Semantic theories of truth: Lecture 4 Semantic dependence and Leitgeb's theory of truth

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Outline

- Summary of Lecture 3.
- Semantic dependence approach.
- Philosophical discussion: Dependence and grounding.
- Leitgeb's theory of truth.

Partial predicates and fixed-point semantics

Which kind of predicate is 'true'?

'true' is a partial predicate which applies to a fragment of the object language.

Which kind of semantics for the truth language?

Since 'true' is a partial predicate the truth language is to be endowed with a partial semantics.

What characterises 'true' among partial predicates?

Since the truth schema characterises truth, 'true' is to be interpreted by fixed points.

Partial predicates and classical semantics

Which kind of predicate is 'true'?

'true' is a partial predicate which applies to a fragment of the object language.

Which kind of semantics for the truth language?

Since we want to use 'true', the truth language is to be endowed with a classical semantics

What characterises 'true' among partial predicates?

Since the truth schema is inconsistent with classical semantics, it is to be restricted to the fragment 'true' applies to.

Partial predicates vs partial semantics

Partial sets and extensions

A partial set is a pair $(Z^{\mathbf{t}}, Z^{\mathbf{f}})$ of disjoint subsets of some domain D. A partial extension is a pair (Z, X) of subsets of D such that $Z \subseteq X$.

Partial interpretations

T is partially interpreted by a partial extension (Z, X).
— X is the range of significance of T.
— Z is the extension of T in the classical sense.

Restricted Convention T

Restricted Convention T

Fix an interpretation I^{-} of the names. A classic admissible valuation v of the truth language $\mathcal{L}_{\mathrm{T}}(P, N)$ has to satisfy, for every name $a \in N$,

(α) $v(\mathrm{T}a) = v(I^{-}(a))$, if $I^{-}(a)$ belongs to the range of significance of T.

(β) $v(\mathrm{T}a) = \mathbf{f}$, otherwise.

Semantic dependence approach

1. Start with a notion of semantic dependence between sentences of the truth language.
2. Define in terms of this notion of dependence the range of significance of the truth predicate.
3. Define a partial interpretation of the truth predicate which respects Restricted Convention T.

Supervenience and groundedness

Supervenience thesis

The interpretation of the semantical constants of the language is determined by the interpretation of the non-semantical constants. Expressed in terms of our formal theory, the intuition becomes: for any given base model there is exactly one correct interpretation of the truth predicate. I call this principle "the supervenience of semantics".

Groundedness thesis

The range of significance of the truth predicate is given by the set of sentences which are grounded in the non-semantic states of affairs.

Grounding and dependence

Semantic groundedness

A sentence of the truth language is grounded if and only if its truth value only depends on the non-semantic states of affairs.

Direct and indirect dependence

The truth value of a sentence can depend either directly or indirectly from the non-semantic states of affairs.

Local determinability of truth

The truth value of a sentence can depend on a proper subset of non-semantic states of affairs.

Base semantics

Base language $\mathcal{L}(P, N)$

— Set P of propositional letters : $p, q, \ldots$
— Set N of names N: $a, b, c, \ldots$

Ground model $\mathcal{M} = (D, I^-)$

— Domain: A non-empty set D.
— Interpretation of the propositional letters: $I^- : P \to \{\mathbf{t}, \mathbf{f}\}$.
— Interpretation of the names: $I^- : N \to D$.

Base admissible valuations

- $\mathrm{Val}^{\tau}_{\mathcal{M}}(p) = I^-(p)$.
- $\mathrm{Val}^{\tau}_{\mathcal{M}}(\neg\phi) = -^{\tau}\mathrm{Val}^{\tau}_{\mathcal{M}}(\phi)$.
- $\mathrm{Val}^{\tau}_{\mathcal{M}}(\phi \wedge \psi) = \mathrm{Val}^{\tau}_{\mathcal{M}}(\phi) *^{\tau} \mathrm{Val}^{\tau}_{\mathcal{M}}(\psi)$.

