Recent work on paraconsistent logic (3)

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The menu for Day 3

1. Recap (short!)
2. Degrees of inconsistent involvement
3. Some contradictory logics
    - A list by Heinrich Wansing
    - Some logics from that list (**CP**, $\mathbf{BL}_{\supset}$, **MC**)
    - A logic not in the list, explicitly designed to be contradictory: contradictory syllogistic

Recap

- Béziau on genuine paraconsistency: inspired by da Costa, requiring some heavy stuff from the logic (Detachment, IpE, Deduction Property, etc.); dislikes **LP**; likes (bits of) Boolean negation. (But seemingly Boolean negation does not make sense.)

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- The recapture strategies; quasi-validity: you have all the classically valid arguments in **LP** in arrow form.

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- The recapture strategies; quasi-validity: you have all the classically valid arguments in **LP** in arrow form.
- Default validity: either the classical arguments hold or one of the premises is contradictory; be classical if you have reasons to reject the contradictory premises.

Recap (ctd)

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- Express non-inconsistency in **LP** expanding it with **f**, $>_d$ and a classicality postulate.
- Non-inconsistency connectives: expand your paraconsistent logic with unary connectives expressing non-inconsistency (consistency, classicality).

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(At least) Four grades of paraconsistent involvement (Beall and Restall 2006):

- Gentle-strength paraconsistency: Explosion is invalid, and inconsistent but non-trivial theories are not interesting or important, they should be replaced by a consistent theory asap.

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- Dialetheic paraconsistency: Explosion is invalid and some inconsistent but non-trivial theories are true.

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7. Some contradictions (but not all) are true in every world.

Dunn moving from 2 to something else

1976: Intuitive Semantics for First-Degree Entailments and 'Coupled Trees'.

> *(...) when I wrote my dissertation [in 1966] (...) in conversation then already talked of sentences being both true and false, and also neither, but I lacked the philosophical nerve to embrace this as a serious way of talking for about another year. This was because I somehow thought that it required that it should be <u>possible</u> for sentences to <u>really</u> be <u>both</u> true and false, or really be <u>neither</u>, and this seemed plain mad. I hope my presentation in Section 3 avoids that.*

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>
> *It is at least as difficult to speculate about the impossible without seeming to talk of it as possible as it is to speculate about the merely possible without seeming to talk of it as actual.*

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Contradictory logics

A logic **L** is contradictory iff, for some A, $\oslash$ and N:

- $\models_{\mathbf{L}} A$ and $\models_{\mathbf{L}} NA$, or
- $\models_{\mathbf{L}} A \oslash NA$

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Note that some authors call these N-inconsistent logics.

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None of these logics were motivated by the idea of obtaining a non-trivial contradictory logic. Contradictoriness was obtained as a side effect of something else.

A logic with a demi-negation

Expand **FDE** with the following connective:

- $1 \in \sigma({\sim_K} A)$ iff $0 \in \sigma(A)$
- $0 \in \sigma({\sim_K} A)$ iff $1 \notin \sigma(A)$

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$\{1\}$	$\{1,0\}$
$\{0\}$	$\{\,\}$
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Omori and Wansing (2018) highlighted that this logic validates both $\sim_K (A \wedge \sim_K\sim_K A)$ and $\sim_K\sim_K (A \wedge \sim_K\sim_K A)$.

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${\sim_K}$ is a demi-negation, since $\sigma({\sim_K}{\sim_K} A) = \sigma(\neg A)$, for all σ.

$\mathbf{BL}_{\supset}$

$\mathbf{BL}_{\supset}$ is **FDE** expanded with the informational connectives

$A \otimes B$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1\}$	$\{\ \}$	$\{\ \}$
$\{1,0\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{\ \}$	$\{\ \}$	$\{\ \}$	$\{\ \}$	$\{\ \}$
$\{0\}$	$\{\ \}$	$\{0\}$	$\{\ \}$	$\{0\}$

$A \oplus B$	{1}	{1,0}	{ }	{0}
{1}	{1}	{1,0}	{1}	{1,0}
{1,0}	{1,0}	{1,0}	{1,0}	{1,0}
{ }	{1}	{1,0}	{ }	{0}
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$\{1\}$	$\{1\}$	$\{1\}$	$\{\,\}$	$\{\,\}$
$\{1, 0\}$	$\{1\}$	$\{1, 0\}$	$\{\,\}$	$\{0\}$
$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$
$\{0\}$	$\{\,\}$	$\{0\}$	$\{\,\}$	$\{0\}$

$A \oplus B$	{1}	{1,0}	{ }	{0}
{1}	{1}	{1,0}	{1}	{1,0}
{1,0}	{1,0}	{1,0}	{1,0}	{1,0}
{ }	{1}	{1,0}	{ }	{0}
{0}	{1,0}	{1,0}	{0}	{0}

