

2IL76

Algorithms for Geographic Data

Spring 2015

Lecture 4: Schematization

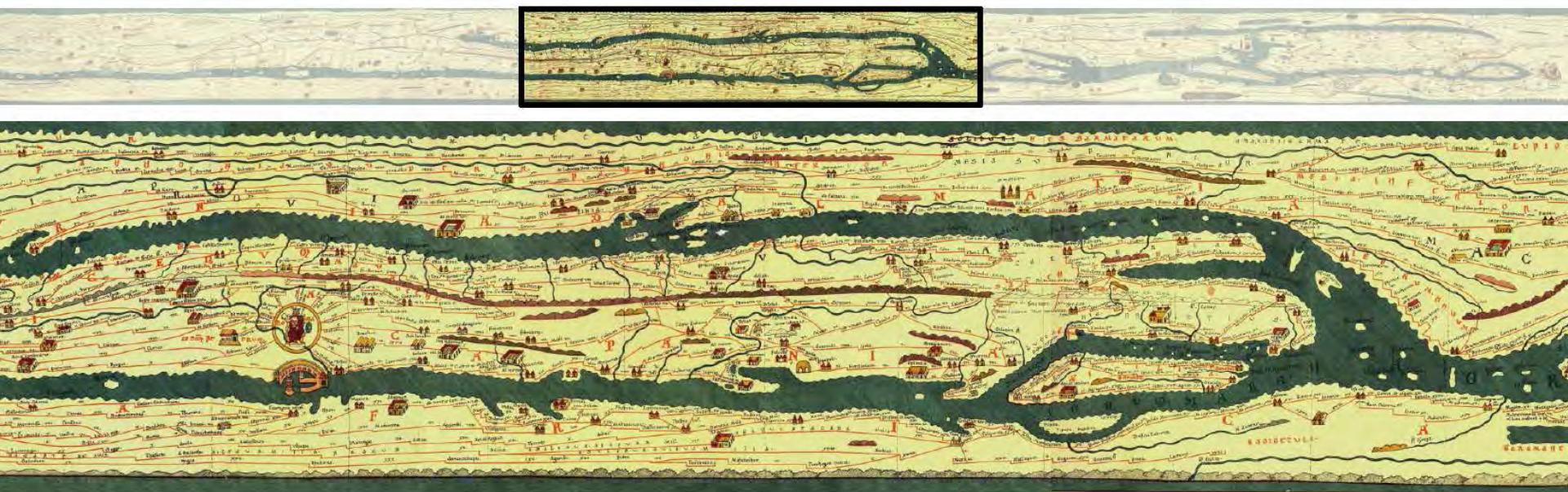
Schematic maps



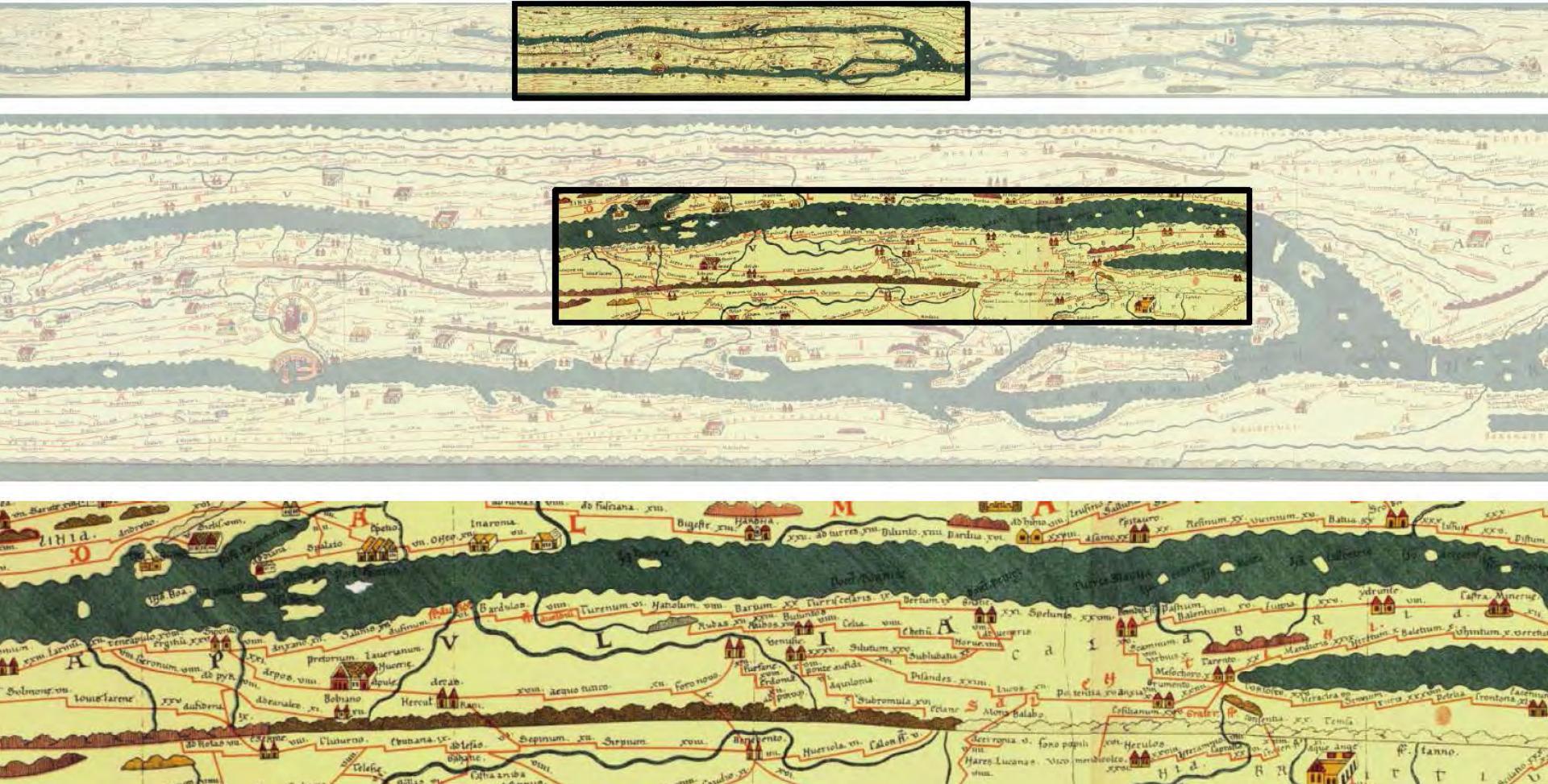
Ancient schematic maps



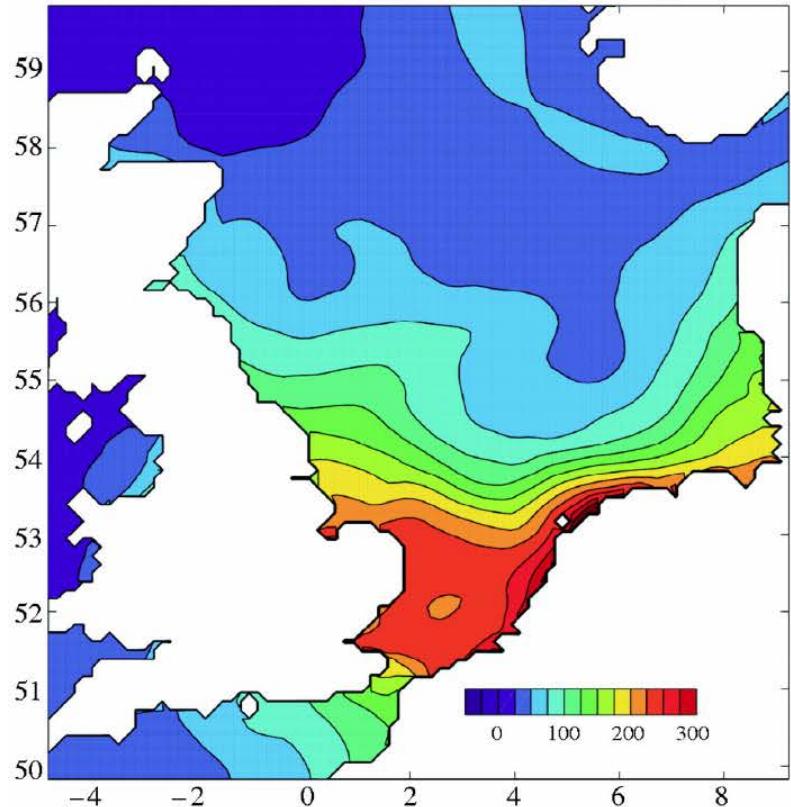
Ancient schematic maps



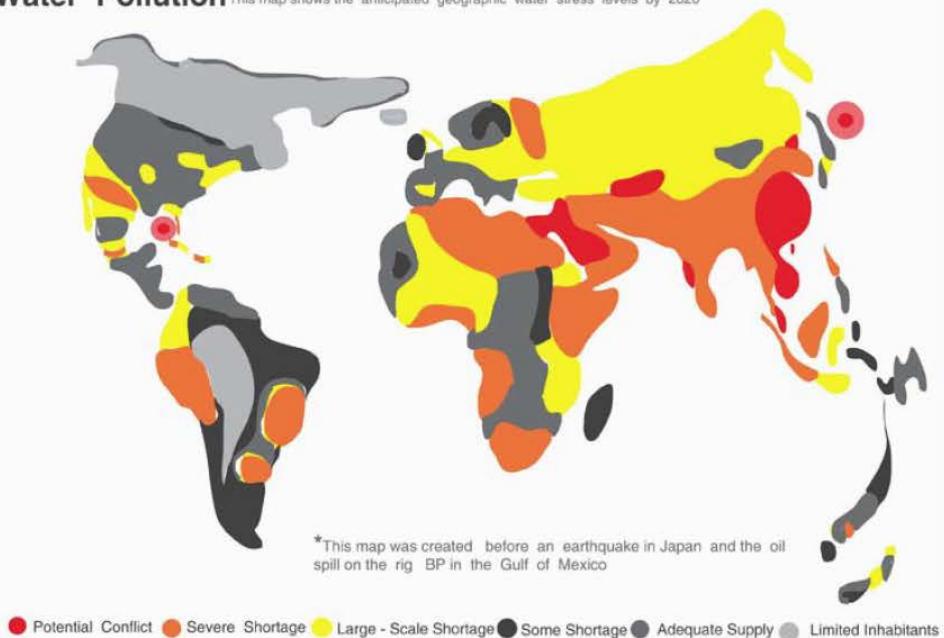
Ancient schematic maps



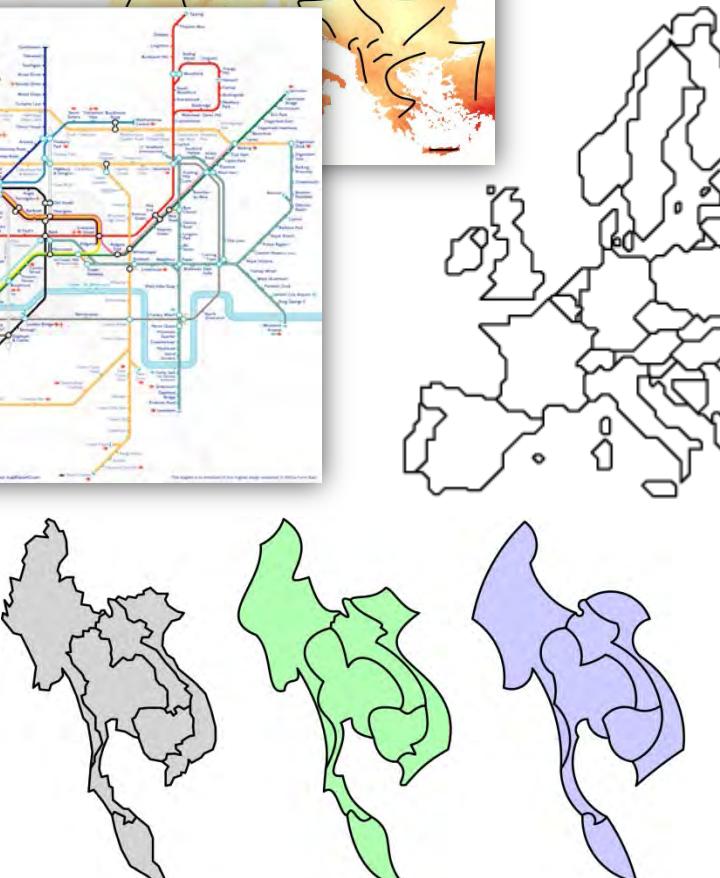
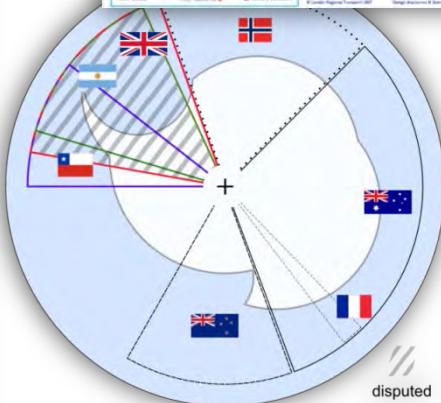
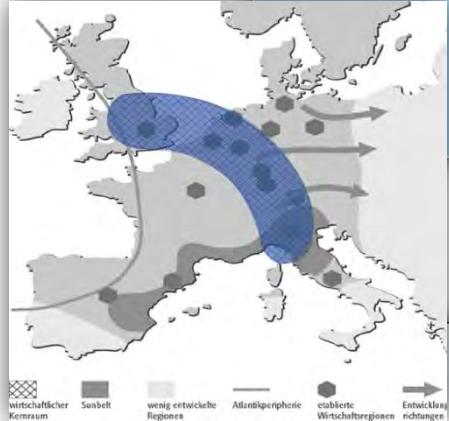
Not just for transport ...



Water Pollution This map shows the anticipated geographic water stress levels by 2020



More schematic maps



Schematic?



Schematization

What is schematization?

a stylized, abstract representation

- usually simpler than input (relation to simplification)
- iconic: few directions of lines, specific curves, ...
- might preserve topology
- some visual resemblance to input

Most commonly schematized

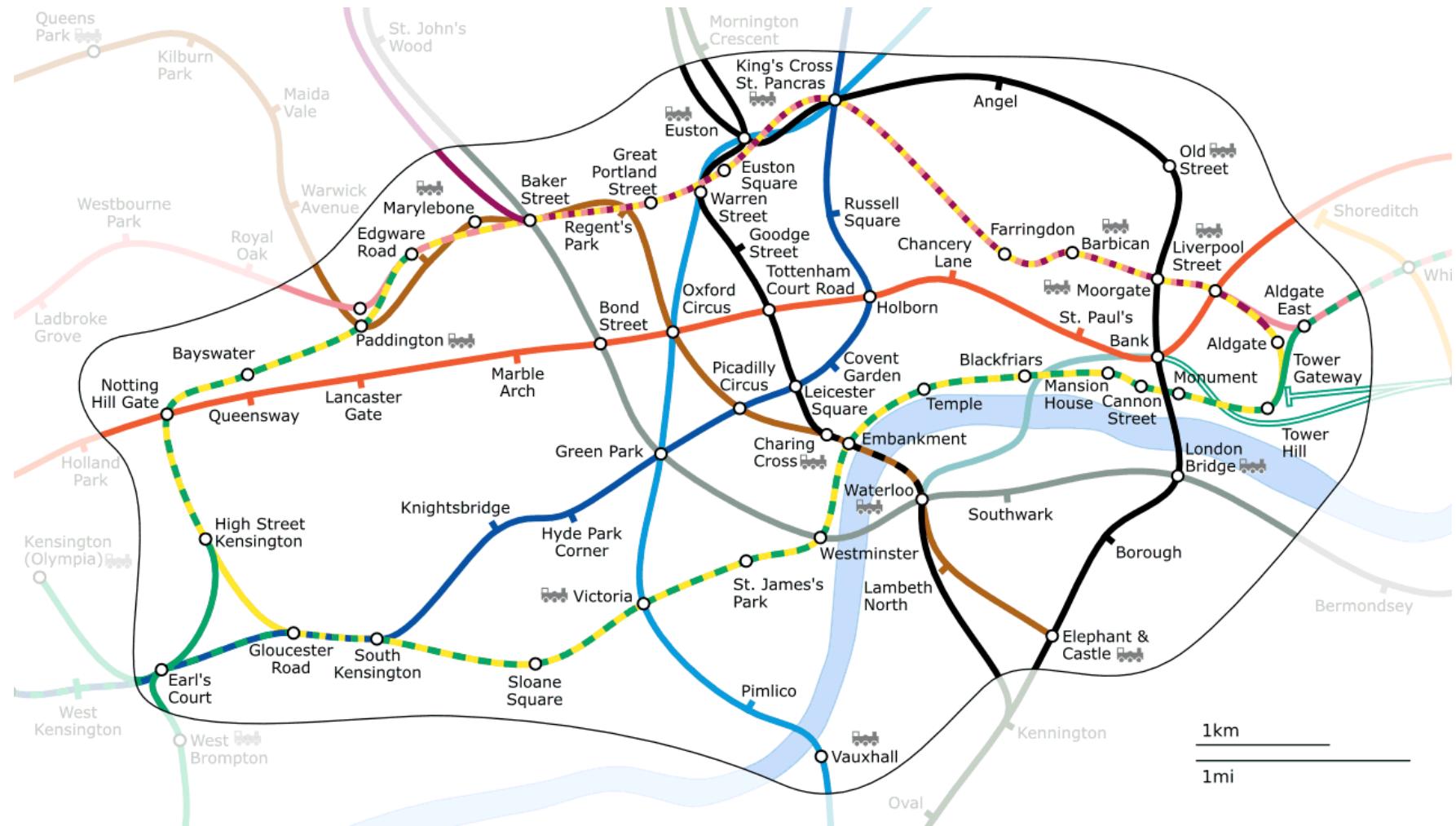
- subdivisions
- networks

vertex-restricted or non-vertex-restricted

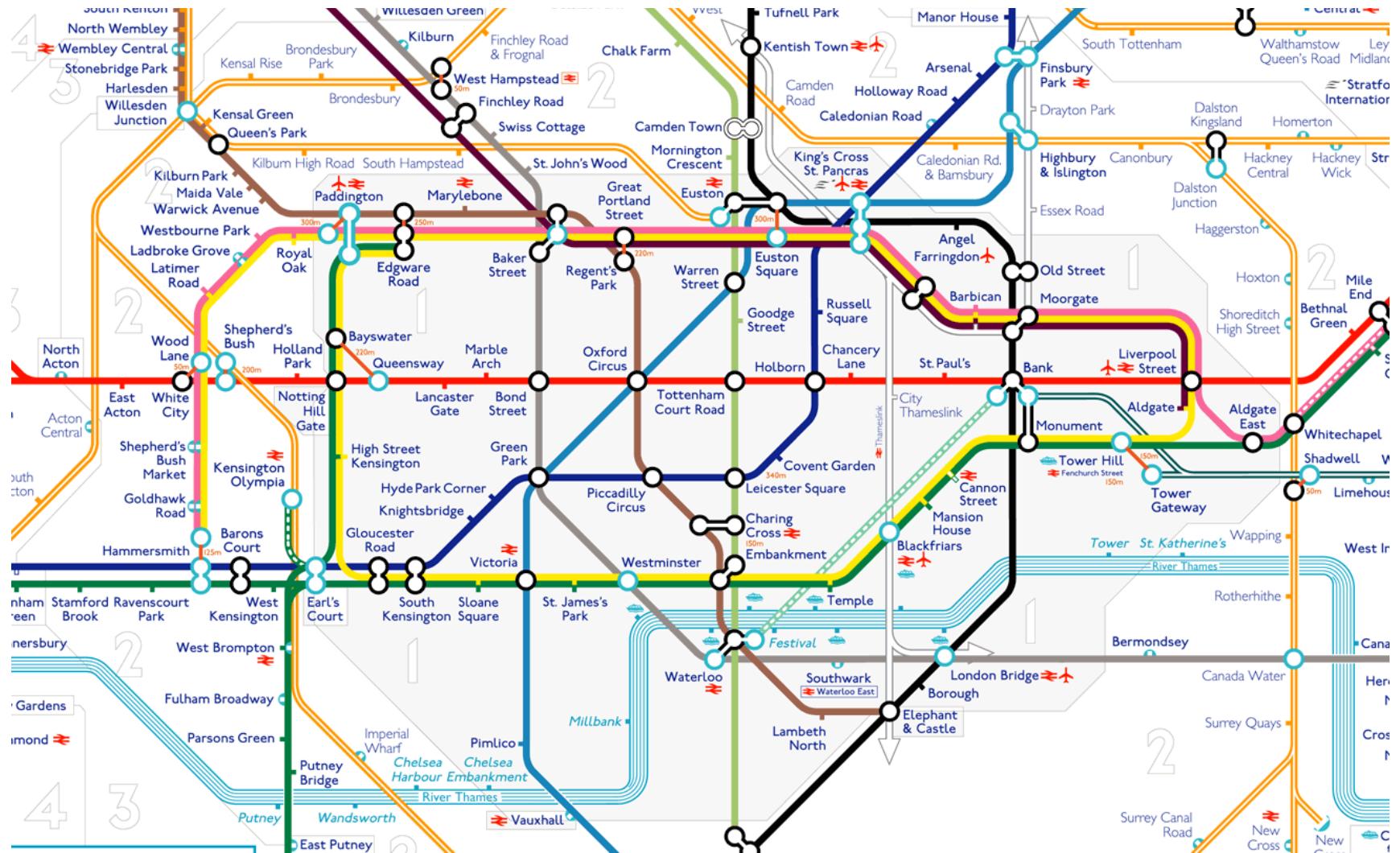
area preserving, topology preserving, using curves or straight lines ...

Network Schematization

Network schematization

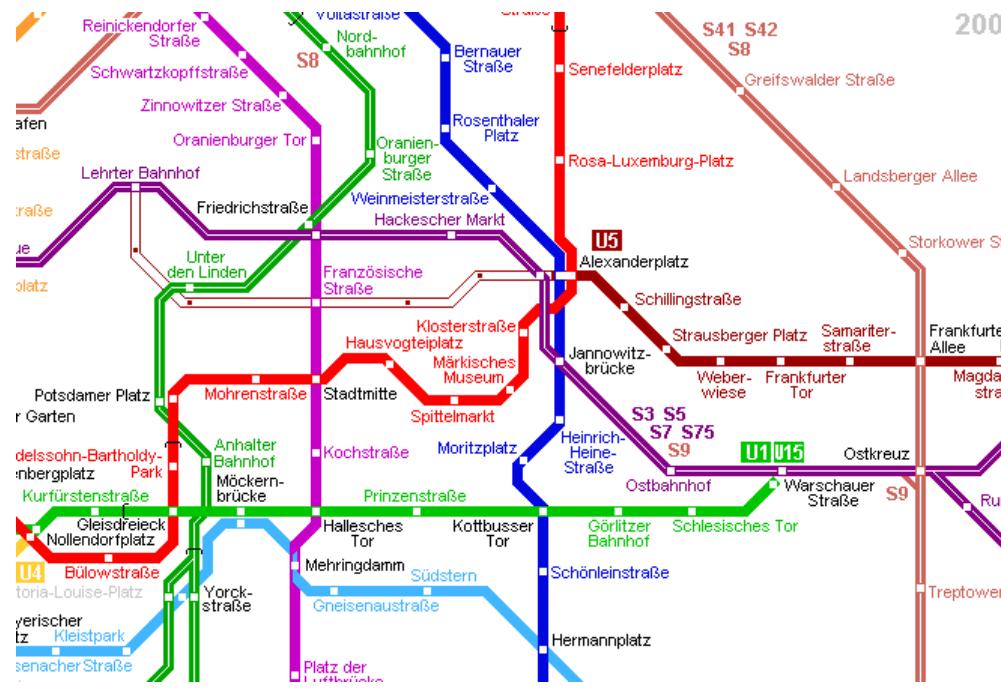


Network schematization



Common criteria

- many stations aligned horizontally, vertically or diagonally
 - sufficient spacing between different lines
 - connections have at most two bends
 - stations are not displaced too much
maximum displacement can be different for different stations



Algorithmic solutions

- many stations aligned horizontally, vertically or diagonally
- sufficient spacing between different lines
- connections have at most two bends
- stations are not displaced too much
maximum displacement can be different for different stations

Many iterative approaches

solution quality and convergence cannot be guaranteed

Algorithmic solutions

- many stations aligned horizontally, vertically or diagonally
- sufficient spacing between different lines
- connections have at most two bends
 - stations are not displaced too much
 - maximum displacement can be different for different stations*

[Cabello, de Berg, van Kreveld, 2005]

combinatorial approach:

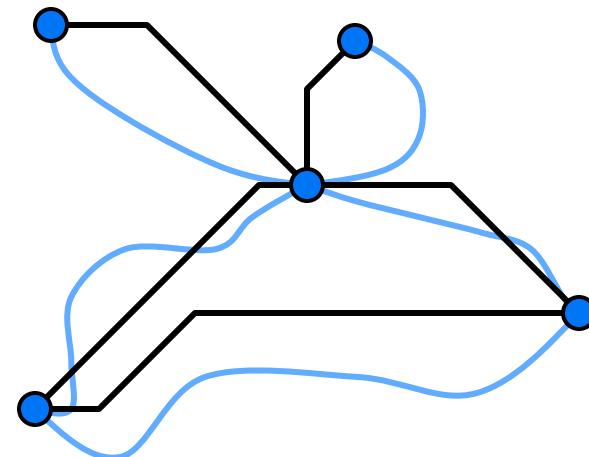
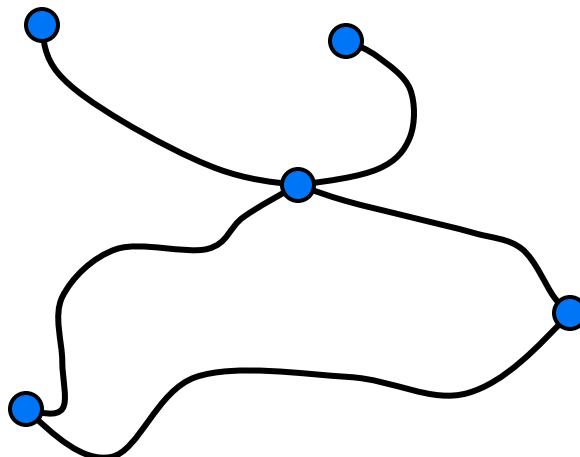
- replace every connection by one of a schematic type
- define a top-to-bottom placement order on connections
- place each connection in its topmost position that gives no overlap

Algorithmic solutions

[Cabello, de Berg, van Kreveld, 2005]

combinatorial approach:

- replace every connection by one of a schematic type
- define a top-to-bottom placement order on connections
- place each connection in its topmost position without overlap

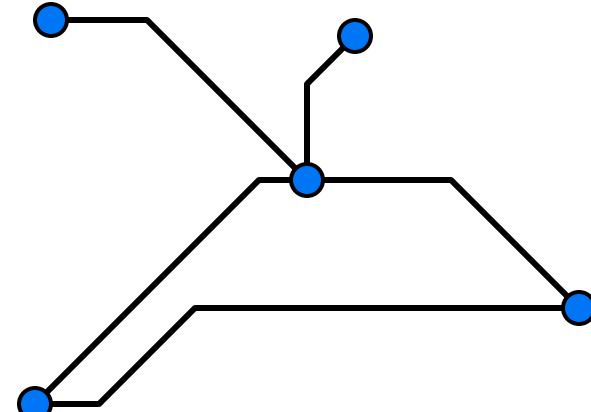
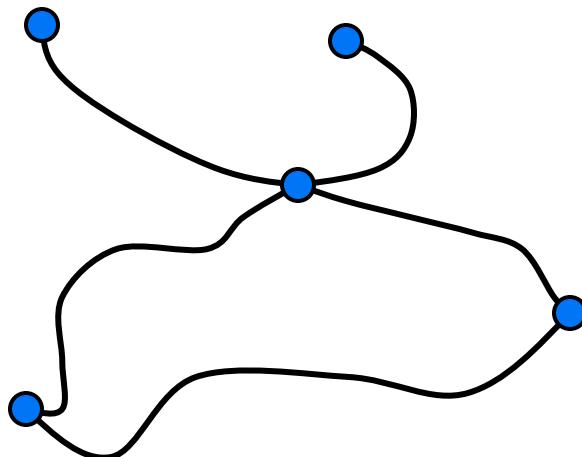


Formalization

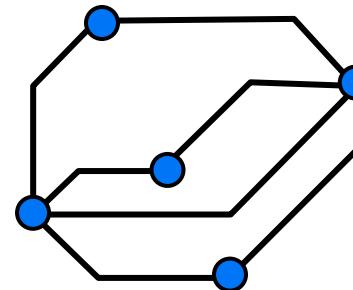
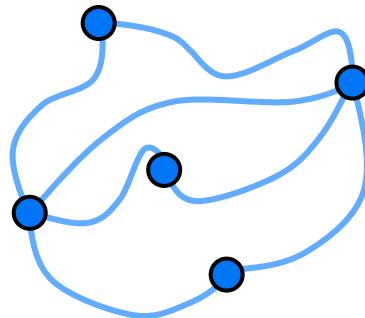
Definitions

A (polygonal) **map M** is a set of simple polygonal paths $\{c_1, \dots, c_m\}$ such that two paths do not intersect except at shared endpoints.

