Simple Analytical Equations for Estimating Ground Water Inflow to a Mine Pit

Article *in* Ground Water · March 2000

DOI: 10.1111/j.1745-6584.2000.tb00342.x

Walter Niccoli
Telesto Solutions Inc.

Simple Analytical Equations for Estimating Ground Water Inflow to a Mine Pit

by Fred Marinelli[a,b] and Walter L. Niccoli[a]

Abstract

Steady-state analytical solutions are presented for estimating the ground water inflow rate to a mine pit that may contain a pit lake of finite depth. The solutions consider (1) the effect of decreased saturated thickness near the pit walls; (2) distributed recharge to the water table; and (3) upward flow through the pit bottom. While the solutions are not appropriate for all hydrogeologic situations, they are relevant to conditions encountered at many mine sites. An example calculation is presented for an actual mine pit containing a pit lake. The analytical solutions provide an estimated ground water inflow rate that is similar to the rate determined independently from a detailed water balance study.

Introduction

Mine feasibility and environmental evaluations can benefit from the use of simple analytical tools for predicting ground water inflow to a mine pit. While numerical modeling may be required at advanced stages of mine planning, simple analytical equations for estimating pit inflow rates can be informative during the initial stages of mine development. Equations for predicting ground water inflows to open pits and underground excavations are presented in Goodman et al. (1965), Verma and Brutsaert (1970, 1971), Sing and Atkins (1984), Atkinson et al. (1989), Naugle and Atkinson (1993), and Hanna et al. (1994). The solutions presented in these technical papers apply to ground water inflow problems with particular sets of boundary conditions and simplifying assumptions. The applicability of a solution depends on the extent to which the real problem under consideration is consistent with assumptions used to derive the mathematical equations. If a solution is judged to be applicable, its accuracy is generally dictated by the appropriateness of the bulk hydraulic conductivity value used to perform the calculations.

This paper presents two analytical solutions useful for predicting the ground water inflow to a mine pit excavated below the water table. A pit lake of finite depth may or may not exist within the pit. While the equations apply to a fairly specific physical problem, they are relevant to hydrogeologic conditions encountered at many mine sites. The solutions, however, are not appropriate for all mining situations. They should therefore be used only after carefully comparing the associated mathematical assumptions with the known or inferred site conditions.

[a]Shepherd Miller Inc., 3801 Automation Way, Ste. 100, Fort Collins, CO 80525; fmarin@lamar.colostate.edu/ wniccoli@shepmill.com

[b]Affiliate Faculty, Department of Civil Engineering, Colorado State University, Fort Collins, CO 80523

Received October 1998, accepted October 1999.

Theoretical Development

The solutions in this paper address the pit inflow conceptual model shown in Figure 1. Important assumptions of the conceptual model are:

- Lowering the water table decreases the saturated thickness of rock materials providing pit inflow.
- Relative to seepage from the pit walls, significant inflow occurs through the pit bottom. Most previously published analytical solutions have not considered this component of the pit inflow.
- The rock formation is semi-infinite below the pit and there exists no impermeable boundary at depth.
- Steady-state flow conditions exist near the mine pit. This assumption is reasonable for moderate to high permeability materials and mine pits that are excavated over a period of years.

For the purpose of calculations, the conceptual flow model in Figure 1 is approximated by the analytical models shown graphically in Figure 2. The flow region is divided into two zones. Zone 1 exists above the base of the pit and represents flow to the pit walls. Zone 2 extends from the bottom of the pit downward and considers flow to the pit bottom. Both analytical models assume that there is no ground water flow between Zones 1 and 2.

