Drift-flux model programming in a Matlab environment

PET E 631Advanced Production Engineering:

Individual course project

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1. Introduction

Reservoir performance and surface facilities are significantly impacted by multiphase flow effects in wellbore and pipe. Pressure drop in some types of wells may result in increase or decrease the flowrate. To make the reservoir simulators models and well performance more efficient and reliable, we need to create accurate multiphase pipeflow models.

The three types of pipeflow models that are most frequently used in petroleum engineering are empirical correlations, homogenous, and mechanistic. Curve fitting of experimental data serves as the foundation for empirical correlations, their suitability is limited by the variables range discovered in the experiments. Both pattern-specific and flow-pattern-independent correlations are possible for these relationships. With homogeneous models, the mixture is assumed to be a single phase and the fluid properties are assumed to be represented by the mixture properties. Allowing slip between the phases is another feature of these models, and it necessitates several empirical parameters. The term driftflux models refers to homogeneous models with slip.

Commonly, mechanistic models are the most reliable because they show models based on comprehensive physics for all different flow patterns. On the other hand, usage of mechanistic models in the case of the reservoir simulation may cause challenges because they may display pressure drop and holdup interruptions at some flow-pattern transitions. Convergence might be decreased by these problems. There are two different approaches to reduce this error, either to smooth transitions or apply a homogeneous pipeflow model. When a bubble or slug flow pattern is expected, the drift-flux model is employed within more general mechanistic models.

The benefits of homogeneous models include their relative simplicity, continuity, and differentiability. They are therefore good for usage in reservoir simulators. The simplest homogeneous models are not suitable for use in reservoir simulators because they do not capture the complex relationship between the in-situ volume fraction and the input volume fraction. These models neglect slip between the fluid phases, meaning that the fluid phases move at the same velocity. Conversely, since drift-flux models take into consideration the slip between the fluid phases, they are a good option for use in reservoir simulators.

A variety of empirical parameters are needed for drift-flux models. The majority of the parameters found in current simulators were found through experiments conducted in vertical pipes with a diameter of no more than 2 inches. However, because the flow mechanisms in small pipes can differ qualitatively from those in large pipes, these parameters might not apply directly to wellbore flows. Determining the drift-flux parameters for wellbores and pipes with sizes of practical interest is important.

The work in this area focuses at developing drift-flux models that are optimized for use in reservoir simulators. Large-scale experimental and modeling efforts were started in order to achieve this. Large diameter (6 in.) pipe flow experiments were conducted as part of the experimental work at various phase flow rates and pipe inclinations. These special experimental data will be used in our current work to evaluate the drift-flux models that are currently in use and to calculate drift-flux parameters for steady-state two-phase flows of oil/water and water/gas in large-diameter pipes or wellbores. An optimization method that minimizes the difference between experimental and model predictions is used to determine the parameters. It is demonstrated that the default settings currently in use are usually not ideal.

2. Theoretical introduction of drift-flux model

It this work we consider the drift-flux model for two-phase flow. The slip between liquid and gas phase is explained as a combination of two mechanisms. The phase distribution and velocity across the pipe have irregular patterns as shown in figure 1. According to the figure, the difference in densities causes the gas to tend to pass through the liquid. Since the fluid velocity is highest in the center of the wellbore, this is the location where gas bubbles tend to concentrate.

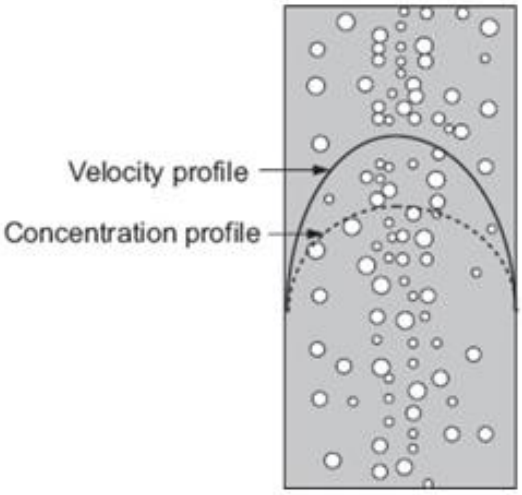

Figure 1. Profile and local slip mechanisms in the drift-flux model. (Jensen, 2017)

The total effect of this mechanism can be expressed in the equation for gas velocity v_g.

$$v_g = v_d + C_0 v_{ms} \tag{1}$$

where v_d is drift velocity known as the difference between liquid and gas velocity, C_0 is profile parameter (typical values range from 1.0 to 1.5 depending on flow regime, pipe diameter and gas fraction), v_{ms} is superficial mixture velocity.

The superficial mixture velocity could be defined in two ways:

$$v_{ms} = v_{sg} + v_{sl} \tag{2}$$

where v_{sg} and v_{sl} are gas and liquid superficial velocities respectively.

$$v_{sg} = \frac{q_g}{A} \tag{3}$$

$$v_{sl} = \frac{q_l}{A} \tag{4}$$

where q_g and q_l are gas and liquid flowrates in standard conditions respectively.

