Assignment 2, Computational Physics Sumit Ghosh, Roll No. 20201222

Solution to Q1a.

timus@timus-Vostro-3590:~/Desktop/Computational Physics/assign 2\$ gfortran trapz_pi.f90 timus@timus-Vostro-3590:~/Desktop/Computational Physics/assign 2\$./a.out Program to evaluate $I = Int 0 1 (4/(1+x^2))dx - by trapezoidal method$

Value of the integral = 3.1415926535897927

timus@timus-Vostro-3590:~/Desktop/Computational Physics/assign 2\$

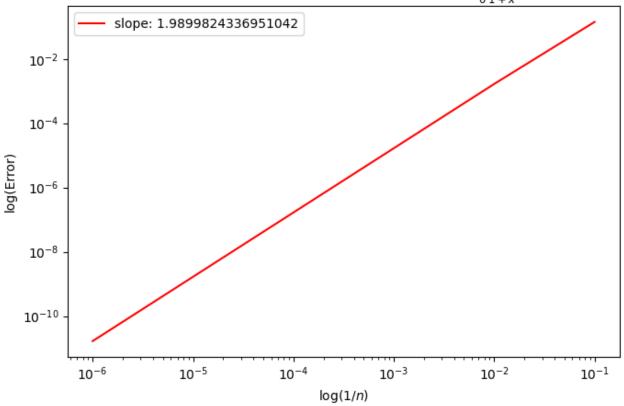
Solution to Q1b.

Using Trapezoidal Method

Source file: "trap_pi.dat"

No. of grid points(n)	Value of the integral	Absolute Error
10	3.00000000000000	0.141592653589793
100	3.13992598890716	0.00166666468263443
1000	3.14157598692313	1.66666666641113E-05
10000	3.14159248692312	1.66666668910409E-07
100000	3.14159265192314	1.66665303780178E-09
1000000	3.14159265357315	1.66404667822917E-11

Q1b. log-log graph of error vs. 1/n: Integral = $\int_0^1 \frac{4}{1+x^2} dx = \pi$

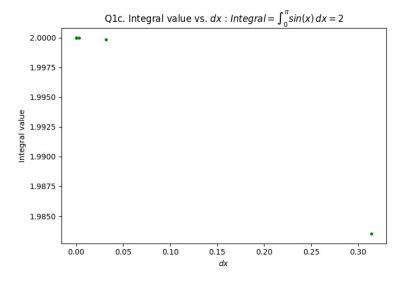


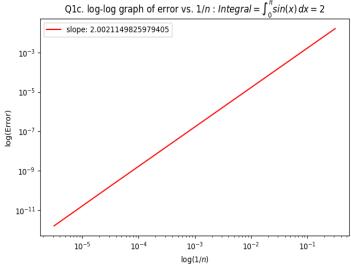
Solution to Q1c.

Using Trapezoidal Method

Source file: "trap_sinx.dat"

dx	Value of the integral	Absolute error
0.314159265358979	1.98352353750945	0.0164764624905454
0.0314159265358979	1.99983550388744	0.000164496112556423
0.00314159265358979	1.99999835506566	1.64493433763013E-06
0.000314159265358979	1.99999998355066	1.64493392240672E-08
3.14159265358979E-05	1.99999999983548	1.64520841394733E-10
3.14159265358979E-06	1.9999999999841	1.5922818619174E-12





Solution to Q1d.

Normalized Gaussian function :

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

As it is normalized the integral value from $-\infty$ to $+\infty$ should be 1 But we integrate it from -3 to +3 and we get:

Source file: trap_gauss.dat

	No. of grids(n)	Value of the integral
10		0.026591090471628
100		0.996530893052922
1000		0.997292229481190
10000		0.997300124163754
100000		0.997300203139005
1000000		0.997300203928767

