Harmonic Analysis of Interacting Pendulum Dynamics

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For first-year physics students, the notion of linked oscillators frequently revolves around solving difficult coupled differential equations, delving into matrix eigenvalues, and grappling with substantial algebraic calculations. While these mathematical tools are certainly important for describing the behavior of coupled systems, they can occasionally lack instant physical intuition, making it difficult for students to grasp the fundamental principles at work.

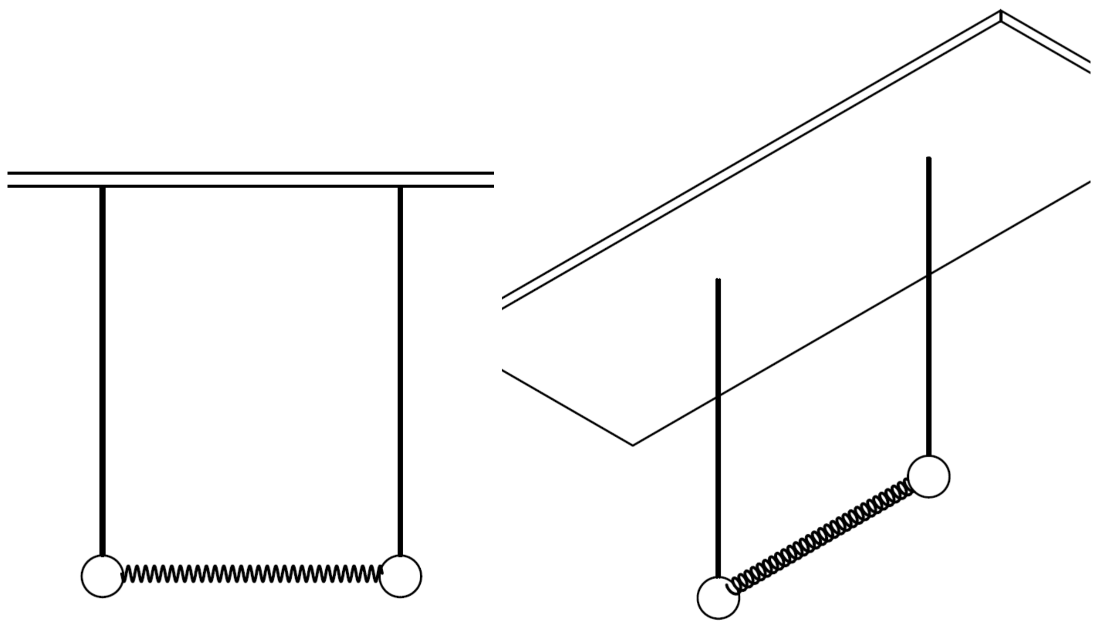
However, the power of the superposition principle, one of the fundamental pillars of physics, can be vividly illuminated through early exposure to real-life systems exhibiting coupled oscillations. Consider, for instance, a row of pendulums connected by springs. When set into motion, this system showcases multiple modes of oscillation, each with its distinct frequency. As energy gracefully transfers between the pendulums through the springs, students witness the superposition principle in action – the idea that complex motion can be understood as the sum of simpler, individual oscillations.

This tangible example bridges the gap between abstract mathematics and physical reality, offering students a deeper appreciation of how the superposition principle permeates various branches of physics. From molecular vibrations in chemistry to coupled quantum systems in quantum mechanics and even the intricate dynamics of musical instruments in acoustics, the principle of superposition emerges as a common thread that weaves together diverse phenomena. Early exposure to such systems not only fosters a richer understanding of the physical world but also instills a sense of wonder and curiosity that drives scientific exploration.

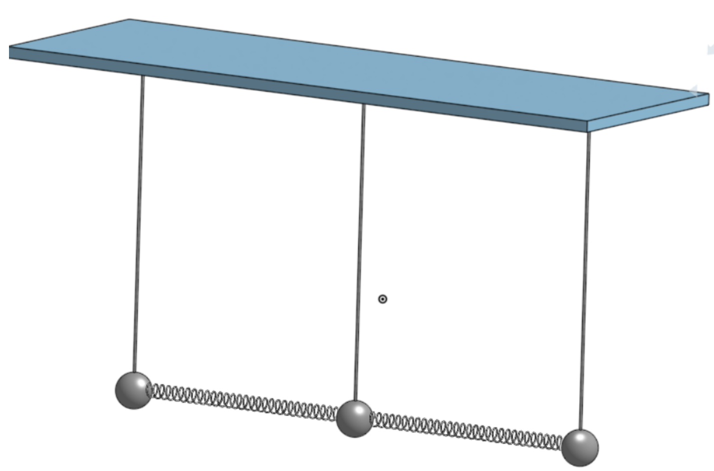
Through extremely simple system like a pair of identical pendulums coupled through a mechanical spring can probe into various of results specific to coupled systems:

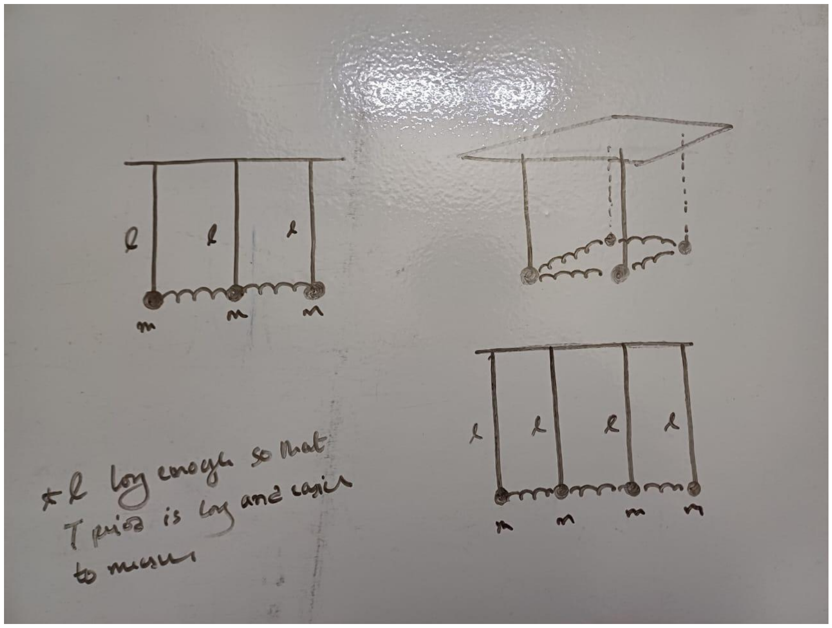
- Existence of normal modes which are pure repetitive motions with fixed frequencies (like stationary states in QM)
- Any random motion of the pendulums being a superposition of these modes (like a superposition of two eigenstates in Quantum Mechanics)
- No. of modes present equal to the no. of oscillators.
- Difference in the frequencies of the independent and coupled oscillator (if the systems energy is dependent on frequency, the splitting of energies upon coupling which is a important phenomena in Quantum Mechanics)

Further such an analogous system can also be built in the lab with coupled LC circuits.



This is just a basic model and due to lack of time we couldn't really add more things which we wished to include. We will definitely revise our experiments and make it more interactive in the future revision of the same.





Measurement tools to be used - Vernier callipers, Meter scale, weighing balance, Stop watch.