Another way to find the superficial mixture velocity is using following equation

$$v_{ms} = H_g v_g + (1 - H_g)V_l \tag{5}$$

where V_l is averaged liquid phase flow velocity across the pipe area defined as

$$V_l = \frac{1-H_g C_0}{1-H_g} v_{ms} - \frac{H_g}{1-H_g} v_d \tag{6}$$

where H_g is the averaged in-situ gas volume fraction across the pipe area, or gas holdup.

2.1 Profile parameter

Profile parameter C_0 value for bubble flow regime is 1.2, for annular flow 1.0. And also this parameter depends on pipe diameter. For large pipe diameter C_0 is could be found by using the following equation

$$C_0 = \frac{C_{0,bub}}{1+(C_{0,bub}-1)\gamma^2} \tag{7}$$

where $C_{0,bub}$ is equal to C_0 in the bubble flow regime, γ value ranges from 0 to 1 and might be obtained from

$$\gamma = \min\left[\max\left(0, \frac{\beta-\bar{\beta}}{1-\bar{\beta}}\right), 1\right] \tag{8}$$

where $\beta > 0$, $\bar{\beta}$ is the value of β at which C_0 starts to drop below $C_{0,bub}$ (figure 2).

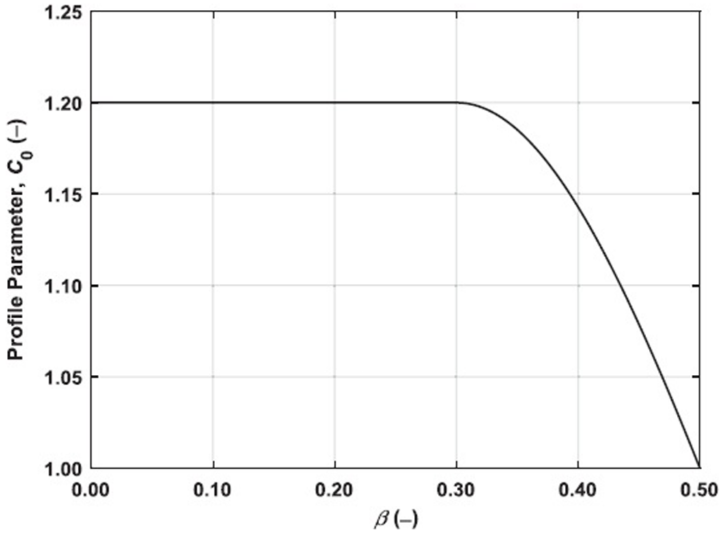

Figure 2. Profile parameter as function of β for $C_0 = 1.2$ and $\bar{\beta} = 0.6$, these parameters are considered for small pipe diameters. (Jensen, 2017)

Parameter β is defined as

$$\beta = \max\left(H_g, H_g \frac{v_{ms}}{v_{g,fld}}\right) \tag{9}$$

where $v_{g,fld}$ is the gas flooding velocity described as the gas velocity that is necessary for keeping a thin annular layer such that it does not flow back down along the wellbore as shown in figure 3.

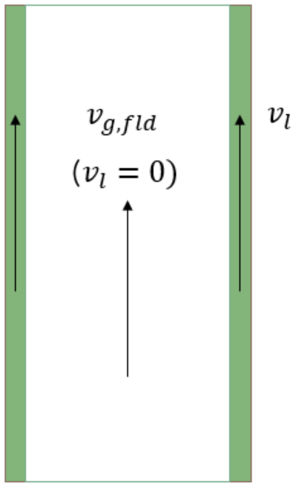

Figure 3. Gas flooding velocity representation.

$v_{g,fld}$ might be obtained from the equation

$$v_{g,fld} = N_{Ku}\sqrt{\frac{\rho_l}{\rho_g}}v_c \tag{10}$$

where N_{Ku} is the critical Kutateladze number, which is the function of a modified pipe diameter number N_d, v_c is the critical velocity.

$$N_d = d\sqrt{\frac{g(\rho_l - \rho_g)}{\sigma_{gl}}} \tag{11}$$

where σ_{gl} is gas/liquid interfacial tension.

Critical velocity could be obtained from the following equation. Note that in this work we define upward velocities as negative.

$$v_c = \sqrt[4]{\frac{\sigma_{gl} g(\rho_l - \rho_g)}{\rho_l^2}} \tag{12}$$

The relationship between N_{Ku} and N_d is shown in the figure 4.