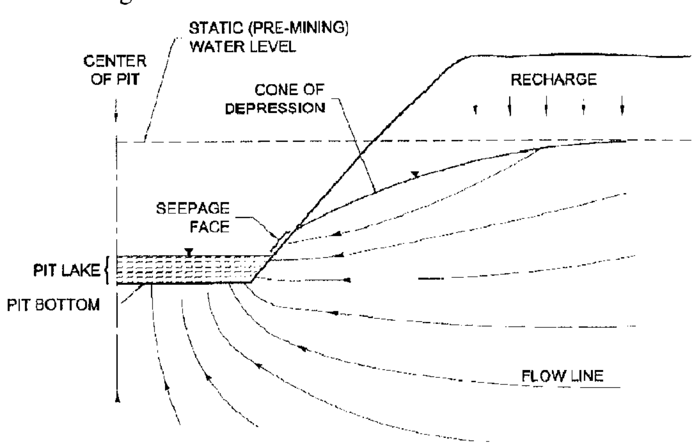

Figure 1. Pit inflow conceptual model.

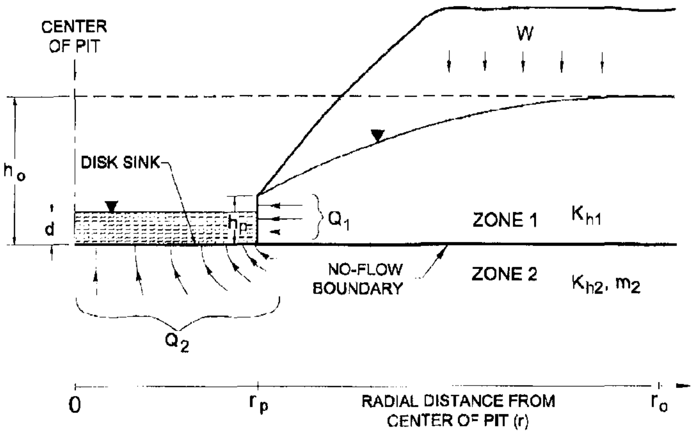

Figure 2. Pit inflow analytical model.

Zone 1 Analytical Solution

The analytical solution for Zone 1 considers steady-state, unconfined, horizontal radial flow, with uniformly distributed recharge at the water table. Additional assumptions of this solution are as follows:

- The pit walls are approximated as a right circular cylinder.
- Ground water flow is horizontal. The Dupuit-Forchheimer approximation (McWhorter and Sunada 1977) is used to account for changes in saturated thickness due to depression of the water table.
- The static (premining) water table is approximately horizontal.
- Uniform distributed recharge occurs across the site as a result of surface infiltration. All recharge within the radius of influence (cone of depression) of the pit is assumed to be captured by the pit.
- Ground water flow toward the pit is axially symmetric.

As derived in the Appendix, the following equation applies for these conditions:

$$(1)\; h_o = \sqrt{h_p^2 + \frac{W}{K_{h1}}\left[r_o^2 \ln\left(\frac{r_o}{r_p}\right) - \frac{(r_o^2 - r_p^2)}{2}\right]}$$

where W is the distributed recharge flux, K_{h1} is the horizontal hydraulic conductivity of materials within Zone 1, r_p is the effective pit radius, h_p is the saturated thickness above the base of Zone 1 at r_p (i.e., saturated thickness at the pit wall), r_o is the radius of influence (maximum extent of the cone of depression), and h_o is the initial (premining) saturated thickness above the base of Zone 1. An equivalent equation that applies to the same physical model is presented in Bear (1979).

Given input values of W, K_{h1}, r_p, h_p, and h_o, the radius of influence (r_o) is determined from Equation 1 by iteration. Once r_o is determined, the pit inflow rate from Zone 1 is computed by

$$(2)\; Q_1 = W\pi(r_o^2 - r_p^2)$$

where Q_1 is the inflow from the pit walls. Note that the pit inflow rate is maximized when h_p is set to zero.

In cross section, the hydraulic head contours for horizontal flow are vertical lines described by the following equation:

$$H_1(r) = H_o - h_o + \sqrt{h_p^2 + \frac{W}{K_{h1}}\left[r_o^2 \ln\left(\frac{r}{r_p}\right) - \frac{(r^2 - r_p^2)}{2}\right]} \quad (3)$$

where H_o is the initial (premining) water table elevation, H_1 is the steady-state hydraulic head elevation, and r is radial distance from the pit center.

