Recent work on paraconsistent logic (2)

Luis Estrada-González
Institute for Philosophical Research, UNAM

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The menu for Day 2

1. Recap (short!)
2. Béziau on genuine paraconsistency
3. The recapture project(s)
    - Quasi-validity and Default validity
    - Shrieking (and shrugging)
    - Non-inconsistency connectives

Recap

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- Several useful notions: logical validity (truth preservation), tautology, antilogy, etc.
- **FDE**: evaluation conditions homophonical to the classical ones; some validities (lattice principles, etc.) and some invalidities (Detachment, DS, Explosion, Implosion, etc.). Definable arrow.

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- First working definition of ‘paraconsistent logic’: failure of Explosion (with respect to some *N* and some premise-binder).
- **LP**: obtained from **FDE** by dropping $\{\ \}$; some validities (all classical tautologies, etc.) and some invalidities (Detachment, DS, Explosion, Transitivity of the arrow, etc.)
- Genuine paraconsistency: demanding the failure of both Explosion and LNC. Examples. da Costa style: the classical desideratum. Less classical: **LP** evaluations with a tweaked notion of logical validity.

Béziau on the invalidity of $N(A \oslash NA)$

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- A special case:
  If $A \models_{\mathbf{L}} B$ then $NB \models_{\mathbf{L}} NA$ (Contraposition)
- $A, A > B \models_{\mathbf{L}} B$ (Detachment)
- If $\Gamma, A \models_{\mathbf{L}} B$ then $\Gamma \models_{\mathbf{L}} A > B$ (1/2 Deduction Property)

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Do we want the negation of a paraconsistent logic to be Boolean-ish?

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It cannot be expressed with the three interpretations of **LP**!
(And similarly for **K3**.)

Very much like `tonk` and classical logic.

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- Example: Peano Arithmetic (PA). Close its axioms under a logic **L**, say, a paraconsistent logic. Classical PA seemingly gets things right when it comes to natural numbers, but ff **L** lacks disjunctive syllogism or certain forms of contraposition or reductio ad absurdum, the resulting theory is likely not strong enough.

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How?

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Nonetheless, anyone inclined enough to draw a sharp distinction between logical consequence and implication would feel uneasy about this maneuver.

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- If we can discard (reject, show as untrue, etc.) the contradiction, we can apply Detachment.

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- Then $A_1, \ldots, A_n \models_{\mathbf{LP}} B \vee (A_i \wedge \sim A_i)$ is valid, not merely quasi-valid.
- If we can discard all the As for which that happens, then we can apply the argument without caring about them.

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If **FDE** or **LP** is our background logic, logic alone does not exclude the possibility of gluts for P.

However, one can force it to be non-glutty by shrieking it, in the terminology of Beall (2013, etc.).

Shrieking

To shriek an n-ary predicate P in the language of a theory $\mathcal{T}$ is to impose the following constraint on $\mathcal{T}$'s closure relation:

$$P(x_1, \ldots, x_n), \sim P(x_1, \ldots, x_n) \vdash_{\mathcal{T}} \bot$$

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Briefly, to shriek P is to exclude the possibility that something in the domain satisfies both P and its negation.

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Similarly, if there are (non-logical) theoretical reasons to exclude the possibility of gaps for a certain non-logical n-ary predicate P in a theory $\mathcal{T}$, then one can shrug it by imposing the following constraint on $\mathcal{T}$'s closure relation:

$$\top \vdash_{\mathcal{T}} P(x_1, \ldots, x_n) \vee \sim P(x_1, \ldots, x_n)$$

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Thus, to shrug P is to exclude the possibility that something neither satisfies P nor its negation.

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If one thinks, like Beall, that arithmetic is entirely classical, then we can shriek and shrug it, even if the background logic is **FDE**.