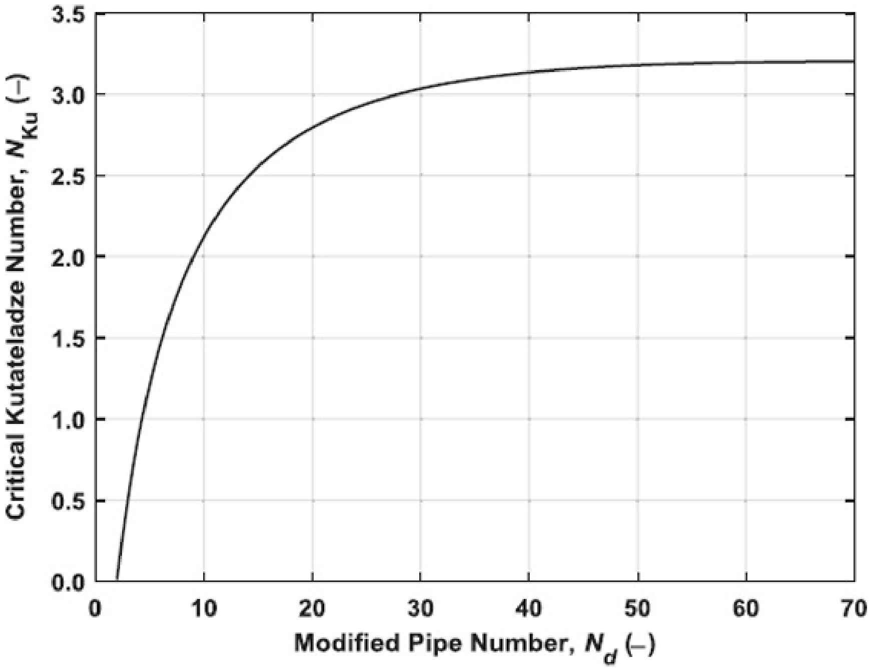

Figure 4. Critical Kutateladze number N_{Ku} as function of modified pipe number N_d. (Jensen, 2017)

2.2 Drift velocity

The flow regime also affects the drift velocity. The gas velocity for bubble flow, which corresponds to very low gas fractions, is found using the following expression by Harmathy (1960), who found that the rise velocity of small bubbles through a stationary liquid is as follows

$$v_{g,bub} = 1.53 v_c \tag{13}$$

At high gas holdup values annular flow is considered as a function of gas holdup defined as

$$v_d = \frac{m_\alpha (1 - H_g C_0) C_0 K v_c}{H_g C_0 \sqrt{\frac{\rho_g}{\rho_l}} + 1 - H_g C_0} \tag{14}$$

where $K = \frac{1.53}{C_0}$ if $H_g < a_1$;

$K = N_{Ku}$ if $a_2 < H_g$

The parameter m_α takes the account of the effect of the wellbore inclination α

$$m_\alpha = m_0 (cos\alpha)^{n_1} (1 + sin\alpha)^{n_2} \tag{15}$$

The parameters a_1, a_2, n_1, n_2, m_0 are determined for large and small pipe diameter (table 1)

Table 1. Optimal parameters for large and small pipe diameters (Shi et al., 2005b)

	a_1	a_2	n_1	n_2	m_0
Small pipe diameter ($d < 0.1\ m$)	0.06	0.12	0.24	1.08	1.27
Large pipe diameter ($d > 0.1\ m$)	0.06	0.21	0.21	0.95	1.85

2.3 Holdup calculation

Gas Holdup H_g is related to the profile parameter C_0 and the drift velocity v_d

$$H_g = \frac{v_{sg}}{v_g} = \frac{v_{sg}}{v_d(H_g^k) + C_0(H_g^k) v_{ms}} \tag{16}$$

where k is the number of the iteration and where the gas volume fraction forms convenient starting value $H_g^0 = \lambda_g$

2.4 Drift-flux and Mukherjee and Brill models comparison

The Mukherjee & Brill method is a simplified model that is flow pattern dependent, and it can be applied to deviated wells.

Drift-flux correlations may be more versatile and applicable in different conditions, but accuracy may depend on the specific flow type and physical parameters.

By comparing these two models, it could be concluded that if simplicity and universality are desired, drift-flux correlations may be preferable. If precise matching to specific conditions is required, Mukherjee and Brill might be more suitable.

3. Methodology

3.1 Holdup calculation in Matlab

Drift-flux correlation model for gas and liquid holdup calculation could be programmed in a Matlab environment by creating a script or function and following the procedure mentioned in chapter 2. The variable names are listed in Table 2. To show the calculation procedure through the equations (1) – (16), we could use "for" loop, "if…else" conditions and Picard iterations. In the model we considered upward flow as positive flow.