The Dupuit-Forchheimer approximation assumes horizontal flow within Zone 1. The success of this approximation in describing ground water flow to wells in water table aquifers (Bear 1979) suggests that it can be appropriate for modeling inflows to a mine pit. However, errors may occur if significant vertical flow components exist in the Zone 1 region, particularly near the pit walls.

An initially horizontal (premining) water table is not likely to exist in a real aquifer system. In fact, for steady-state flow with distributed recharge, there must be a hydraulic gradient (i.e., sloped water table) to allow for lateral flow of accumulated ground water. As with similar analytical solutions in well hydraulics, the computed inflow rate to the mine pit (Equation 2) is not likely to be sensitive to the initial hydraulic gradient provided that the variation in initial head on opposite sides of the pit is small compared to the pit geometry. However, if the initial water table slope is relatively steep, the computed inflow rate may not be accurate. In addition, the predicted hydraulic head distribution about the pit (Equation 3) may not be reliable if the initial water table has a significant slope. Although not strictly valid, due to the nonlinear nature of the preceding equations, a first-order approximation of the ultimate hydraulic head distribution can be determined by superposition of the preceding solution with the initial head distribution:

$$H_1(x,y,r) = H_o(x,y) - h_o + \sqrt{h_p^2 + \frac{W}{K_{h1}}\left[r_o^2 \ln\left(\frac{r}{r_p}\right) - \frac{(r^2 - r_p^2)}{2}\right]} \quad (4)$$

where x and y are horizontal map coordinates. Equation 4 becomes less reliable as the slope of the initial water table increases.

Zone 2 Analytical Solution

The analytical solution for Zone 2 is based on steady-state flow to one side of a circular disk sink of constant and uniform drawdown. The sink represents the bottom of the pit. This solution is based on the following assumptions:

- Hydraulic head is initially uniform (hydrostatic) throughout Zone 2. Initial head is equal to the elevation of the initial water table in Zone 1.
- The disk sink has a constant hydraulic head equal to the elevation of the pit lake water surface. If the pit is completely dewatered, the disk sink head is equal to elevation of the pit bottom.
- Flow to the disk sink is three-dimensional and axially symmetric.
- Materials within Zone 2 are anisotropic, and the principal coordinate directions for hydraulic conductivity are horizontal and vertical.

The steady-state inflow rate to one side of the disk sink is given by the following equations:

$$Q_2 = 4\, r_p \left(\frac{K_{h2}}{m_2}\right)(h_o - d) \quad (5)$$

$$m_2 = \sqrt{\frac{K_{h2}}{K_{v2}}} \quad (6)$$

where Q_2 is the pit inflow rate from Zone 2 (through the pit bottom),

Table 1
Hydraulic Conductivities Measured at the Mine Site

Test Well	Measured Hydraulic Conductivity	
1	1.5×10^{-7} m/s	(0.043 ft/d)
2	9.9×10^{-8} m/s	(0.028 ft/d)
3	1.5×10^{-6} m/s	(0.43 ft/d)
4	4.2×10^{-6} m/s	(1.2 ft/d)
5	3.9×10^{-7} m/s	(0.11 ft/d)
Geometric Mean	5.2×10^{-7} m/s	(0.15 ft/d)

K_{h2} and K_{v2} are the horizontal and vertical hydraulic conductivity of materials within Zone 2, m_2 is an anisotropy parameter, and d is the depth of the pit lake. The preceding equations are based on Carslaw and Jaeger (1959), with coordinate transformations described in Bear (1979) to account for anisotropy in the vertical plane. Note that in Equation 5, $h_o - d$ is the hydraulic drawdown along the pit bottom.

