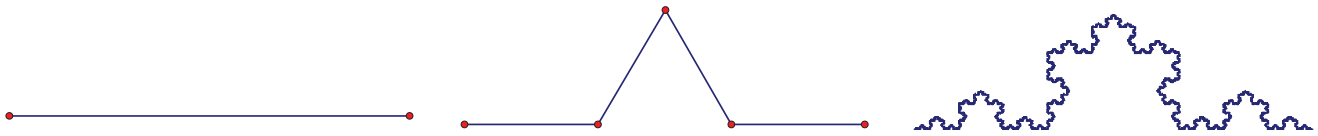
CREATING THE KOCH CURVE AND SNOWFLAKE WITH SKETCHPAD



Initiator Generator Approximation of fractal

1.) Start with a blank sketch and construct a line segment using the straightedge tool. This is the initiator of Koch's curve.

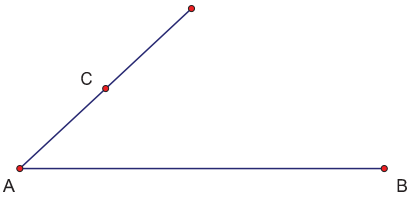
2.) Using the text tool, click on endpoints of our segment to label the points as "A" and "B"

We need to trisect segment AB. To do this, we will utilize the proposition that parallel lines intersecting a triangle will partition the sides into proportional lengths. Thusly, we will construct a triangle such that one side is our original segment and a new side consists of three collinear congruent segments. The third side of the triangle will be the segment connecting the remaining endpoints.

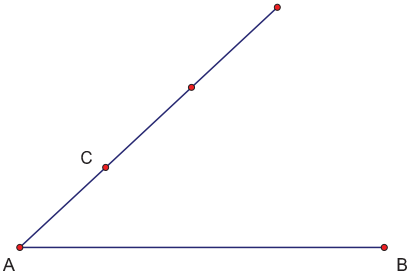
3.) Using the straightedge tool, construct a new segment starting at A and creating an acute angle with vertex A. The acute angle is not necessary, but does help to keep the construction more compact. Use the text tool to label the new endpoint of the segment as "C".

4.) We now will mark a vector for a segment translation. Using the selection arrow tool, deselect all objects, and then ***in order*** select points A and then C. Now from the transformation menu, choose mark vector. You should see dots race briefly from point A to point C.

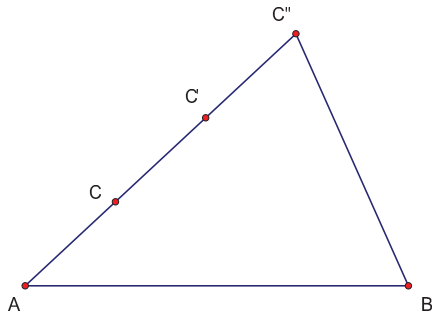
5.) Now select the AC segment and point C. From the transform menu, choose translate. When the translate menu box appears, make sure the marked radio button is selected (it should be by default) and then press the translate button.



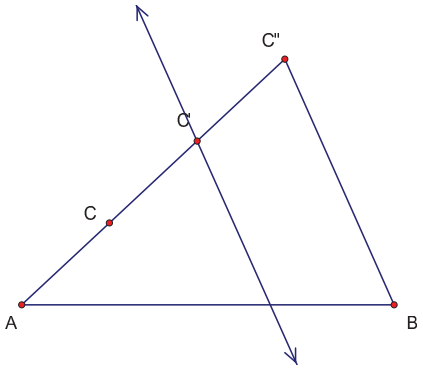
6.) Without changing the current selections, from the transform menu, choose translate and press the translate button. Your image should now look as follows.



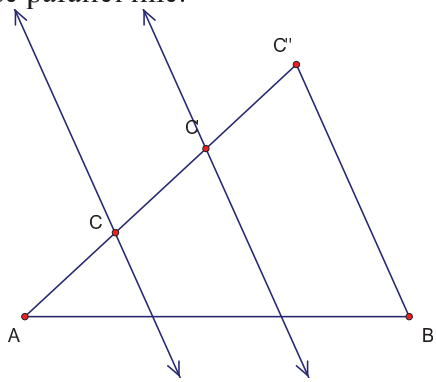
7.) Use the text tool to label the new points, in order, as C' and C''. Then, using the selection arrow tool, deselect all objects, and then select C'' and B. From the construct menu, choose segment.