Table 2. Matlab variables

Variable	Parameter	Variable in Matlab	Units	Value
d	Tubing inside diameter	d	m	0.0623
e	Tubing roughness	e	m	7.62×10^{-5}
α	Well inclination angle	alpha	Degrees	60
μ_g	Gas viscosity	mu_g	Pa×s	10^{-6}
μ_l	Liquid viscosity	mu_l	Pa×s	0.015
T	Temperature	T	Celsium degrees	60
q_g	Gas flowrate, sc	q_g	m^3/s	-0.03
q_o	Oil flowrate, sc	q_o	m^3/s	-0.01
ρ_o	Oil density, sc	rho_o	kg/m^3	762
ρ_g	Gas density, sc	rho_g	kg/m^3	69
σ_{gl}	local gas-liquid interfacial tension	sigma_gl	*N/m*	0.01
v_{sg}	Gas superficial velocity	v_sg	*m/s*	
v_{sl}	Liquid superficial velocity	v_sl	*m/s*	
v_{ms}	Mixture superficial velocity	v_ms	*m/s*	
N_d	Dimensionless pipe diameter	N_d	-	
v_c	Critical velocity	V_c	*m/s*	
$v_{g,fld}$	Gas flooding velocity	V_g	*m/s*	
λ_g	Gas no-slip holdup	lambda_g	-	
λ_l	Liquid no-slip holdup	lambda_l	-	

The flowchart to determine gas and liquid holdups is represented in the figure 5:

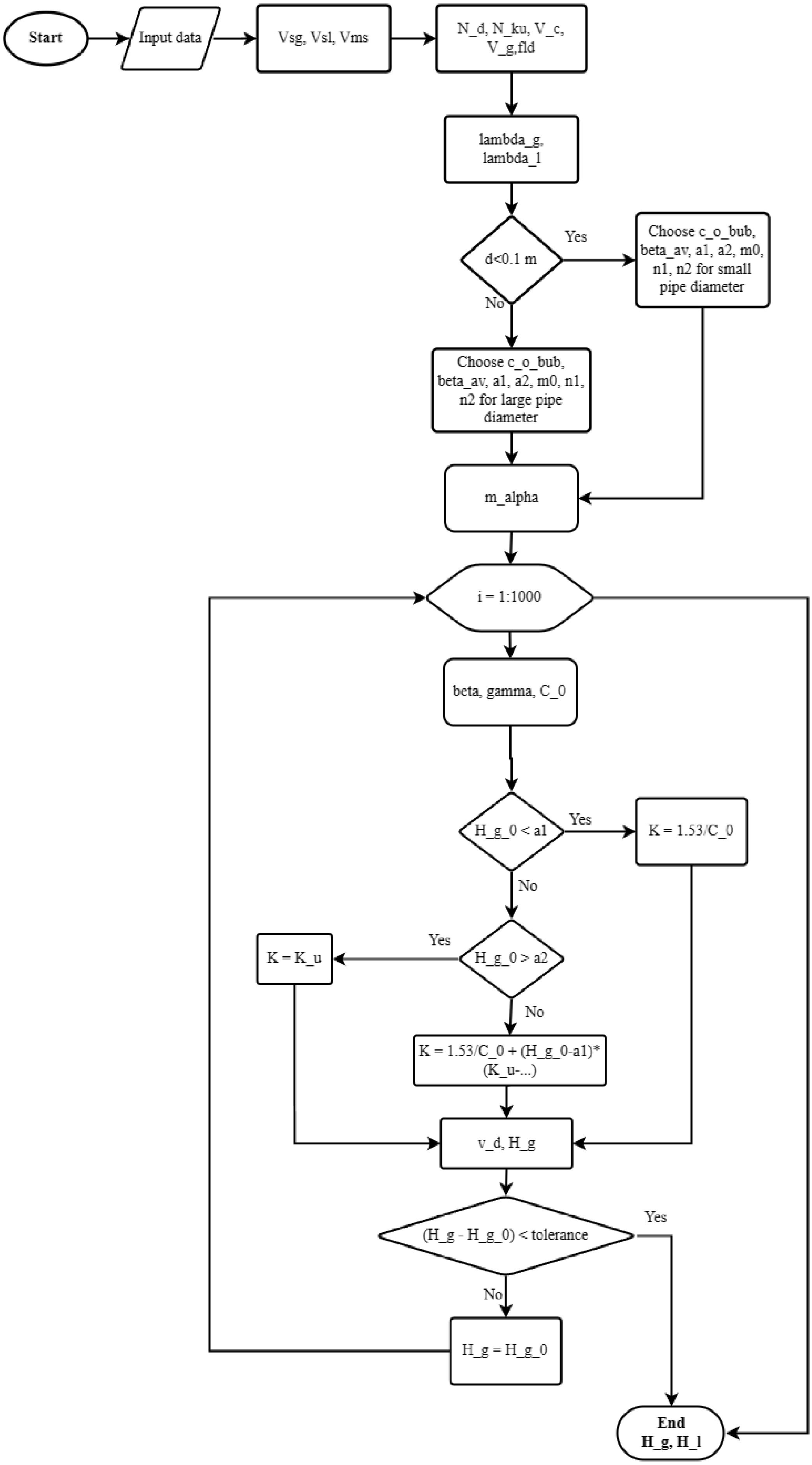

Figure 5. Flow chart to determine holdup in Matlab environment.

4. Results and discussion

4.1 Holdup calculation results

Input data for holdup calculation requires variables alpha, d, q_g, q_l, rho_l, rho_g, sigma_gl (could be taken from table 2). The function Drift_Flux_Holdup.m could be run through the Matlab command window as follows:

```
>> [H_g,H_l,lambda_l,lambda_g] = Drift_Flux_Holdup(from_deg_to_rad(60),0.0623,-0.03,-0.01,762,69,0.01)
```

Figure 6. Running function Drift_Flux_Holdup.m.

