Computational Physics

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Assignment – II

Year - 2023

By trapezoidal method calculate $\int_0^1 \frac{4}{1+x^2} dx$

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timus@timus-Vostro-3590:~/Desktop/Computational Physics/assign 2$ gfortran trapz_pi.f90
timus@timus-Vostro-3590:~/Desktop/Computational Physics/assign 2$ ./a.out

 Program to evaluate I = Int_0_1 (4/(1+x^2))dx - by trapezoidal method

 Value of the integral =    3.1415926535897927

timus@timus-Vostro-3590:~/Desktop/Computational Physics/assign 2$
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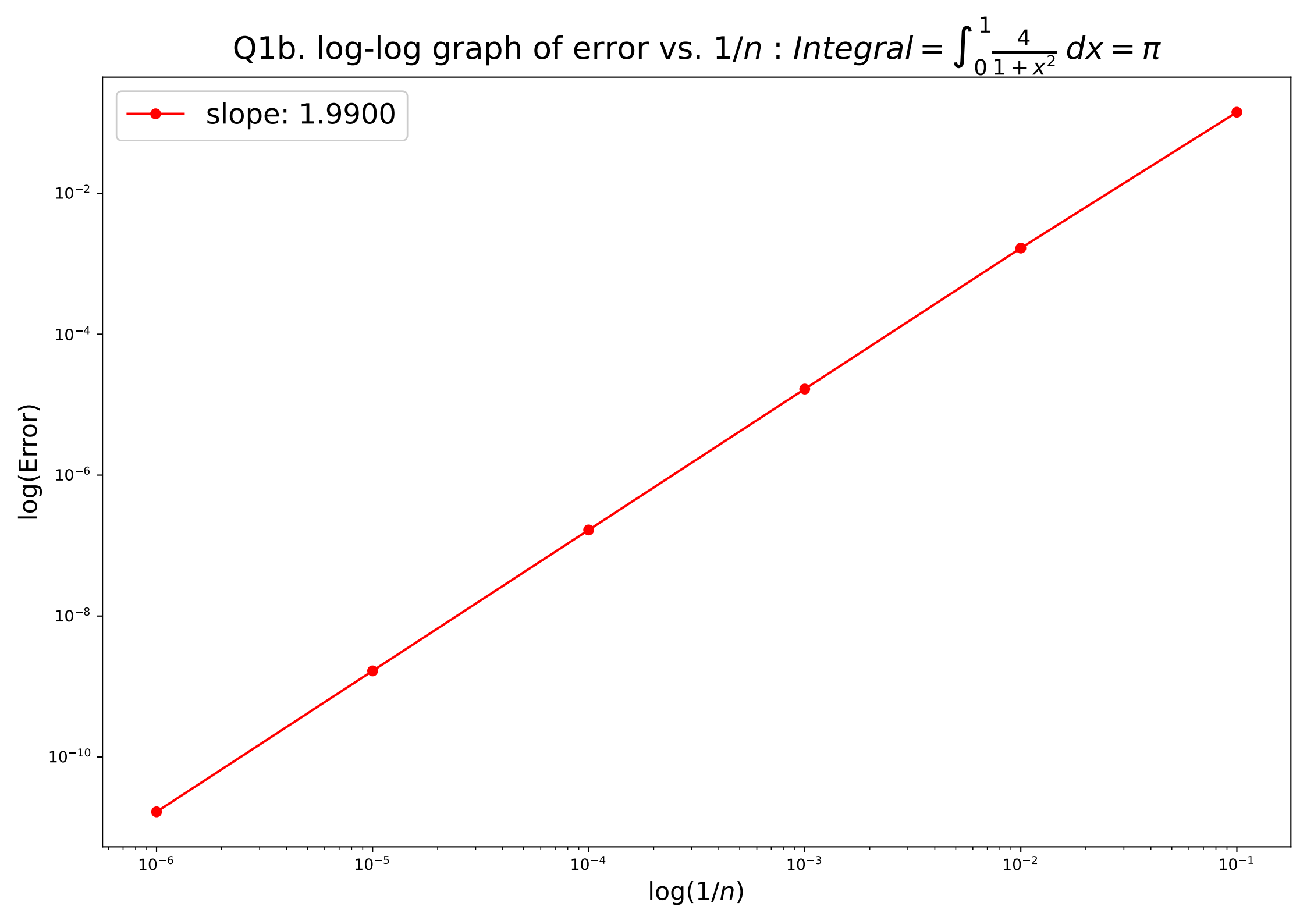
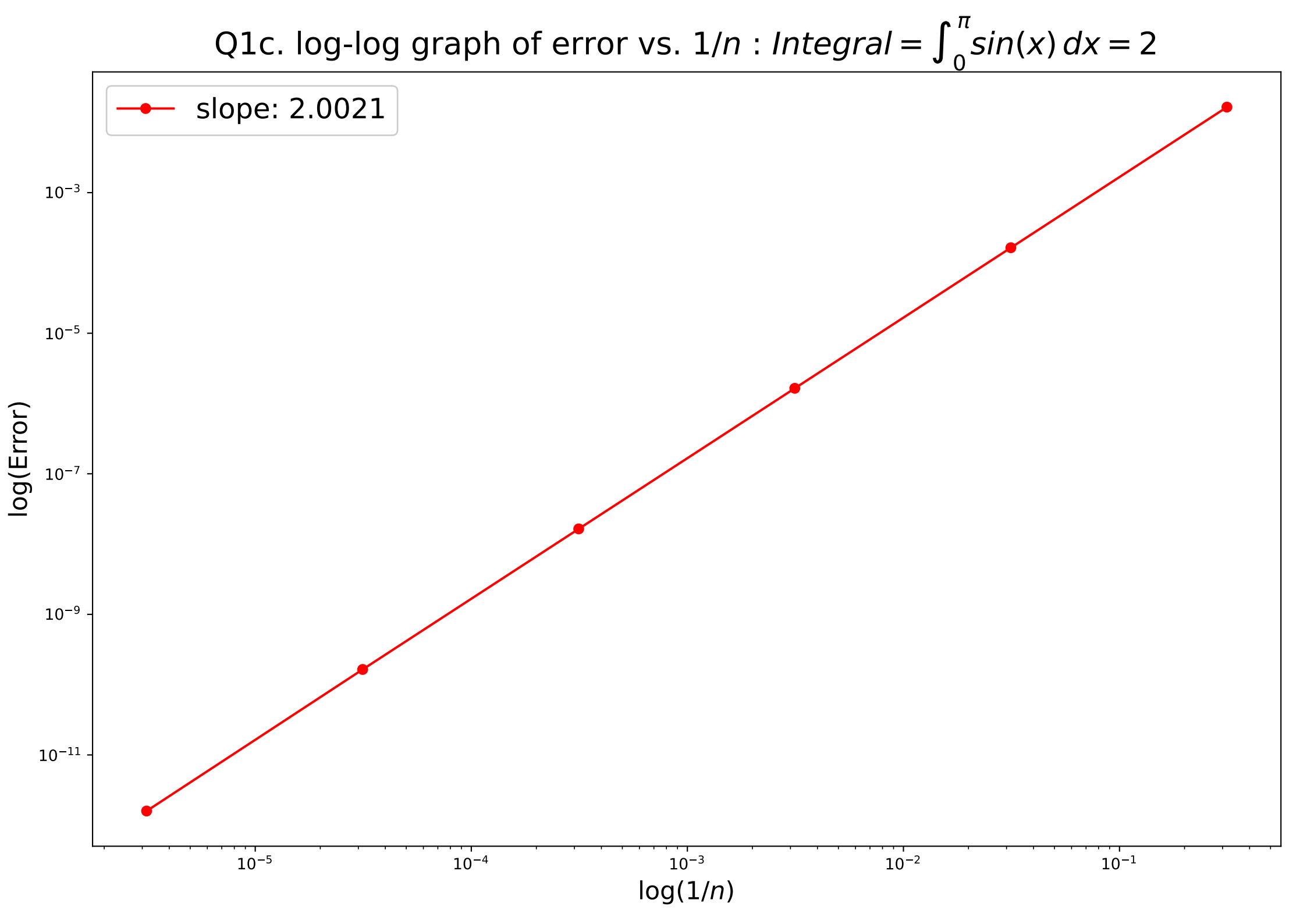
Trapezoidal method, Actual value = π		
No. of grid points (n)	Value of the integral	Absolute_error
10	3.00000000000000	0.141592653589793
100	3.13992598890716	0.0016666468263443
1000	3.14157598692313	1.66666666641113E-05
10000	3.14159248692312	1.66666668910409E-07
100000	3.14159265192314	1.66665303780178E-09
1000000	3.14159265357315	1.66404667822917E-11