A **monotone map** is a map where all paths are **x-monotone**.



Potential issues



- keep cyclic order of paths around endpoints
- each path of the schematic map is a deformation of the original path, without passing over endpoints
- or, each path in the original map is a deformation of a path in the schematic map, without passing over the endpoints

more formally ...

Formalization

Definitions

Two maps M and M' are equivalent if and only if

- they have the same endpoints
- each path of M can be continuously deformed to a path of M' without passing over the endpoints

schematic path: axis-aligned, x -monotone, at most 3-links ...

Formal problem statement

Given a polygonal map M , compute an equivalent schematic map M' whose paths are schematic.

optional: minimum vertical distance, shared (pieces of) paths ...

Formalization

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[Cabello, de Berg, van Kreveld, 2005]

combinatorial approach:

- replace every connection by one of a schematic type
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[Cabello, de Berg, van Kreveld, 2005]

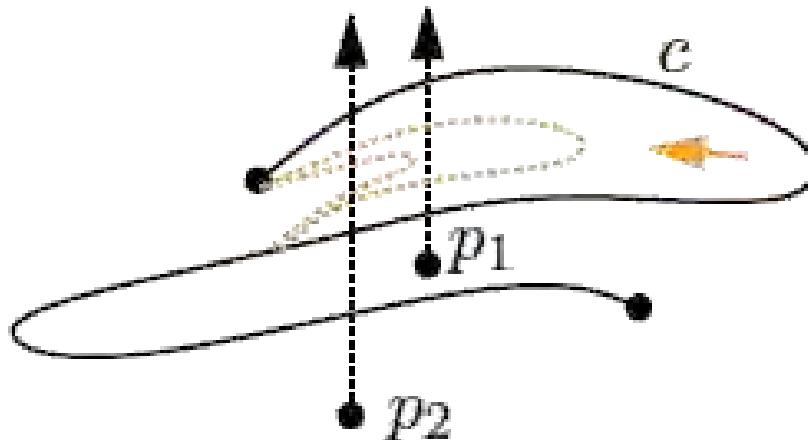
combinatorial approach:

- replace every connection by one of a **schematic type**
- define a **top-to-bottom placement order** on connections
- place each connection in its **topmost position** without overlap

Below and above a curve

Definition

Point p is **below/above** path c if any continuous deformation of c , that does not pass over p , intersects the vertical **upper/lower** ray from p .

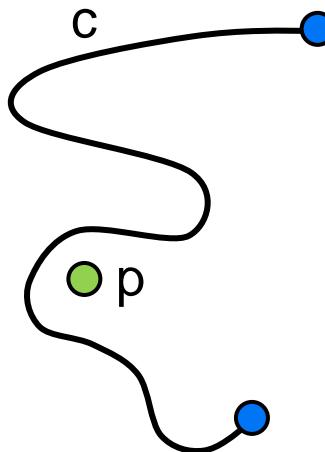


- to decide whether a point is above or below a path, we do not consider other points

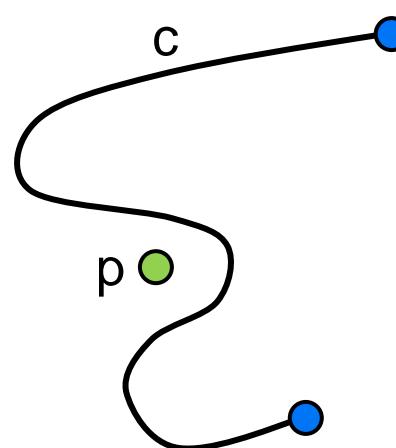
Below and above a curve

Definition

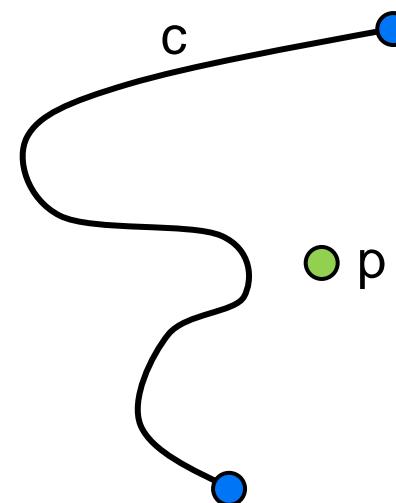
Point p is **below/above** path c if any continuous deformation of c , that does not pass over p , intersects the vertical **upper/lower** ray from p .



p is both above
and below c



p and c have
no relation

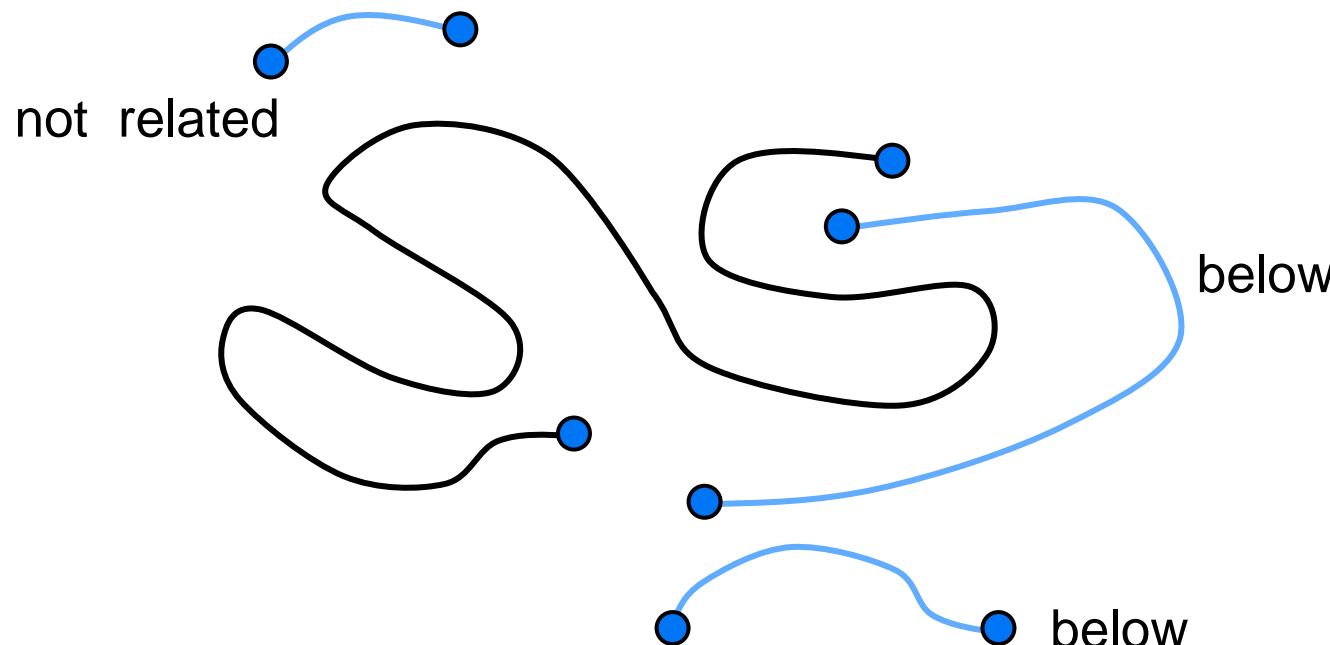


p is below c

Order among paths

Definition

Path c is **above** path c' if any endpoint of c is above c' or any endpoint of c' is below c .



Lemma

The above-below relation among paths is invariant between equivalent maps.

Order among paths

Lemma

The above-below relation among paths is invariant between equivalent maps.

- the above-below relation is preserved in the schematic map
- if the above-below relation is **acyclic**, extend to order and use to place schematic connections **topmost**

Remaining questions

Is there always an order?

If there is an order, can we compute it efficiently?

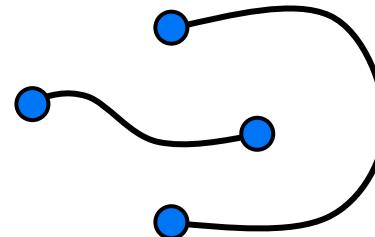
Does an order imply a schematic map exists?

At least for certain types of connections?

Order among paths

Is there always an order?

No



Lemma

For a monotone map M , the above-below relation among paths is acyclic. Furthermore, if M has complexity n , a total order extending this relation can be computed in $O(n \log n)$ time.

Definition

Path c is **above** path c' if any endpoint of c is above c' or any endpoint of c' is below c .

for x -monotone paths equivalent to

Path a is above path b (denoted $a \triangleright b$) if and only if there are points $(x, y_a) \in a$ and $(x, y_b) \in b$ with $y_a > y_b$.

Computing above-below relationships

If there is an order, can we compute it efficiently?

Theorem

For a map M of total complexity n , we can decide in $O(n \log n)$ time whether an equivalent, monotone map exists.

Proof

uses canonical sequences, rectified maps, universal cover ...

Schematization of networks

Sergio Cabello, Mark de Berg, and Marc van Kreveld

Computational Geometry 30:223–238, 2005

Testing Homotopy for Paths in the Plane

Sergio Cabello, Yuanxin Liu, Andrea Mantler, and Jack Snoeyink

Discrete & Computational Geometry 31(1):61-81, 2004

Schematic maps

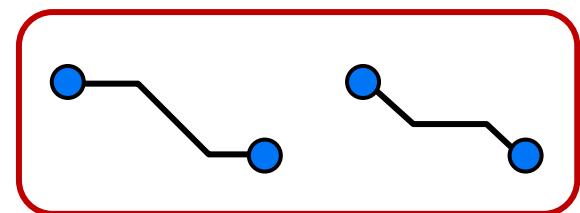
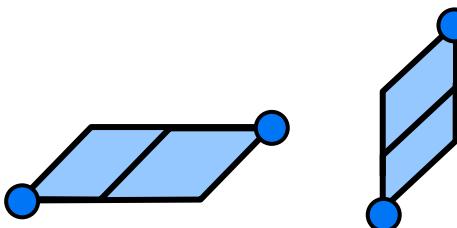
Does an order imply a schematic map exists? **No**

At least for certain types of connections? **No**

But if we specify the types of schematic connections, we can test ...

Intuition

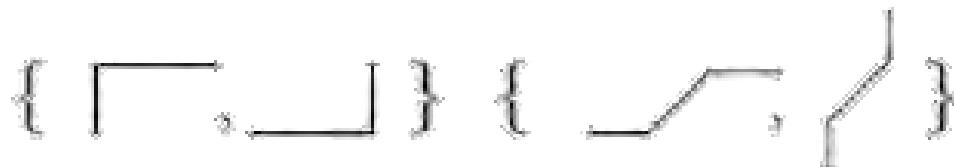
we can only use schematic connections together that have a clear topmost placement ...



x-monotone ordered schematic map model

Schematic map models

x-monotone ordered schematic map models



Algorithm

Input: a map M and an x -monotone ordered schematic map model

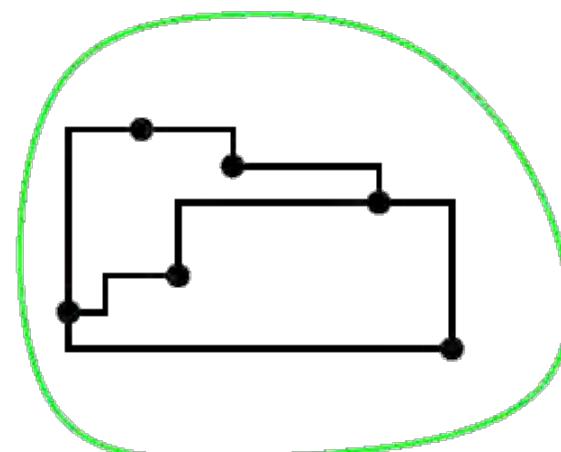
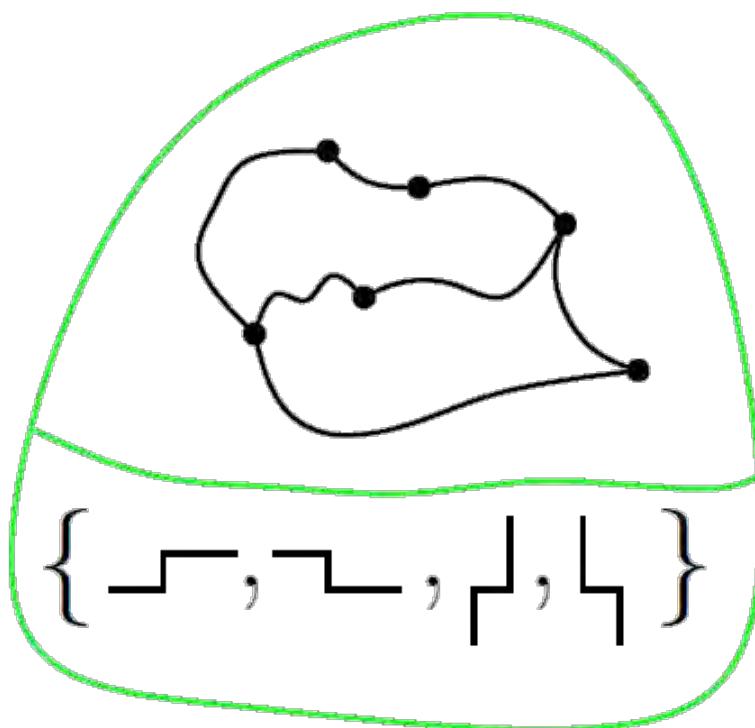
Output: an equivalent schematic map M' or “does not exist”

1. compute above-below relations among paths of M
2. if acyclic, complete to order, otherwise return “does not exist”
3. place paths (topmost each) in order
return “does not exist” if placement is not possible

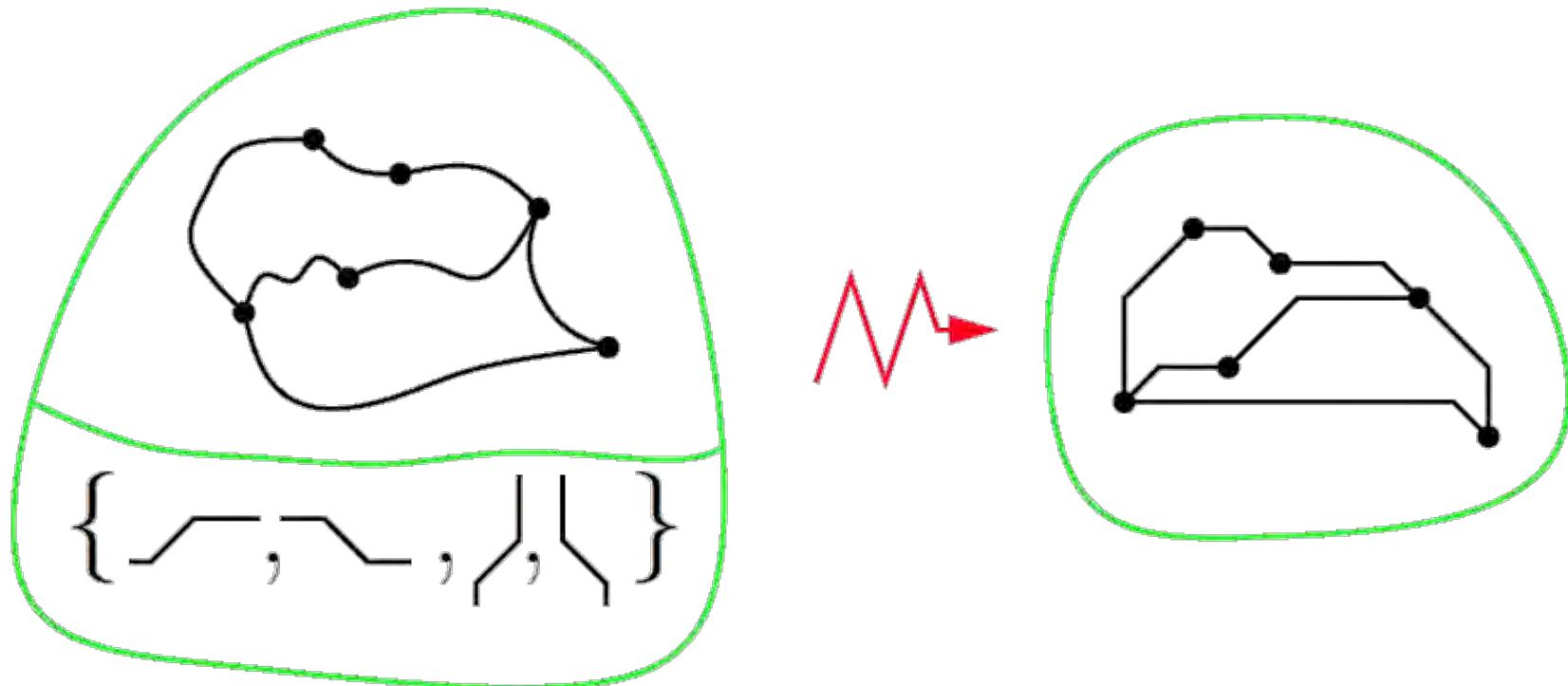
Running time: $O(n \log n)$ where n is the complexity of M

maintain lower envelop of already-placed connections ...

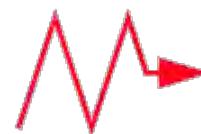
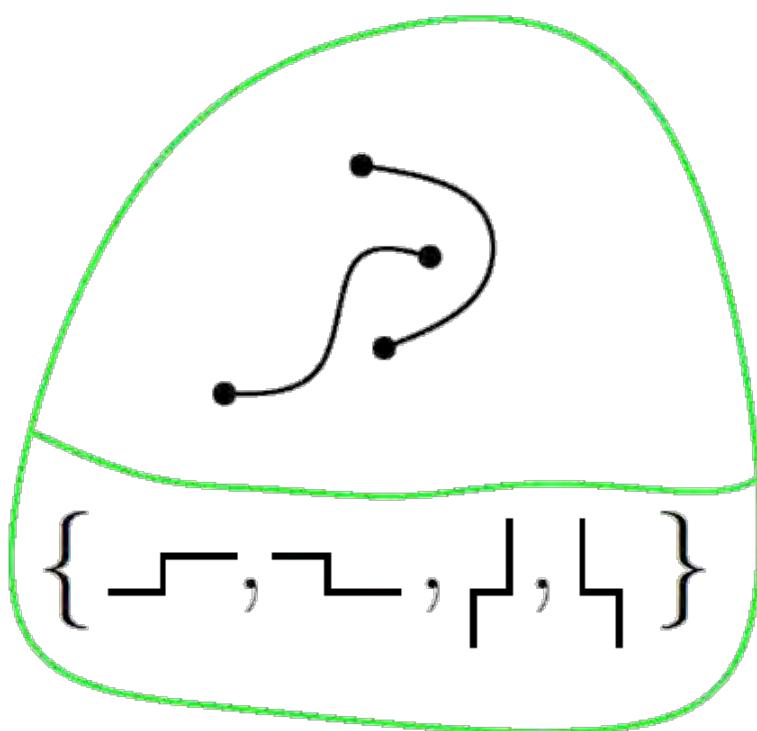
Results



Results

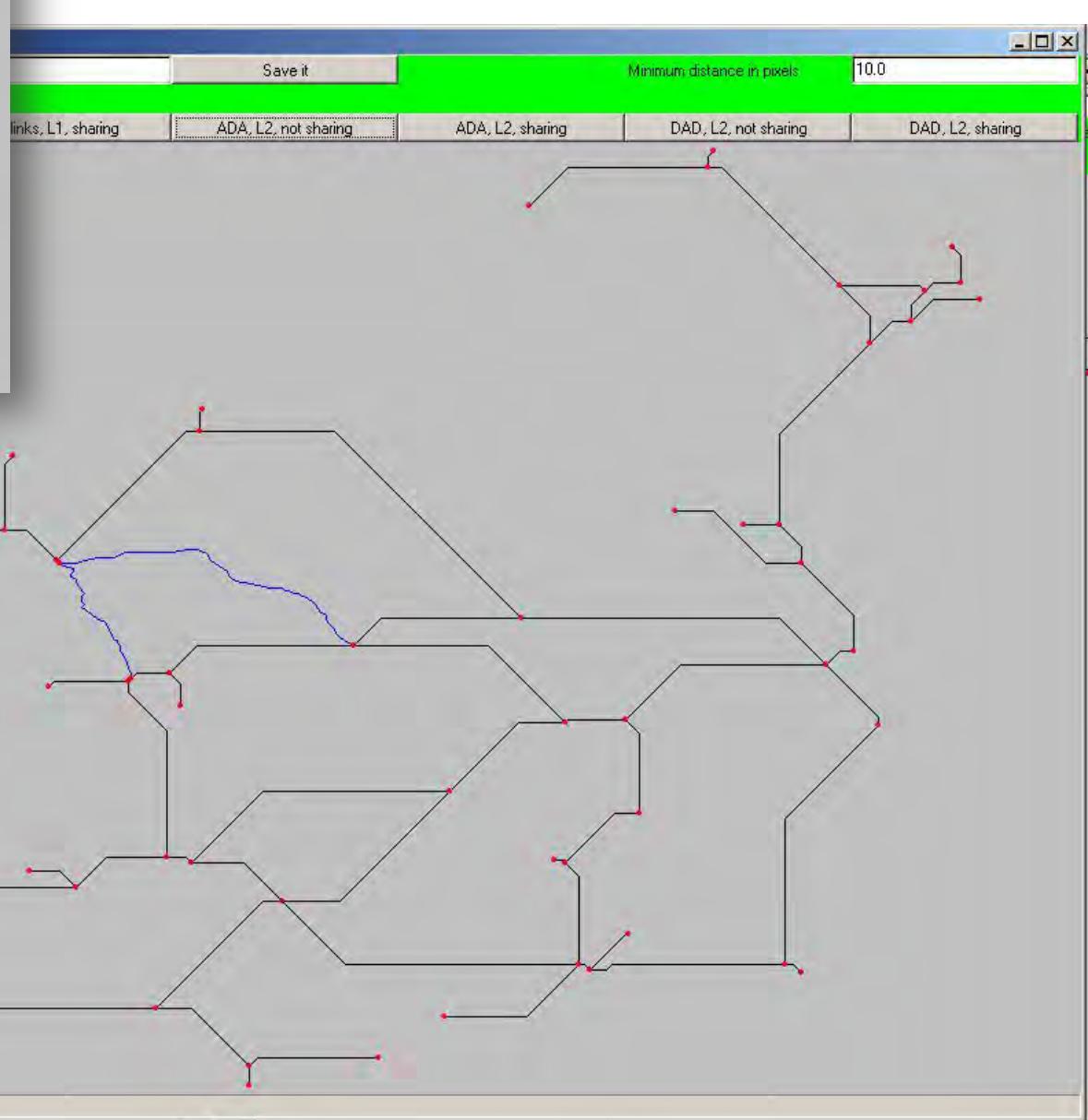
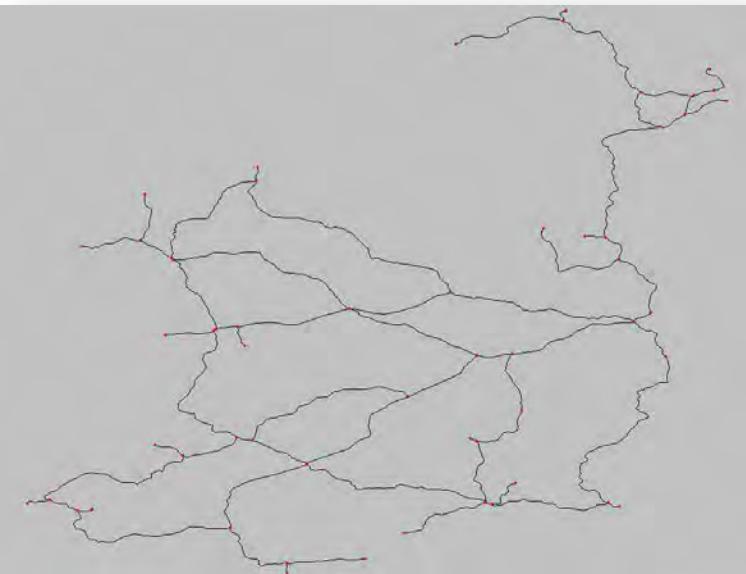