Lets increase the range : -5 to +5

Source file : trap_gauss.dat

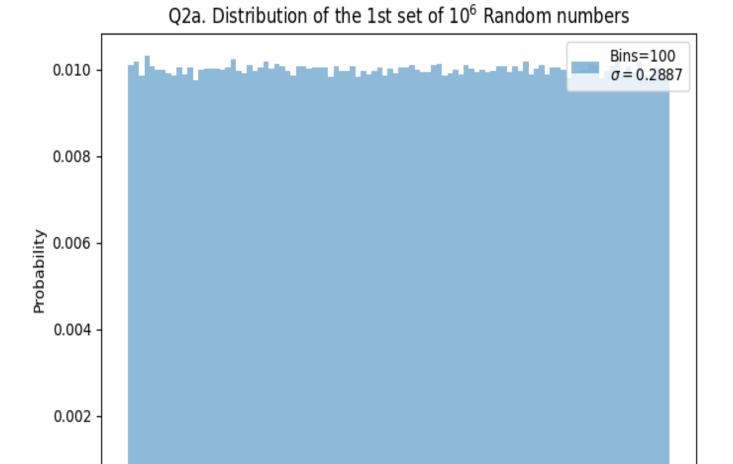
0.000 -

0.0

0.2

No. of §	grids(n)	Value of the integral
10		1.4867195147343E-05
100		0.999998506461016
1000		0.999999414352764
10000		0.999999426572969
100000		0.999999426695614
1000000		0.999999426696839

As we increase the range the value of the integral approaches 1 as expected. Solution to Q2a.



0.4

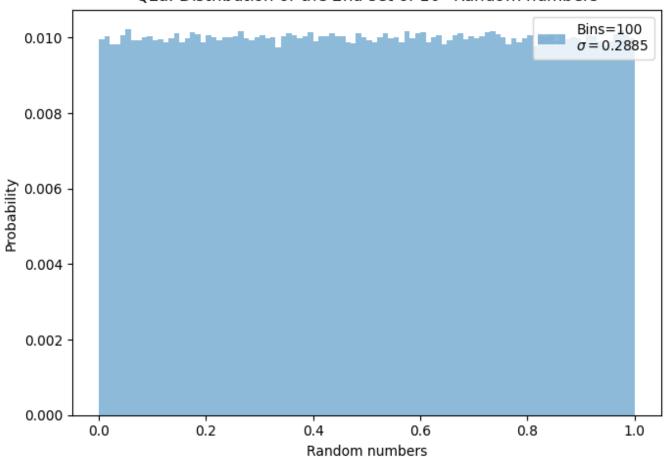
Random numbers

0.6

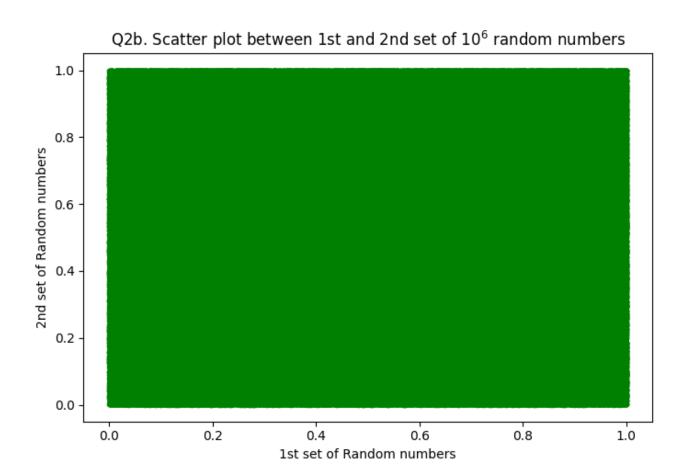
0.8

1.0

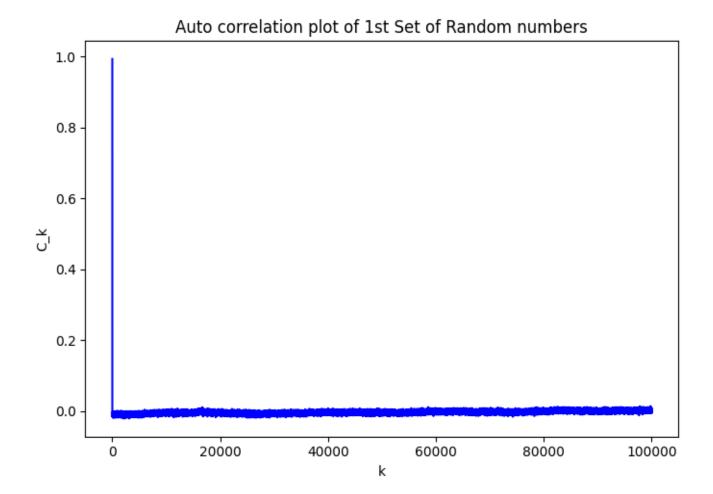
Q2a. Distribution of the 2nd set of 10⁶ Random numbers



Solution to Q2b.