Q1b. log-log graph of error vs. 1/n : Integral = $\int_0^1 \frac{4}{1+x^2}\,dx = \pi$
slope: 1.9900
log(Error)
log(1/n)
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}

By trapezoidal method integrate Sin(x) within limits 0 to π.

Trapezoidal method, Actual value = 2		
dx	Value of the integral	Absolute_error
0.314159265358979	1.98352353750945	0.0164764624905454
0.0314159265358979	1.99983550388744	0.00016449612556423
0.00314159265358979	1.99999835506566	1.64493433763013E-06
0.000314159265358979	1.99999998355066	1.64493392240672E-08
3.14159265358979E-05	1.99999999983548	1.6452084139473 3E-10
3.14159265358979E-06	1.99999999999841	1.5922818619174E-12

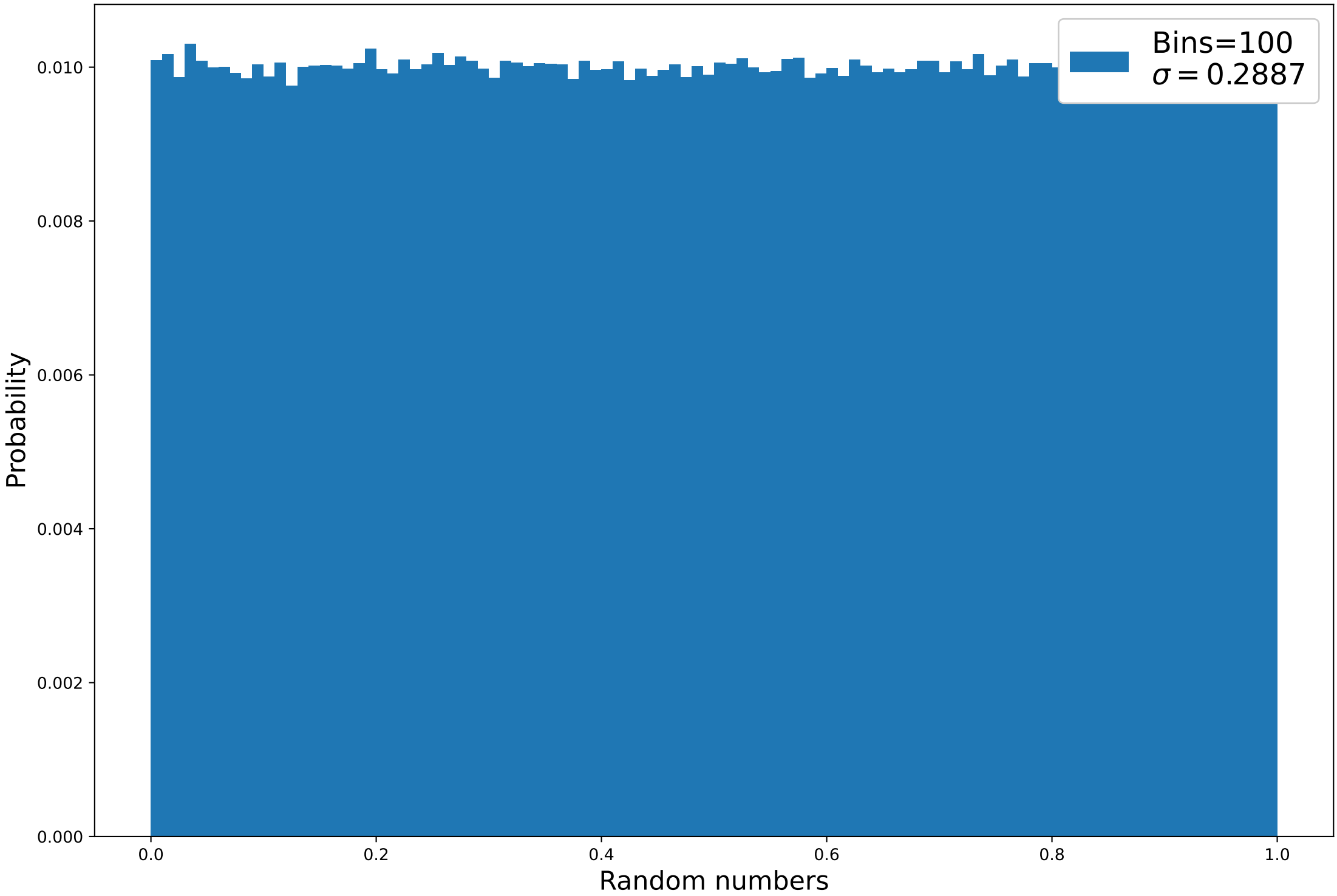

Q1c. log-log graph of error vs. 1/n : Integral = $\int_0^{\pi} sin(x)\,dx = 2$
slope: 2.0021
log(Error)
log(1/n)
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}

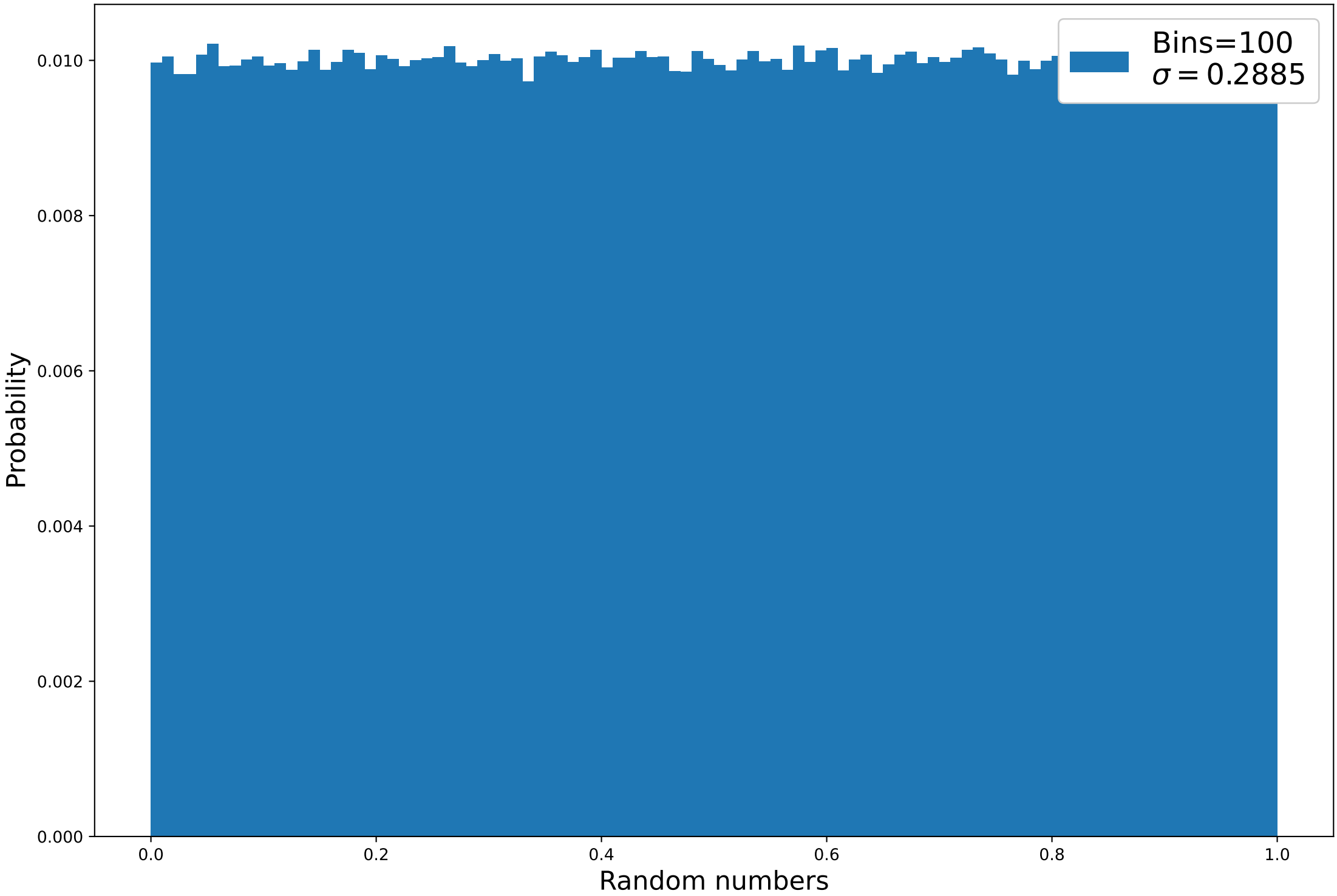
By using trapezoidal mathod we integrate the normalized gaussian function :