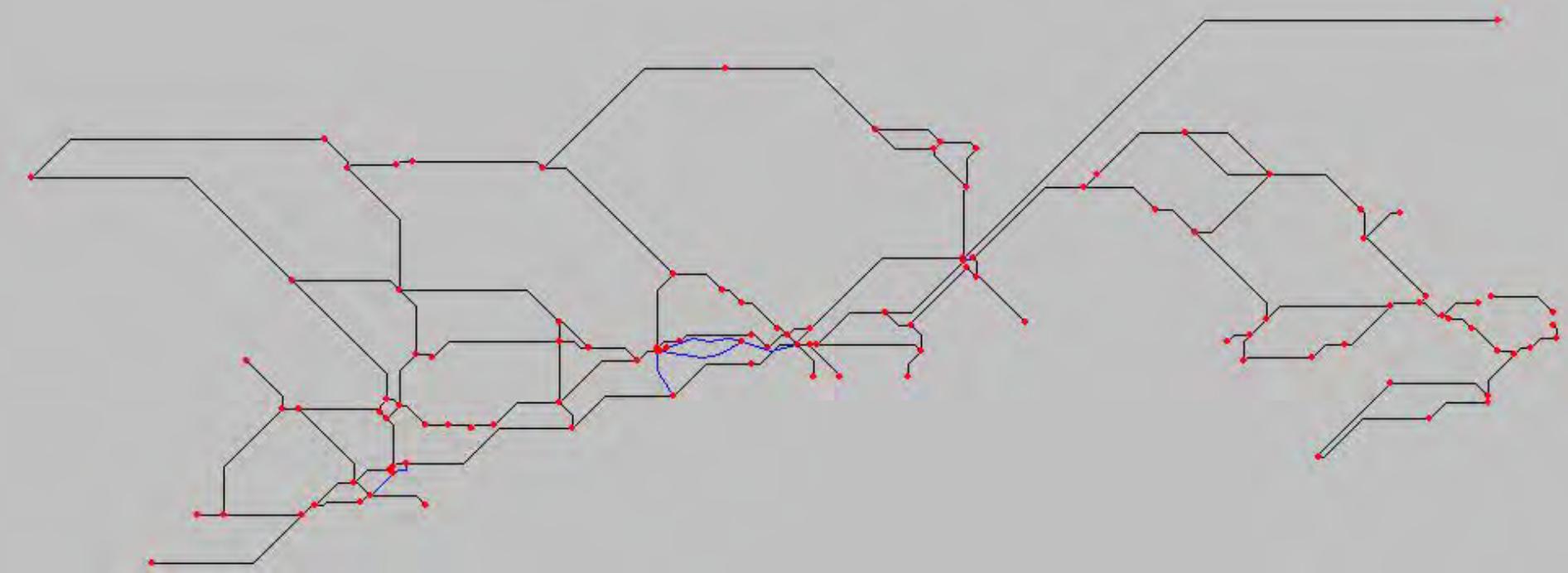
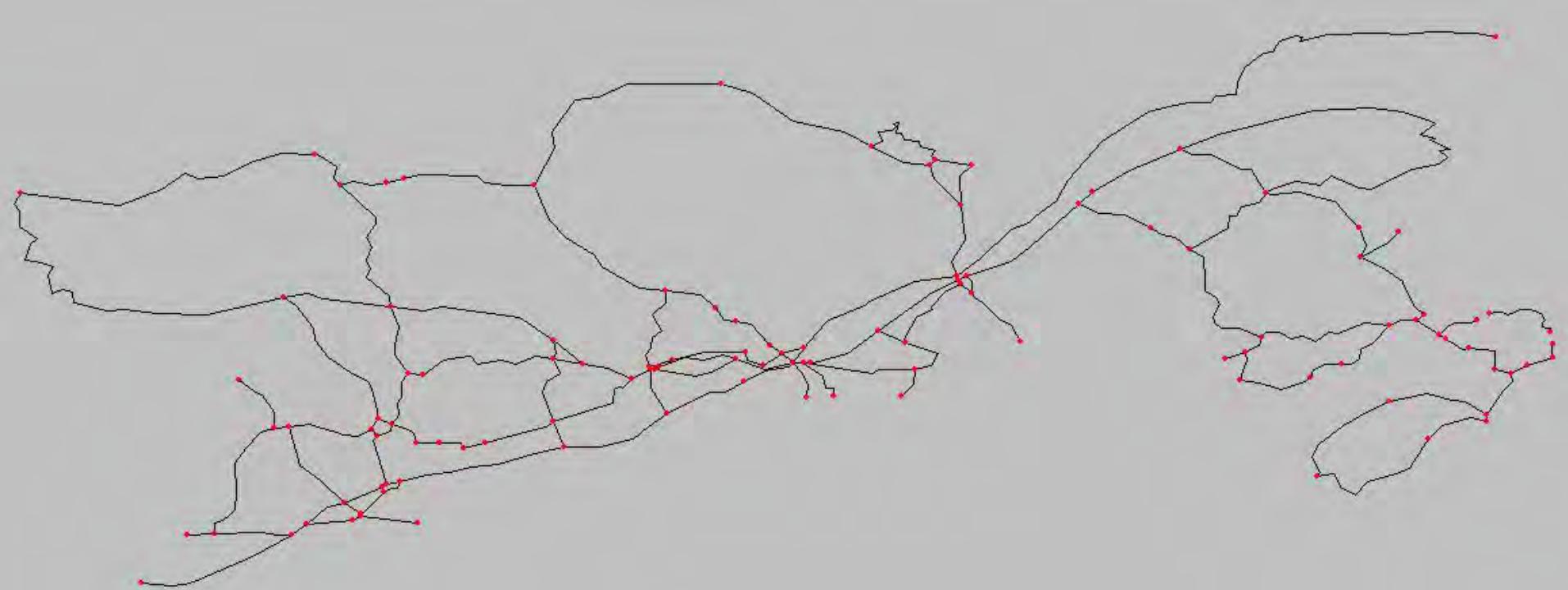
Results



It is not
possible

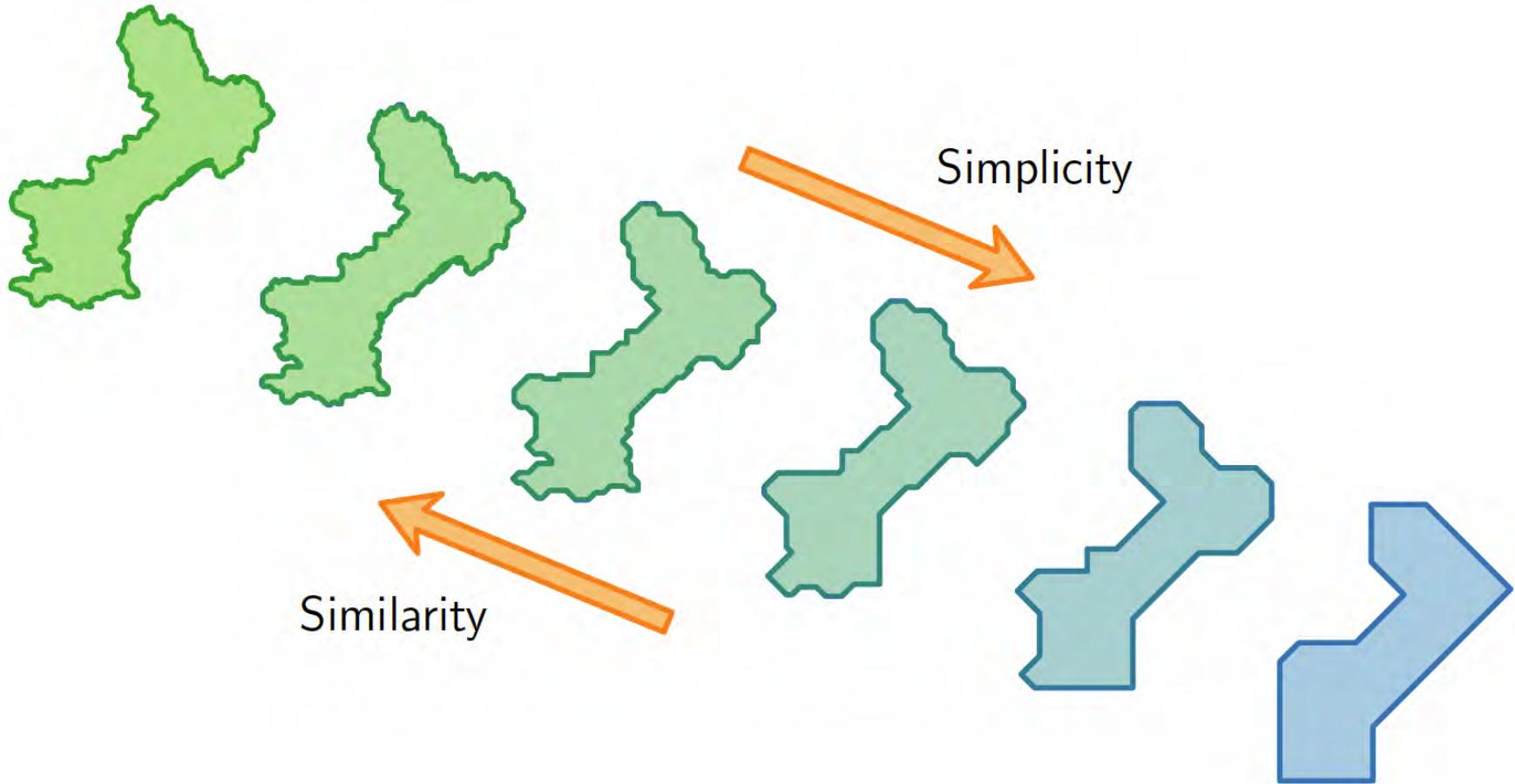
Output



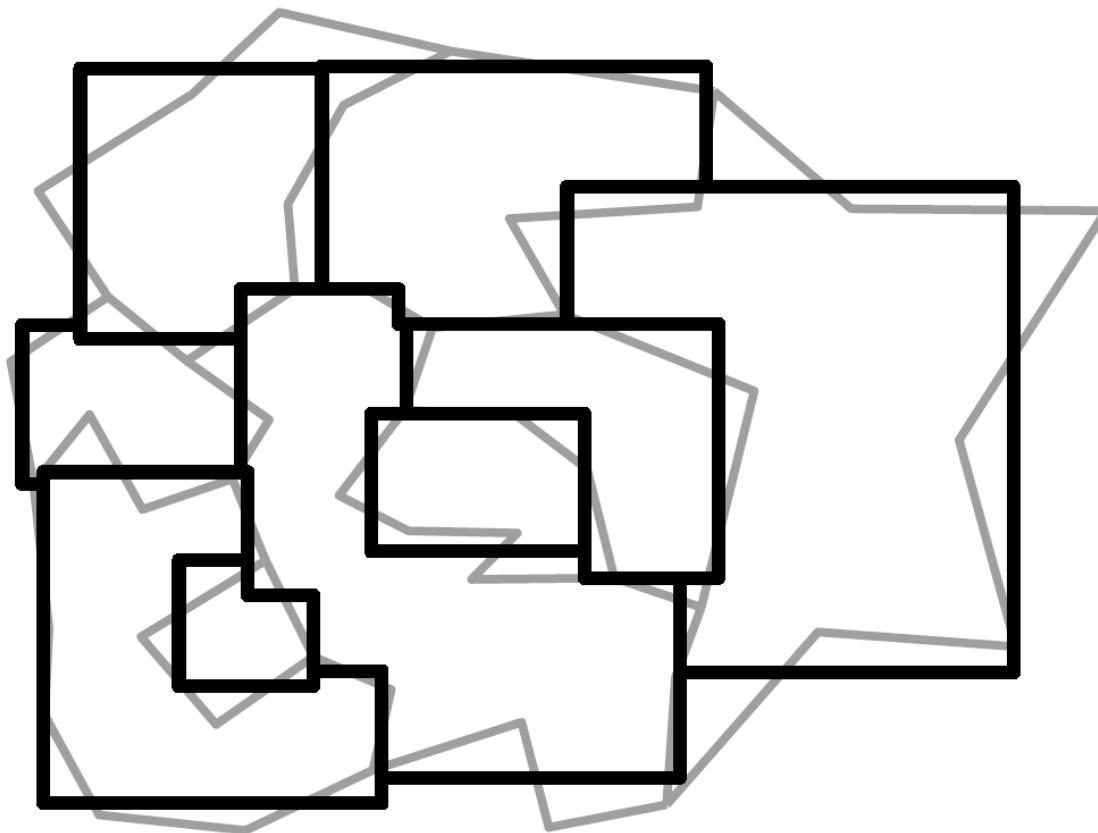


Territorial Outlines

Area-preserving schematization

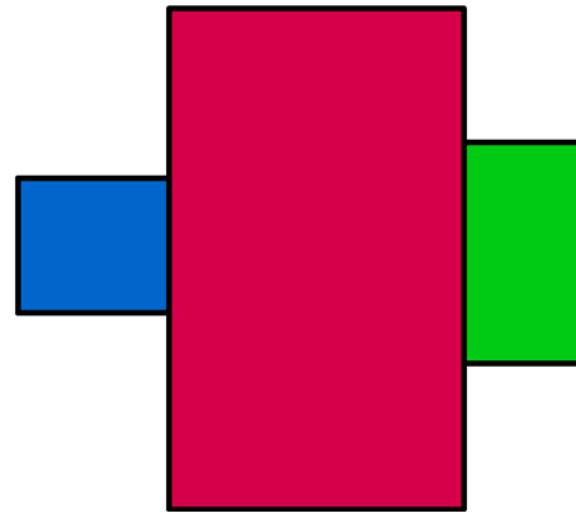
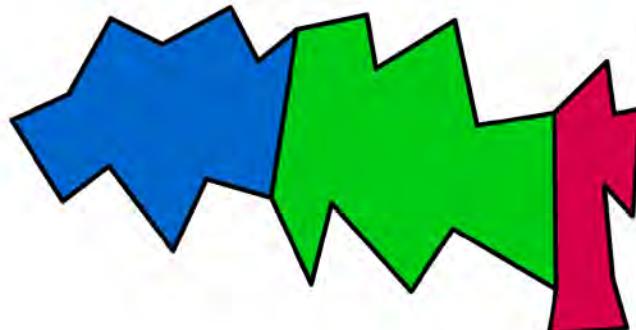


Area-preserving schematization



Requirements

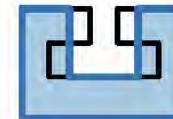
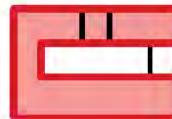
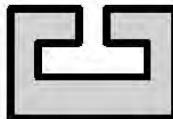
- Few orientations
 - Few lines
 - Preserve “shape”
 - Area
 - Topology
- }
- Single operation
 - Complete for polygons



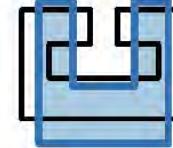
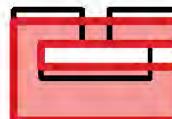
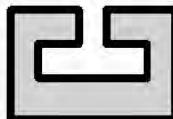
- Minimize distance function?

Distance measures

Symmetric difference



Hausdorff distance



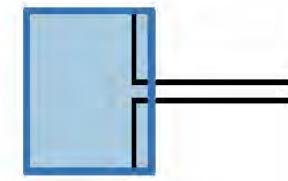
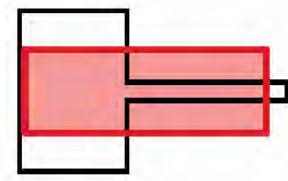
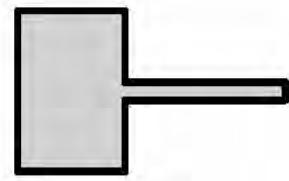
Turning angle distance



CDTW distance



Fréchet distance

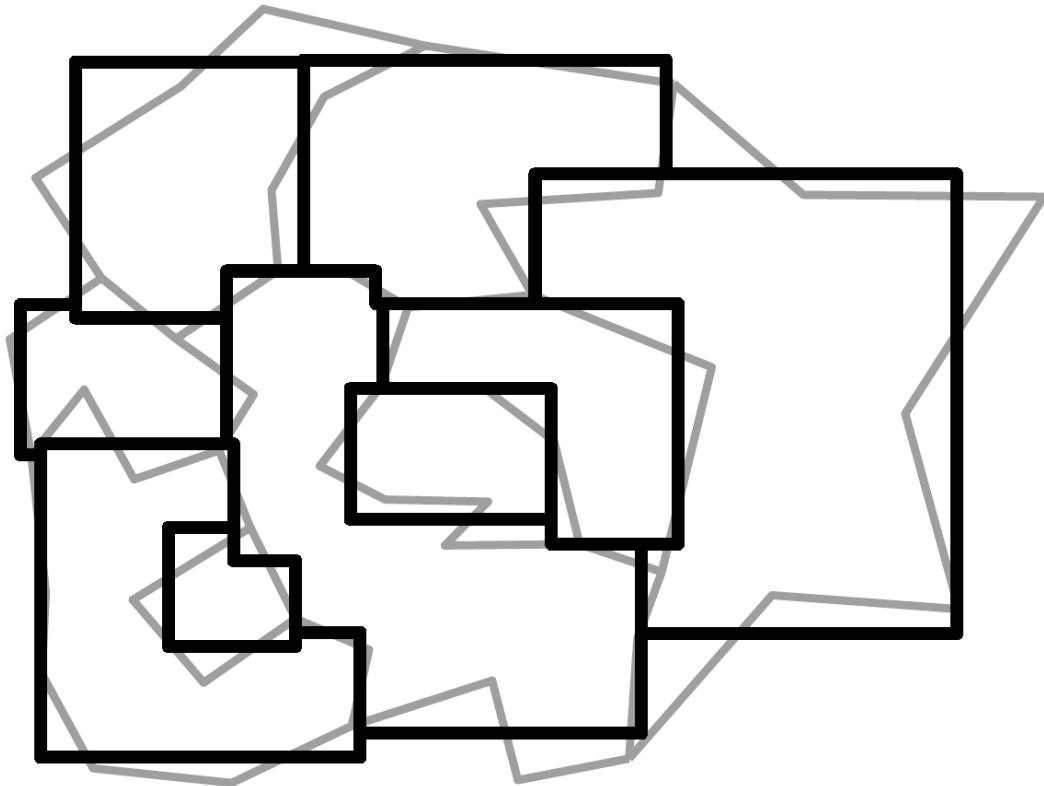


The idea

Given a simple subdivision

1. Convert to **rectilinear**
 - area-preserving

2. **Contract configuration**
 - choose greedily
 - until satisfied

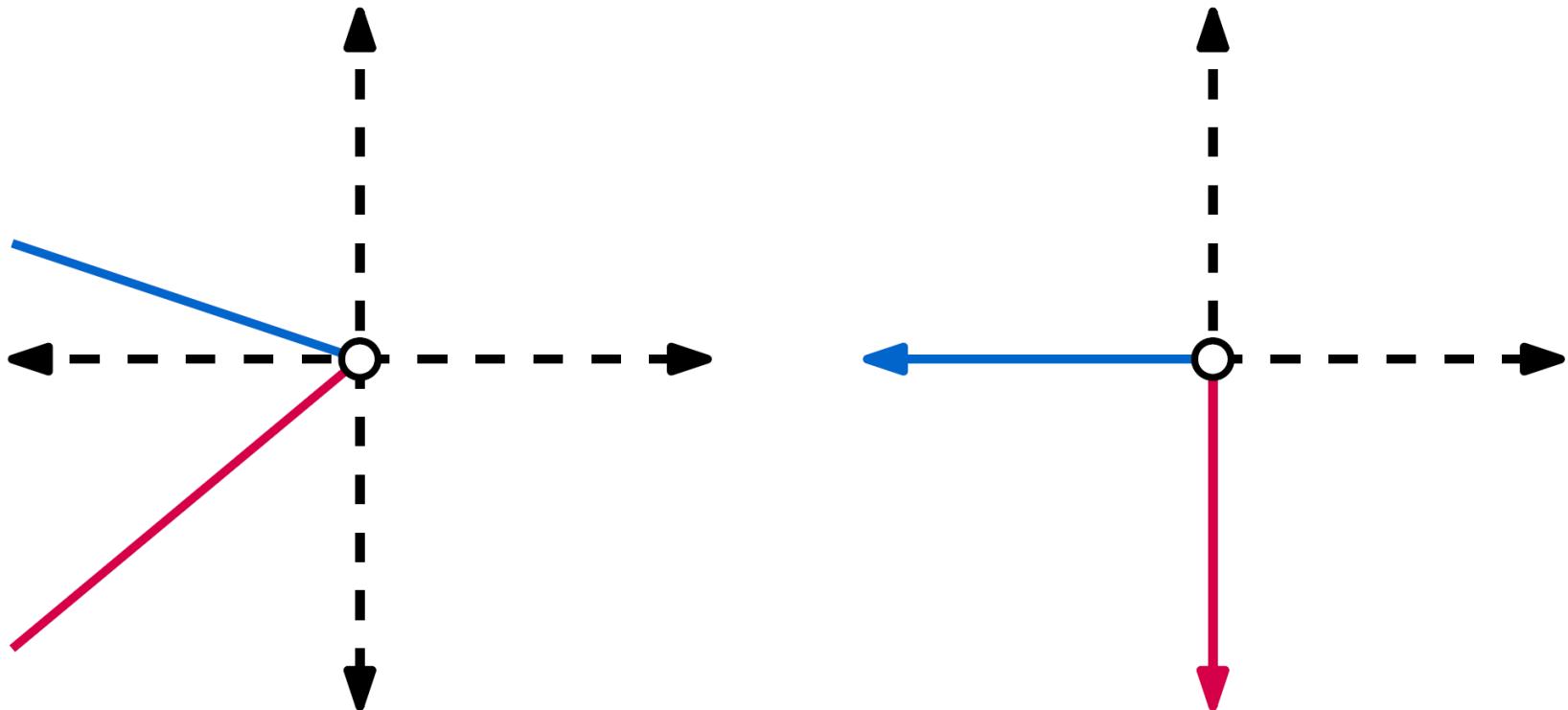


Rectilinearization

- Convert simple subdivision into simple rectilinear subdivision
 - Area-preserving
 - Preserve adjacencies
 - Minimize angular deviation