A bi-valued valuation $v^- : \mathcal{L}(P) \to \{\mathbf{t}, \mathbf{f}\}$ is classically admissible iff $v^- = \mathrm{Val}^{\tau}_{\mathcal{M}}$.

Classical semantics

Truth language $\mathcal{L}_{\mathrm{T}}(P, N)$

— $\mathcal{L}(P, N)$ augmented by a unary predicate T.

Partial model $\mathcal{M} + Z$

— The ground model $\mathcal{M}$ expanded by an extension $I(\mathrm{T}) = Z$ interpreting T.

Classical admissible valuations

- $\mathrm{Val}^{\tau}_{\mathcal{M}+Z}(p) = I^{-}(p)$.
- $\mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\mathrm{T}a) = I(\mathrm{T})(I^{-}(a))$.
- $\mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\neg\phi) = -^{\tau}\mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\phi)$.
- $\mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\phi \wedge \psi) = \mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\phi) *^{\tau} \mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\psi)$.

A bi-valued valuation $v : \mathcal{L}(P) \to \{\mathbf{t}, \mathbf{f}\}$ is classically admissible iff $v = \mathrm{Val}^{\tau}_{\mathcal{M}+Z}$.

Leitgeb's semantic dependence

Fix a ground model $\mathcal{M} = (D, I^-)$ such that $\mathcal{L}_\mathsf{T}(N, P) \subseteq D$. The Tarskian operator is the function τ on subsets of D defined by

$$\tau(Z) = \{\phi \in \mathcal{L}_\mathsf{T}(N, P) \mid \mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\phi) = \mathbf{t}\}.$$

Leitgeb's dependence operator

Let Φ be a set of sentences of the truth language:

$$\phi \in \Delta(\Phi) \Leftrightarrow$$

$$\forall Z, Z' \, (Z \cap \Phi = Z' \cap \Phi \Rightarrow \mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\phi) = \mathrm{Val}^{\tau}_{\mathcal{M}+Z'}(\phi)).$$

Leitgeb's grounded sentences

A sentence ϕ is grounded in Leitgeb's sense iff

$$\phi \in \mathrm{lfp}(\Delta).$$

Leitgeb's semantics

Characterisation "from below" of the grounded sentences

- $\Phi_0 = \emptyset$;
- $\Phi_{\alpha+1} = \Delta(\Phi_\alpha)$;
- For α limit, let $\Phi_\alpha = \bigcup\{\Phi_\beta \mid \beta < \alpha\}$.

Since Δ is monotonic, $\mathrm{lfp}(\Delta) = \Phi_{\alpha_*}$.

Truth definition for the grounded sentences

- $\Gamma_0 = \emptyset$;
- $\Gamma_{\alpha+1} = \{\phi \in \Phi_{\alpha+1} \mid \mathrm{Val}_{\Gamma_\alpha}(\phi) = \mathbf{t}\}$;
- For α limit, let $\Gamma_\alpha = \bigcup\{\Gamma_\beta \mid \beta < \alpha\}$.

The partial interpretation of the truth predicate is the partial extension $(\Gamma_{\alpha_*}, \Phi_{\alpha_*})$.

Leitgeb's theory of truth

Fix a ground model $\mathcal{M} = (D, I^-)$ such that $\mathcal{L}_\mathsf{T}(N, P) \subseteq D$ and for every sentence ϕ there is a name a such that $I^-(a) = \phi$.

Leitgeb's truth definition

Suppose that $I^-(a) = \phi \in \Phi_{\alpha_*}$. Then

$$\mathrm{Val}_{\mathcal{M}+\Gamma_{\alpha_*}}(\mathsf{T}a) = \mathbf{t} \Leftrightarrow \phi \in \Gamma_{\alpha_*} \Leftrightarrow \mathrm{Val}_{\mathcal{M}+\Gamma_{\alpha_*}}(\phi) = \mathbf{t}.$$

Hence, Leitgeb's partial interpretation $(\Gamma_{\alpha_*}, \Phi_{\alpha_*})$ of T satisfies Restricted Convention T.