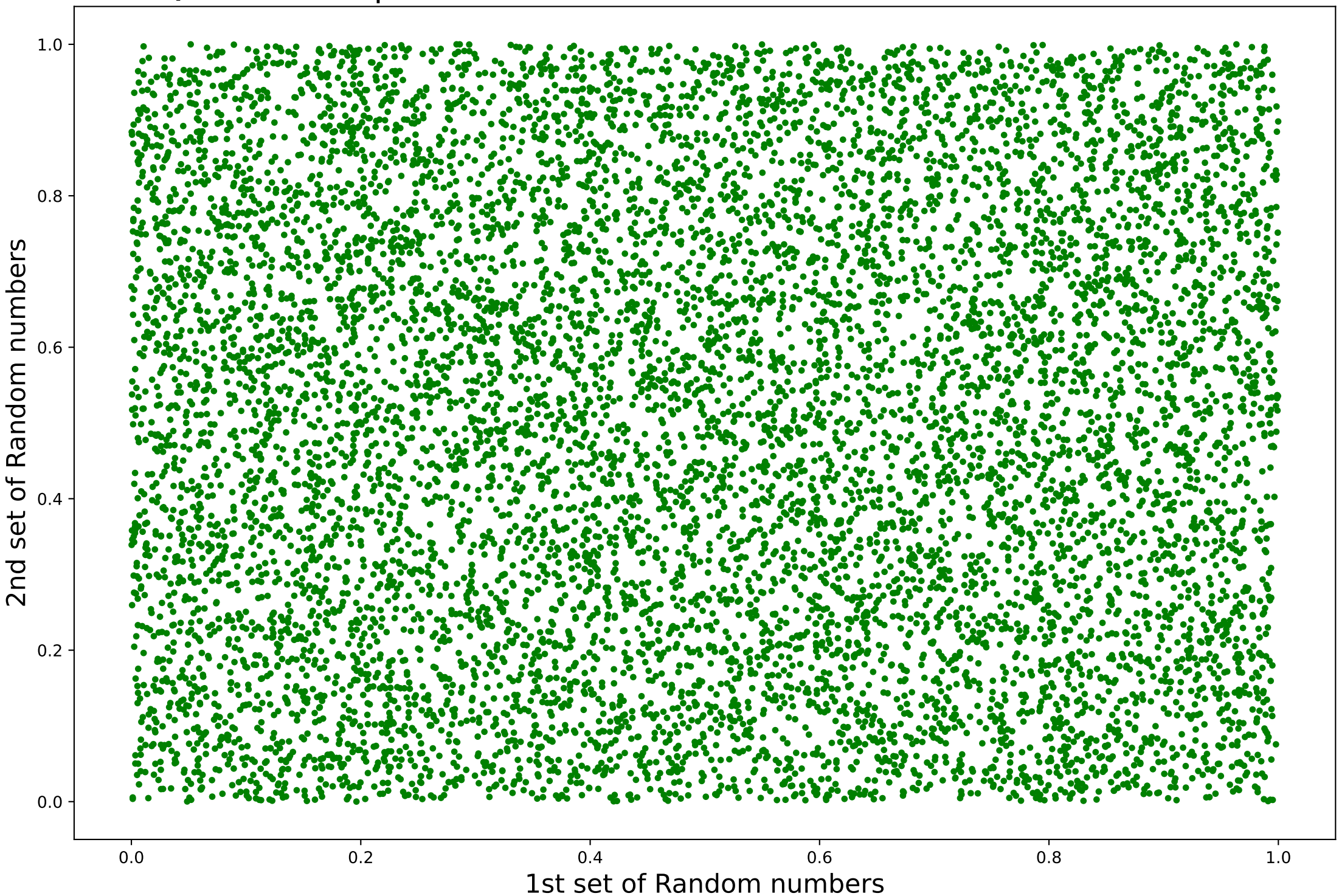
$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

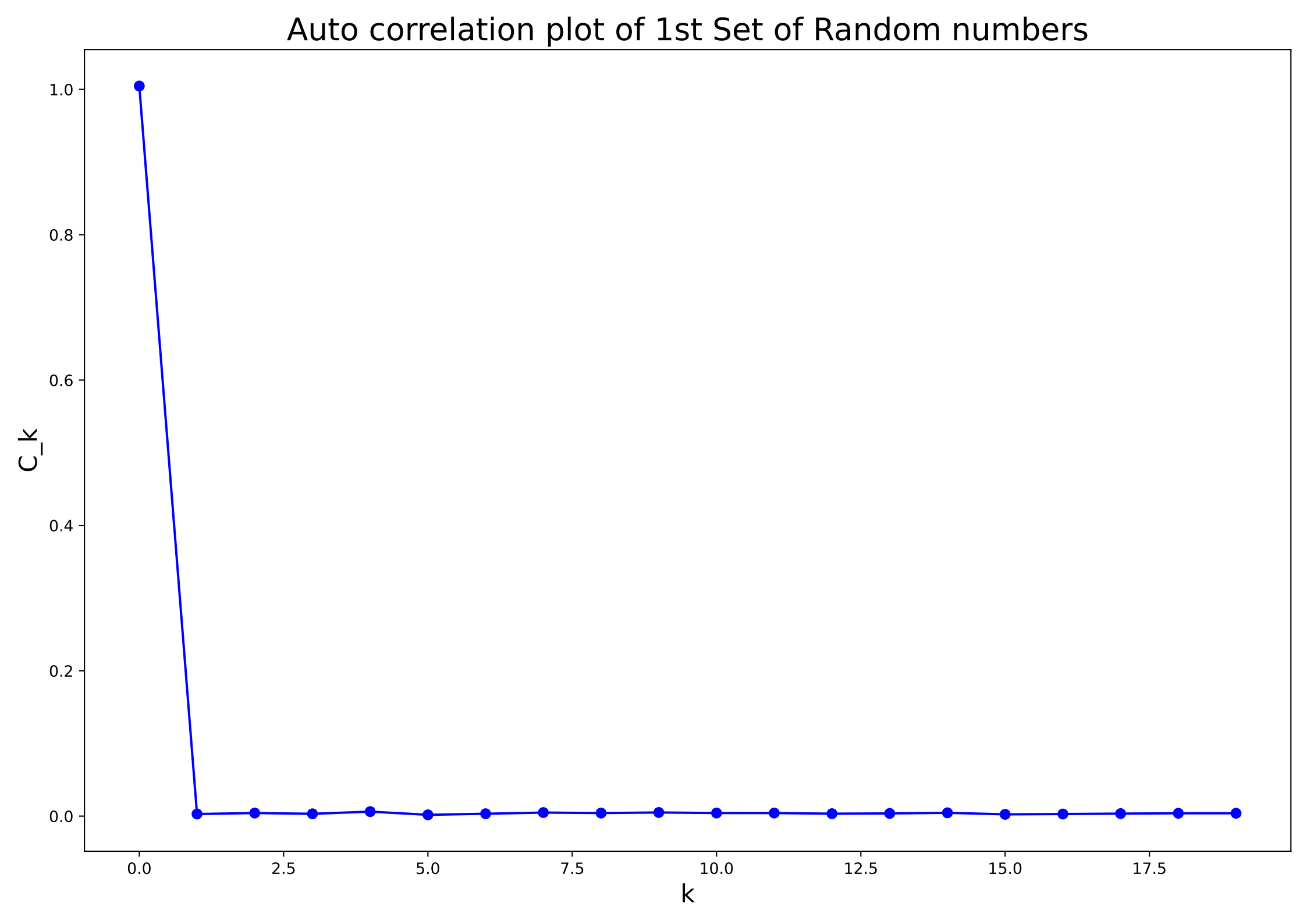
Limits -3 to +3	
No. of gridpoints (n)	Value of the integral
10	0.026591090471628
100	0.996530893052922
1000	0.99729222948119
10000	0.997300124163754
100000	0.997300203139005
1000000	0.997300203928767