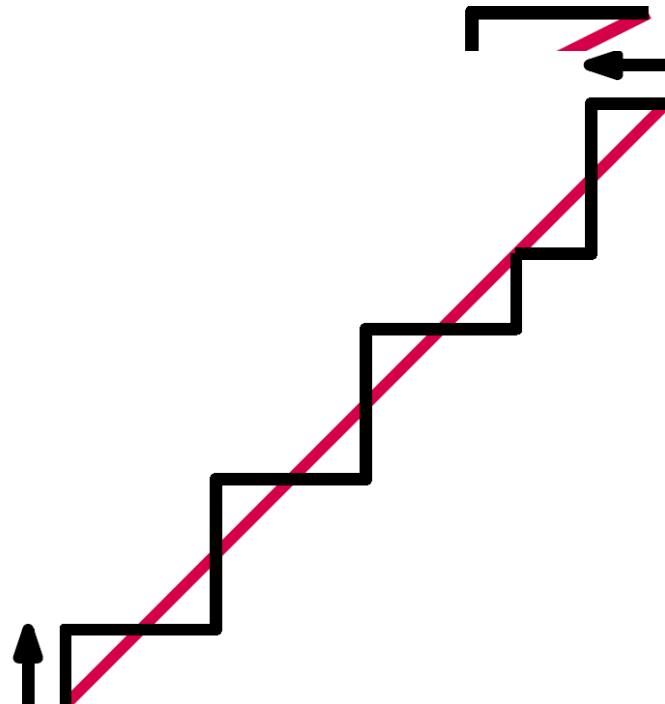
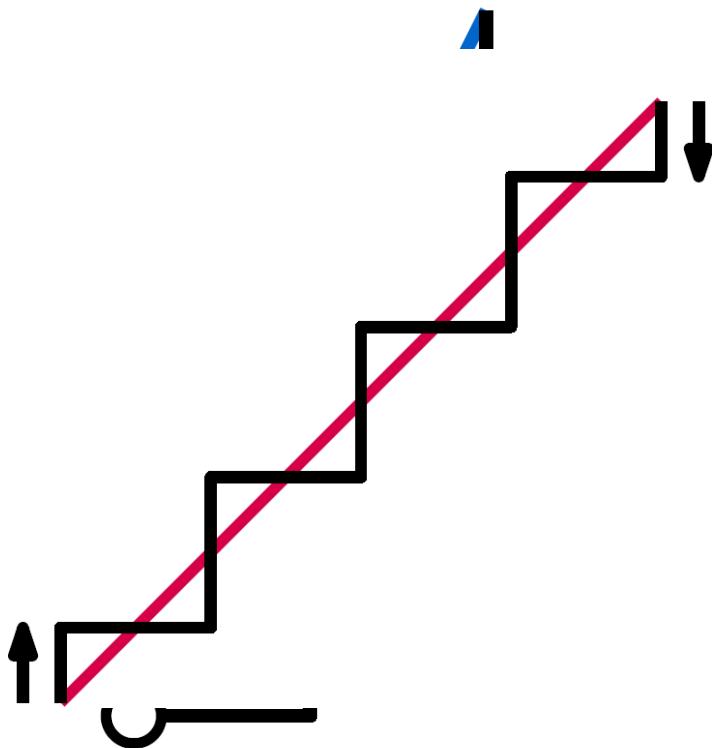
Rectilinearization

- Assign direction to each vertex of each edge
 - Minimize angular deviation
 - Sharp endpoints



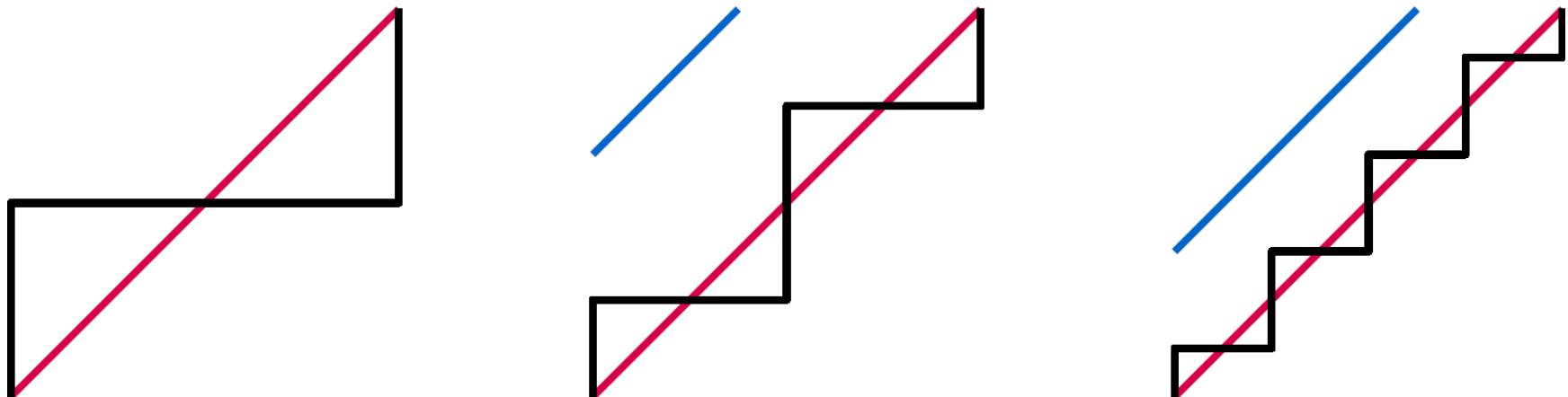
Rectilinearization

- For each edge
 - Create staircase
 - Use **evasive behaviour** for sharp endpoints

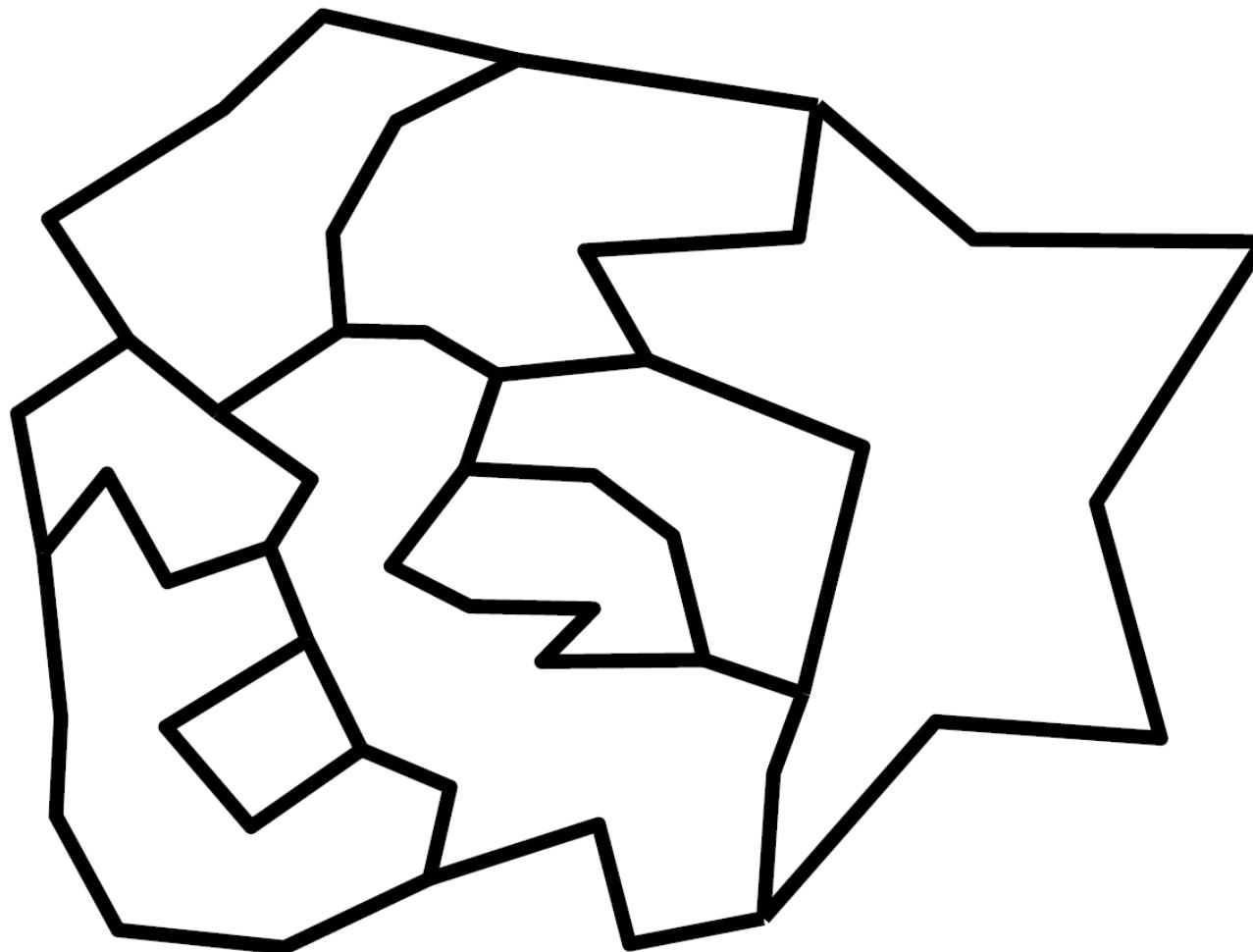


Rectilinearization

- Increase in complexity
 - Depends on distance between non-adjacent edges
 - Depends on angle between adjacent edges



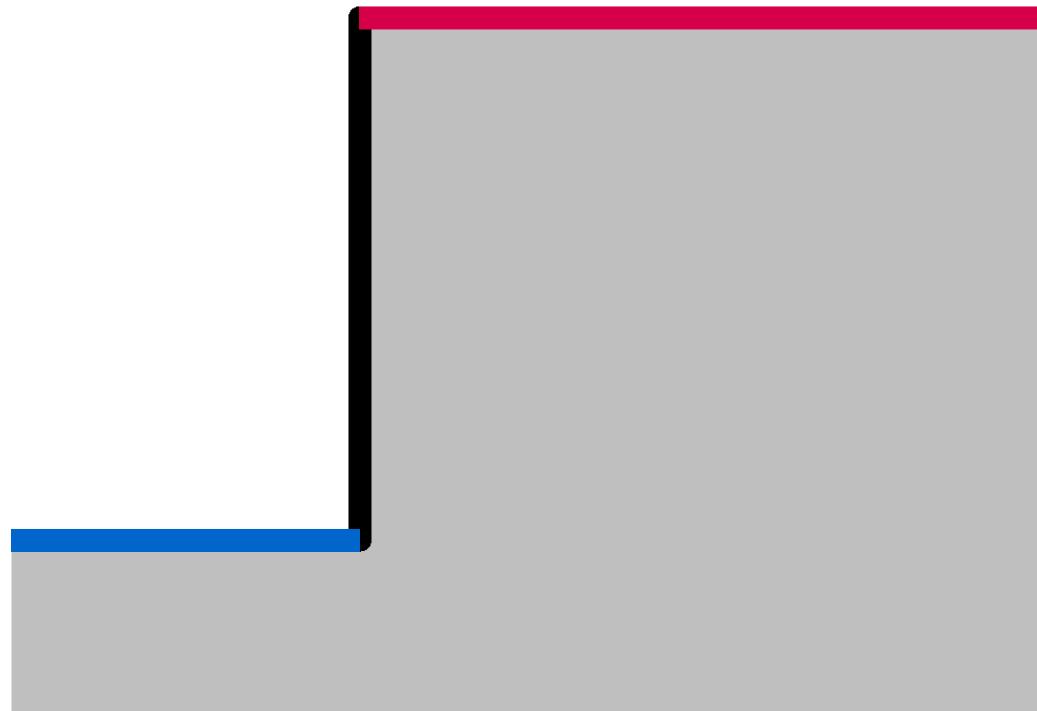
Rectilinearization



S-configuration

- S-configuration

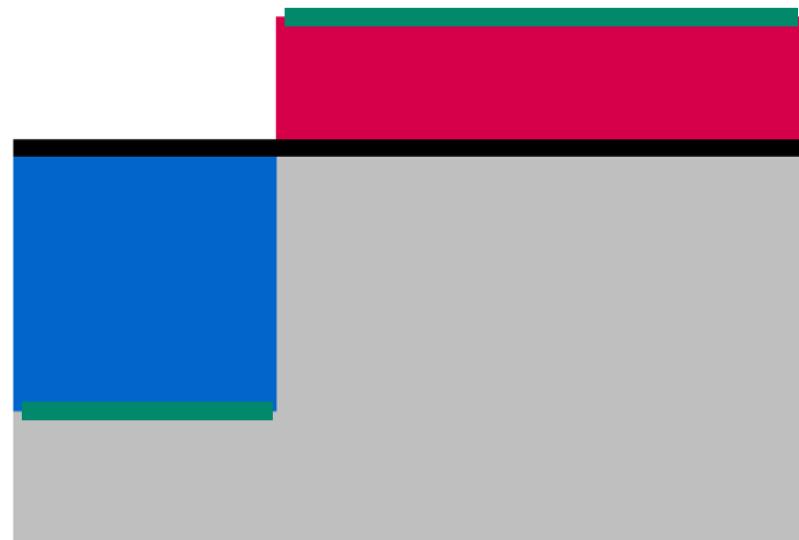
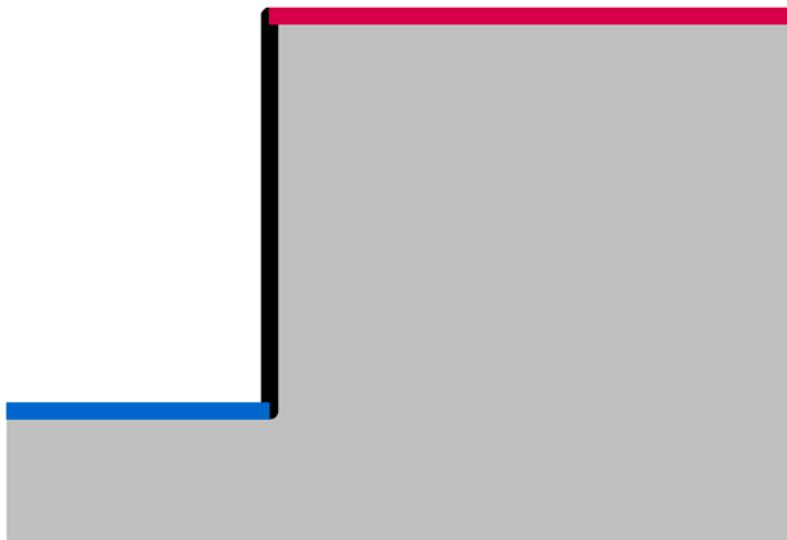
- 3 consecutive edges
- 2 different turns



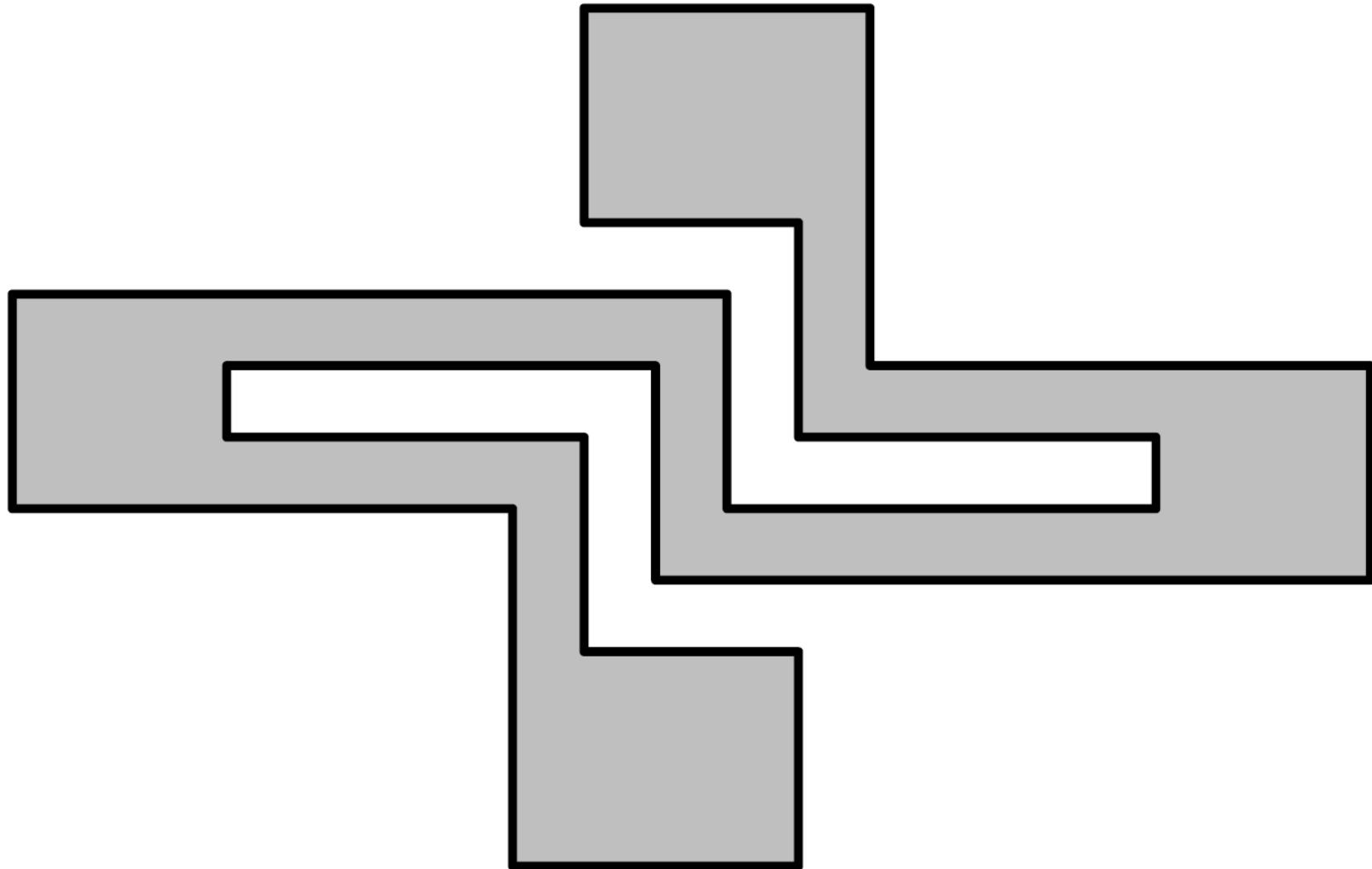
S-contraction

- S-contraction
 - Replace by 1 edge
 - Weighted average

- Feasible
 - Contraction area is empty



Deadlock



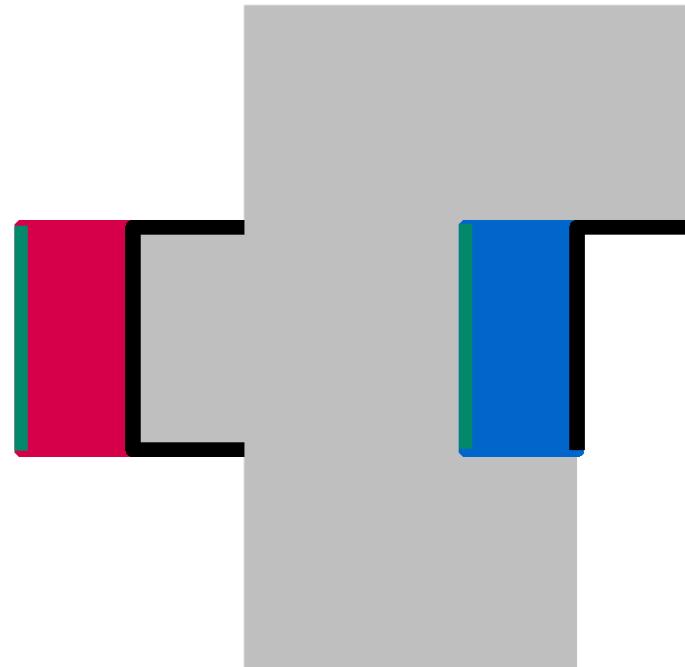
C-contractions

□ C-configuration

- 3 consecutive edges
- 2 similar turns
- Inner & outer C-configuration

□ C-contraction

- Inner & outer C-configuration
- Remove smallest
- Compensate for area change



Completeness for polygons

Theorem

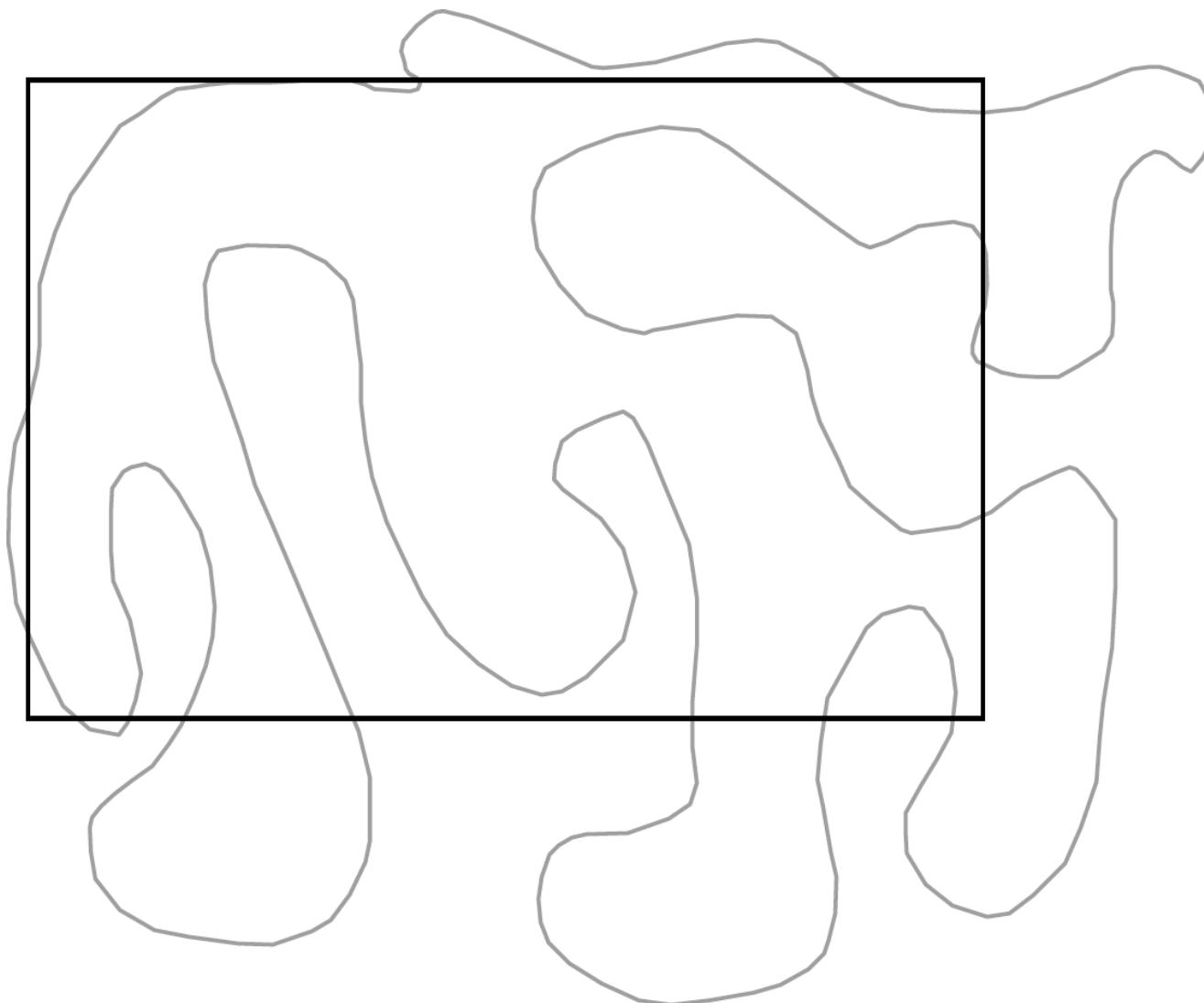
A rectilinear polygon with at least 6 edges has

- a feasible inner C-configuration
- a feasible S-configuration
or a feasible outer C-configuration

A contraction is always possible

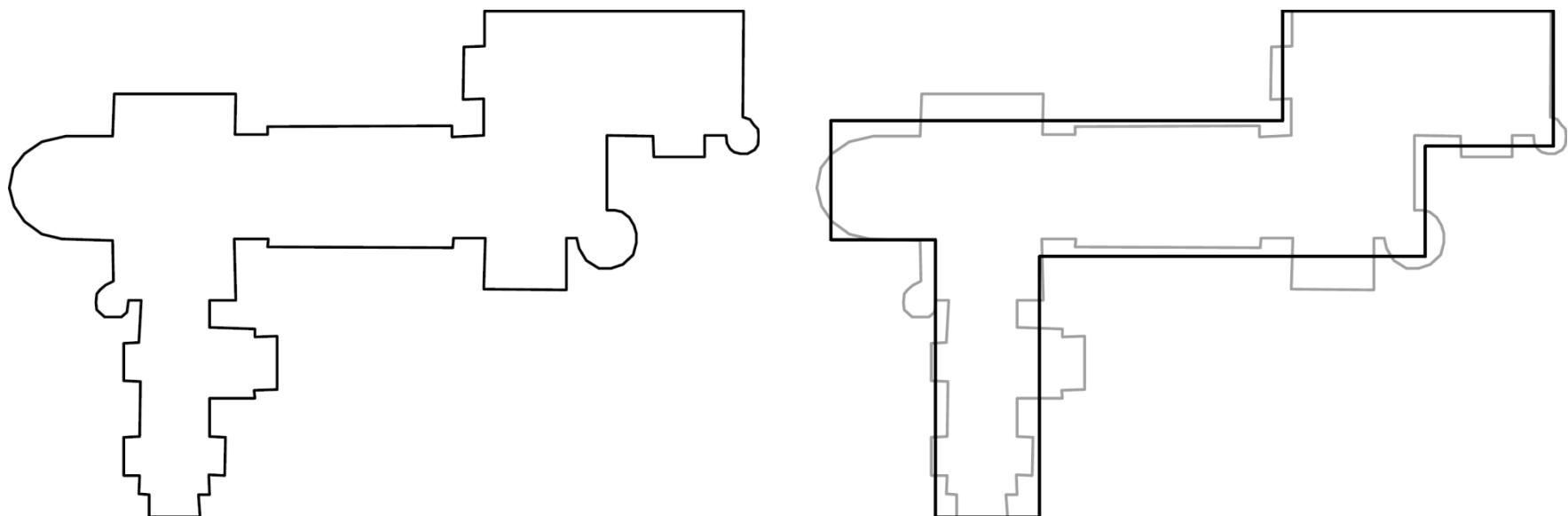
Generalizes to C-oriented polygons ...

