\theta) E + (1 - p(\theta)) U \right]$$

where $p(\theta) = \theta q(\theta)$.

- Nash bargaining. Use often justified by appealing to results from sequential bargaining games: Binmore, Rubinstein and Wolinski (1986).
- Workers share of the match surplus: $E - U$.
- Firm's share of the match surplus: J. (Recall $V = 0$ under free entry.)
- Total match surplus: $S = E - U + J$.

Nash bargaining:

$$\max_{(E-U),J} (E-U)^{\phi} J^{1-\phi}$$

subject to $S = E - U + J$.
The FONC yields

$$E - U = \phi S \text{ and } J = (1-\phi) S$$

where $S = E - U + J$.
Each party's share is increasing in their bargaining strength.

From worker's value of employment we get

$$E = \frac{w + \beta s U}{1 - \beta\left(1 - s\right)}$$

and together with $J = \frac{c}{\beta q(\theta)}$ into the FONC, we get

$$w = \frac{r}{1+r}U + \phi\left(y - \frac{r}{1+r}U\right)$$

Wage rate under Nash bargaining is the *annuity value of being unemployed* plus *worker's share of one-period match surplus.*
We can show that *annuity value of being unemployed* is

$$\frac{r}{1+r}U = z + \frac{\phi\theta c}{1-\phi}$$

And we can re-write wage rate:

Wage-setting Curve

$$w = (1-\phi)\, z + \phi\, (y + \theta c) \tag{WS}$$

Note WS is upward sloping in (θ, w) space.

Nash bargaining with $\phi \in (0, 1)$ results in wage that is a *weighted average* of

- compensation for outside opportunity z of worker, and,
- worker's MPL plus average vacancy/hiring cost for each unemployed worker.

A very strong firm, $\phi \to 0$, in bargaining terms, will pay w that only takes account of worker's outside option, $w \to z$.

Now we are ready to collect our optimality conditions for aggregate unemployment (BC), firms' "labor demand" (JC), and worker's optimal decisions and wage bargaining outcome (WS).
Steady-state equilibrium is the triple (u, θ, w) that satisfies:

$$u = \frac{s}{s + \theta q(\theta)} \tag{BC}$$

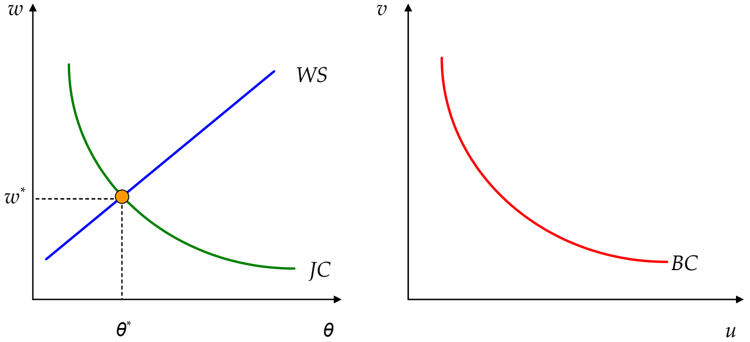
$$w = y - \frac{c}{q(\theta)}(r + s) \tag{JC}$$

$$w = (1 - \phi) z + \phi (y + \theta c) \tag{WS}$$

Notice that (θ, w) can be solved independently of u using (JC) and (WS).
Equating these two curves we have:

$$y - z = \frac{r + s + \phi\theta q(\theta)}{(1-\phi) q(\theta)} c. \tag{JC = WS}$$

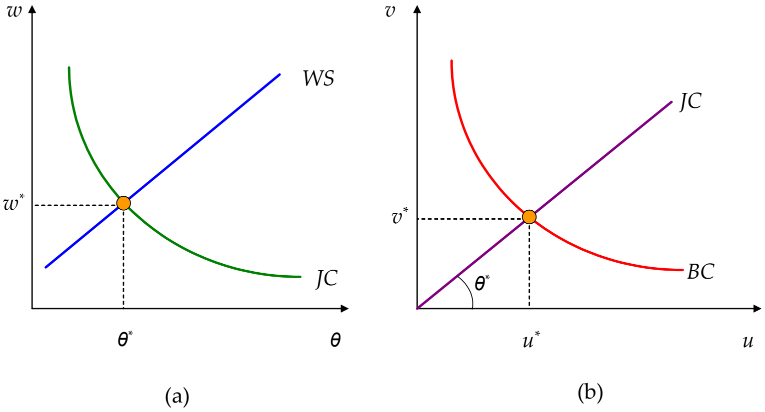
So we can solve for θ^* and then solve for w^* using either (JC) or (WS).



(a)

(b)

(a) Steady-state (θ, w) solved independently of u, using (JC) and (WS).



(b) θ^* then determines steady-state unemployment rate, u^* on the BC.

Exercise

Explain using the model and show in the diagrams what happens when:

1. *Worker's unemployment benefits is increased. You may interpret* z *as including unemployment benefits.*
2. *Hiring cost* c *is increased.*
3. *The rate of job destruction* s *increases.*

Welfare Analysis

- Recall we said traders create trading or congestion externalities.
- These externalities are not internalized by traders so that the equilibrium is not efficient.
- What would a benevolent social planner do?
- A benchmark to find a socially optimal outcome that internalizes the externality.