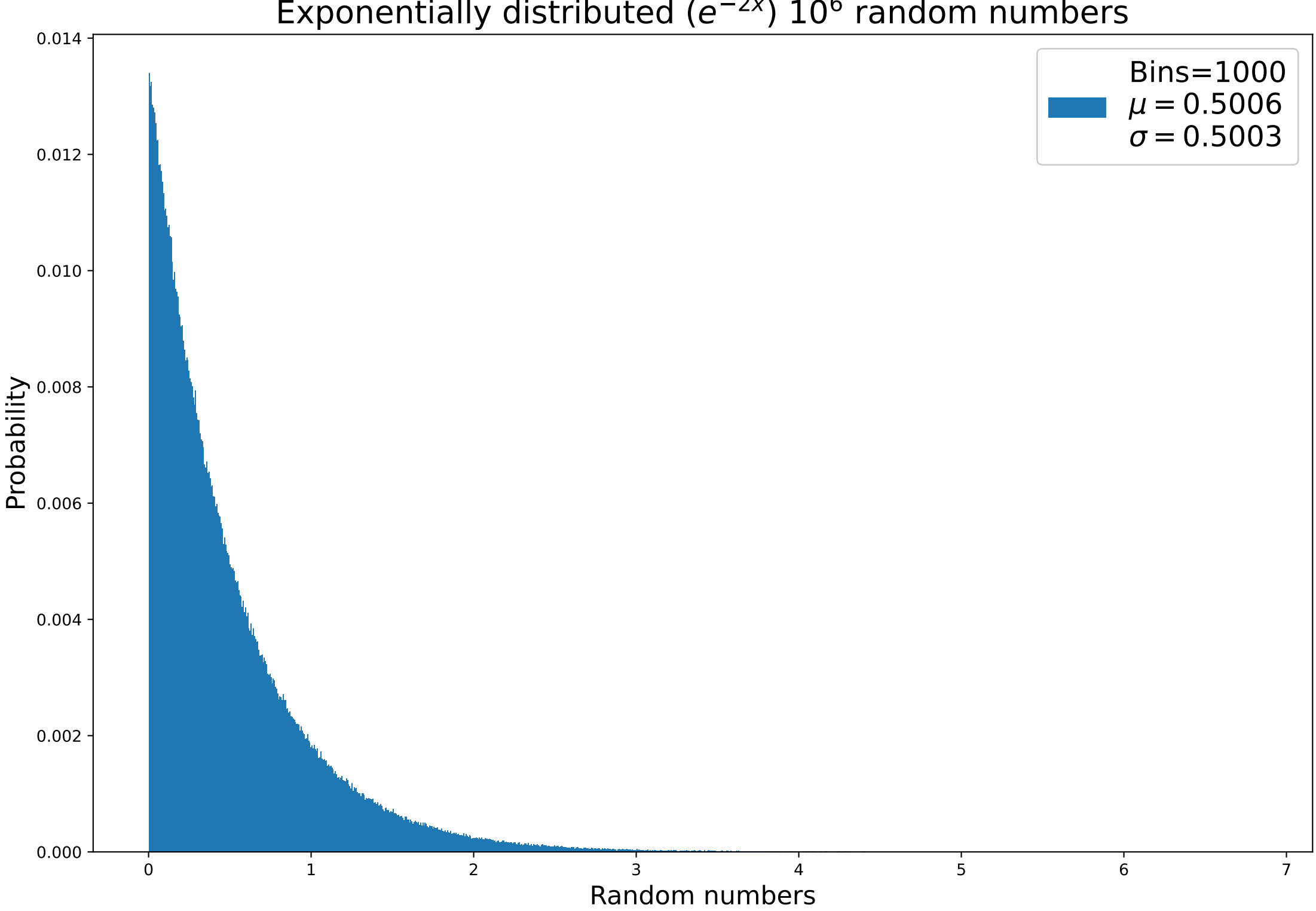
Limits -5 to +5	
No. of grid points (n)	Value of the integral
10	1.4867195147343E-05
100	0.999998506461016
1000	0.999999414352764
10000	0.999999426572969
100000	0.999999426695614
1000000	0.999999426696839