For these input parameters, the function gives the result as shown in figure 7

```
H_g =

    0.7287


H_l =

    0.2713


lambda_l =

    0.2500


lambda_g =

    0.7500

fx >>
```

Figure 7. Function Drift_Flux_Holdup.m results for data provided in table 2.

4.2 Drift-flux pressure loss calculation

Function Drift_Flux_dpds.m computes the pressure drop over an inclined well using drift-flux correlation model. The function is based on the Hagedorn and Brown correlation for multiphase flow – Hag_Brown_dpds. The gas and liquid holdups are calculated by calling the function drift_flux_holdup.m as follows:

```
% Compute hold-ups (slip) and in-situ volume fractions (no-slip):
[H_g,H_l] = Drift_Flux_Holdup(alpha,d,q_g,q_l,rho_l,rho_g,sigma_gl)
```

Figure 8. Calling Drift_Flux_Holdup.m to compute gas and liquid holdup.

Using modified example_traverse.m script, we could build the pressure traverse for both Mukherjee and Brill (figure 9) and Drift-Flux models (figure 10). In the script, for MB model we should choose variable fluid $= 4$ (multi-phase gas-oil-water flow, Mukherjee and Brill correlation). For Drift-Flux model – fluid $= 6$ (multi-phase gas-oil-water flow, Shi et al. drift flux model). For both models we used wellbore inclination angle alpha $= 60^{\circ}$, oil flowrate q_o_sc $= -0.0001$ m^3/s. Other parameters remain unchanged.

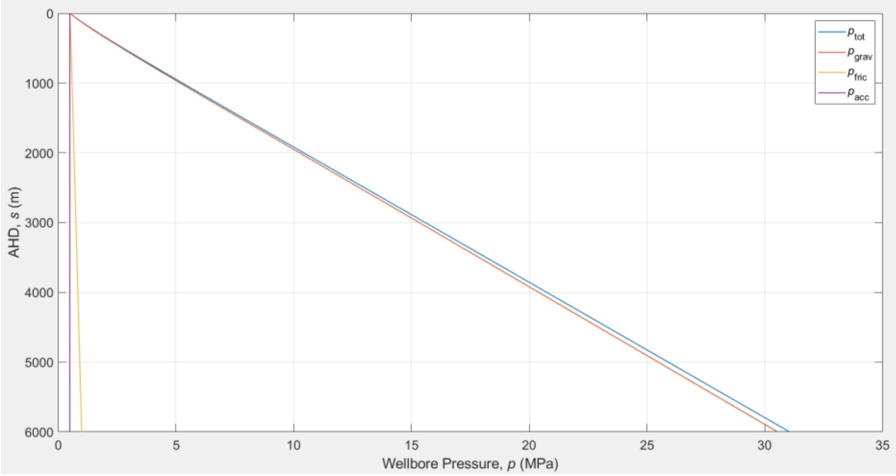

Figure 9. The pressure traverse – Mukherjee and Brill model.

The pressure loss for Mukherjee and Brill correlation: $p_{tot} \approx 31.04\ MPa$, $p_{grav} \approx 30.54\ MPa$, $p_{fric} \approx 1\ MPa$, $p_{acc} \approx 0.5\ MPa$.

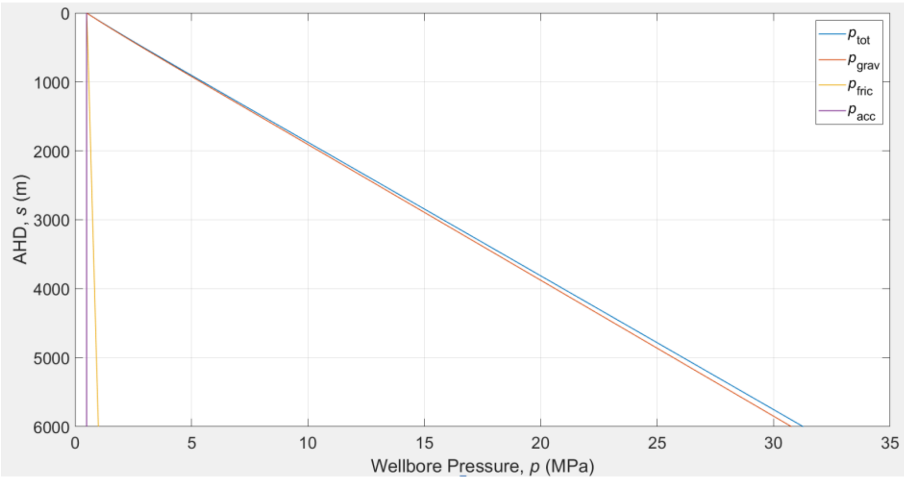

Figure 10. The pressure traverse plot – Drift-Flux model.