A planner would choose allocations to maximize discounted social value (value of output and leisure net of hiring costs). Let total employment $n_t = 1 - u_t$. Social planner solves:

$$\max_{\{v_t, n_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[y n_t + z\left(1 - n_t\right) - c v_t \right]$$

s.t.

$$n_{t+1} = \left(1 - s\right) n_t + q\left(\theta_t\right) v_t$$

$$\theta_t = \frac{v_t}{1 - n_t} \text{ and } n_0 \text{ given.}$$

Planner's problem is nonstochastic because the probabilities of successful and destroyed matches, respectively, are equal to their aggregate fractions.

Planner's Lagrangian:

$$\max \sum_{t=0}^{\infty} \beta^t \Big\{ \left[y n_t + z\left(1 - n_t\right) - c v_t \right] + \mu_t \left[\left(1 - s\right) n_t + q\left(\theta_t\right) v_t - n_{t+1} \right] \Big\}$$

s.t.

$$\theta_t = \frac{v_t}{1 - n_t}.$$

FONC:

$$-c + \mu_t \left[q'(\theta_t)\,\theta_t + q(\theta_t) \right] = 0$$

$$-\mu_t + \beta (y - z) + \beta \mu_{t+1} \left[(1 - s) + q'(\theta_{t+1})\,\theta_{t+1}^2 \right] = 0$$

Notice we have $\mu_t :=$ "marginal value of vacancy" and simplifying FONCs:

$$\frac{c}{q'(\theta_t)\,\theta_t + q(\theta_t)}$$
$$= \beta\left\{(y-z) + \frac{c}{q'(\theta_{t+1})\,\theta_{t+1} + q(\theta_{t+1})}\left[(1-s) + q'(\theta_{t+1})\,\theta_{t+1}^2\right]\right\}$$

an Euler equation relating marginal value of vacancy this period to marginal value of vacancy next period.

Evaluating at steady state:

$$y - z = \frac{c\left\{1 - \beta\left[(1-s) + q'(\theta)\,\theta^2\right]\right\}}{\beta\left[q'(\theta)\,\theta + q(\theta)\right]}$$

Example

Suppose we have a Cobb-Douglas matching function as before. So then $q'(\theta) = -\alpha q(\theta)/\theta < 0$. Then we have, under the **efficient planner's outcome,** θ^{**} solves

$$\begin{aligned} y - z &= \frac{c\left\{1 - \beta\left[(1-s) - \alpha q(\theta)\theta\right]\right\}}{\beta(1-\alpha)q(\theta)} \\ &= \frac{r + s + \alpha q(\theta)\theta}{(1-\alpha)q(\theta)}c \qquad \text{(Pareto JC = WS)} \end{aligned}$$

Example

Contrast with the **decentralized equilibrium** we found earlier, where θ^* solved

$$y - z = \frac{r + s + \phi\theta q(\theta)}{(1-\phi)\, q(\theta)} c. \qquad \text{(JC = WS)}$$

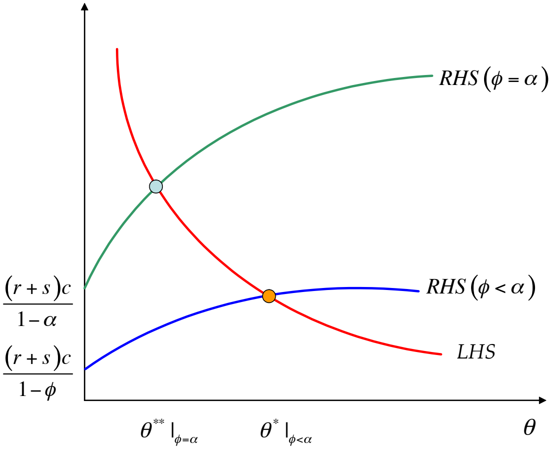
Note α is the –ve of elasticity of $q(\theta)$ with respect to θ.

Consider (JC = WS) re-written as

$$(y-z)\,q(\theta)=\left(\frac{r+s}{1-\phi}\right)c+\theta q(\theta)\left(\frac{\phi}{1-\phi}\right)c.$$

Note:

- LHS is a function of θ, with slope $(y-z)\,q'(\theta)<0$
- RHS is a concave function of θ with slope $(\theta q'(\theta)+q(\theta))\,\phi\,(1-\phi)^{-1}c=(1-\alpha)\,q(\theta)\,\phi\,(1-\phi)^{-1}c>0.$
- So there is a unique solution θ^*. Comparative statics: θ^* decreases with the parameter ϕ.



Unique fixed-point θ and θ is higher for lower worker bargaining strength ϕ.

$$\phi \begin{cases} \leq \alpha & \Rightarrow \theta^* \geq \theta^{**} \\ \geq \alpha & \Rightarrow \theta^* \leq \theta^{**} \end{cases}$$

- A high (low) α means for a small % increase in vacancy (θ) results in a large (small) % decline in probability of filling a job $q(\theta)$.
- A low (high) ϕ means low (high) worker bargaining power.
- A social planner internalizes externality arising from a decentralized outcome that yields sub-optimal θ^*.
- Planner would like to increase (decrease) worker's bargaining power ϕ to equal α to decrease (increase) the number of vacancies to an optimal level.