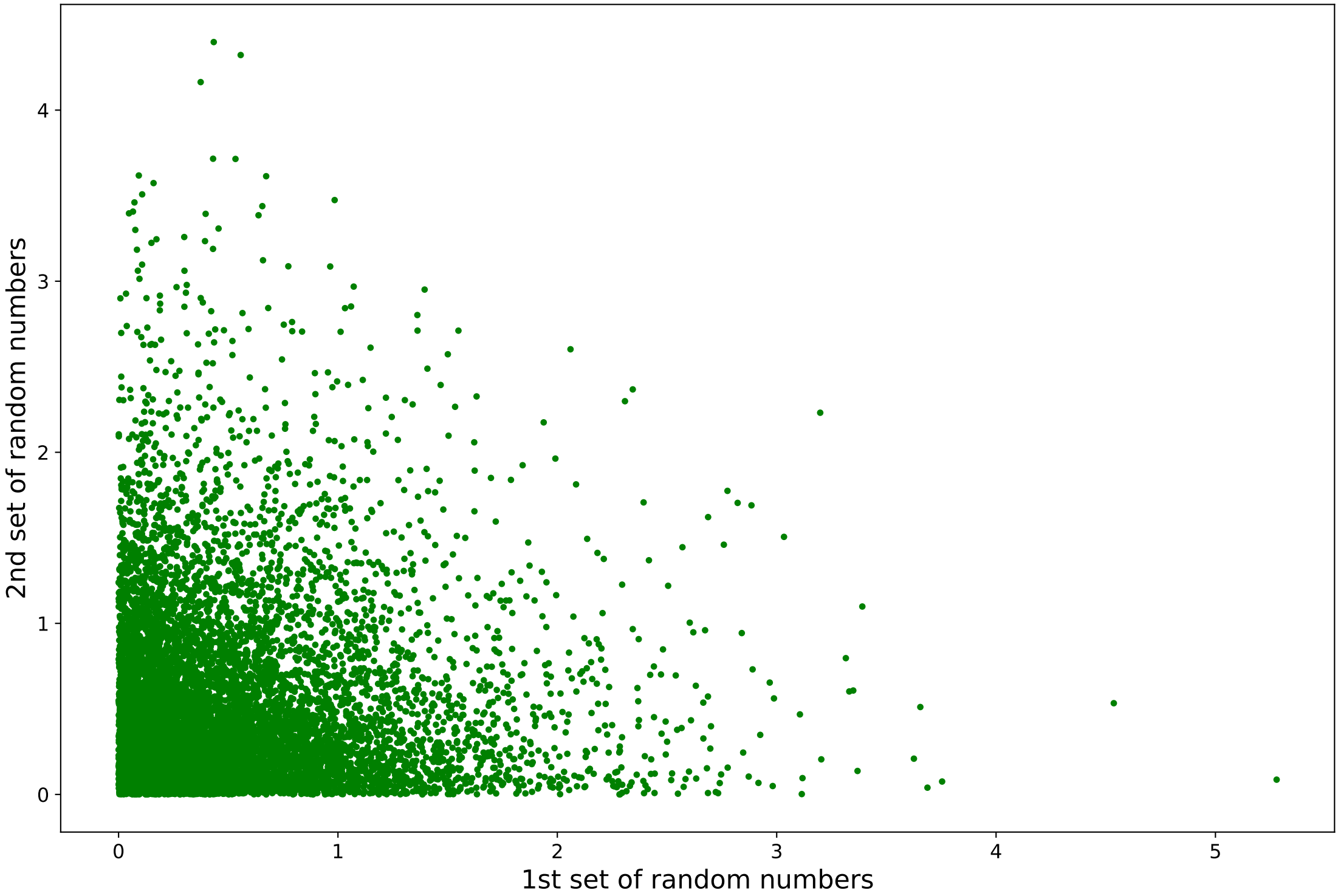

Q2a. Distribution of the 1st set of 10^6 Random numbers
Bins=100
$\sigma = 0.2887$
Probability
0.000
0.002
0.004
0.006
0.008
0.010
0.0
0.2
0.4
0.6
0.8
1.0
Random numbers