Expressing consistency

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Expressing the consistency of A by $\sim(A \wedge \sim A)$, will not do, for $\sim(A \wedge \sim A)$ is logically valid in **LP**. Thus, any countermodel to DS will also be a countermodel to

$$\sim(A \wedge \sim A), \sim A, A \vee B \models_{\mathbf{LP}} B$$

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This way of expressing consistency is hopeless. There is no formula $F(p)$ in the language of **LP**, whose only propositional parameter is p and such that

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Actually, not even expanding **LP** with a detachable implication is enough. Let me call $\mathbf{LP}_{>_d}$ an expansion of **LP** with an implication such that

- If $1 \in \sigma(A >_d B)$ and $1 \in \sigma(A)$ then $1 \in \sigma(B)$

or, equivalently (since the semantics is not inconsistent),

- If $1 \notin \sigma(B)$ then $1 \notin \sigma(A >_d B)$ or $1 \notin \sigma(A)$

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The story is different if we expand $\mathbf{LP}_{>_d}$ with propositional constants, and in particular with the propositional constant $\mathbf{f}$, characterized by the schema $\mathbf{f} >_d A$. (In terms of interpretations, $\mathbf{f}$ is false in all of them, and true in none.)

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Let $\mathcal{T}$ be a theory built on top of **CL** and $\mathcal{T}^+$ be $\mathcal{T}$ together with the axiom schema $(A \wedge \sim A) >_d \mathbf{f}$. (Priest calls this “the classical postulate” —about contradictions, presumably.)

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Suppose that $\mathcal{T} \models_{\mathbf{CL}} A$. Then, $\mathcal{T} \models_{\mathbf{LP}_{>_d,\mathbf{f}}} A \vee (B \wedge {\sim}B)$, for some B.

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Expressing consistency (ctd)

The story is different if we expand $\mathbf{LP}_{>_d}$ with propositional constants, and in particular with the propositional constant $\mathbf{f}$, characterized by the schema $\mathbf{f} >_d A$. (In terms of interpretations, $\mathbf{f}$ is false in all of them, and true in none.)

Let $\mathcal{T}$ be a theory built on top of **CL** and $\mathcal{T}^+$ be $\mathcal{T}$ together with the axiom schema $(A \wedge \sim A) >_d \mathbf{f}$. (Priest calls this "the classical postulate" —about contradictions, presumably.)

Then, any classical consequence of $\mathcal{T}$ is a consequence of $\mathcal{T}^+$ with $\mathbf{LP}_{>_d,\mathbf{f}}$.

Suppose that $\mathcal{T} \models_{\mathbf{CL}} A$. Then, $\mathcal{T} \models_{\mathbf{LP}_{>_d,\mathbf{f}}} A \vee (B \wedge \sim B)$, for some B. Hence, $\mathcal{T}^+ \models_{\mathbf{LP}_{>_d,\mathbf{f}}} A \vee \mathbf{f}$, by the classical postulate, and therefore $\mathcal{T}^+ \models_{\mathbf{LP}_{>_d,\mathbf{f}}} A \vee A$, by the properties of $\mathbf{f}$, and hence $\mathcal{T}^+ \models_{\mathbf{LP}_{>_d,\mathbf{f}}} A$, by the idempotence of disjunction.

LFIs and the expression of non-inconsistency

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How come?

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How come? Exactly like that: A (unary) connective $\circledast$ is a non-inconsistency connective iff

$$\text{If } \sigma(A) = \{1, 0\} \text{ then } 1 \notin \sigma(\circledast A)$$

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A	$\circ A$	$\odot A$	$\circledcirc A$
$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
$\{1, 0\}$	$\{0\}$	$\{0\}$	$\{0\}$
$\{\ \}$	$\{1\}$	$\{\ \}$	$\{0\}$
$\{0\}$	$\{1\}$	$\{1\}$	$\{1\}$

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Read them, respectively

A is not inconsistent

A is consistent (if it has a value at all)

A is classical.

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One cannot distinguish them in, say, **LP**, yet, with any of them in the language, $A, \sim A, \circledast A \models_{\mathbf{LP}_{\circledast}} B$.

Thanks, see you tomorrow!

loisayaxsegrob@comunidad.unam.mx