Results for polygons

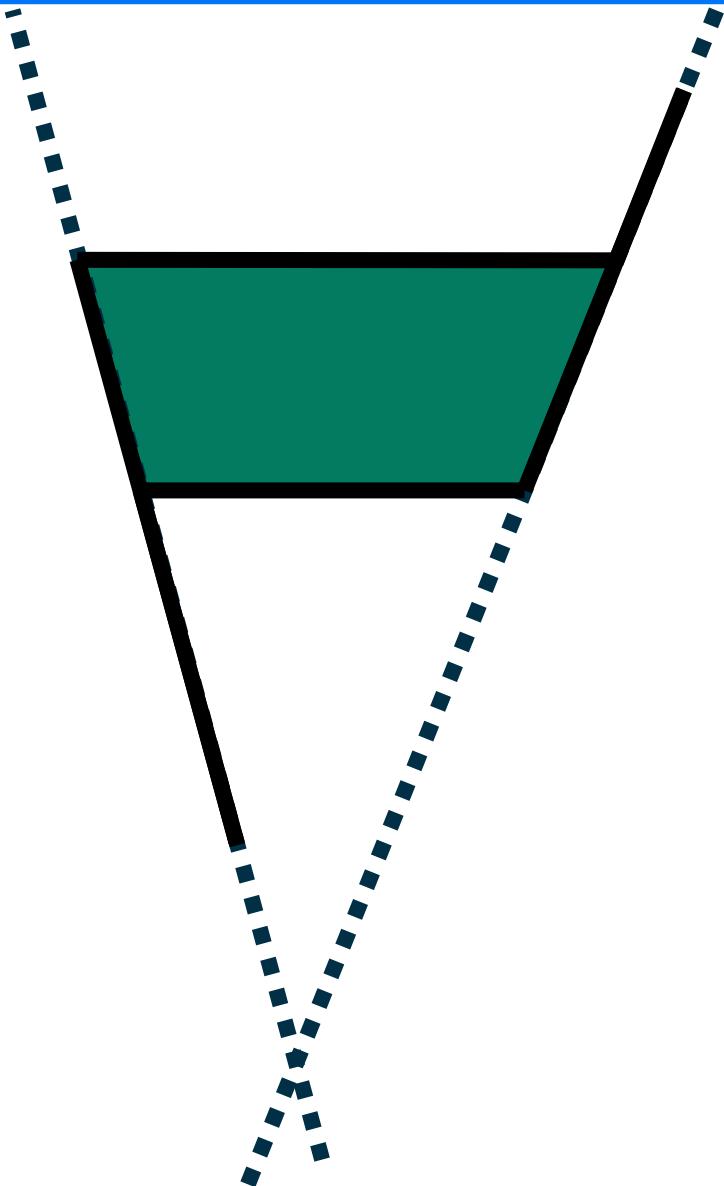


Results for polygons

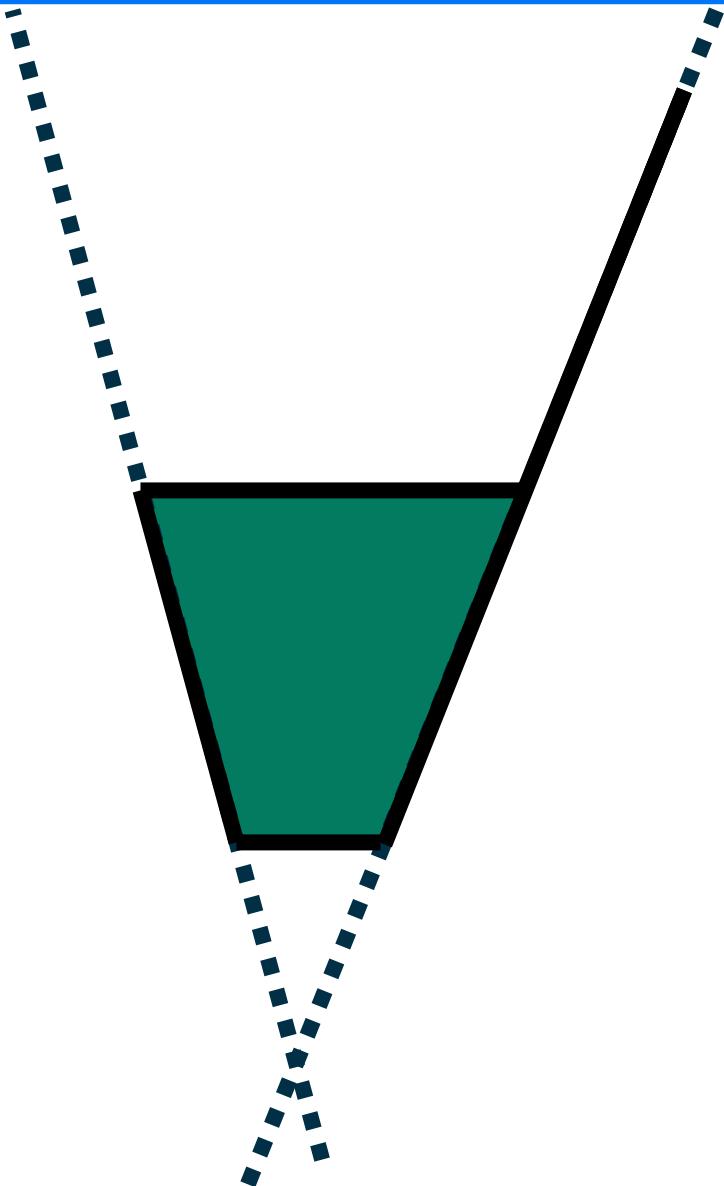
- Building generalization



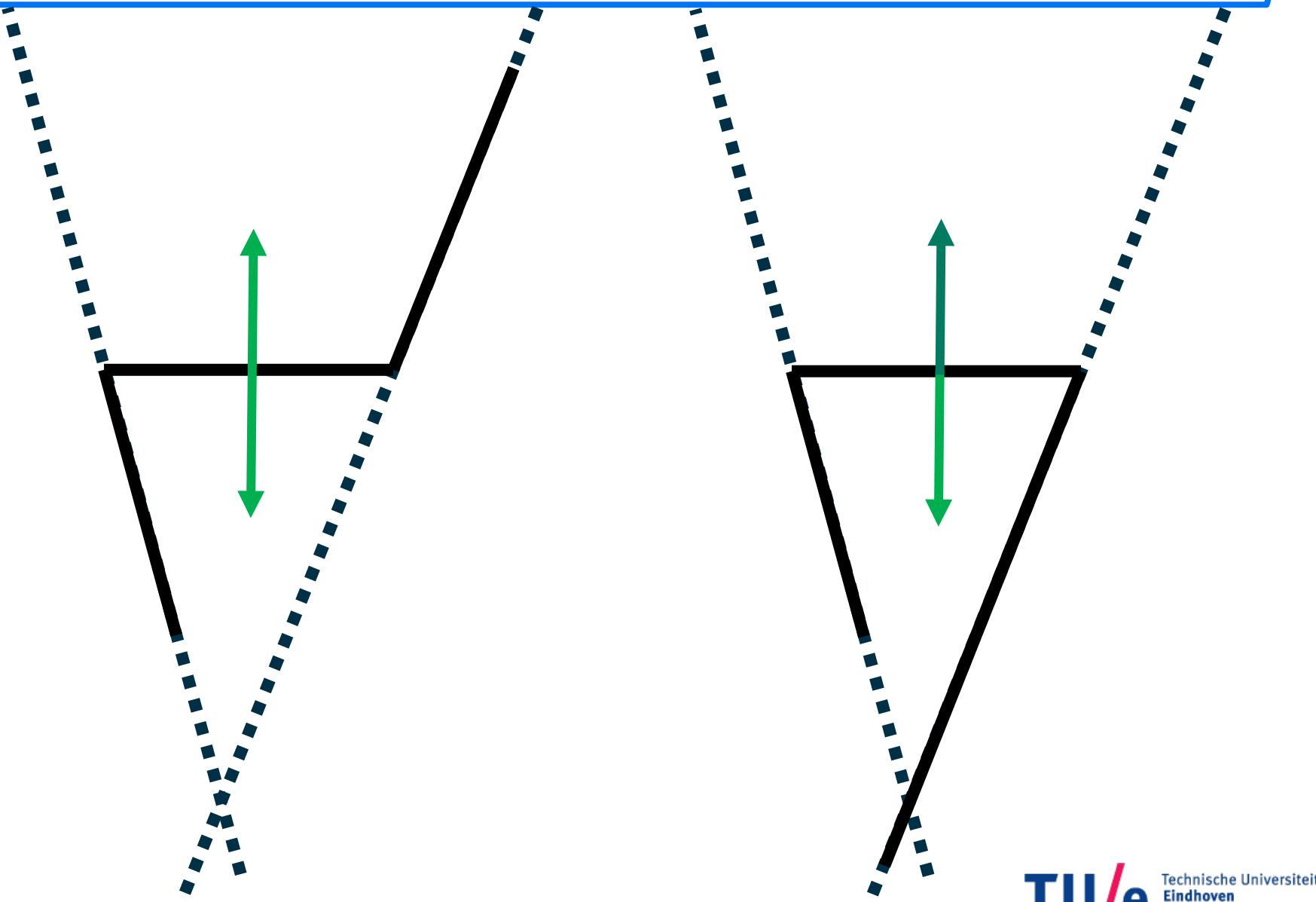
C-oriented polygons: edge moves



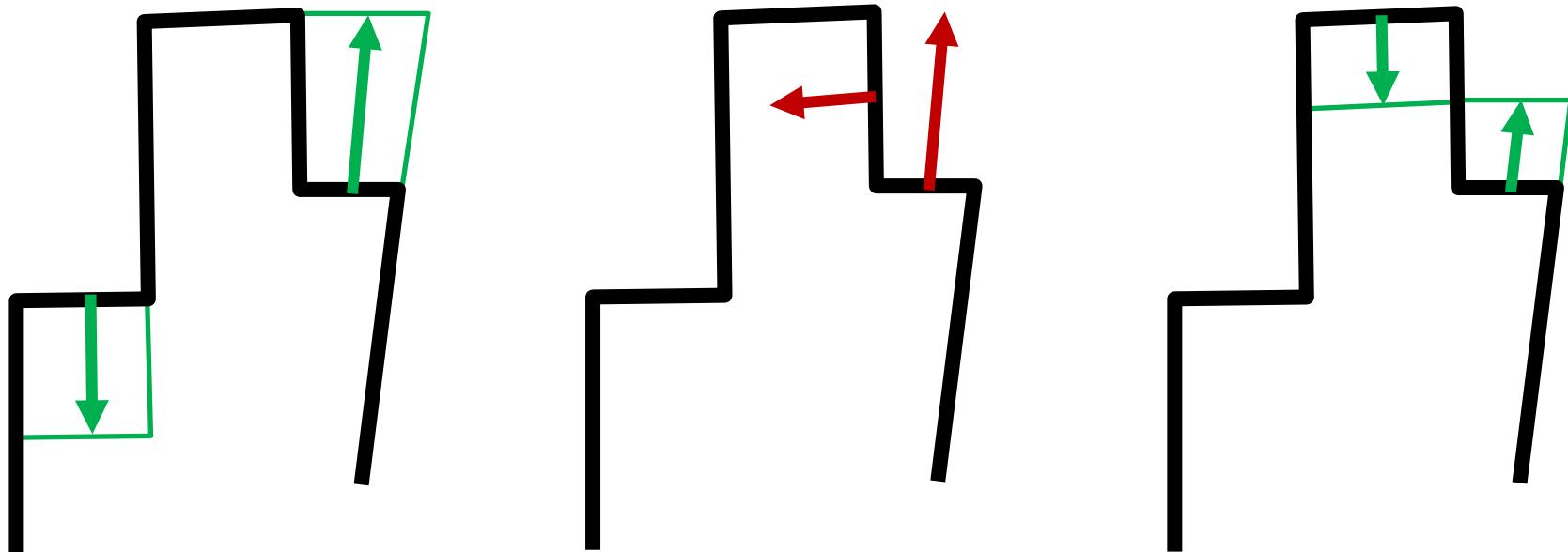
C-oriented polygons: edge moves



Edge-moves



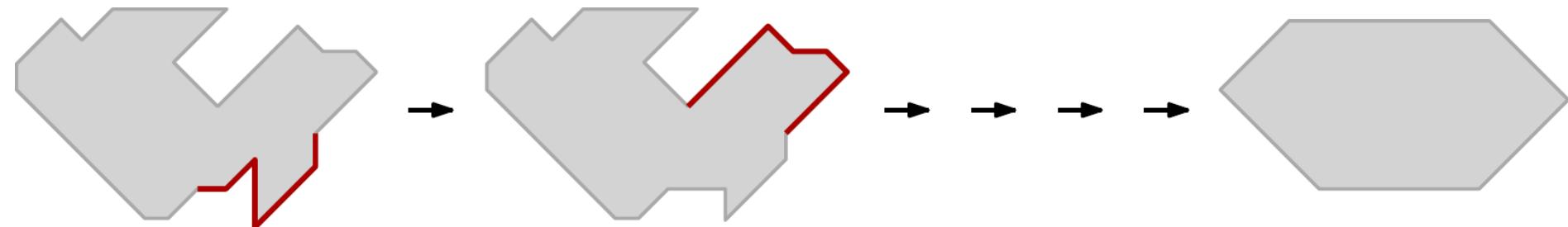
Combining edge-moves



Completeness

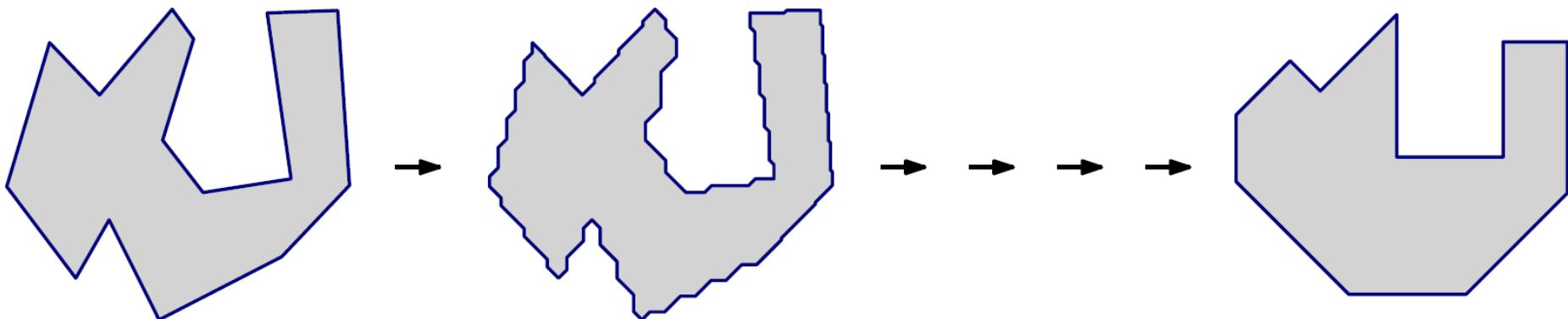
Theorem. Every simple non-convex polygon has a non-conflicting pair of complementary feasible contractions.

Corollary. Every simple C-oriented polygon can be schematized area-preservingly with at most $2|C|$ edges using only edge-moves.

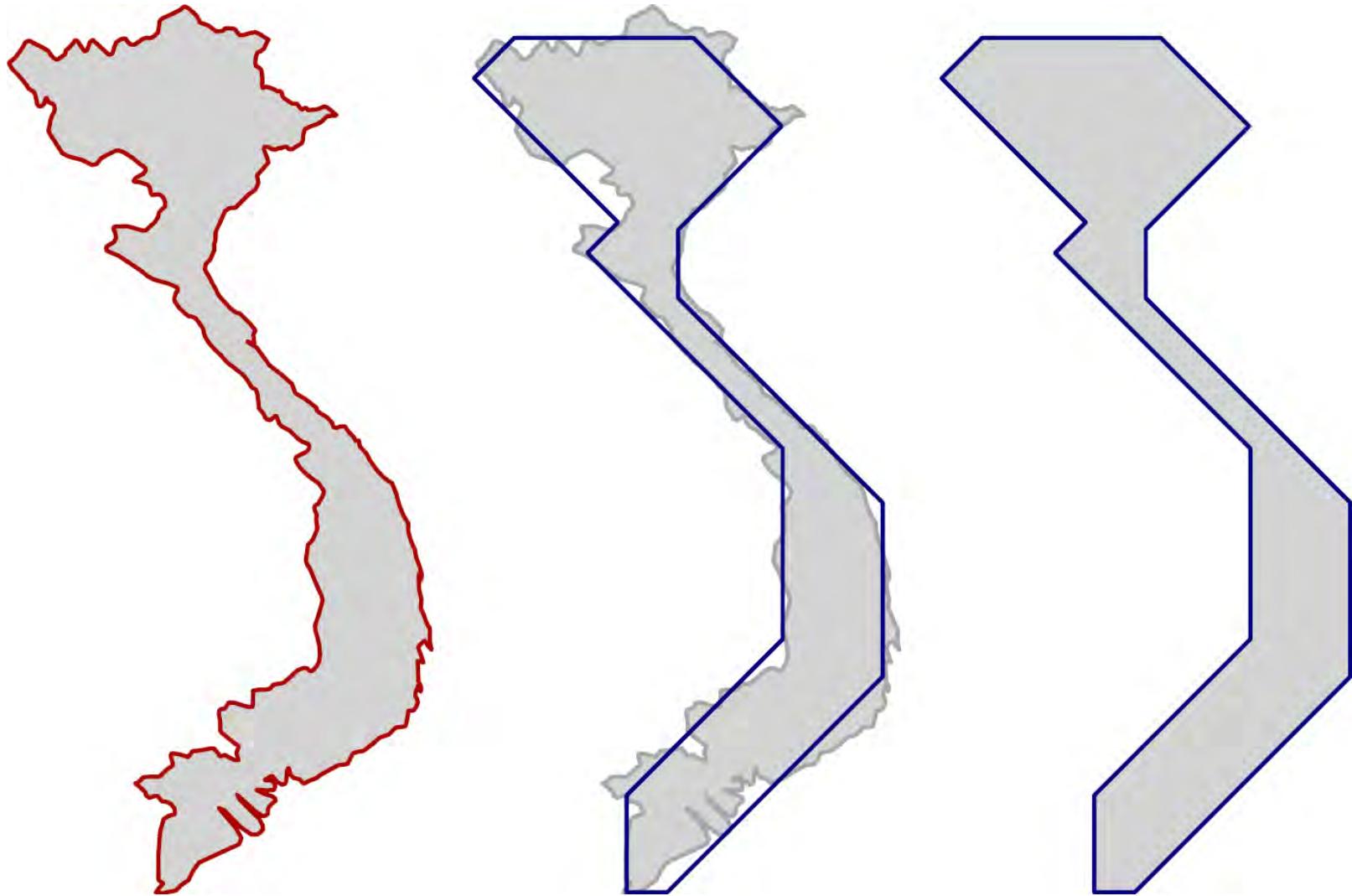


Schematization algorithm

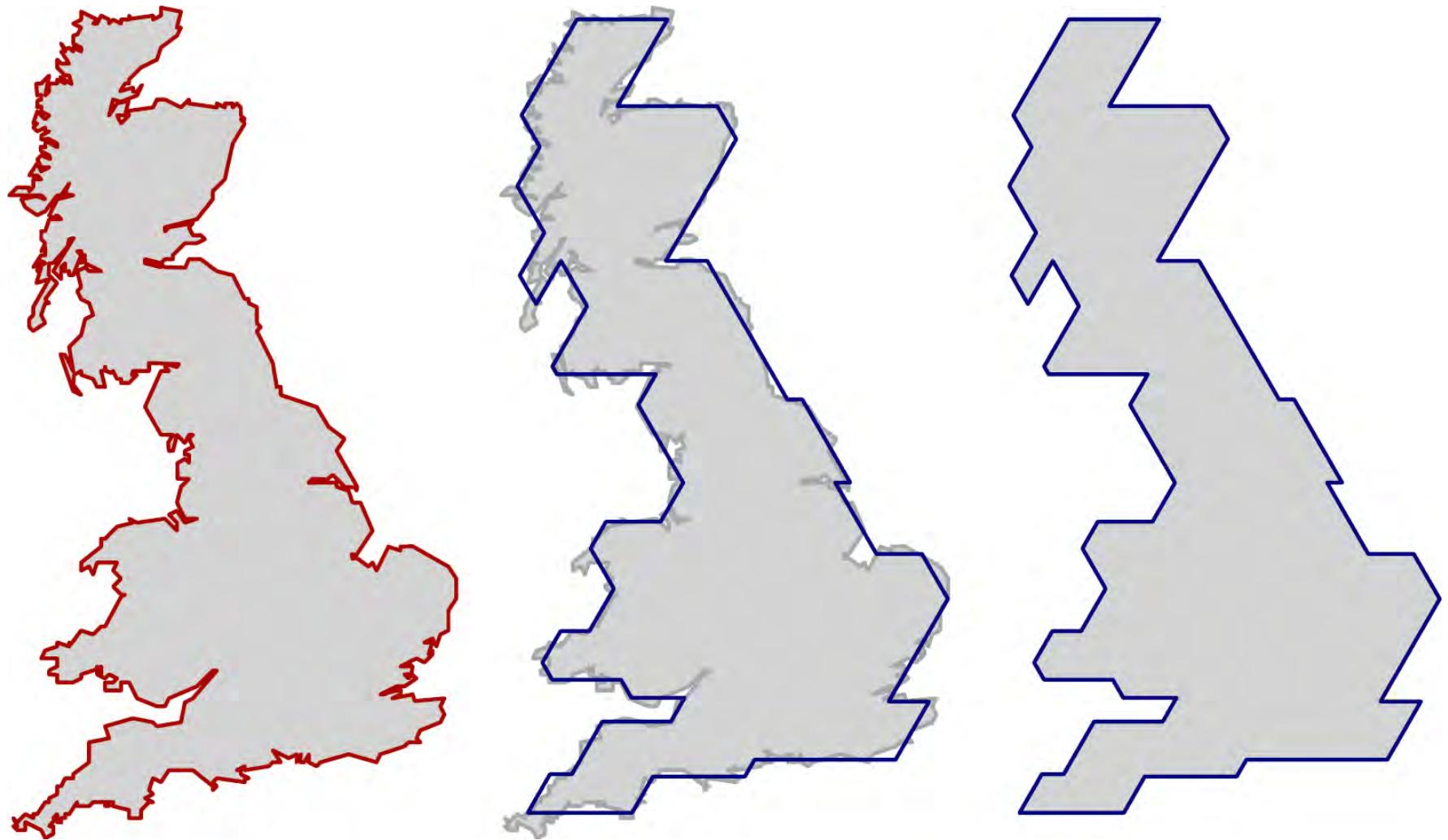
- Convert input to C-oriented subdivision
- Until satisfied
 - Execute pair of edge-moves with smallest contraction area