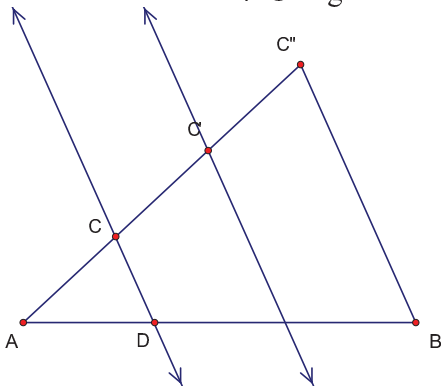
8.) Now select point C', *note:* $\overline{C''B}$ *is already selected*, and, from the construct menu, choose parallel line.



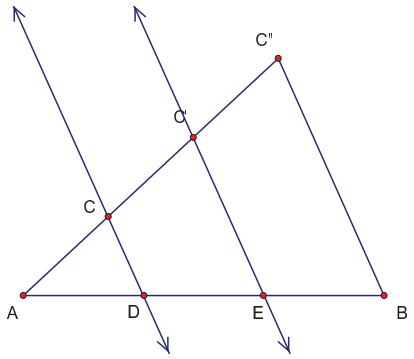
9.) Now select point C, *note: the previously constructed parallel line is still selected*, and, from the construct menu, choose parallel line.



10.) We now want to select $\overline{AB}$, *note: the previously constructed parallel line is still selected*, and from the construct menu choose intersection. Using the text tool label this new point D.



11.) Using the selection arrow tool, deselect all objects, and then select $\overline{AB}$ and the parallel line passing through C′. From the construct menu choose intersection and then, using the text tool, label this new point E.



12.) Using the selection arrow tool, deselect all objects, and then select all objects other than the points A, D, E, B, and from the display menu choose hide objects.

13.) Congratulations! You have successfully trisected $\overline{AB}$. If you want to verify this, select points A and D and from the measure menu choose distance. Repeat this for points D, E and E, B. *Note: all three distances are equal, but may not necessarily be same as those shown here.*

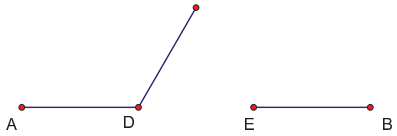
AD = 2.13 cm
DE = 2.13 cm
EB = 2.13 cm

14.) If you choose to measure the above distances, select them and press the delete key as we don't really need them and they will just clutter up the image. If you didn't measure the distances, just go on to the next step.

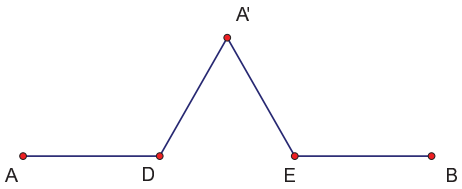
15.) Using the selection arrow tool, deselect all objects, and then select points A and D. From the construct menu, choose segment.

16.) Using the selection arrow tool, deselect all objects, and then select points E and B. From the construct menu, choose segment.

17.) Now double click on point D to make it as a center of rotation for a transformation. You should see a bulls-eye animate briefly around point D. Using the selection arrow tool, deselect all objects, and then select point A and $\overline{AD}$. From the transform menu choose rotate. Make sure the fixed angle radio button is selected (it should be by default) and enter -120 into the degrees entry space. You should see a lighter image of the rotated segment in the sketch. If everything looks correct, press the rotate button and obtain the following image.

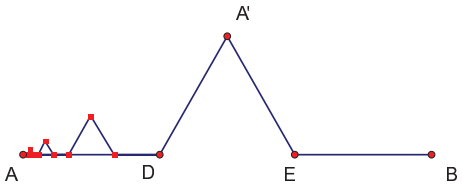


18.) Using the text tool, label the new point as A'. Now, using the selection arrow tool, deselect all objects, and then select points A' and E. From the construct menu, choose segment.



We have now constructed the generator of Koch Curve (Snowflake).
To approximate the fractal we now need to iterate the generator construction process.

13.) Using the arrow selection tool, deselect all objects and then select points A and B. In the transform menu, select iterate. A new menu appears. We have to tell Sketchpad where we want the iterated process mapped or placed. We want point A to be mapped to point A and point B to be mapped to point D. This will iterate the process on $\overline{AD}$. So with the A $\Rightarrow$ box highlighted, click on point A in the sketch. Now the B $\Rightarrow$ and point B should highlight. We want to map point B to point D, so now click on point D in the sketch. *Note: you may have to move the iterate menu box to have access to point D in your sketch.* When you have done this you should see a lighter image that looks like the following

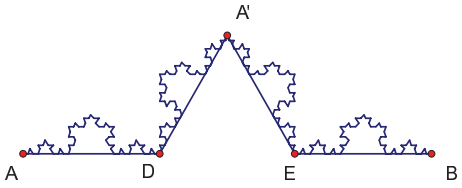


14.) We still need to map the iterated process onto $\overline{DA'}$, $\overline{A'E}$, and $\overline{EB}$. In the iterate menu box, click on *structure* and then select *add new map*. This time map A to D and B to A'. So first click on point D and then on point A' in the sketch.