As we could see from the plot, the pressure loss for Drift-Flux correlation $p_{tot} \approx 31.23\ MPa$, $p_{grav} \approx 30.76\ MPa$, $p_{fric} \approx 1\ MPa$, $p_{acc} \approx 0.5\ MPa$.

4.3 Drift-flux and Mukherjee and Brill intake curves comparison

Tubing intake curves for both models could be combined with IPR and compared to choose optimal well deliverability (figures 11-12).

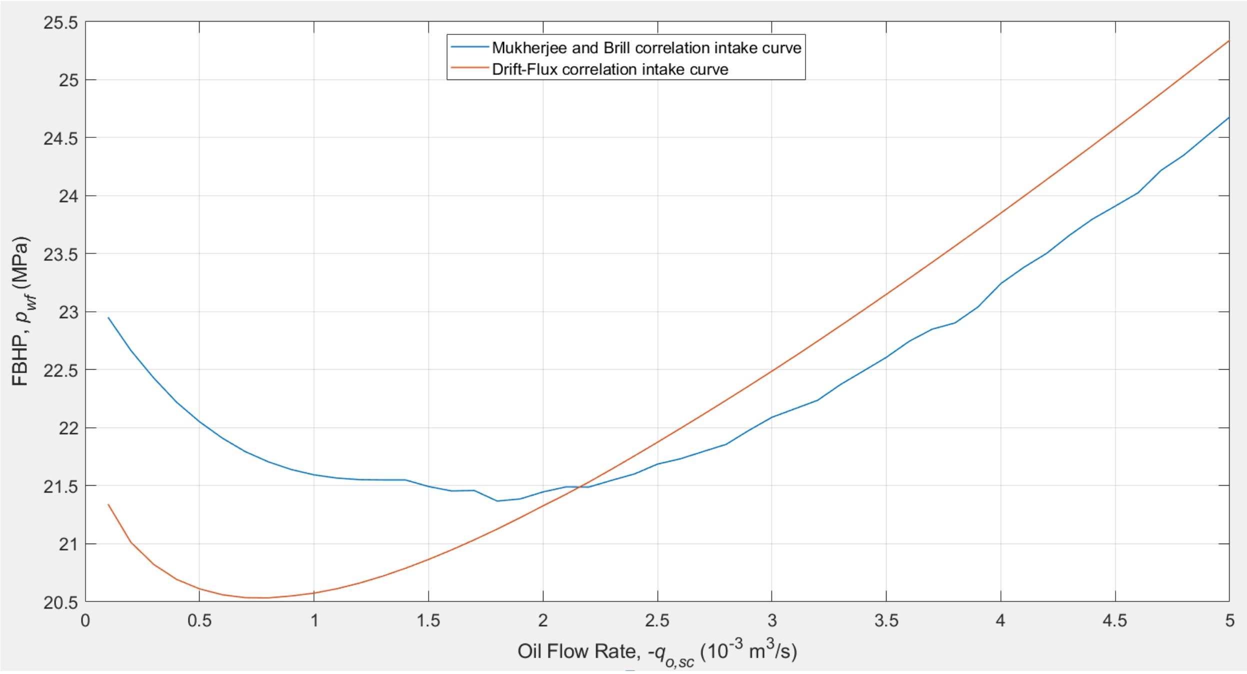

Figure 11. The intake curves for Mukherjee and Brill and Drift-Flux correlation models.