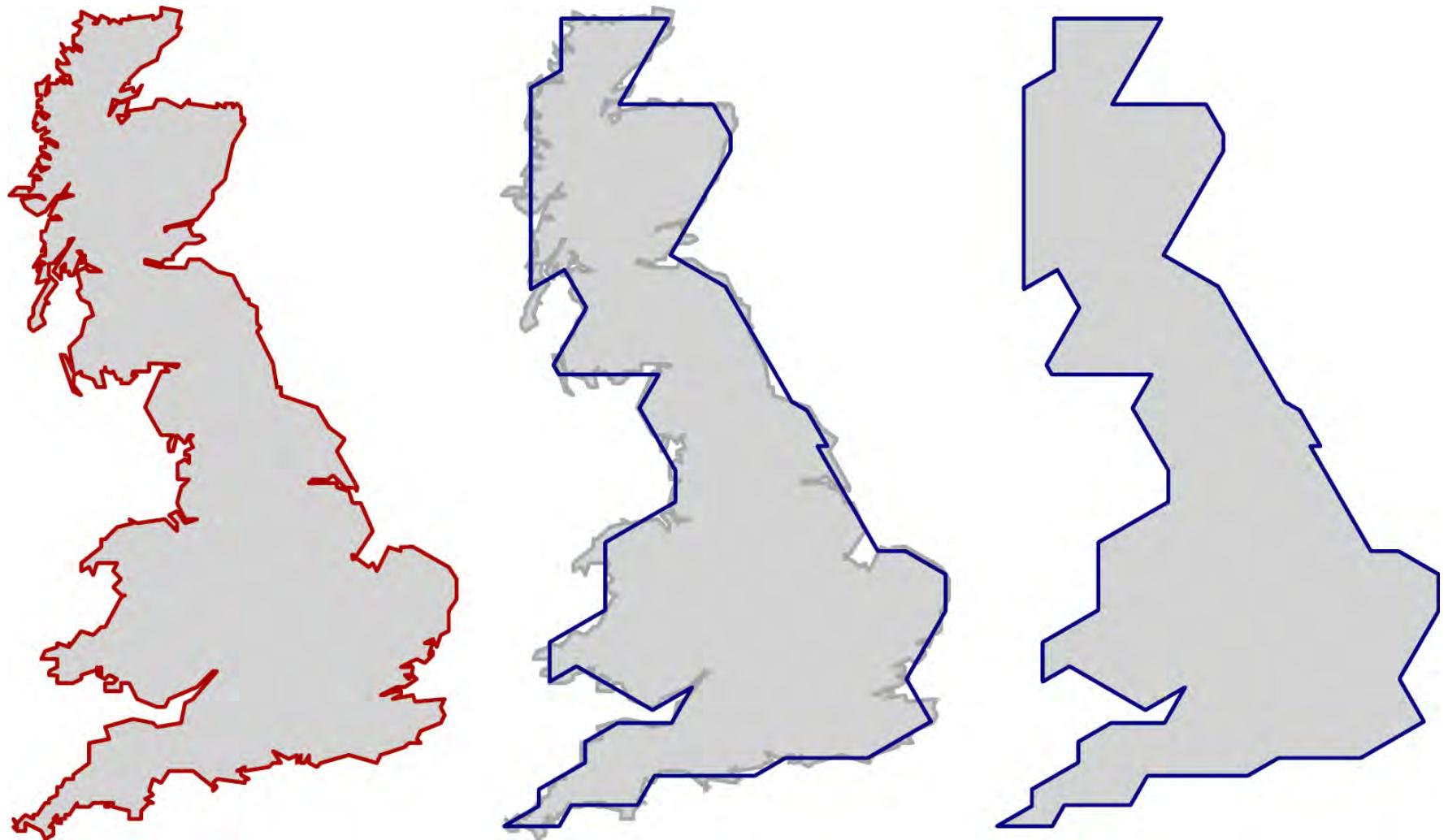
Experimental results



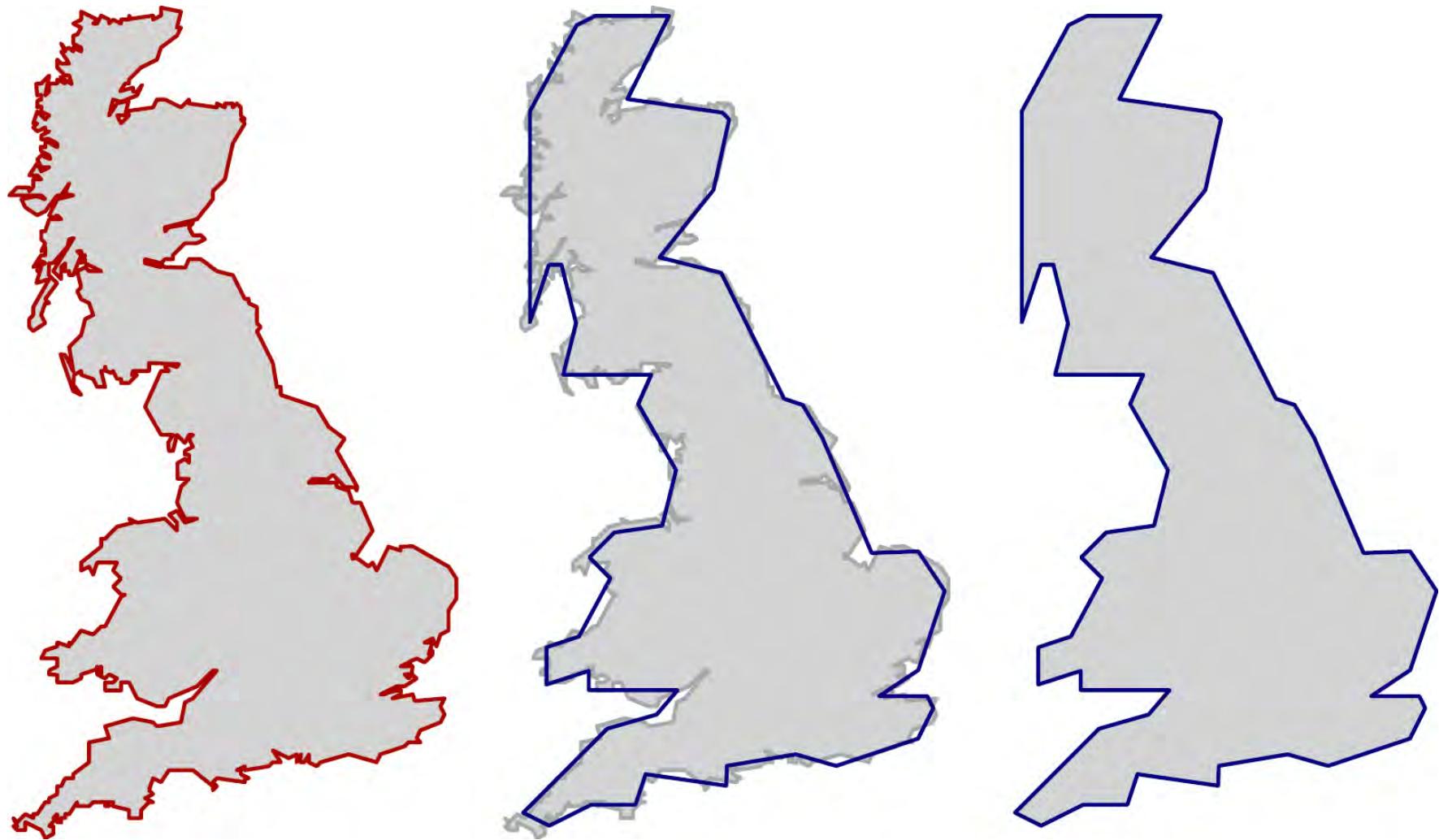
Experimental results



Experimental results



Experimental results



Experimental results

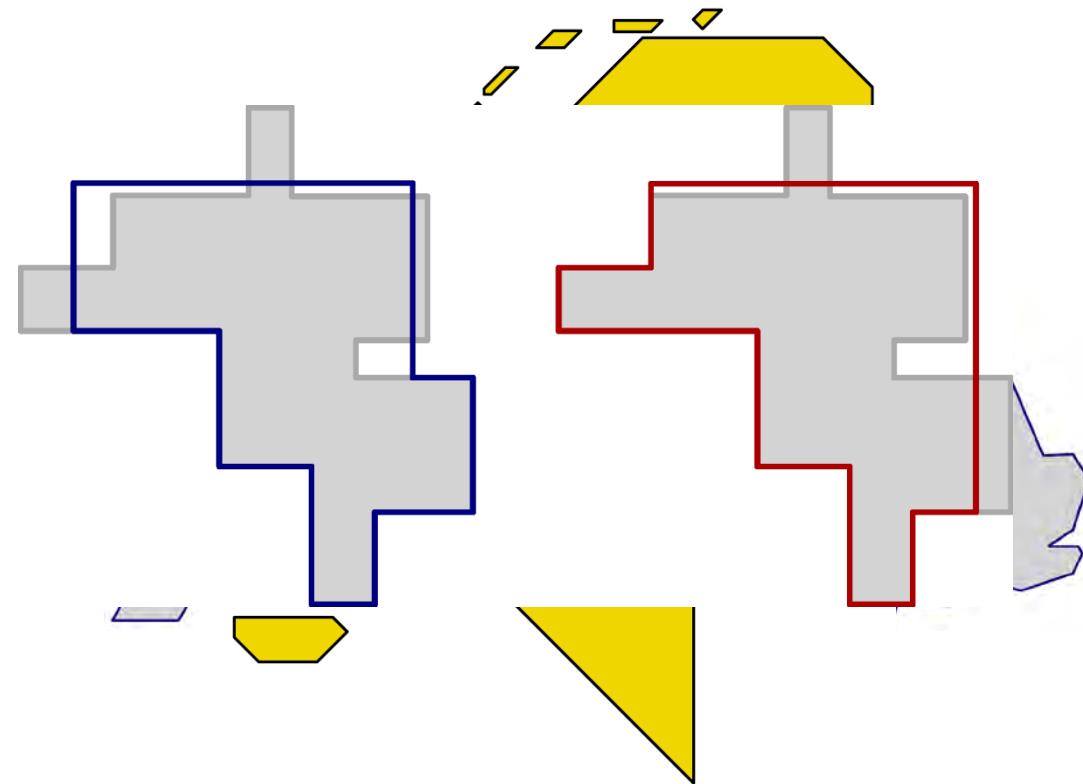


Experimental results



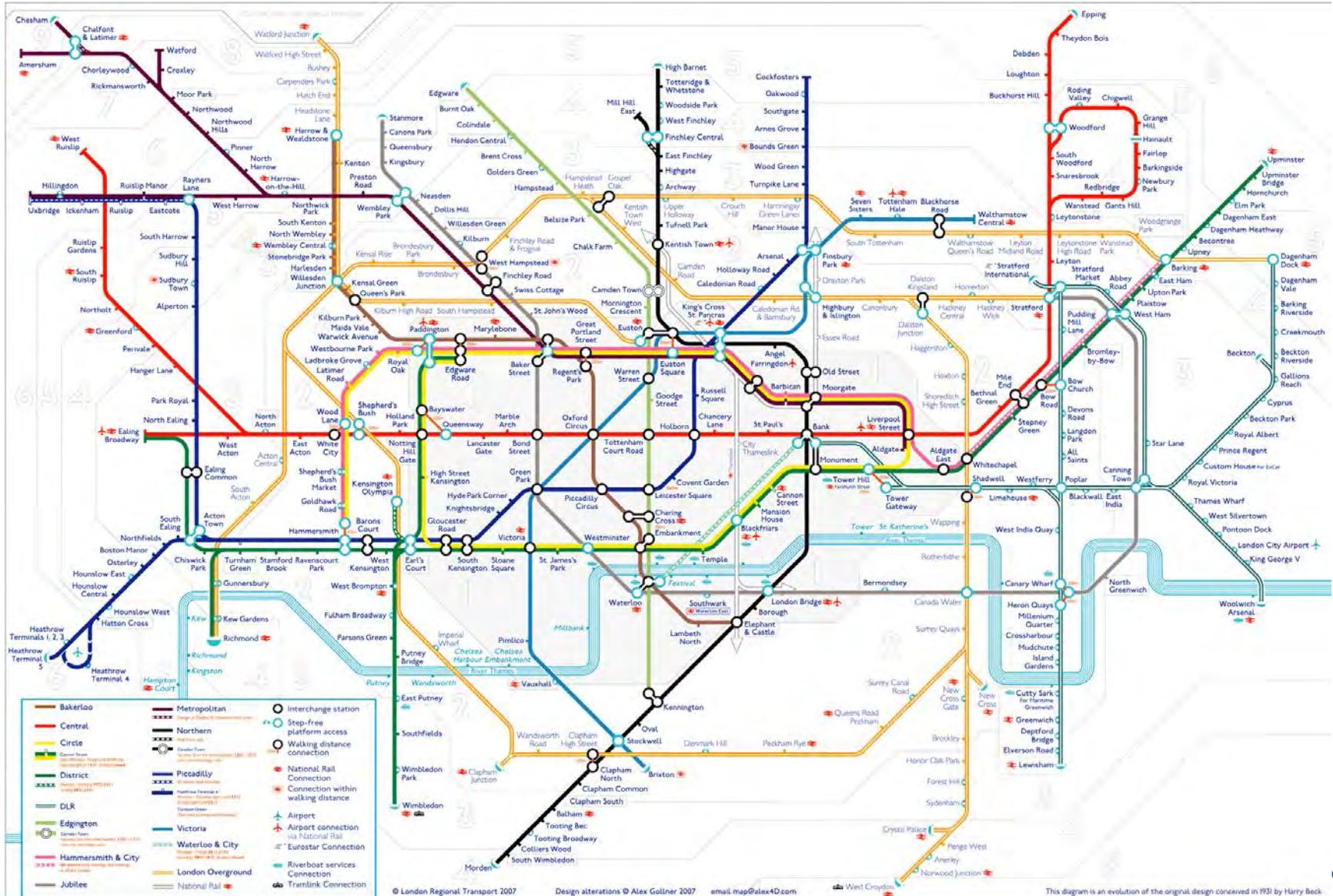
Still open ...

- Orientation selection
- Edge-move selection
- Adding orientations
- When to stop?