- $1 \in \sigma(A \otimes B)$ iff $1 \in \sigma(A)$ and $1 \in \sigma(B)$
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$\{0\}$	$\{\ \}$	$\{0\}$	$\{\ \}$	$\{0\}$

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- $1 \in \sigma(A \oplus B)$ iff $1 \in \sigma(A)$ or $1 \in \sigma(B)$
- $0 \in \sigma(A \oplus B)$ iff $0 \in \sigma(A)$ or $0 \in \sigma(B)$

$\mathbf{BL}_{\supset}$ (ctd)

…and a material conditional, $A \supset B$, evaluated as follows:

- $1 \in \sigma(A \supset B)$ iff $1 \notin \sigma(A)$ or $1 \in \sigma(B)$
- $0 \in \sigma(A \supset B)$ iff $1 \in \sigma(A)$ and $0 \in \sigma(B)$

$A \supset B$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1,0\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
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$A \supset B$	$\{1\}$	$\{1,0\}$	$\{\,\}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1,0\}$	$\{\,\}$	$\{0\}$
$\{1,0\}$	$\{1\}$	$\{1,0\}$	$\{\,\}$	$\{0\}$
$\{\,\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
$\{0\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$

$\mathbf{BL}_{\supset}$ is contradictory, since it validates both $((A \supset A) \vee_{AA} \sim(A \supset A))$ and $\sim((A \supset A) \vee_{AA} \sim(A \supset A))$.

Material connexive logic

Expand **FDE** with the following implication:

- $1 \in \sigma(A \to_W B)$ iff
  $1 \notin \sigma(A)$ or $1 \in \sigma(B)$
- $0 \in \sigma(A \to_W B)$ iff
  $1 \notin \sigma(A)$ or $0 \in \sigma(B)$

$A \to_W B$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1,0\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
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$\{1\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
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Then one obtains **MC**, 'material connexive logic', introduced by Wansing in his SEP entry.

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$A \rightarrow_W B$	$\{1\}$	$\{1,0\}$	$\{\,\}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1,0\}$	$\{\,\}$	$\{0\}$
$\{1,0\}$	$\{1\}$	$\{1,0\}$	$\{\,\}$	$\{0\}$
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$\{0\}$	$\{1,0\}$	$\{1,0\}$	$\{1,0\}$	$\{1,0\}$

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MC is contradictory; as witnesses, take $(A \wedge \sim A) \rightarrow_W A$ and $\sim ((A \wedge \sim A) \rightarrow_W A)$.

Contradictory syllogistic

Meyer and Martin (2019) wanted to provide a logic for Aristotle's syllogistic. Their logic, **S**, is axiomatized as follows:

$$\vdash_{\mathbf{S}} (C \to D) \to ((A \to C) \to (A \to D)) \tag{B}$$

$$\vdash_{\mathbf{S}} (A \to C) \to ((C \to D) \to (A \to D)) \tag{B'}$$

$$\text{If } \vdash_{\mathbf{S}} C \to D \text{ then } \vdash_{\mathbf{S}} (A \to C) \to (A \to D) \tag{(Strict) Rule B}$$

$$\text{If } \vdash_{\mathbf{S}} A \to C \text{ then } \vdash_{\mathbf{S}} (C \to D) \to (A \to D) \tag{(Strict) Rule B'}$$

$$\vdash_{\mathbf{S}} C \to D \text{ entails that if } \vdash_{\mathbf{S}} A \to C \text{ then } \vdash_{\mathbf{S}} A \to D \tag{(Strict) Rule BB}$$

Contradictory syllogistic (ctd)

Meyer and Martin extend **S** into a new logic, **SI∼I**, with the following axiom schemas:

$$\vdash A \rightarrow A \tag{I}$$

$$\vdash{\sim}(A \rightarrow A) \tag{$\sim$ I}$$

Meyer and Martin: $A \rightarrow A$ is a borderline case of implication.

It is invalid (it is the paradigmatic case of circular reasoning, etc.),

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It is invalid (it is the paradigmatic case of circular reasoning, etc.), but for that reason it is not only invalid, but unsatisfiable: no instance of it can be true. Hence, $\sim(A \to A)$ is valid.

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Meyer and Martin: $A \rightarrow A$ is a borderline case of implication.

It is invalid (it is the paradigmatic case of circular reasoning, etc.), but for that reason it is not only invalid, but unsatisfiable: no instance of it can be true. Hence, ${\sim}(A \rightarrow A)$ is valid.

Nonetheless, because of the truth-preservation account of entailment, $A \rightarrow A$ is also valid.

Contradictory syllogistic (ctd)

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Formalizing the meta-linguistic part (and forgetting about the initial turnstile), (Strict) Detachment looks as follows:

$$(A \to B) \Rightarrow (A \Rightarrow B) \qquad \text{((Strict) Detachment} \Rightarrow\text{)}$$

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and if one does not buy the language/metalanguage divide, (Strict) Detachment with $\Rightarrow$ would amount to

$$(A \to B) \to (A \to B)$$

Thanks, see you tomorrow!

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