Recent work on paraconsistent logic (1)

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The menu for today

1. Technical and terminological preliminaries (lengthy!)
2. Our base logic: **FDE**
3. First working definition of ‘paraconsistent logic’
4. Another nice logic: **LP**
5. Genuine paraconsistency

Technical preliminaries: language

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- The first capital letters of the Latin alphabet, 'A', 'B', 'C'$\ldots$: variables ranging over arbitrary formulas.
- Some capital Greek letters, 'Γ', 'Δ'$\ldots$, for sets of such formulas.

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- For simplicity, a (Dunn) valuation is a function $v : Atom \longrightarrow \{\{\ \}, \{0\}, \{1\}, \{1, 0\}\}$. Any valuation v can be then extended to an interpretation σ to cover all formulas.

Technical preliminaries: some useful notions

Now, let Γ be a set of formulas, and A and B formulas of the base language of a logic **L**. Then

- A is a logical consequence of Γ in **L**, $\Gamma \models_{\mathbf{L}} A$, if and only if (hereafter, 'iff'), for every evaluation σ, $1 \in \sigma(A)$ if $1 \in \sigma(B)$ for every $B \in \Gamma$.

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- A is a logical truth (in **L**, according to a Dunn semantics) iff, for all σ, $1 \in \sigma(A)$.

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- A is a logical falsity (in $\mathbf{L}$, according to a Dunn semantics) iff, for all σ, $1 \notin \sigma(A)$.
- An argument is invalid in $\mathbf{L}$ iff there is an evaluation in which the premises are true, i.e. $1 \in \sigma(B)$ for every $B \in \Gamma$, but the conclusion is not, i.e. $1 \notin \sigma(A)$.

FDE (first-degree entailment): evaluation conditions

- $\sigma(p) = V(p)$, for every $p \in Atom$
- $1 \in \sigma({\sim}A)$ iff $0 \in \sigma(A)$
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FDE: tables

The above model-theoretic semantics for **FDE** can be represented in a tabular way as follows:

A	$\sim A$
$\{1\}$	$\{0\}$
$\{1, 0\}$	$\{1, 0\}$
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$\{1\}$	$\{1\}$	$\{1,0\}$	$\{\,\}$	$\{0\}$
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$A \vee B$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
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and in tabular form:

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FDE: useful facts

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- The following arguments are valid:

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$A \models_{\mathbf{FDE}} A \vee B$

$B \models_{\mathbf{FDE}} A \vee B$

$A \wedge (B \vee C) \models_{\mathbf{FDE}} (A \wedge B) \vee C$

$\sim\sim A \models_{\mathbf{FDE}} A$

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If $A \models_{\mathbf{FDE}} B$, $B \models_{\mathbf{FDE}} C$ then $A \models_{\mathbf{FDE}} C$

If $A \models_{\mathbf{FDE}} B$, $A \models_{\mathbf{FDE}} C$ then $A \models_{\mathbf{FDE}} B \wedge C$

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$A, A \rightarrow B \models_{\mathbf{FDE}} B$

$A \rightarrow B, B \rightarrow C \models_{\mathbf{FDE}} A \rightarrow C$

$A \models_{\mathbf{FDE}} (A \wedge B) \vee (A \wedge \sim B)$

$\sim A, A \vee B \models_{\mathbf{FDE}} B$

$A \models_{\mathbf{FDE}} B \vee \sim B$

$A, \sim A \models_{\mathbf{FDE}} B$

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FDE: more useful facts

- The following arguments are invalid:

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  $A, A \to B \models_{\mathbf{FDE}} B$

  $A \to B, B \to C \models_{\mathbf{FDE}} A \to C$

  $A \models_{\mathbf{FDE}} (A \wedge B) \vee (A \wedge \sim B)$

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  If $A, B \models_{\mathbf{FDE}} C$ then $A, \sim C \models_{\mathbf{FDE}} \sim B$ Why?

- There are neither logical truths nor logical falsities in **FDE**.

Paraconsistent logic: first working definition

- A logic **L** is paraconsistent iff there are at least two formulas A and B, a negation N and a premise-binder © such that $A © NA \not\models_{\mathbf{L}} B$.

(I am not assuming that, in each logic, there is a single best candidate for each of the roles of negation and premise-binder.).

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- Johansson's minimal logic (where $A \owedge NA \not\models_{\mathbf{L}} B$ but $A \owedge NA \models_{\mathbf{L}} NB$).
- Some of Arruda and da Costa's **J** logics, where $A \owedge NA \not\models_{\mathbf{L}} B$ but $A \owedge NA \models_{\mathbf{L}} B > C$, for any B and C, with $>$ an implication in such logics.

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Two independent forms of paraconsistency worth having in mind:

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- A logic **L** is ,-paraconsistent iff there are at least two formulas A and B, a negation N and a comma , such that $A, NA \not\models_{\mathbf{L}} B$.

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Two independent forms of paraconsistency worth having in mind:

- A logic **L** is $\otimes$-paraconsistent iff there are at least two formulas A and B, a negation N and a conjunction $\otimes$ such that $A \otimes NA \not\models_{\mathbf{L}} B$.
- A logic **L** is ,-paraconsistent iff there are at least two formulas A and B, a negation N and a comma , such that $A, NA \not\models_{\mathbf{L}} B$.

In a logic for which $A \otimes B \models_{\mathbf{L}} C$ implies $A, B \not\models_{\mathbf{L}} C$, conjunctive-paraconsistency implies collective-paraconsistency; in a logic for which the reverse holds, the reverse is true.

