Semantic theories of truth: Lecture 5 Gupta's puzzle and revision-theoretic semantics

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Outline

- Restricted Convention T.
- Revision theory of truth.
- Conclusion

Restricted truth approach

Which kind of predicate is ‘true’?

‘true’ is a partial predicate which applies to a fragment of the object language, its range of significance.

Which kind of semantics for the truth language?

Since we want to use ‘true’, the truth language is to be endowed with a classical semantics

What characterises ‘true’ among partial predicates?

Since the truth schema is inconsistent with classical semantics, it is to be restricted to the range of significance of ‘true’.

Restricted Convention T

Restricted Convention T

Fix an interpretation I^- of the names. A classic admissible valuation v of the truth language $\mathcal{L}_\mathrm{T}(P, N)$ has to satisfy, for every name $a \in N$,

(α) $v(\mathrm{T}a) = v(I^-(a))$, if $I^-(a)$ belongs to the range of significance of T.

(β) $v(\mathrm{T}a) = \mathbf{f}$, otherwise.

Range of significance

- The set of sentences of the base language.
- Leitgeb's set of sentences that depend on non-semantic states of affairs.
- The "closed off" version of the Strong Kleene least fixed point.
- The "closed off" version of the Strong Kleene greatest intrinsic fixed point.
- The "closed off" version of the Weak Kleene least fixed point.
- ...

Base semantics

Base language $\mathcal{L}(P, N)$

— Set P of propositional letters : $p, q, \ldots$
— Set N of names N: $a, b, c, \ldots$

Ground model $\mathcal{M} = (D, I^{-})$

— Domain: A non-empty set D.
— Interpretation of the propositional letters: $I^{-} : P \to \{\mathbf{t}, \mathbf{f}\}$.
— Interpretation of the names: $I^{-} : N \to D$.

Base admissible valuations

- $\mathrm{Val}^{\tau}_{\mathcal{M}}(p) = I^{-}(p)$.
- $\mathrm{Val}^{\tau}_{\mathcal{M}}(\neg\phi) = -^{\tau}\mathrm{Val}^{\tau}_{\mathcal{M}}(\phi)$.
- $\mathrm{Val}^{\tau}_{\mathcal{M}}(\phi \wedge \psi) = \mathrm{Val}^{\tau}_{\mathcal{M}}(\phi) *^{\tau} \mathrm{Val}^{\tau}_{\mathcal{M}}(\psi)$.

A bi-valued valuation $v^{-} : \mathcal{L}(P) \to \{\mathbf{t}, \mathbf{f}\}$ is classically admissible iff $v^{-} = \mathrm{Val}^{\tau}_{\mathcal{M}}$.

Classical semantics

Truth language $\mathcal{L}_{\mathrm{T}}(P, N)$

— $\mathcal{L}(P, N)$ augmented by a unary predicate T.

Classical model $\mathcal{M} + Z$

— The ground model $\mathcal{M}$ expanded by an extension $I(\mathrm{T}) = Z$ interpreting T.

Classical admissible valuations

- $\mathrm{Val}^{\tau}_{\mathcal{M}+Z}(p) = I^{-}(p)$.
- $\mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\mathrm{T}a) = I(\mathrm{T})(I^{-}(a))$.
- $\mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\neg\phi) = -^{\tau}\mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\phi)$.
- $\mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\phi \wedge \psi) = \mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\phi) *^{\tau} \mathrm{Val}^{\tau}_{\mathcal{M}+Z}(\psi)$.

A bi-valued valuation $v : \mathcal{L}(P) \to \{\mathbf{t}, \mathbf{f}\}$ is classically admissible iff $v = \mathrm{Val}^{\tau}_{\mathcal{M}+Z}$.

The Tarskian operator

Fix a ground model $\mathcal{M} = (D, I^-)$ such that:

- $\mathcal{L}_\mathsf{T}(N, P) \subseteq D$.
- $\forall \phi \in \mathcal{L}_\mathsf{T}(N, P)\, \exists a \in N\, (I^-(a) = \phi)$.