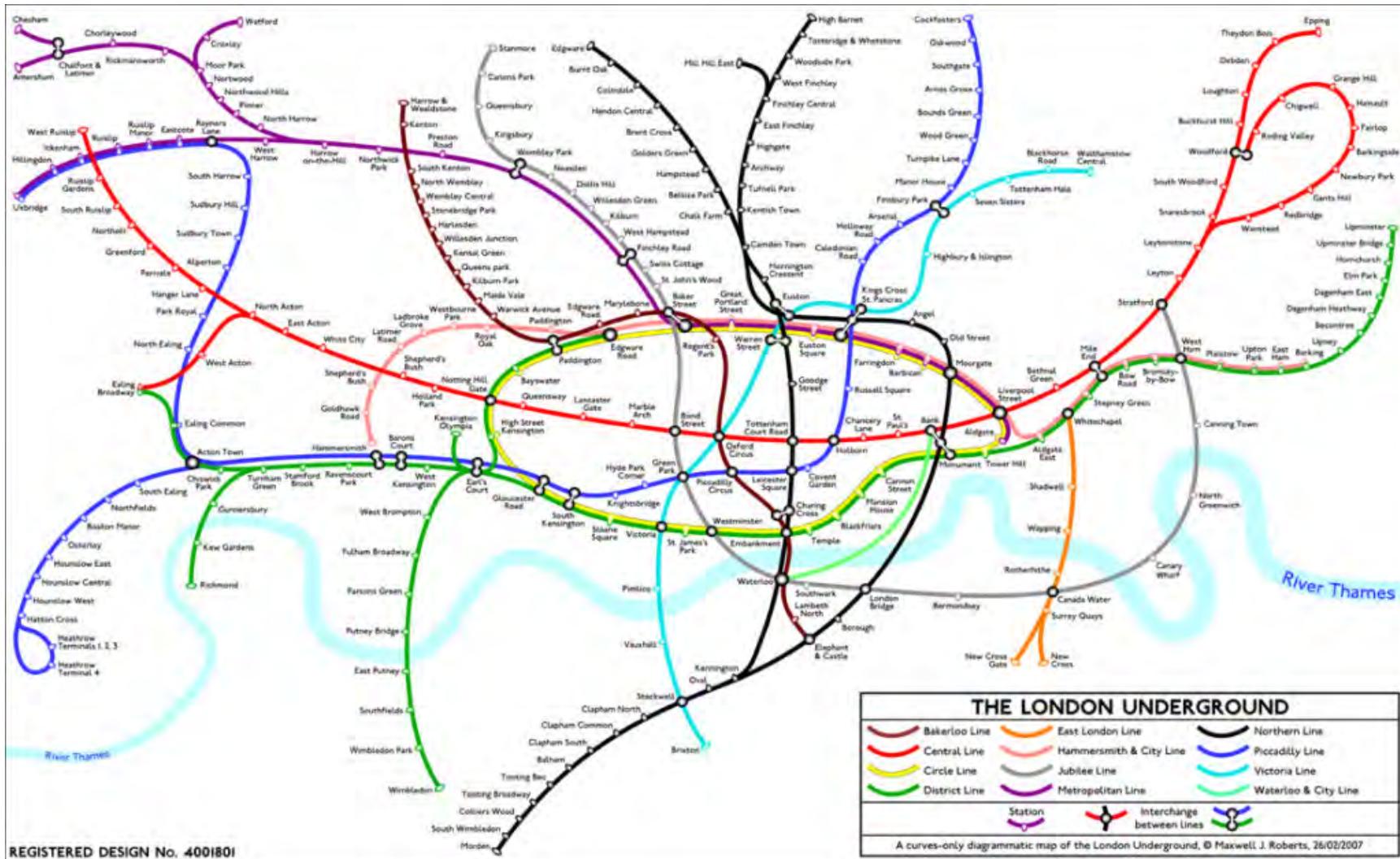


Curved Schematization

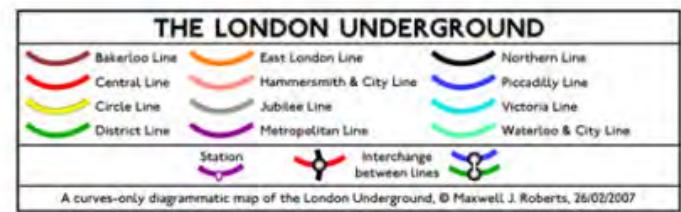
Curved networks



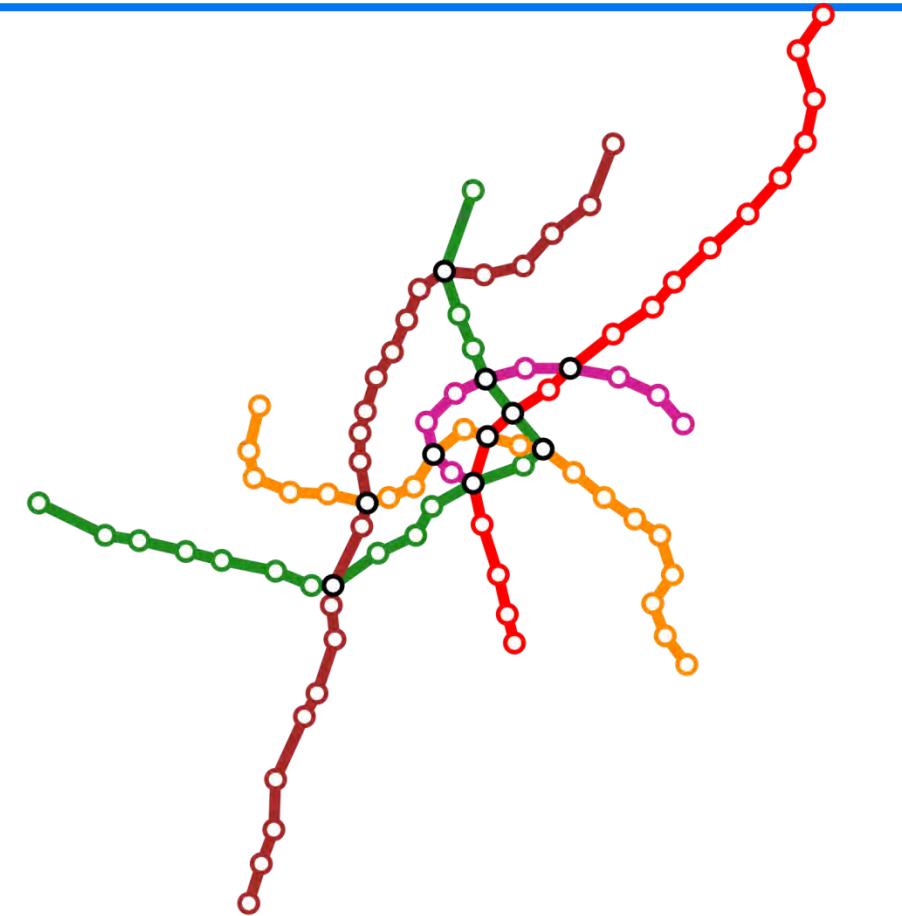
Curved networks



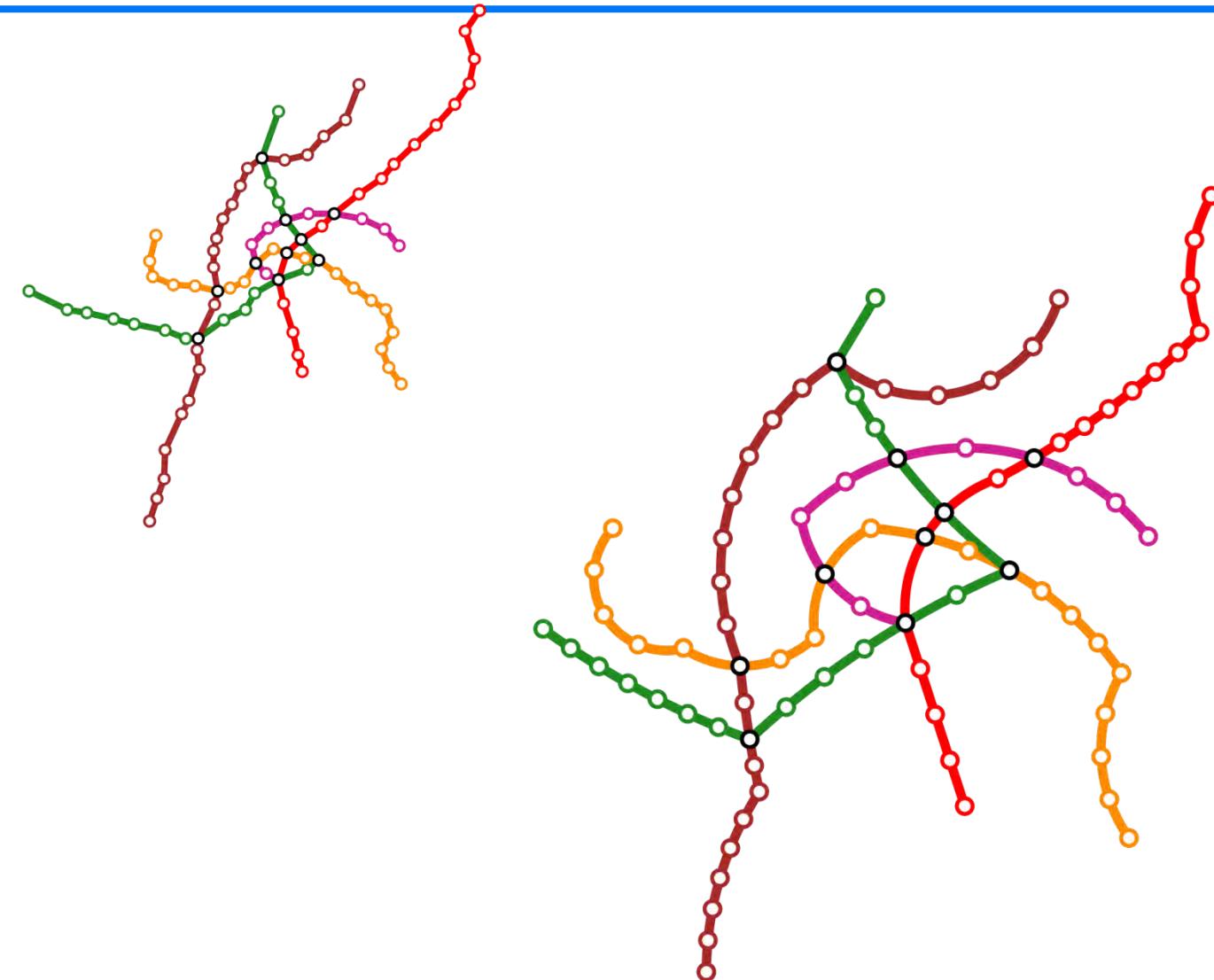
REGISTERED DESIGN No. 4001801



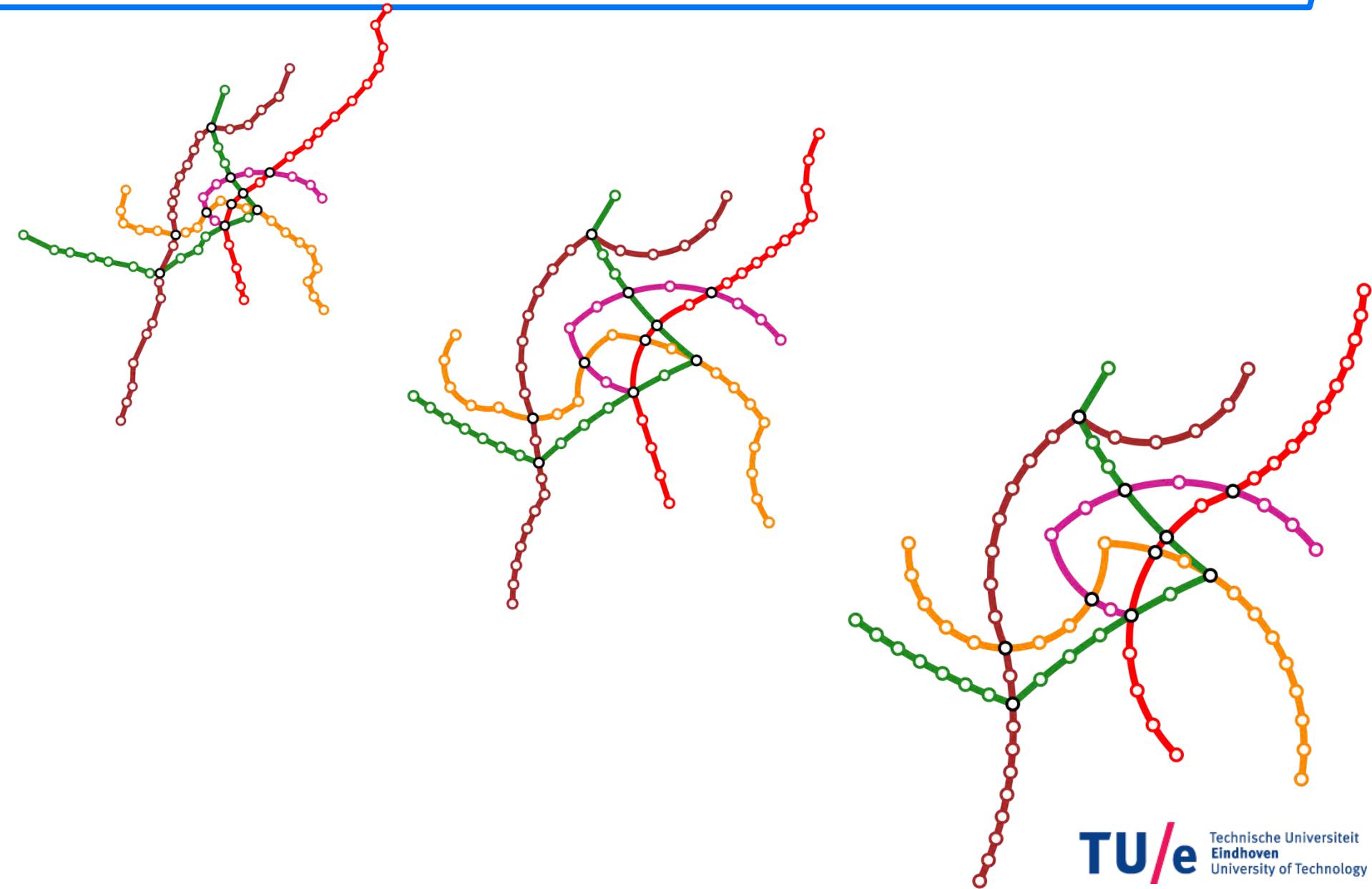
Curved networks



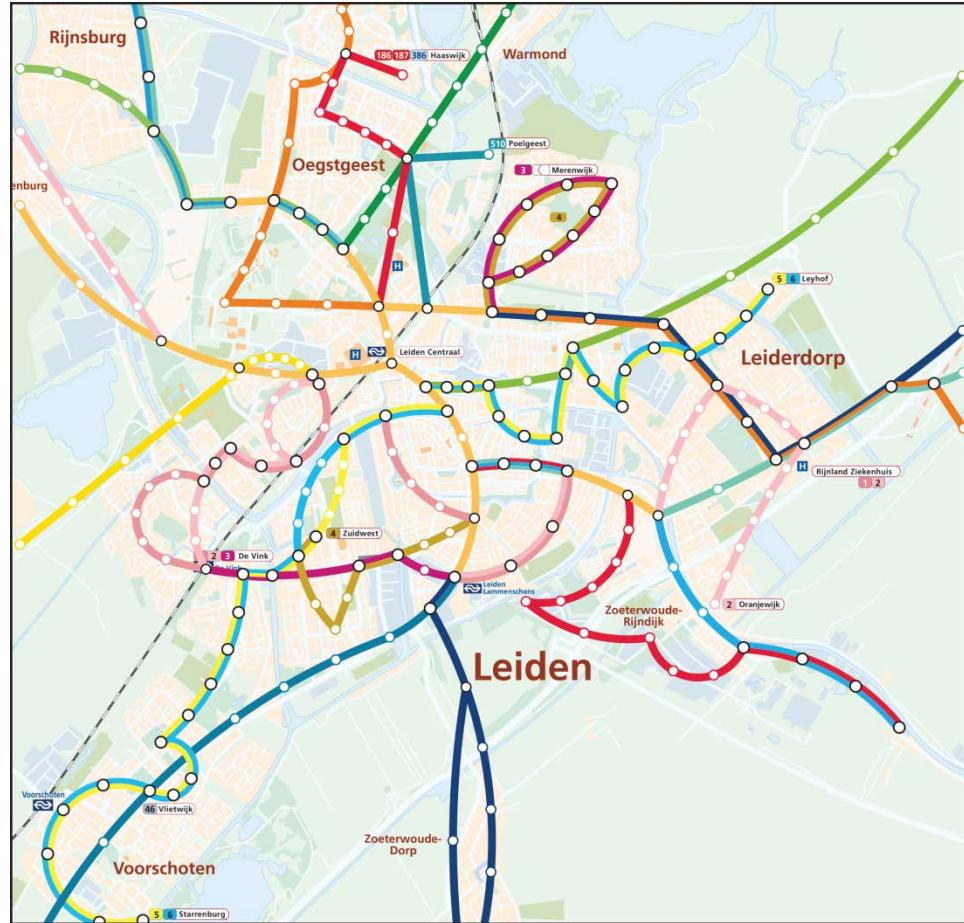
Curved networks



Curved networks

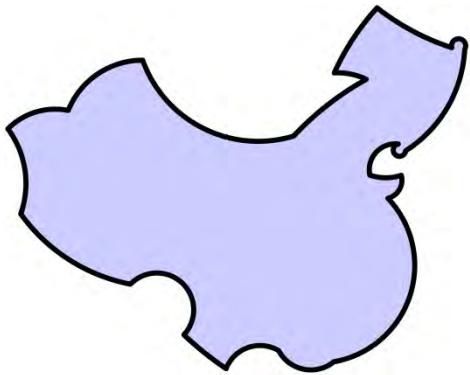


Curved networks



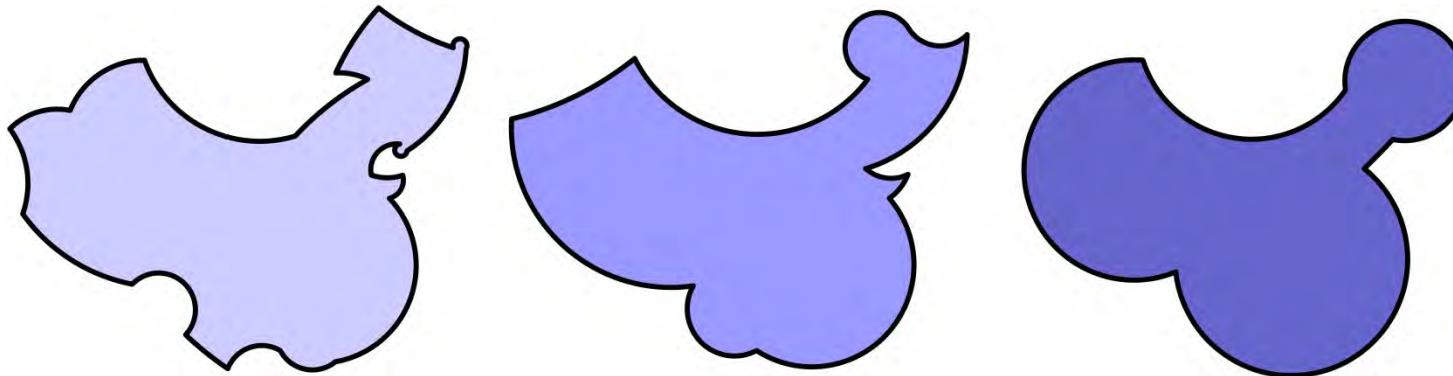
Curved outlines

Area preserving circular arcs



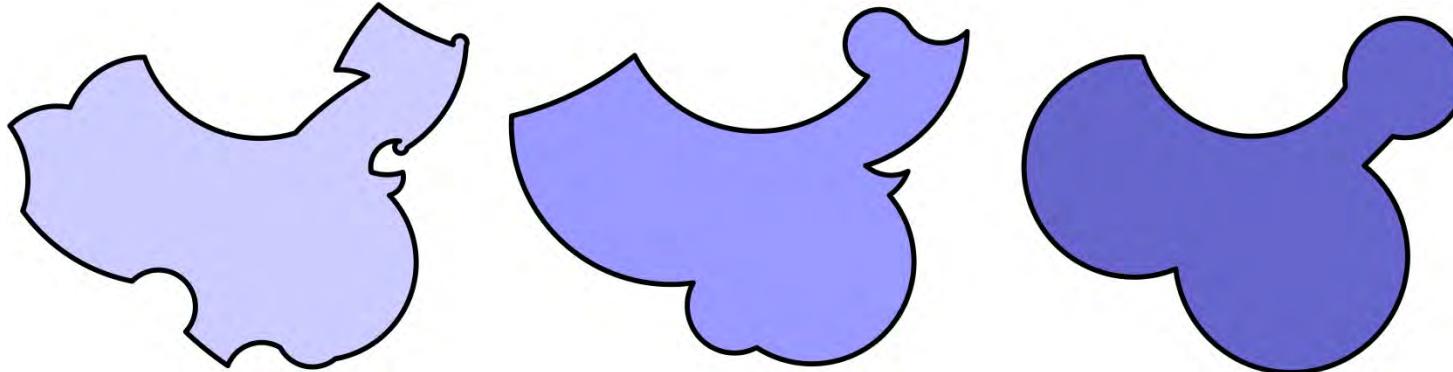
Curved outlines

Area preserving circular arcs

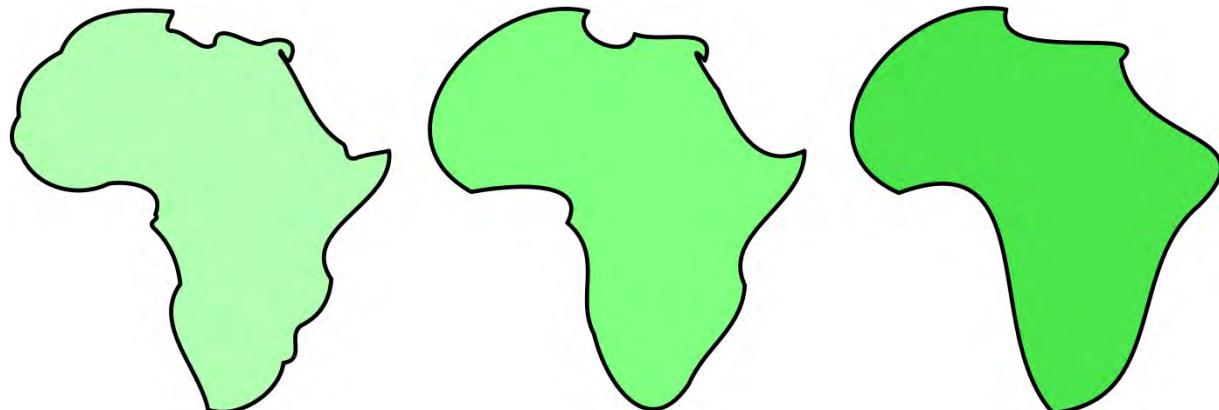


Curved outlines

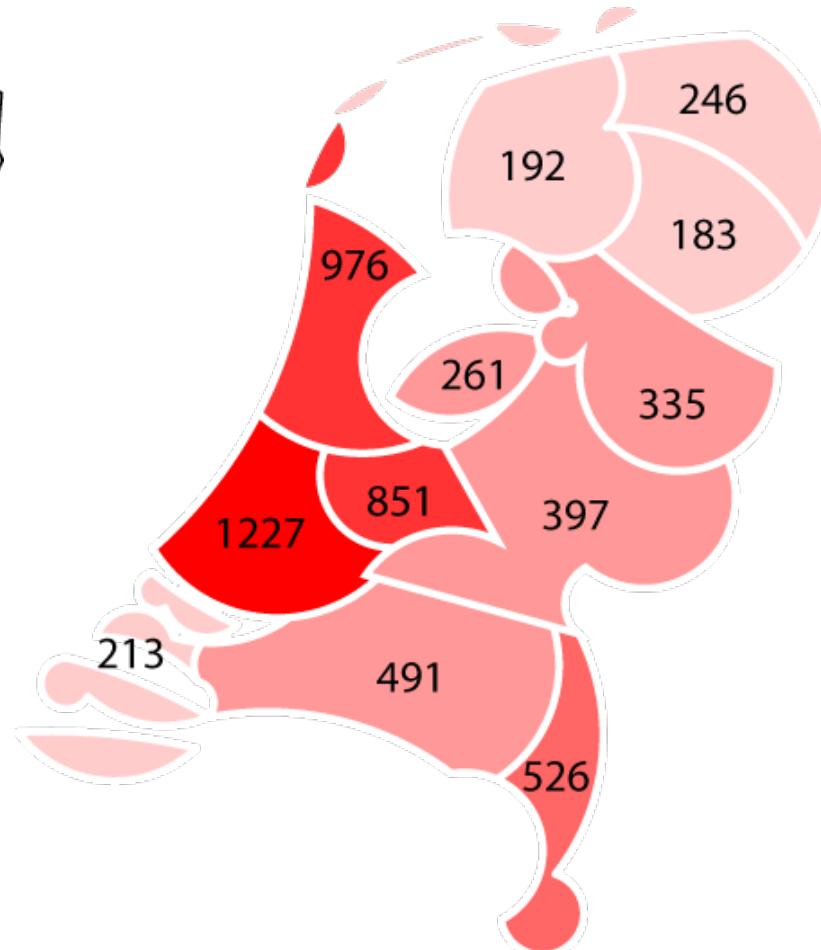
Area preserving circular arcs



Bézier curves



Curved outlines



Extreme Schematization

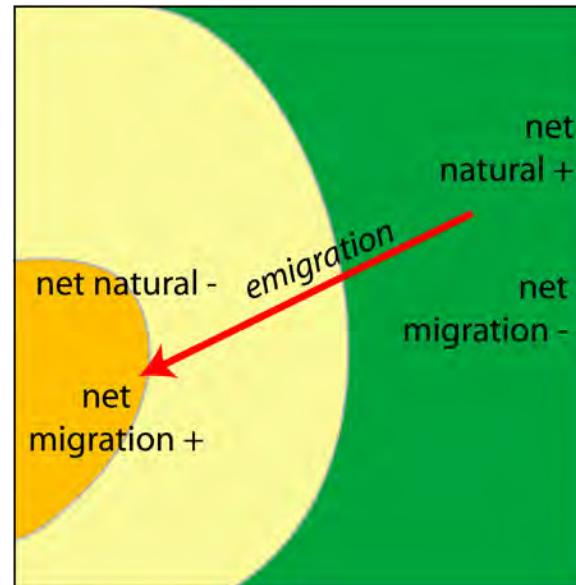
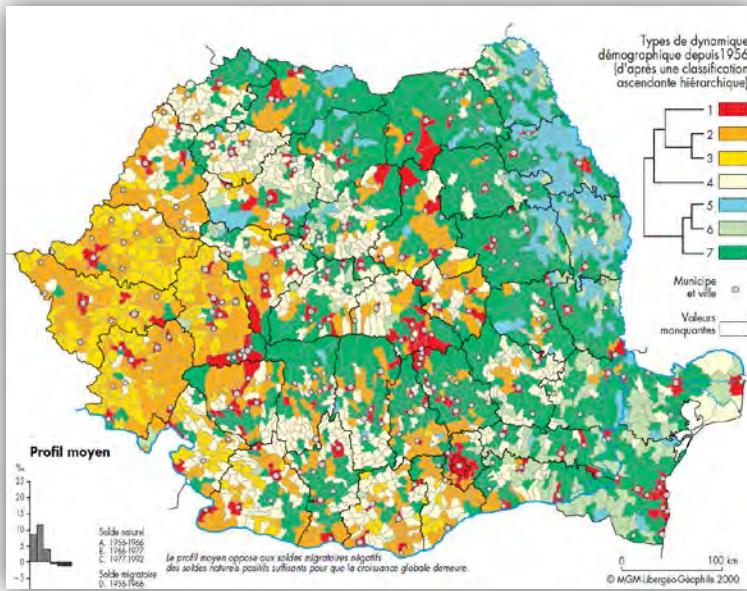
Chorematic diagrams

- Extreme generalization combined with schematized geometries

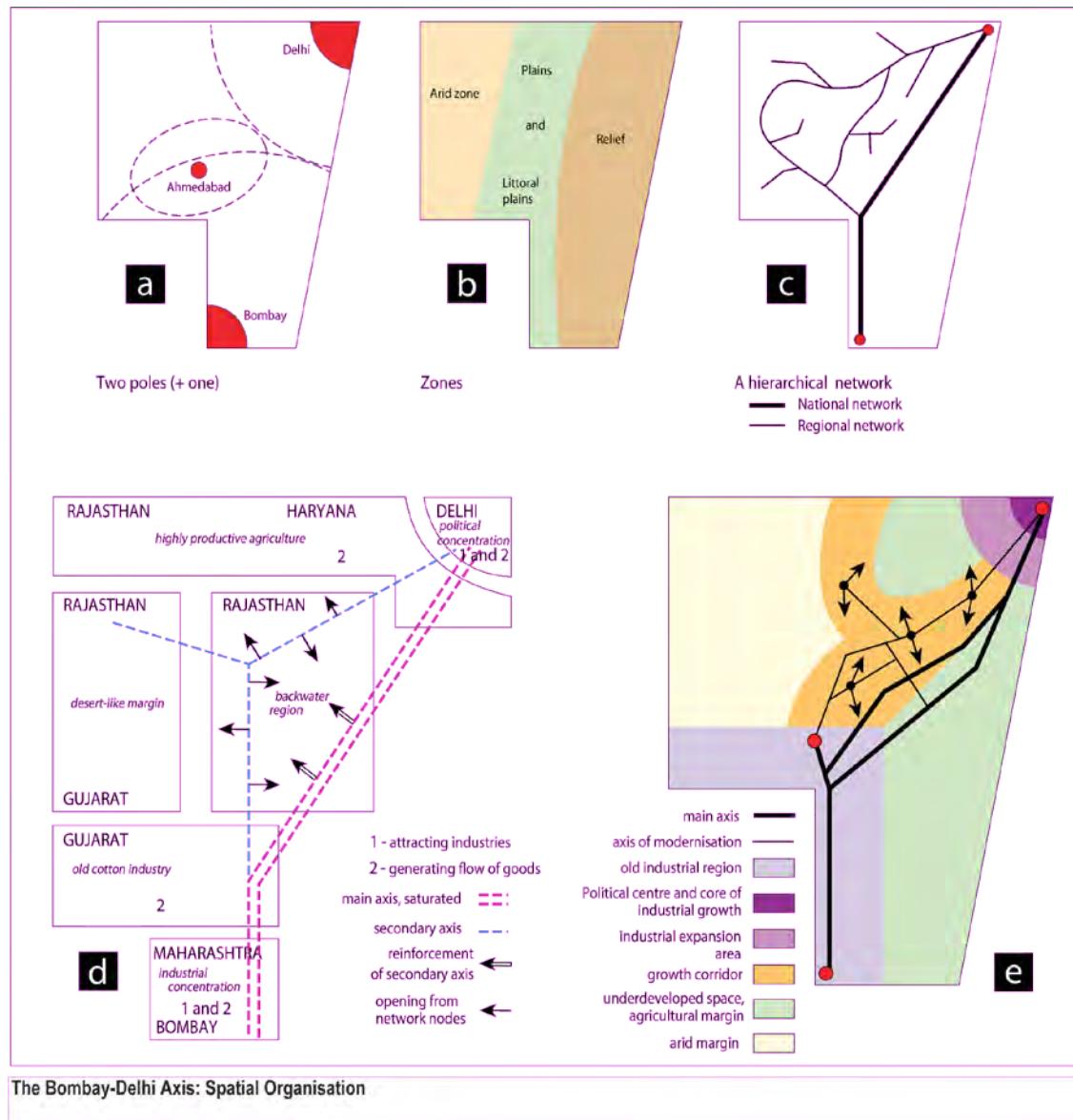
Generalization

“process by which information is selectively removed from a map in order to simplify pattern without distortion of overall content”

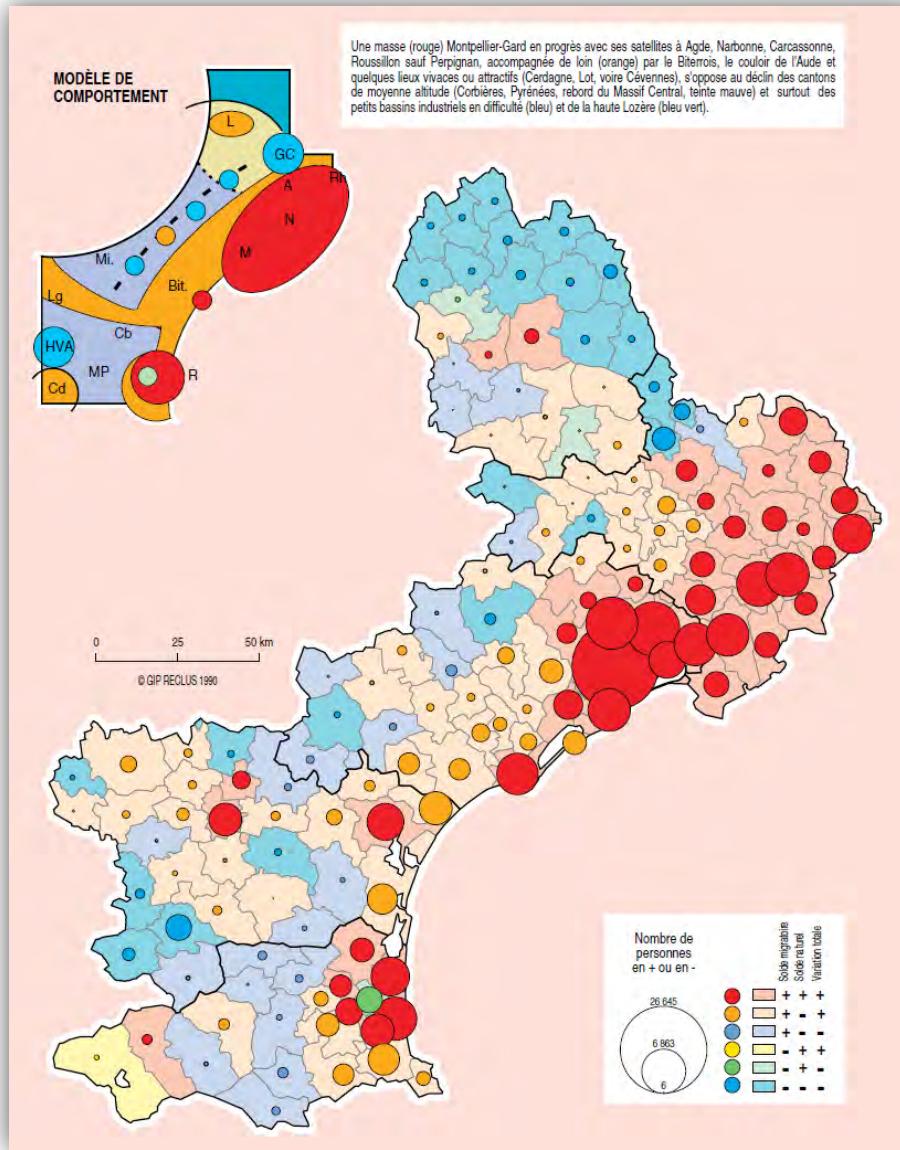
[Heywood et al. '98]