K3 and LP

Model-theoretically, **K3** is obtained by ignoring the interpretation $\{1, 0\}$; **LP** is obtained by ignoring the interpretation $\{\,\}$; by ignoring those two interpretations at once, one obtains classical logic, **CL**.

LP is paraconsistent but **K3** is not.

Axiomatically, **LP** is obtained by adding

$$\Gamma \vdash A \vee \sim A \qquad \text{(Implosion)}$$

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- The following arguments are invalid:

$$A, A \to B \models_{\mathbf{LP}} B \qquad {\sim}A, A \vee B \models_{\mathbf{LP}} B$$

$$A \to B, B \to C \models_{\mathbf{LP}} A \to C \qquad A, {\sim}A \models_{\mathbf{LP}} B$$

If $A, B \models_{\mathbf{LP}} C$ then $A, {\sim}C \models_{\mathbf{LP}} {\sim}B$

Genuine paraconsistency

Let N be some negation and $\owedge$ be some conjunction. According to Béziau and Franceschetto, a logic **L** is *genuinely paraconsistent* iff it satisfies the following two conditions:

$$\not\models_{\mathbf{L}} N(A \owedge NA) \tag{GPcons1}$$

$$A \owedge NA \not\models_{\mathbf{L}} \tag{GPcons2}$$

The insistence on having both (GPcons1) and (GPcons2) is intriguing at first sight, especially because it can be easily proved that they are independent:

- $\models_{\mathbf{LP}} {\sim}(A \wedge {\sim}A)$ but $A \wedge {\sim}A \not\models_{\mathbf{LP}}$.
- $\not\models_{\mathbf{K3}} {\sim}(A \wedge {\sim}A)$ but $A \wedge {\sim}A \models_{\mathbf{K3}}$.

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Example 2. Arieli and Avron's logic $\mathbf{BL}_{\supset}$ is $\mathbf{FDE}_{\supset}$ expanded with two connectives: informational meet, $\otimes$, and informational join, $\oplus$:

$A \otimes B$	$\{1\}$	$\{1,0\}$	$\{\,\}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1\}$	$\{\,\}$	$\{\,\}$
$\{1,0\}$	$\{1\}$	$\{1,0\}$	$\{\,\}$	$\{0\}$
$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$
$\{0\}$	$\{\,\}$	$\{0\}$	$\{\,\}$	$\{0\}$

$A \oplus B$	$\{1\}$	$\{1,0\}$	$\{\,\}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1,0\}$	$\{1\}$	$\{1,0\}$
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It is genuinely paraconsistent, as it contains **FDE**. Also, the following hold good:

$$\not\models_{\mathbf{BL}_{\supset}} {\sim}(A \otimes {\sim} A) \qquad\qquad A \otimes {\sim} A \not\models_{\mathbf{BL}_{\supset}}$$

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> *The Calculi C_n. As C_n, $1 \leq n \leq \omega$, are intended to serve as bases for non-trivial inconsistent theories, it seems natural that they satisfy the following conditions: (i) In these calculi the principle of contradiction, $\neg(A \& \neg A)$, must not be a valid schema; (ii) from two contradictory formulas, A and $\neg A$ it will not in general be possible to deduce an arbitrary formula B; (iii) it must be simple to extend C_n, $1 \leq n \leq \omega$, to corresponding predicate calculi (with or without equality) of first order; (iv) C_n, $1 \leq n \leq \omega$, must contain the most part of the schemata and rules of C_0 (i.e. classical logic) which do not interfere with the first conditions. (Evidently, the last two conditions are vague.)*

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da Costa's classicality (ctd)

An inconsistent but non-trivial theory $\mathcal{T}$ is one with contradictory theorems but where not every formula is a theorem.

That is, there is at least one *s* such that *s* and *Ns* are both true in every interpretation in $\mathcal{T}$. da Costa calls these "bad theorems" of $\mathcal{T}$.

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The classical desideratum is expressed as the fact that complex formulas get, whenever possible, only classically admissible interpretations. This can be expressed in the following tables:

A	$\neg A$
{1}	{0}
{1,0}	{1}
{0}	{1}

$A \sqcap B$	{1}	{1,0}	{0}
{1}	{1}	{1}	{0}
{1,0}	{1}	{1}	{0}
{ 0}	{0}	{0}	{0}

Therefore, $s \sqcap \neg s$ is just true in every interpretation in the theory, and therefore $\neg(s \sqcap \neg s)$ is just false in every interpretation in the theory, which entails the non-theoremhood of $\neg(A \sqcap \neg A)$.

A bit less classical

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Suppose that every formula, including complex ones, can be both true and false, just as in **LP**:

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{1,0}	{1,0}
{0}	{1}

$A \wedge B$	{1}	{1,0}	{0}
{1}	{1}	{1,0}	{0}
{1,0}	{1,0}	{1,0}	{0}
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Suppose also that an argument is logically valid if and only if every interpretation in which the premises are true is one where the conclusion is not false.

Then, $\sim(A \wedge \sim A)$ is not valid, because it is false in at least one interpretation.

Béziau on the invalidity of $N(A \owedge NA)$

In many cases, in a logic in which $N(A \owedge NA)$ holds, some of the following fails too:

- $A \mathrel{=\!\!|}_{\mathbf{L}}\!\models B$ iff $NA \mathrel{=\!\!|}_{\mathbf{L}}\!\models NB$
- $A, A > B \models_{\mathbf{L}} B$
- If $\Gamma, A \models_{\mathbf{L}} B$ then $\Gamma \models_{\mathbf{L}} A > B$

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But this is one of the marks of Boolean-ish negations. Do we want the negation of a paraconsistent logic to be Boolean-ish?

Thanks, see you tomorrow!

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