Hydraulic head contour lines within the Zone 2 region are computed from the following equation, which is based on Carslaw and Jaeger (1959) and appropriate coordinate transformations for anisotropy:

$$H_2(r,z) = H_o - \frac{2\,(h_o - d)}{\pi}\ \sin^{-1}\left\{\frac{2\,r_p}{\sqrt{(r-r_p)^2 + (m_2 z)^2} + \sqrt{(r+r_p)^2 + (m_2 z)^2}}\right\} \qquad (7)$$

where H_2 is the hydraulic head elevation within Zone 2, r is radial distance from the center of the pit, and z is the vertical depth below the pit bottom (positive downward).

The preceding solution assumes that Zone 2 is seemingly infinite in lateral and vertical extent, and receives recharge at an infinite distance from the pit. Obviously, this assumption is not strictly met in real aquifer systems. However, because most of the head loss occurs close to the pit, the solution is not particularly sensitive to aquifer conditions at large radial and vertical distances. Thus, violation of the preceding assumption is not likely to affect the accuracy or applicability of the solution for most hydrologic situations. However, accuracy could be affected if the real system contains a large recharge feature close to the pit (e.g., a large surface water body) or if the aquifer base is not far below the pit bottom. If either of these conditions exist, the applicability of the solution should be questioned.

Because Zone 2 is modeled independently from Zone 1, hydraulic head contours may not match along the interface between the two regions. To evaluate the accuracy of calculations, the head distribution given by Equation 3 for $H_1(r)$ can be compared with that of Equation 7 evaluated at z = 0. If the head distributions match reasonably well along the interface, the accuracy of the inflow calculations is supported. If the head distribution curves are significantly different, the analysis should be questioned.

Example Calculation

As a case history, we consider an actual pit lake that exists at a nonoperating gold mine in northern Nevada. The water level in this small pit lake fluctuates seasonally due to the offsetting effects of water inflows and outflows. For the period of March to April 1996, the mine owner performed a detailed pit lake water balance that considered ground water inflows, piped inflows from other areas of the mine site, direct precipitation onto the pit lake, surface water inflows, evaporation, and changes in the pit lake storage volume. Based on the water balance, it was concluded that during the period of evaluation, the average ground water flow rate into the pit lake was 2.4×10^{-4} m³/s (4.3 gpm).

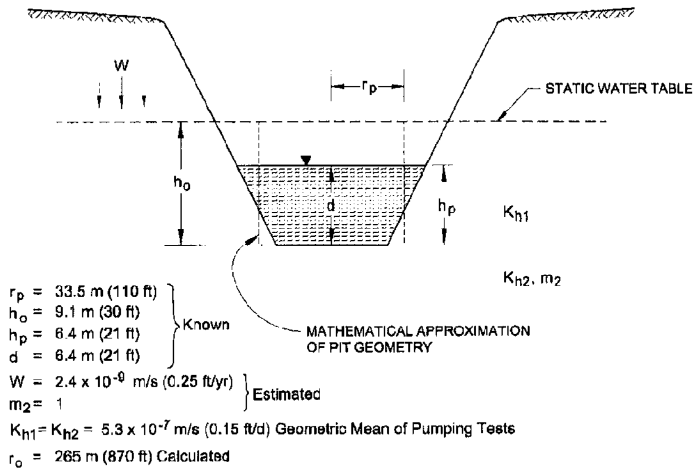

Figure 3. Example calculation.

