Semantic theories of truth: Lecture 1
The Liar and Tarskian Semantics

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Outline

- What are semantic theories of truth?
- Tarski's theory of truth.
- A formal setting for STT.
- Tarskian semantics.
- Interlude: Solving a puzzle vs solving a paradox.
- Conclusion.

Semantic theories of truth

Truth

"Truth" identifies the topic.

- A primary object of study in philosophy.
- A property of linguistic items.

Semantics

"Semantic" identifies the perspective.

- Truth itself as a linguistic object: The truth predicate.
- Semantic vs axiomatic theories of truth.

Theory

"Theory" identifies the method.

- Formalisation in formal languages.
- Mathematical theory about the formal languages.

Logical theories of truth

Logic

- Scope: The logical aspect of truth.
- Method: Philosophical logic.

Theories

- Many aspects of truth, many theories.
- Several mathematical tools serving different philosophical approaches.

Examples and puzzles

The role of puzzles

- A test of adequacy.
- A unifying methodological perspective.

Formalisation

1. Natural language example.
2. (Regimentation).
3. Truth-theoretical puzzle.

Example 1 - The Liar paradox

The displayed sentence is not true.

Assume that the displayed sentence is true. Since the statement asserts of itself the contrary, it is to be not true: Contradiction. Assume that the displayed sentence is not true. Since this is exactly what the statement says of itself, we must conclude that the displayed sentence is to be true. Again, a contradiction. Therefore, we conclude that the statement is paradoxical.

Tarski's theory

1. The criterion of formal correctness and material adequacy for a definition of truth.
2. The definition of truth in the metalanguage for the object language.
3. The indefinability of truth theorem.

Tarski's Convention T

A materially adequate definition of truth must imply the following two conditions

Condition (α) – The truth schema:

For each sentence ϕ of the object language, the following sentence of the metalanguage:

$$\text{“}\phi\text{” is true} \;\Leftrightarrow \phi.$$

Condition (β):

$$\forall x\,(x \text{ is true} \Rightarrow x \text{ is a sentence of the object language}).$$

Tarski's solution of the Liar paradox

A paradox-free theory

The Liar sentence does not belong to the object language.

An impossibility theorem

The truth predicate for the object language is not definable in terms of the object language itself.

The object language

- First-order language of arithmetic augmented by a truth predicate.
- Truth-theoretic propositional languages.

Truth-theoretic propositional languages

Primitive symbols

- A set P of *propositional letters*: $p, q, \ldots$.
- A set N of *names*: $a, b, \ldots$.
- A unary predicate T.

Definition

An atomic sentence is either a propositional letter or a *truth-atomic sentence*, namely, a string of the form $\mathrm{T}a$, where a is a name.

Definition

A truth-theoretic propositional language is a propositional language $\mathcal{L}_{\mathrm{T}}(P, N)$ inductively built-up from the atomic sentences and the connectives $\neg$ (*negation*) and $\wedge$ (*conjunction*).

Base language and semantics

- The base language: The truth-free part $\mathcal{L}(P)$ of $\mathcal{L}_{\mathsf{T}}(P, N)$.
- The truth language: The full language $\mathcal{L}_{\mathsf{T}}(P, N)$.

We assume that the base language is given a classic semantics, namely, that a *bivalent valuation* v of all sentences of $\mathcal{L}(P)$ is admissible iff for all sentences ϕ, ψ,

- $v(\neg\phi) = \mathbf{t} \Leftrightarrow v(\phi) = \mathbf{f}$;
- $v(\phi \wedge \psi) = \mathbf{t} \Leftrightarrow v(\phi) = v(\psi) = \mathbf{t}$.

Model-theoretic semantics

A ground model for $\mathcal{L}_{\mathsf{T}}(P, N)$ is a pair $\mathcal{M} = (D, I^{-})$, where:

- D is a non-empty set, the *domain* of the model.
- For each propositional letter p, $I^{-}(p)$ is a valuation of p into $\{\mathbf{t}, \mathbf{f}\}$.
- For each name a, $I^{-}(a)$ is an element of D.

For every sentence ϕ of the base language

$$\mathrm{Val}_{\mathcal{M}}(\phi) = \mathbf{t} \Leftrightarrow \mathcal{M} \models \phi.$$

Truth-theoretic semantics

A truth-theoretic semantics is a semantics for the truth language such that:

(a) Each admissible valuation when restricted to the sentences of the base language is still admissible;
(b) The semantics makes the predicate T mean "true" at least for a fragment of the truth language.

A classical model for the truth language is denoted by $\mathcal{M}+Z$, where

- $\mathcal{M}$ is a ground model (D, I^{-}).
- $Z \subseteq D$ is the interpretation assigned to the truth predicate.

Truth-theoretic puzzles

Fix a semantics for the truth language.

Definition

A puzzle is a pair (X, π), where

- X is a set (a support) of sentences of the truth language.
- π is a function (a reference list) from the set of names occurring in X into X.

A solution of a puzzle (X, π) is an assignment of truth values to the sentences in X which is admissible according to the given semantics.

Tarskian semantics

Convention T

Fix an interpretation I^- of the names. A classic admissible valuation v of the truth language $\mathcal{L}_\mathrm{T}(P, N)$ has to satisfy, for every name $a \in N$,

(α) $v(\mathrm{T}a) = v(I^-(a))$, if $I^-(a)$ is a sentence of the base language $\mathcal{L}(P, N)$;

(β) $v(\mathrm{T}a) = \mathbf{f}$, otherwise.

Remark

For every admissible valuation v^- of the base language there exists exactly one admissible Tarskian valuation v of the truth language which extends v^-.

Puzzle 1 - The Liar puzzle

Liar sentence

a: $\neg \mathrm{T}a$.

Puzzle 1

Support $X = \{\neg \mathrm{T}a\}$.

Reference list $\pi(a) = \neg \mathrm{T}a$.

Notion of solution $h : X \to \{\mathbf{t}, \mathbf{f}\}$ such that there exists a Tarskian valuation v such that $v \restriction X = h$.

Solution

Let $h(\neg \mathrm{T}a) = \mathbf{t}$. Let v be a Tarskian valuation. Hence,

1. $v(\mathrm{T}a) = \mathbf{f}$ [Convention T (β)]
2. $v(\neg \mathrm{T}a) = \mathbf{t} = h(\neg \mathrm{T}a)$ [Classic rule for negation].

Solving a puzzle vs solving a paradox

Solving a puzzle

A mathematical task, analogous to that of solving an algebraic system of equations.

Solving a paradox

A philosophical task, analogous to that of giving a philosophical account of any other kind of problem.

A Tarskian solution of the Liar

By interpreting T as 'true sentence of the base language', Tarskian semantics is adequate and the paradox dissolves.