Solution to Q2c.



Solution to Q2d.

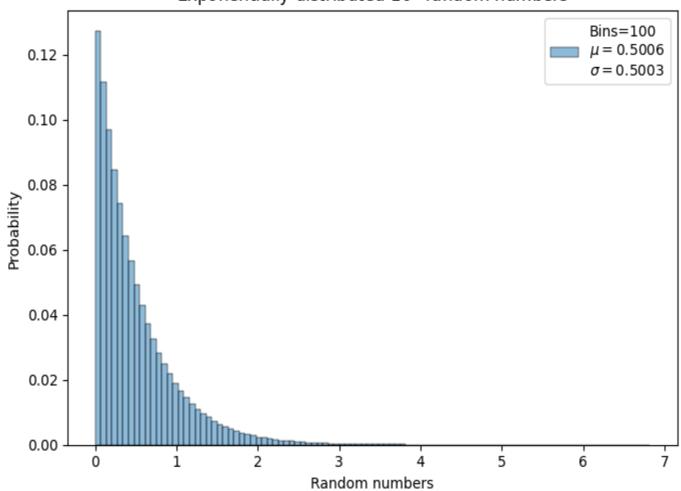
```
timus@timus-Vostro-3590:~/Desktop/Computational Physics/assign 2$ gfortran uniform_ran_dist.f90
timus@timus-Vostro-3590:~/Desktop/Computational Physics/assign 2$ ./a.out
For 1st set of random numbers
Mean =
0.4996768178
Standard deviation =
0.2887332210
For 2nd set of random numbers
Mean =
0.5000312366
Standard deviation =
0.2885413530
timus@timus-Vostro-3590:~/Desktop/Computational Physics/assign 2$
```

Solution to Q4a.

Distribution:

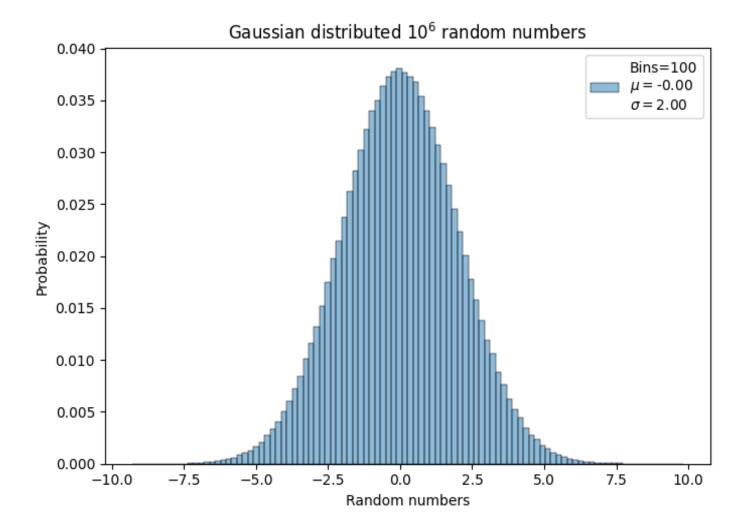
exponential (e^{-2x})

Exponentially distributed 10⁶ random numbers



Solution to Q4b.

Distribution : Gaussian (SD = 2)



Solution to Q5a.

Using Monte Carlo Brute-force method :

No. of grid points (n)	Value of the integral	Standard Deviation
10	1.68760910076265E-20	1.60023495777444E-20
100	0.000529576579474597	0.000526519923600668
1000	12.6166357573585	11.5949100909209
10000	3.99992335540219	1.79929118968529
100000	10.9098085472761	4.02696349282702
1000000	10.5813253767356	1.1141708019767
10000000	10.7973195244708	0.35521805238774
100000000	10.9554093466325	0.11566824719333

Using Monte Carlo importance sampling method :

No. of grids (n)	Value of the integral	Standard deviation
10	12.5763045447924	2.46223381497976
100	10.1134860625641	0.741116595691969
1000	11.1561968784927	0.255905453642664
10000	11.0810475535896	0.0814247155037868
100000	10.9504794798991	0.0254730896630834
1000000	10.9609607593055	0.00805073243564577
10000000	10.9608664942441	0.00254620316641202
10000000	10.9636321759507	0.00080534827219262