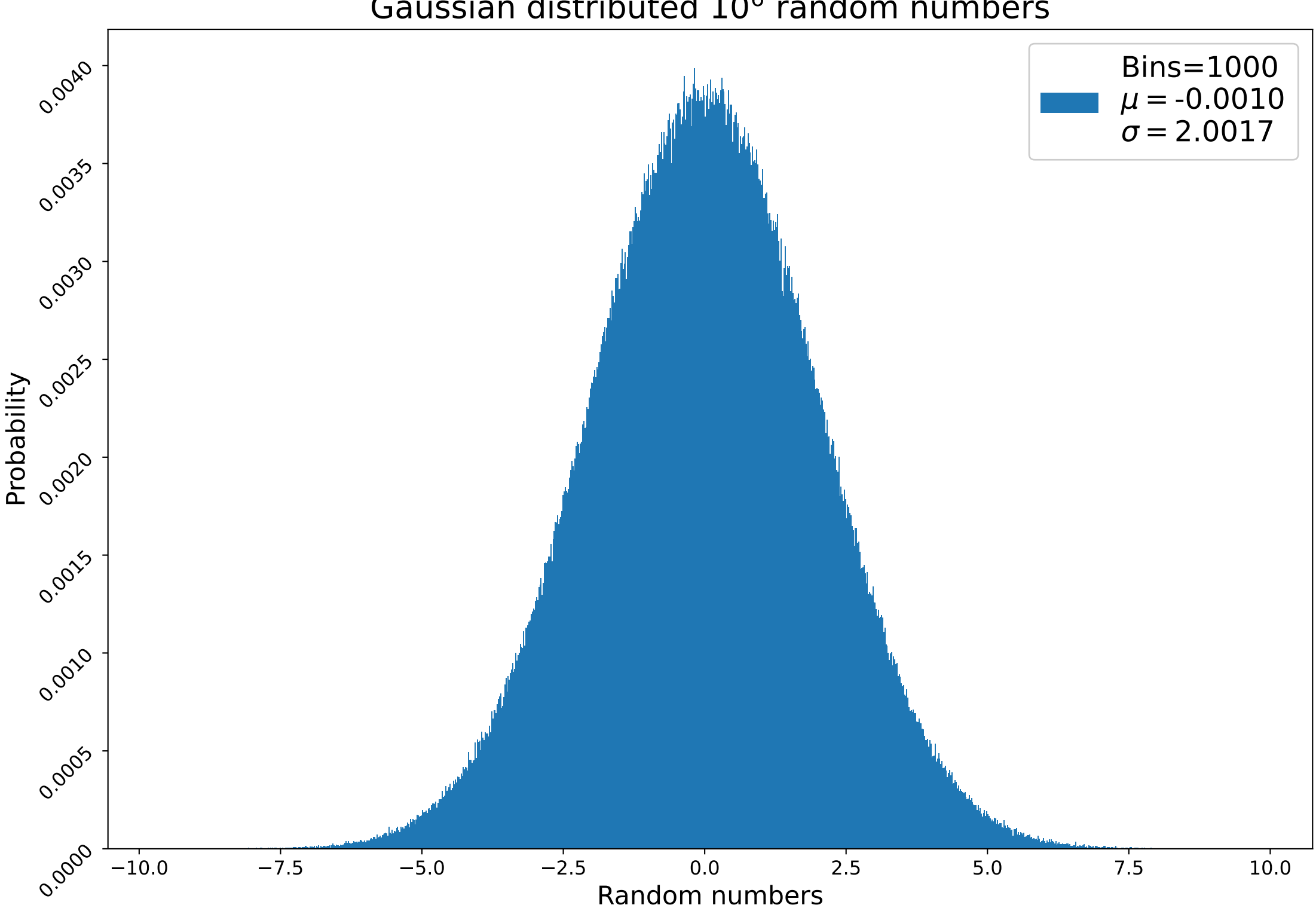

Q2a. Distribution of the 2nd set of 10^6 Random numbers
Bins=100
$\sigma = 0.2885$
Probability
0.000
0.002
0.004
0.006
0.008
0.010
0.0
0.2
0.4
0.6
0.8
1.0
Random numbers

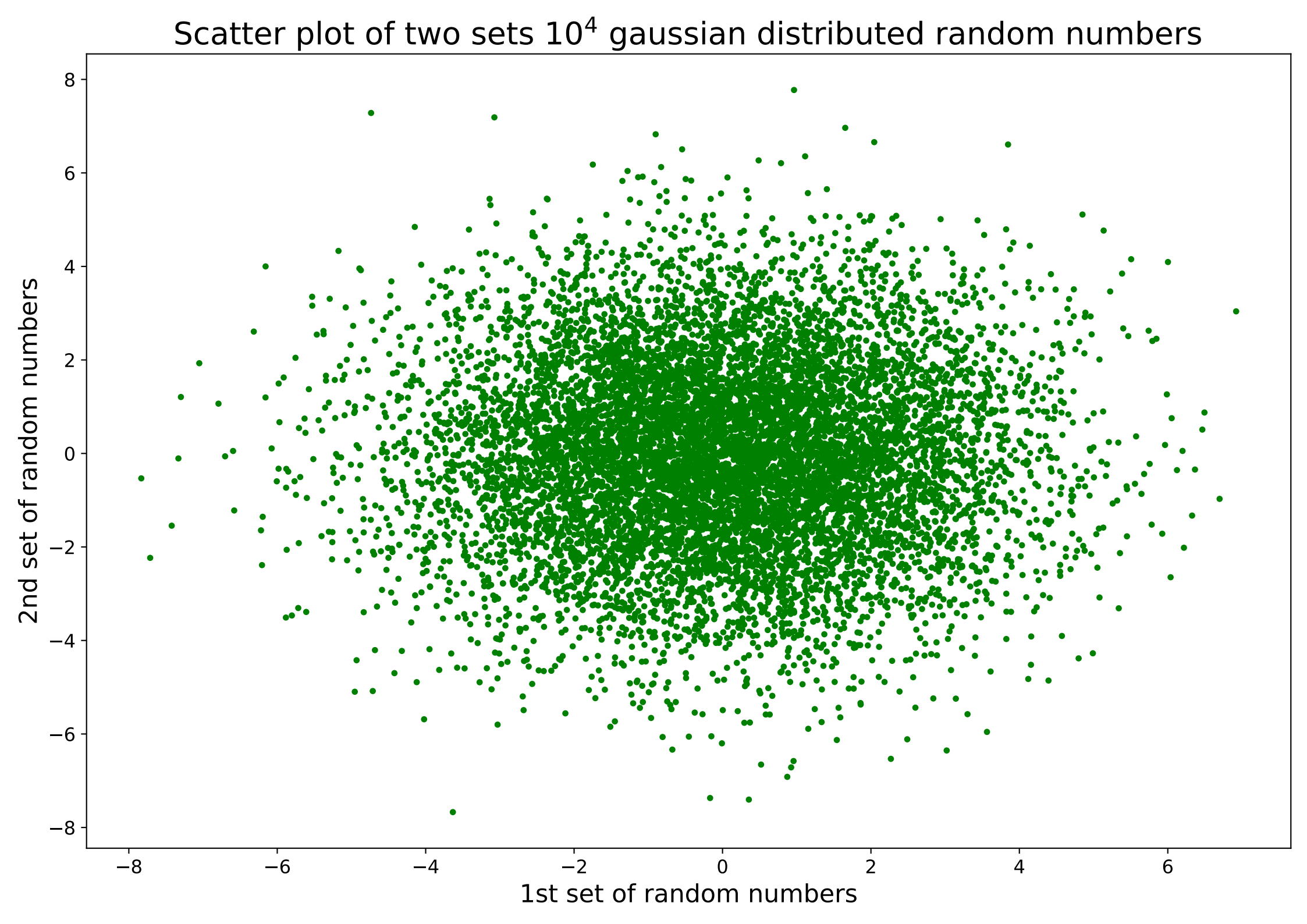

Q2b. Scatter plot between 1st and 2nd set of 10^4 random numbers
2nd set of Random numbers
1st set of Random numbers
0.0
0.2
0.4
0.6
0.8
1.0


Auto correlation plot of 1st Set of Random numbers
C_k
k
1.0
0.8
0.6
0.4
0.2
0.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5