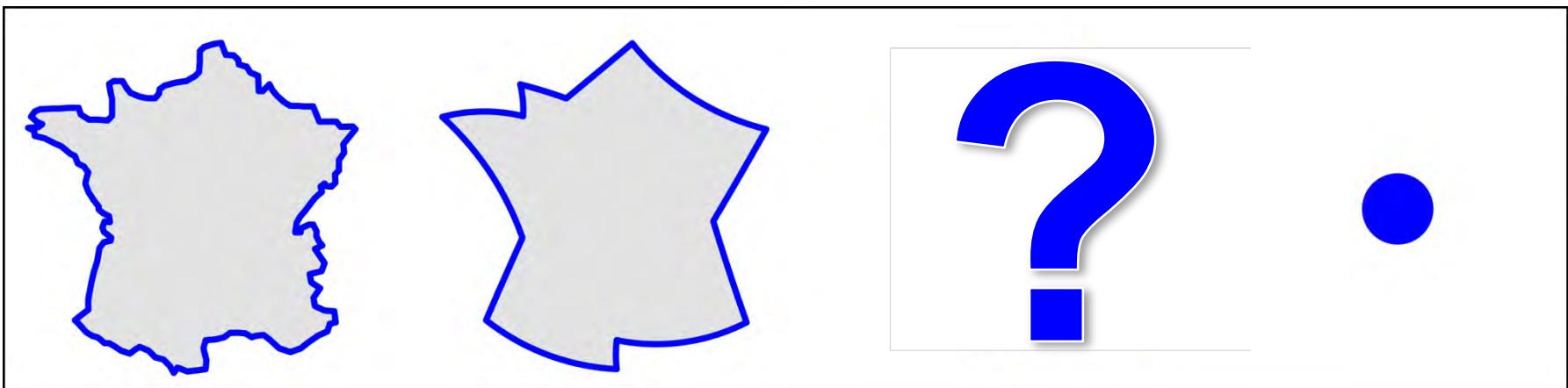
Choreomatic diagrams



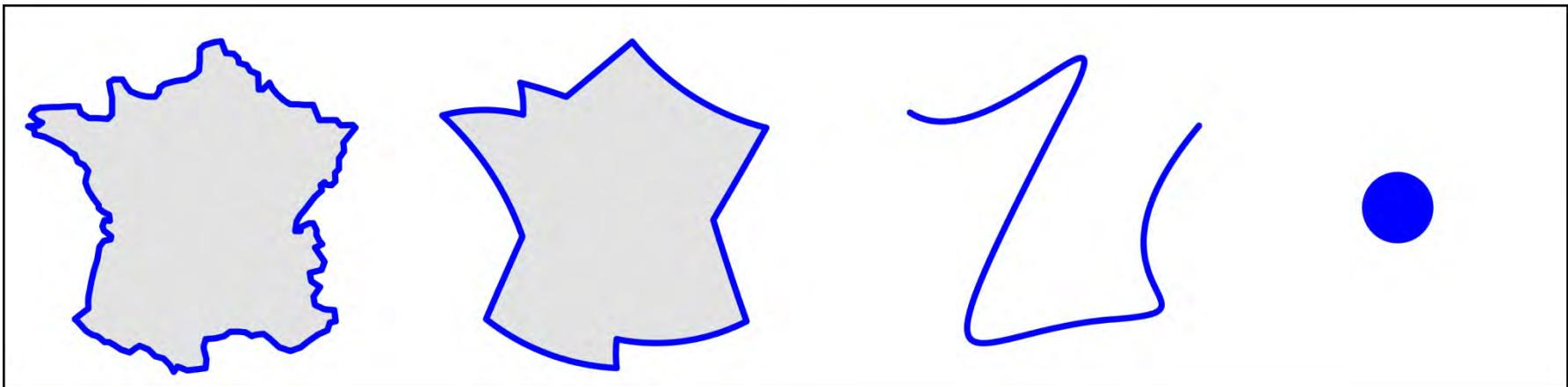
Chorematic diagrams



Extreme Schematization



Extreme Schematization



Inspiration

Stenography

C L N M S

T P H J V X

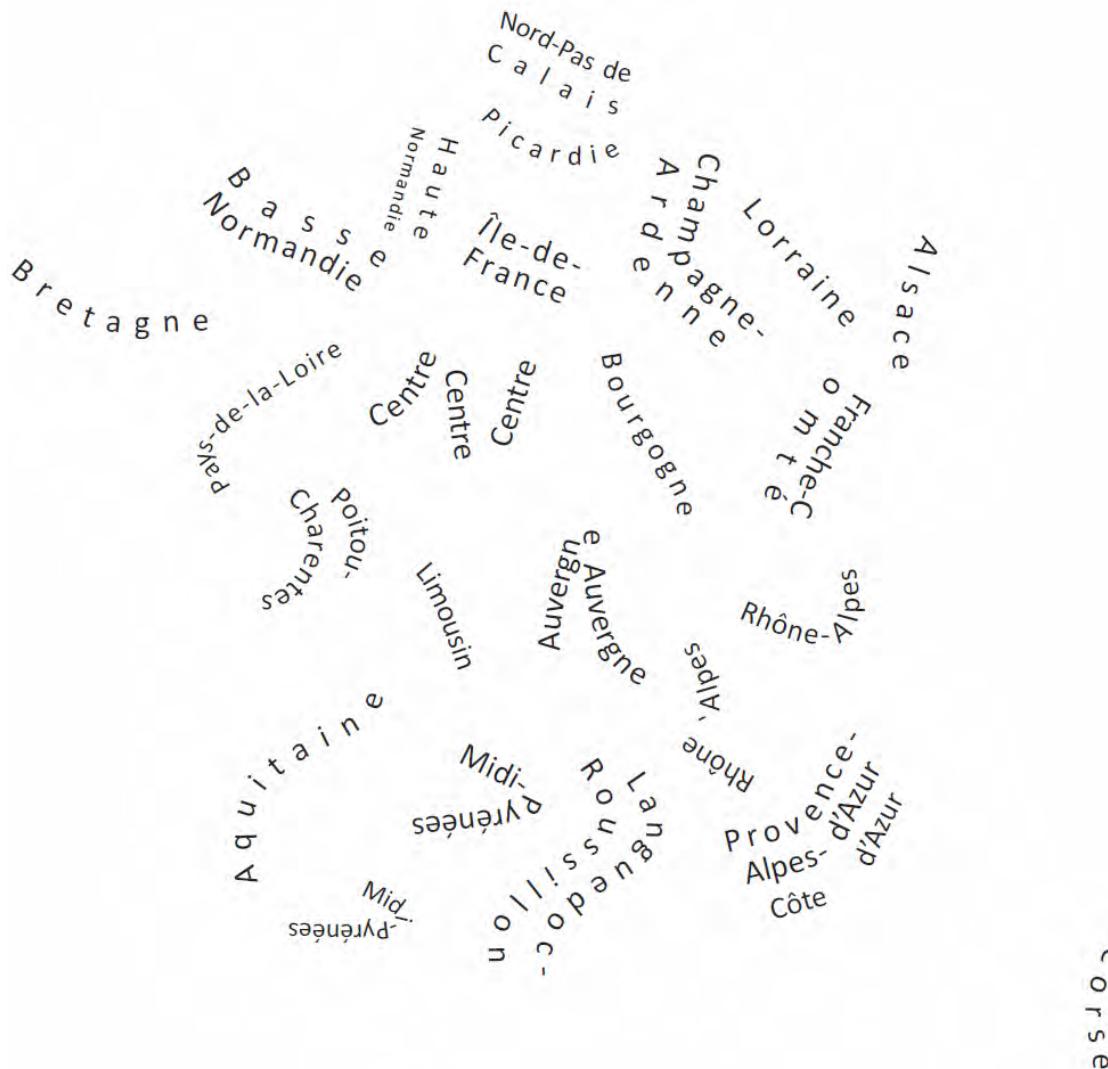
The Dog



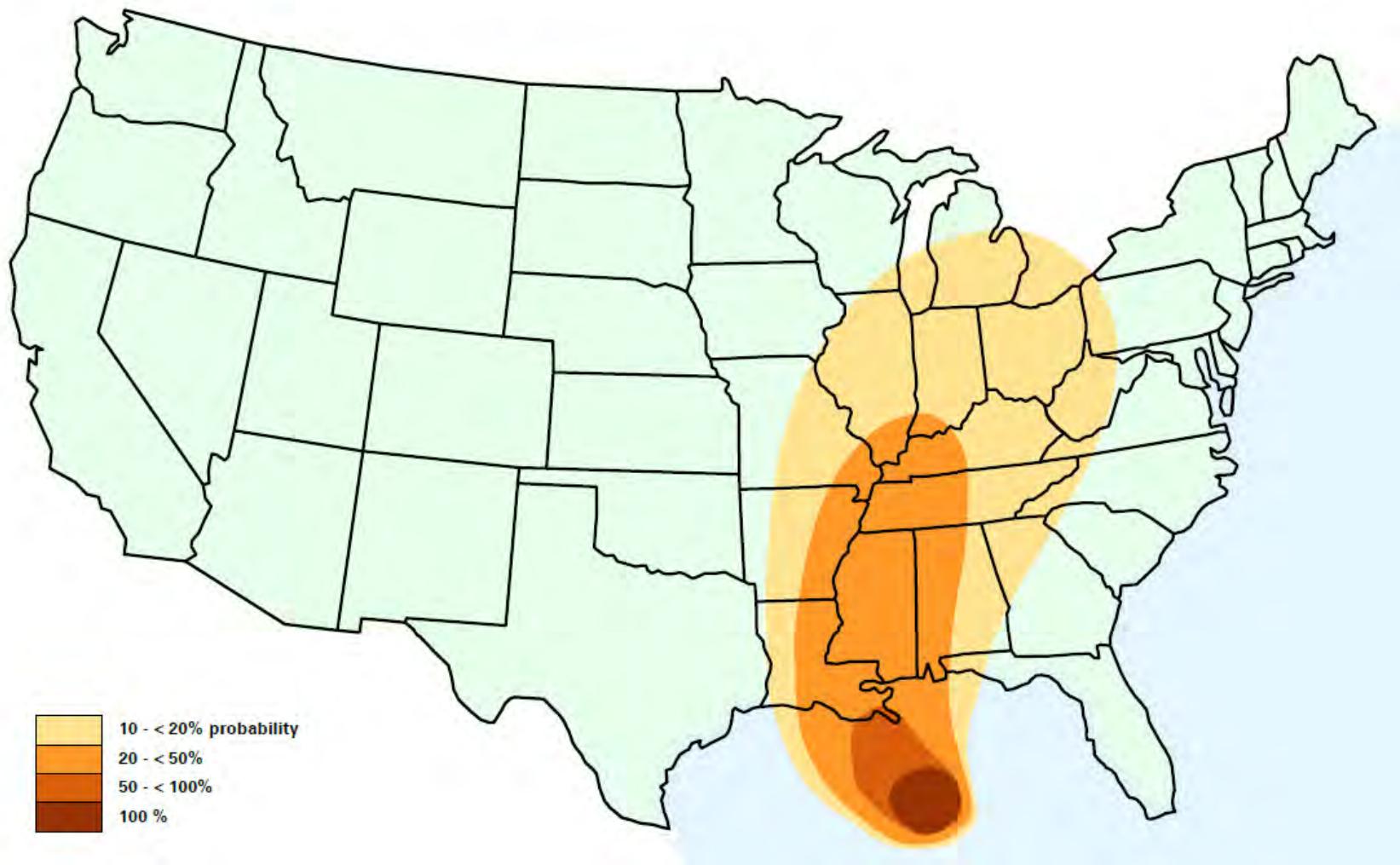
Stenomaps



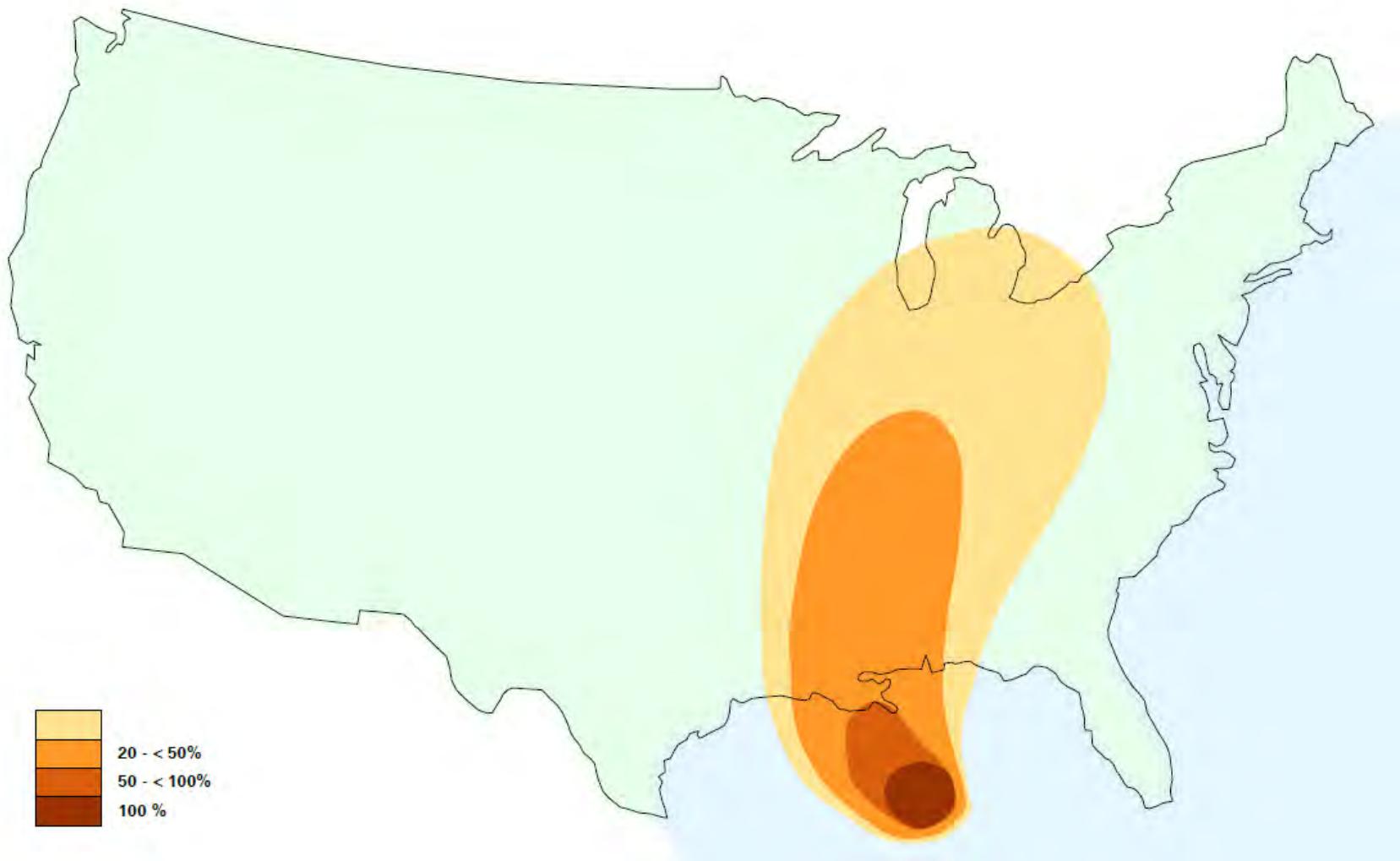
Stenomaps



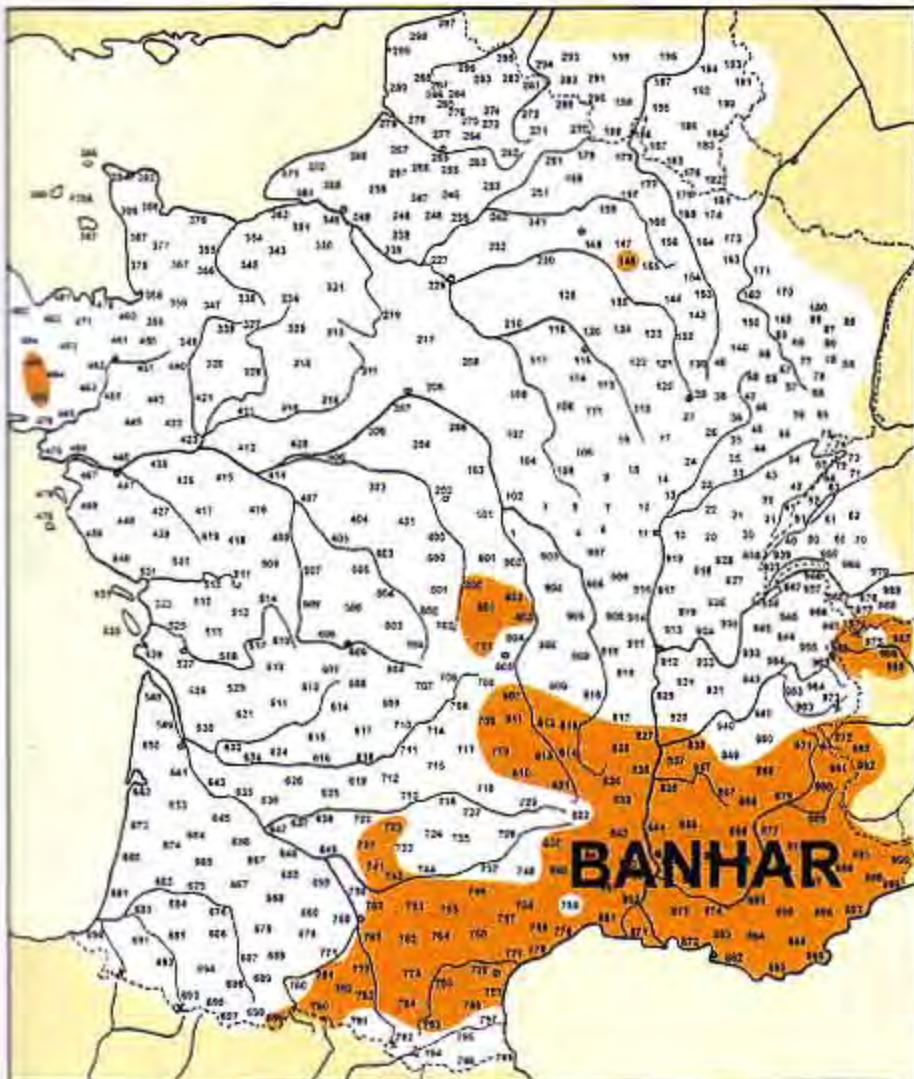
Hurricane path prediction



Hurricane path prediction

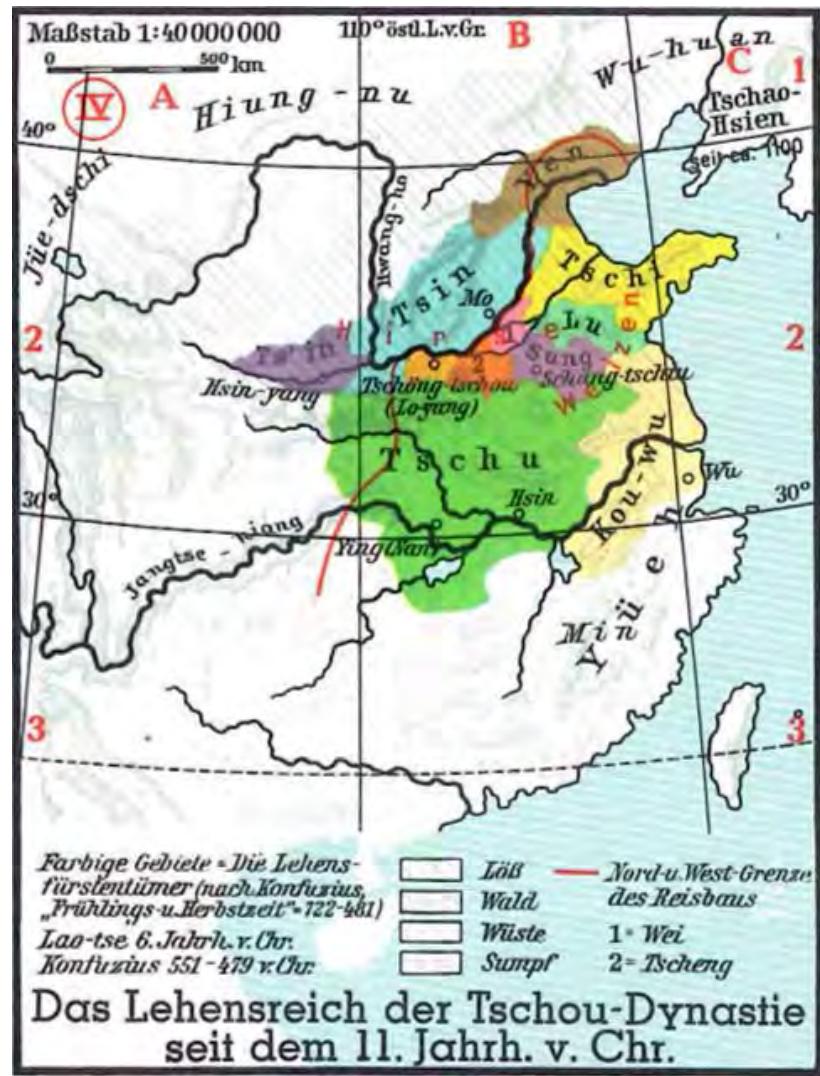


Rivers as locational aid



Carte 454. ALF 880 Mouiller : aire banhar

[Guylaine Brun-Trigaud]

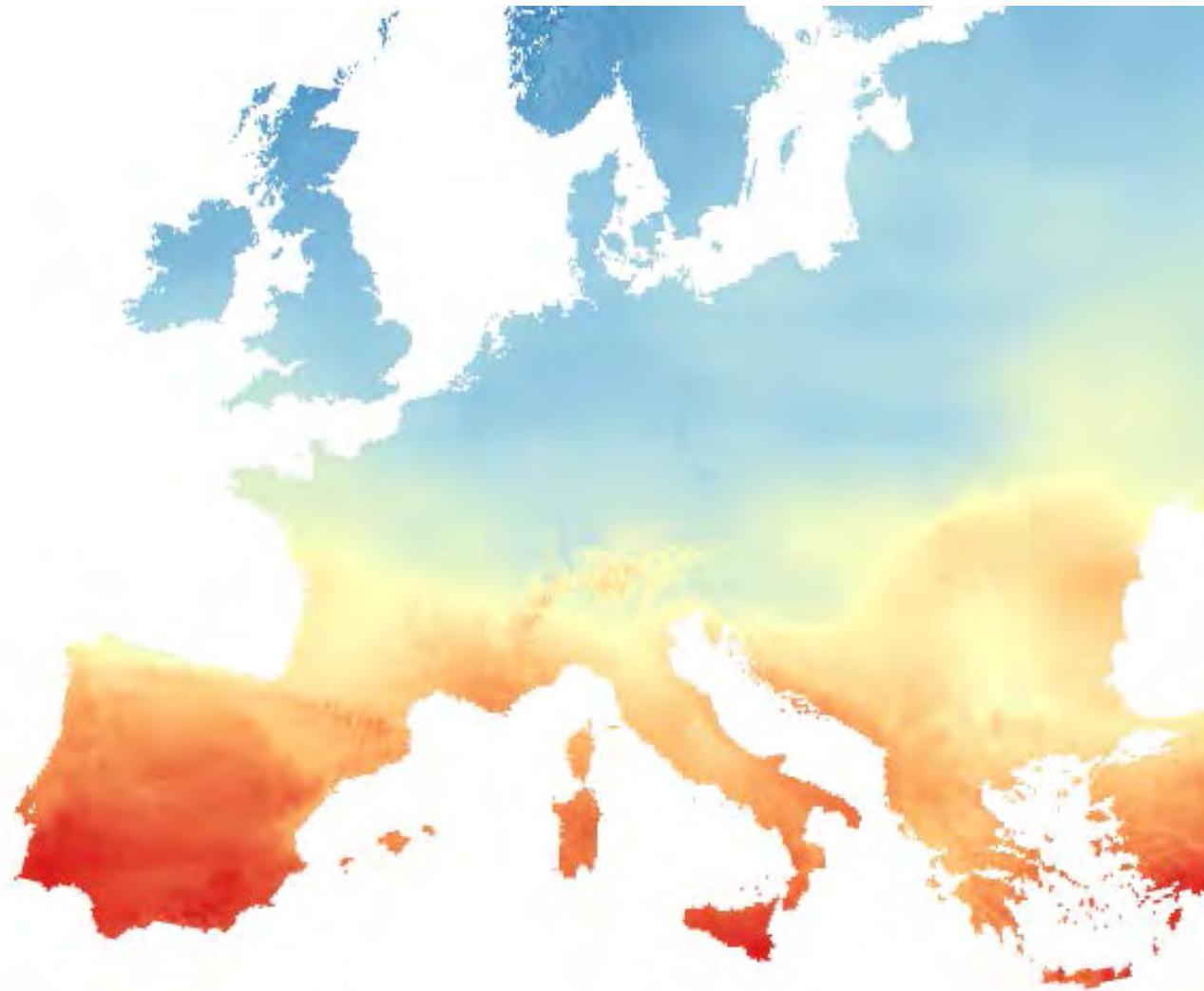


[Grosser Atlas zur Weltgeschichte, 1991]

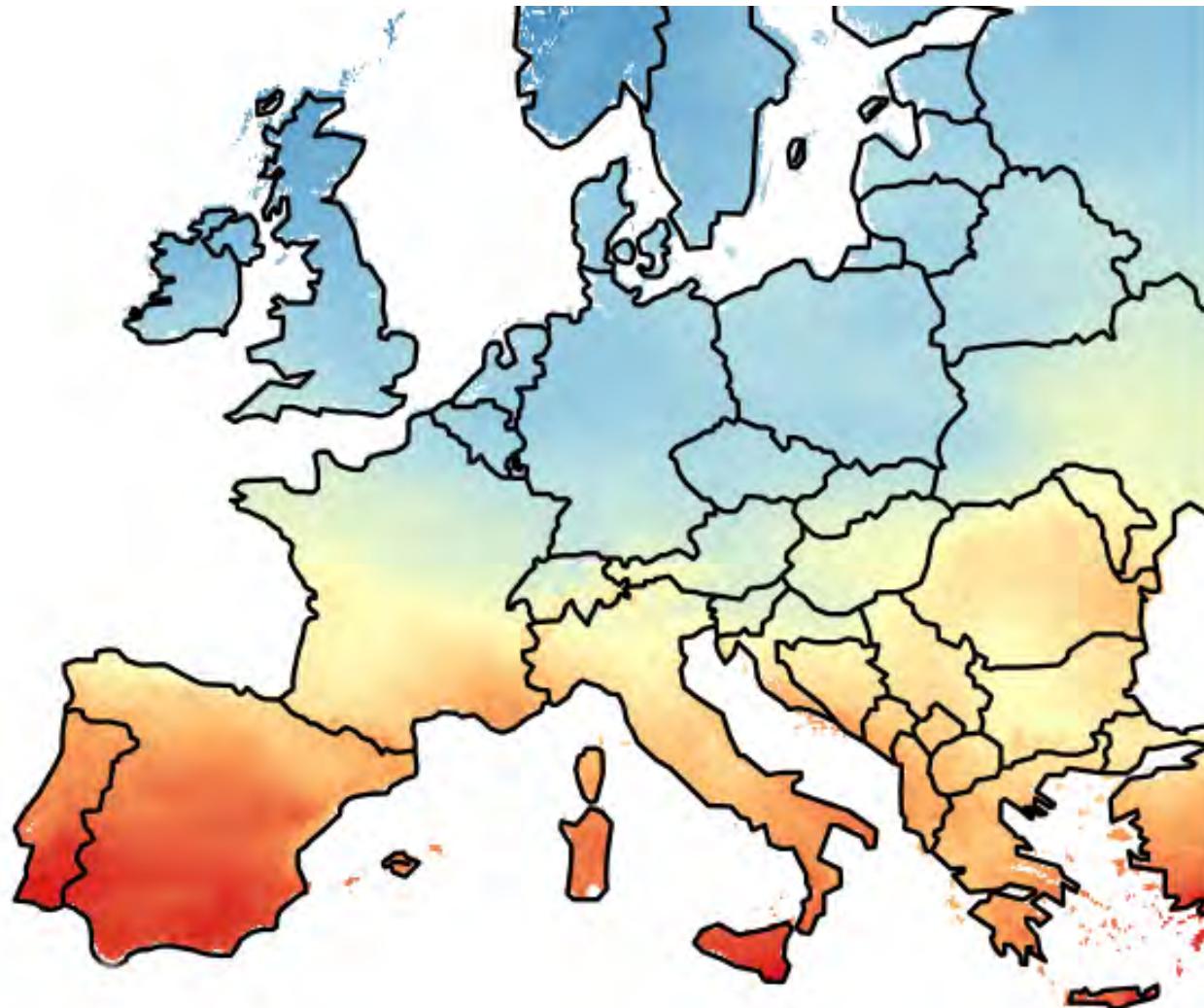
Stenomaps



Solar potential in Europe



Solar potential in Europe



Solar potential in Europe