The Tarskian operator is the function τ on subsets of D defined by

$$\tau(Z) = \{\phi \in \mathcal{L}_\mathsf{T}(N, P) \mid \mathrm{Val}^\tau_{\mathcal{M}+Z}(\phi) = \mathbf{t}\}.$$

ω-revision

ω-iteration

An ω-iteration of τ is a sequence $s = \langle Z_i \mid i \in \omega \rangle$ such that, for every $i \in \omega$,

$$Z_{i+1} = \tau(Z_i).$$

ω-stability

A sentence ϕ of the truth language is

- *s*-stably true iff $\exists i \in \omega \, \forall j > i \, (\phi \in Z_j)$.
- *s*-stably false iff $\exists i \in \omega \, \forall j > i \, (\phi \notin Z_j)$.
- *s*-stable iff it is either *s*-stably true or *s*-stably false.

$Z^s_\omega = \{\phi \in \mathcal{L}_\mathrm{T}(N, P) \mid \phi \text{ is } s\text{-stably true}\}$.
$W^s_\omega = \{\phi \in \mathcal{L}_\mathrm{T}(N, P) \mid \phi \text{ is } s\text{-stably false}\}$.
$X^s_\omega = \{\phi \in \mathcal{L}_\mathrm{T}(N, P) \mid \phi \text{ is } s\text{-stable}\}$.

ω-revision-theoretic semantics

Remark

If $I^{-}(a) = \phi$ belongs to X^{s}_{ω}, then

$$\mathcal{M} + Z^{s}_{\omega} \models \mathrm{T}a \leftrightarrow \phi.$$

ω-revision-theoretic semantics

Let $\mathcal{C}$ be a set of ω-iterations.

- $W_{\omega} = \bigcap\{W^{s}_{\omega} \mid s \in \mathcal{C}\}$ is the anti-extension of T in $\mathcal{M}$.
- $Z_{\omega} = \bigcap\{Z^{s}_{\omega} \mid s \in \mathcal{C}\}$ is the extension of T in $\mathcal{M}$.
- $X_{\omega} = Z_{\omega} \cup W_{\omega}$ is the range of significance of T in $\mathcal{M}$.

Puzzle 5 - The revision-theoretic Liar

Liar sentence

$$a: \neg Ta.$$

Revision-theoretic solution

	Z	$\tau(Z)$
	$\neg Ta$	$\neg Ta$
Z_1	**t**	**f**
Z_2	**f**	**t**
...	...	...
Z_i	**f**	**t**
Z_{i+1}	**t**	**f**
...	...	...
X^s_ω	**n**	

Example 5 - Visser-Chapuis Gupta puzzle

Suppose that speakers A and B make the assertions displayed below.

A says:

(a1) Statement (b) is true.

(a2) Statement (b) is not true.

B says:

(b) Statements (a1) and (a2) cannot both be true.

There is no trouble in evaluating these sentences. (a1) and (a2) contradict each other, hence (b) is to be true. Therefore (a1) is true and (a2) is false.

Puzzle 6 - Revision-theoretic Gupta's puzzle

a: Tc.

b: $\neg Tc$.

c: $\neg(Ta \wedge Tb)$.

Revision-theoretic solution

	Extension Z			Tarskian valuation $\tau(Z)$		
	Tc	$\neg Tc$	$\neg(Ta \wedge Tb)$	Tc	$\neg Tc$	$\neg(Ta \wedge Tb)$
h_1	**f**	**f**	**f**	**f**	**t**	**t**
h_2	**f**	**f**	**t**	**t**	**f**	**t**
h_3	**f**	**t**	**f**	**f**	**t**	**t**
h_4	**f**	**t**	**t**	**t**	**f**	**t**
h_5	**t**	**f**	**f**	**f**	**t**	**t**
h_6	**t**	**f**	**t**	**t**	**f**	**t**
h_7	**t**	**t**	**f**	**f**	**t**	**f**
h_8	**t**	**t**	**t**	**t**	**f**	**f**
Z^s_ω	**t**	**f**	**t**			

Summary of the course

- Lecture 1: The Liar and Tarskian semantics.
- Lecture 2: Nixon-Dean and fixed-point semantics.
- Lecture 3: Fixed-point semantics - Part II.
- Lecture 4: Semantic dependence and Leitgeb's theory of truth.
- Lecture 5: Gupta's puzzle and revision-theoretic semantics.

Lecturer's website

https://unito.academia.edu/EdoardoRivello

References I

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Lecture 4

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