Exponentially distributed (e^{-2x}) 10^6 random numbers
Bins=1000
$\mu = 0.5006$
$\sigma = 0.5003$
Probability
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0
1
2
3
4
5
6
7
Random numbers

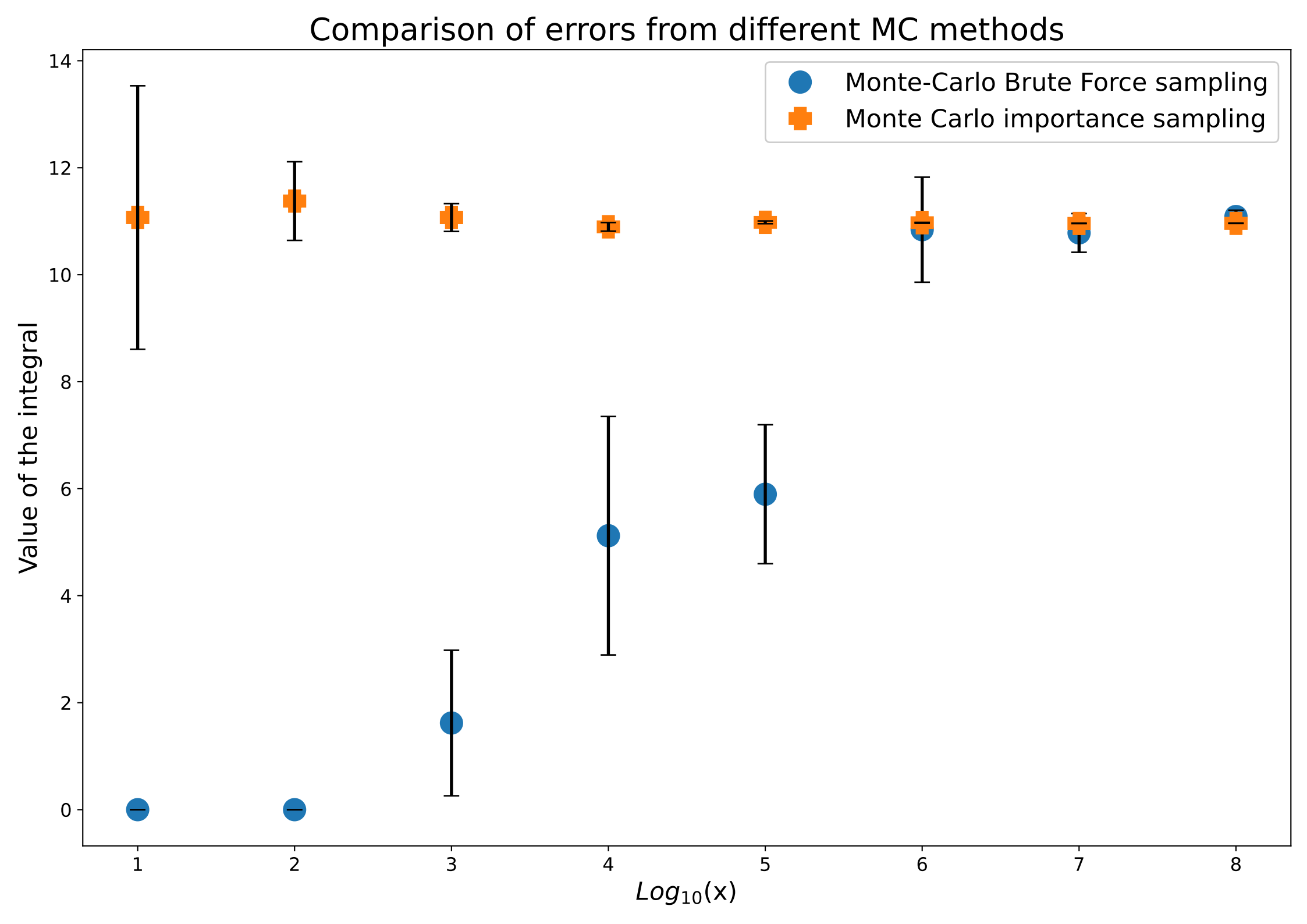

Scatter plot of two sets 10^4 exponentially(e^{-2x}) distributed random numbers
2nd set of random numbers
1st set of random numbers
0
1
2
3
4
5
0
1
2
3
4


Gaussian distributed 10^6 random numbers
Bins=1000
$\mu = -0.0010$
$\sigma = 2.0017$
Probability
0.0000
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
0.0035
0.0040
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
Random numbers


Scatter plot of two sets 10^4 gaussian distributed random numbers
2nd set of random numbers
1st set of random numbers
8
6
4
2
0
−2
−4
−6
−8
−8
−6
−4
−2
0
2
4
6

Using Monte Carlo Brute Force method		
No. of iterations (n)	Value of the integral	Standard deviation
10	9.02253710014757E-07	8.5522350007934E-07
100	7.84976850971433E-06	7.80992654121664E-06
1000	1.61975432572342	1.36019921262025
10000	5.12100379571561	2.22879424933112
100000	5.89670067916585	1.29868842683667
1000000	10.8418267299382	0.981605377194319
10000000	10.7817305203506	0.362123705348325
100000000	11.0877487072685	0.116524406634294

Using Monte Carlo importance sampling method		
No. of iterations (n)	Value of the integral	Standard deviation
10	11.069117498493	2.46316962021154
100	11.3767831158921	0.734888003541454
1000	11.0686458897031	0.259392925244426
10000	10.8941871973983	0.0806824124643543
100000	10.9798037967382	0.0255159419141156
1000000	10.9710404351189	0.0080550620720396
10000000	10.959848601345	0.00254577821592316
100000000	10.9610982518212	0.000805196403344519


Comparison of errors from different MC methods
Monte-Carlo Brute Force sampling
Monte Carlo importance sampling
Value of the integral
Log_{10}(x)
0
2
4
6
8
10
12
14
1
2
3
4
5
6
7
8