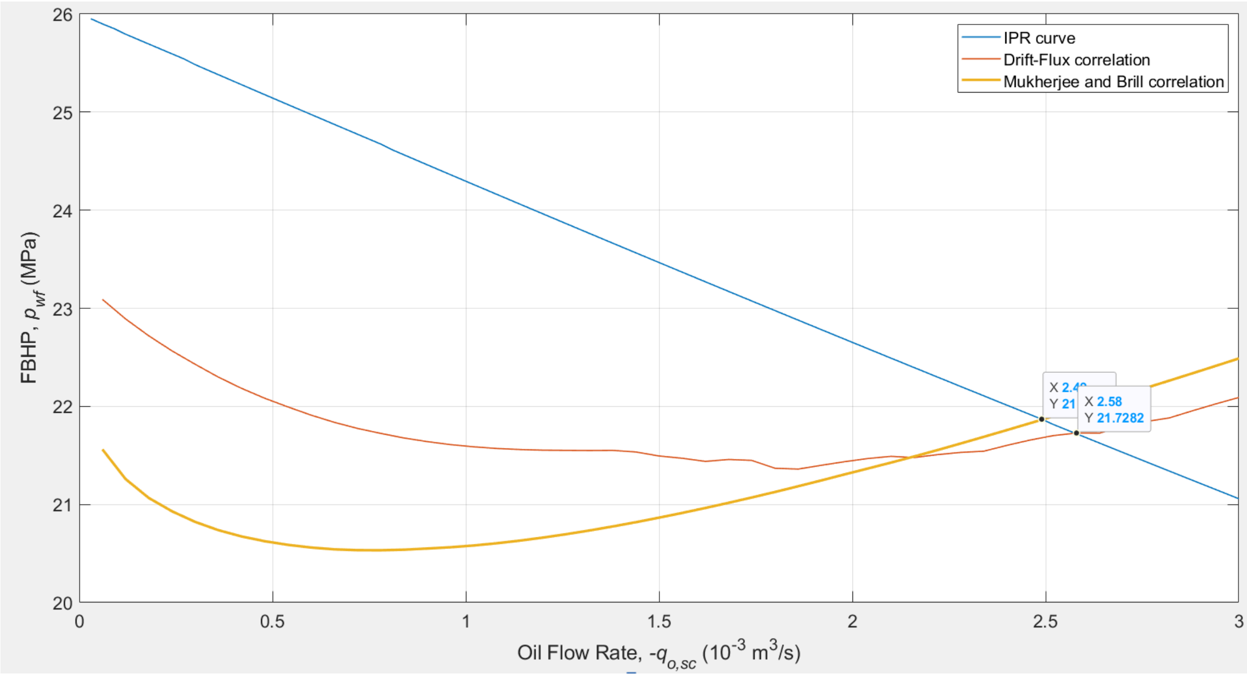

Figure 12. The intake curves and IPR for Mukherjee and Brill and Drift-Flux correlation models, $d = 62\ mm$.

Analyzing two intake curves for both correlations and IPR curve, we could conclude that Drift-Flux correlation model gives better well deliverability for the same input variables. Using Drift-Flux model, we get higher oil flowrate $q_{o,sc} = 2.58 \times 10^{-3}$ m^3/s for the lower bottomhole pressure $p_{wf} = 21.73$ MPa.

We could also investigate the effect of tubing diameter on the well deliverability using the tubing diameter $d = 50.67$ mm (figure 13). The intake was built only for Drift-Flux correlation in this case because Drift-Flux model gives better well deliverability.

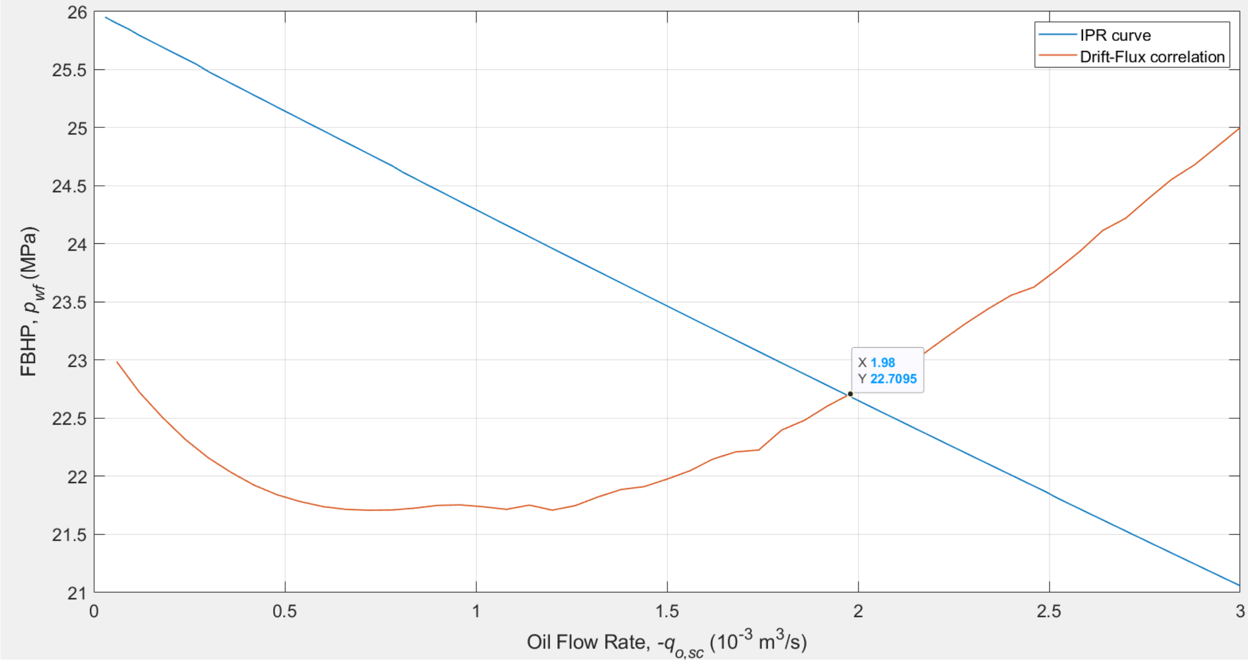

Figure 13. The intake curves and IPR for Drift-Flux correlation model, $d = 50.67$ mm.

As we reduced the tubing diameter, we got lower oil flowrate $q_{o,sc} = 1.98 \times 10^{-3}$ m^3/s for the higher bottomhole pressure $p_{wf} = 22.71$ MPa as compared with 62 mm tubing diameter.