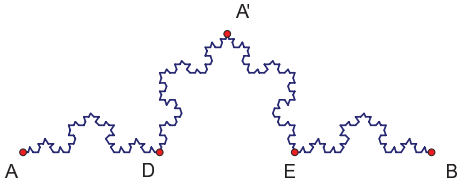
15.) Now click on *structure* and then select *add new map* a second time. This time map A to A' and B to E. So first click on point A' and then on point E in the sketch.

16.) Finally, click on *structure* and then select *add new map* a third time. This time map A to E and B to B. So first click on point E and then on point B in the sketch.

17.) Press the *display* button in the iterate menu box and choose *final orbit only*. Now press the iterate button to obtain the following picture.

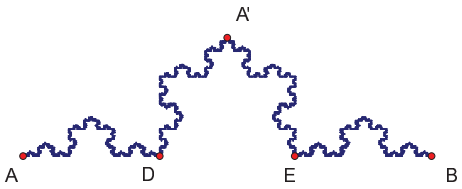


19.) Using the selection arrow tool, deselect all objects, and then select $\overline{AD}$, $\overline{DA'}$, $\overline{A'E}$, and $\overline{EB}$. From the display menu, choose hide segments. Your picture will now appear as follows.



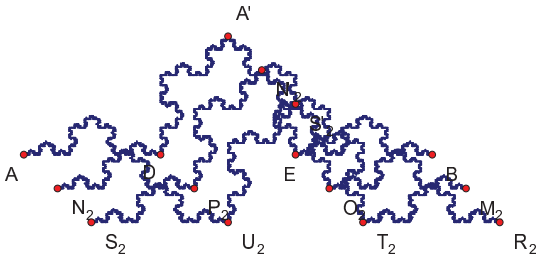
This is an approximation of the Koch curve.

20.) Using the selection arrow tool, select the Koch curve and press the "+" keys to increase the number of iterations displayed. It should work twice. You now have this image.

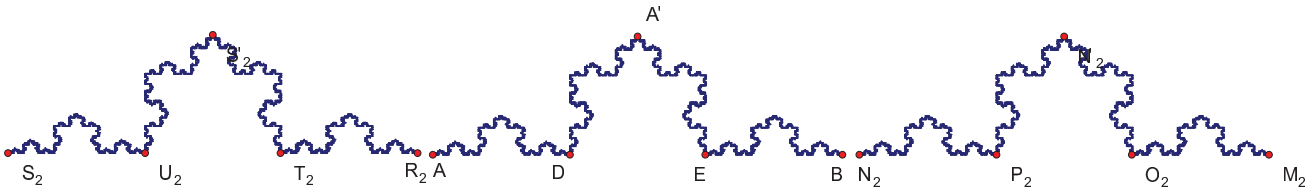


There are many ways to finish the snowflake. This is an easy one.

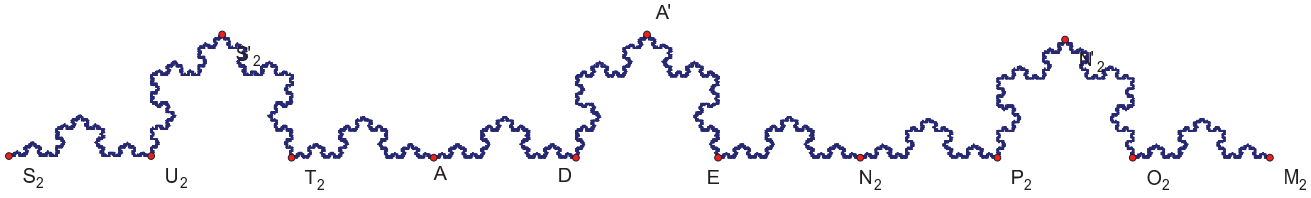
21.) To finish the snowflake, from the edit menu choose select all, then copy, and then paste twice. The image will be a mess and look like this.
Note: if you repeat and undo the copy and paste process, Sketchpad will rename the points differently. Use the point labels given here as references only and do not worry if the points in your copied images are labeled differently.



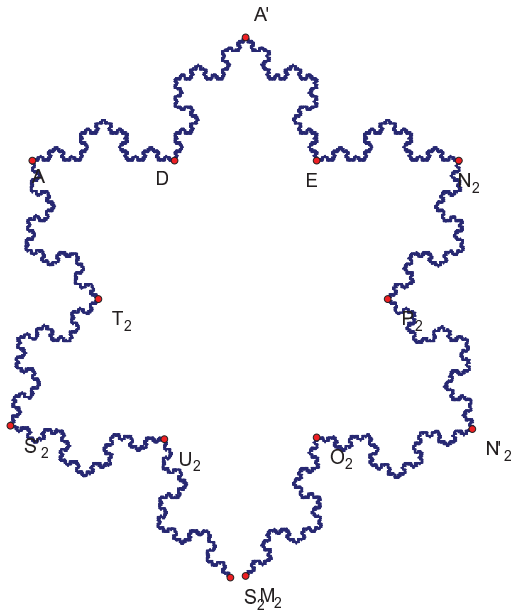
22.) Now move one copy near the end of the original so that points B and N_2 are very close to each other as in the following diagram. Move the second copy near the other end of the original so that points R_2 and A are very close to each other as in the following diagram. *When moving the copies, select the Koch curve, rather than the labeled points, and the curve will move as a unit and not deform.*



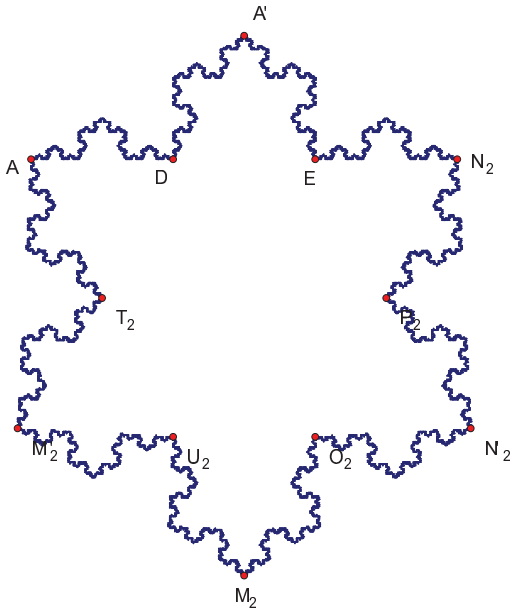
23.) Now, using the arrow selection tool, deselect all objects and then select points B and N_2. From the edit menu select merge. One of the points, in my case B, will move and become one with the other point. Repeat this process to merge points R_2 and A.



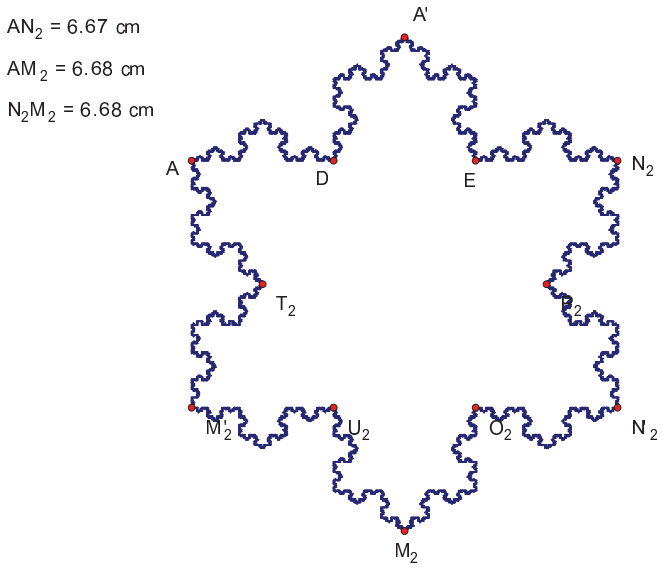
24.) Using the arrow selection tool, deselect all objects and then select point S_2. Drag this point to a location directly underneath A′ so that the distance from S_2 to A is preserved. Don't worry if the distance isn't exact, we can fix it later. Now, deselect all objects and then select point M_2 and move it next to S_2. It should look as follows.



25.) Using the arrow selection tool, deselect all objects and then select points S_2 and M_2. From the edit menu select merge.



26.) Using the point selection tool, deselect all objects, then select points A and N_2. From the measure menu select distance. Repeat this process to also measure the distances from A to M_2 and from N_2 to M_2. Now slightly adjust the locations of point M_2 to try to equalize all three distances. Don't worry, you probably won't be able to make them perfectly equal.



27.) Now to clean up the picture, select only the three measurements and from the display menu choose hide distance measurements.

28.) Here is a neat trick to get rid of the points. Click on the point tool. Now from the edit menu choose select all points. Only the points will be selected. *(Using the tools and select all also works with straight objects and circles!)* Finally, from the display menu choose hide points. Viola! The Koch Snowflake! *Note: The Koch Snowflake lacks true self similarity and, thus, is not a true fractal.*