The next parameter to determine its changing on the well deliverability is tubing head pressure. In the previous figures we used $p_{tf} = 0.5$ MPa and got the oil flowrate $q_{o,sc} = 2.58 \times 10^{-3}$ m^3/s as shown in the figure 14. As we increase tubing head pressure to 1 MPa, we are getting lower oil flowrate $q_{o,sc} = 1.98 \times 10^{-3}$ m^3/s as shown in the figure 15.

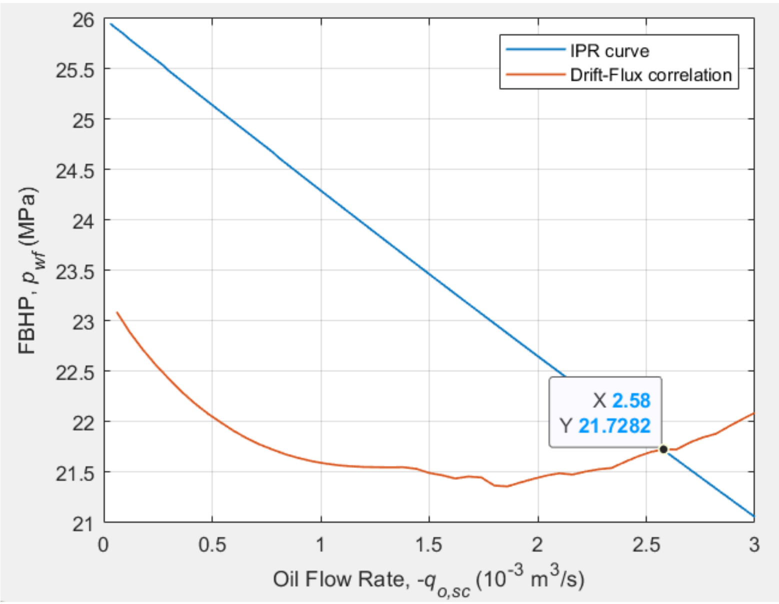

Figure 14. The intake curves and IPR for Drift-Flux correlation model, $d = 62$ mm, $p_{tf} = 0.5$ MPa.

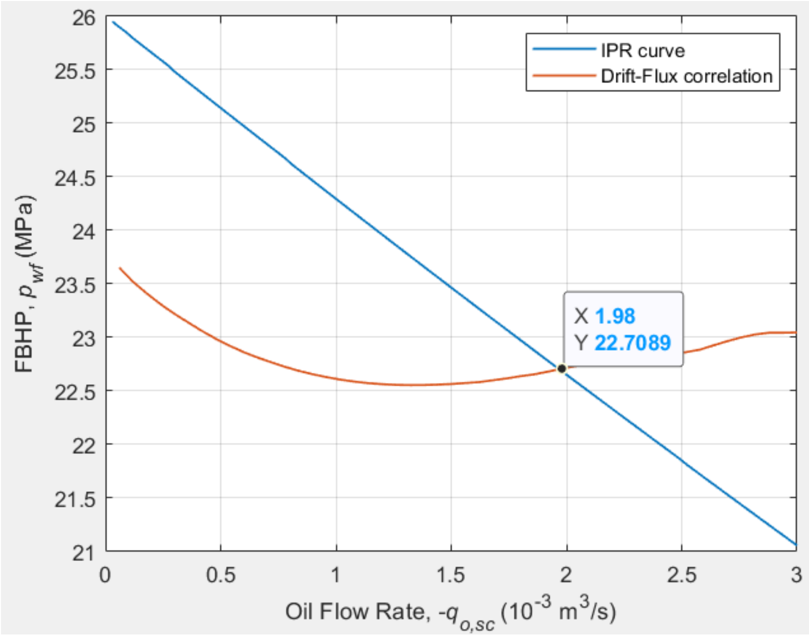

Figure 15. The intake curves and IPR for Drift-Flux correlation model, $d = 62$ mm, $p_{tf} = 1$ MPa.

5. Conclusions

The drift-flux model offers a continuous and differentiable model for multiphase flow in pipes and wells, making it an excellent choice for use in reservoir simulators due to its relative simplicity. The following conclusions can be drawn from this study:

1. The Drift-Flux model correlation could be modeled in the Matlab environment to calculate gas and liquid holdup using Picard iterations and determine pressure loss and well deliverability.

2. Compared to the Mukherjee and Brill correlation, the Drift-Flux model gives better well deliverability for the same input variables; the oil flowrate for the latter is 0.1 m3/s higher. At the same time, the Mukherjee and Brill correlation takes into account the flow regime for calculations and might give more accurate answers, while the Drift-Flux model is more generalized and universal.

3. For pressure loss, both models give almost similar results. The difference in total pressure loss between two models is 0.19 MPa.

4. Combining the intake curve and IPR for the Drift-Flux model, it could be concluded that well deliverability is sensitive to changing tubing size and tubing head pressure. For higher values of tubing head pressure, we observe lower oil flowrates. Also, as we reduce the tubing size, we also observe a reduction in oil flowrate.

References

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