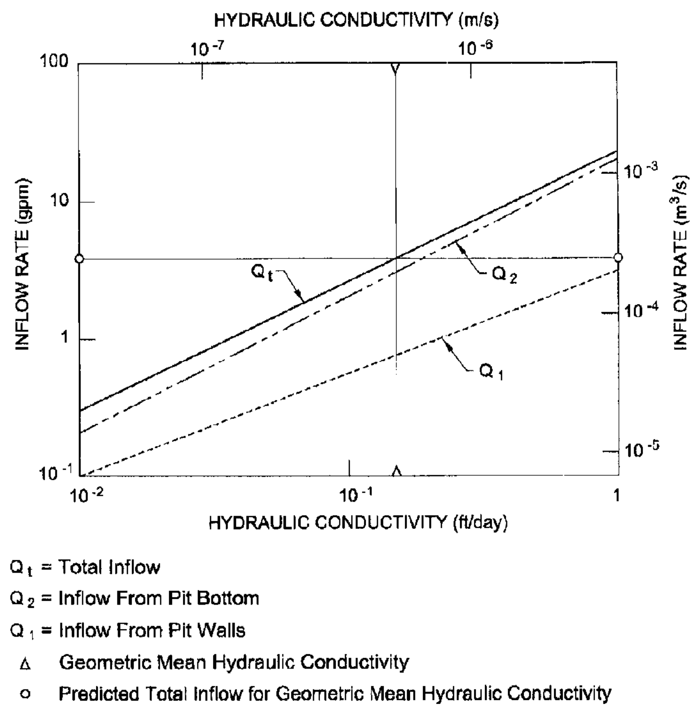

Figure 4. Predicted pit inflow rate versus hydraulic conductivity.

In the general vicinity of the pit, the mine owner conducted single borehole pumping tests in five monitoring wells. As shown in Table 1, the measured hydraulic conductivity values range from 9.9×10^{-8} to 4.2×10^{-6} m/s (0.028 to 1.2 ft/d) and have a geometric mean of 5.3×10^{-7} m/s (0.15 ft/d). The geometric mean was taken as the best-estimate bulk hydraulic conductivity of geologic materials supplying ground water inflow to the pit lake. Because the wells were completed above and below the pit bottom, the geometric mean was applied to both the Zone 1 and Zone 2 analytical solutions. In addition, Zone 2 was assumed to be isotropic with regard to hydraulic conductivity.

A diagrammatic cross section of the pit lake is shown in Figure 3. The position of the static water table was estimated based on interpolation of water levels measured in wells assumed to be located outside the pit lake drawdown cone. In addition, the assumed static water level is similar to the pit lake level measured during June 1996, a period for which a water balance indicated there was little exchange of water between the pit lake and the ground water system. The resulting geometric and hydrologic input parameters are summarized in Figure 3. Also shown is the cylindrical pit geometry assumed by the analytical solutions. Note that the recharge rate (W) is estimated from regional hydrologic information.

Based on the Zone 1 and Zone 2 analytical solutions, the predicted relationship between ground water inflow rates and the bulk hydraulic conductivity is shown in Figure 4. It can be seen that the pit wall and pit bottom inflow rates (Q_1 and Q_2, respectively) are sensitive to the assumed hydraulic conductivity value, and most of the inflow occurs through the pit bottom. For the best-estimate (geometric mean) hydraulic conductivity of 5.3×10^{-7} m/s (0.15 ft/d), the total inflow rate ($Q_t = Q_1 + Q_2$) is computed to be 2.4×10^{-4} m³/s (3.8 gpm), which is similar to the independently estimated water balance value of 2.7×10^{-4} m³/s (4.3 gpm). The relative agreement between these values suggests that the analytical solutions can provide reliable estimates of inflow rates to a mine pit, provided that (1) the assumptions of the solutions conform adequately to the physical setting of the site, and (2) an appropriate bulk hydraulic conductivity value is used in the calculations.

Conclusions

The analytical solutions presented in this paper provide a convenient means for estimating ground water inflows to mine pits, with or without pit lakes. The solutions assume an equivalent porous medium and are based on many simplifying assumptions. The applicability of the solutions to a real mine site is directly related to the consistency of these assumptions with the actual site conditions. We caution that the blind use of these equations may lead to inaccurate and potentially misleading results. Where applicable, however, we have found these analytical equations to be quite robust and provide results that are consistent with water balance and numerical modeling studies. Their primary value is in providing preliminary estimates of pit inflow rates to be used in the initial phases of mine planning.

References

Atkinson, L.C., T.M. Hanna, and J.W. Anthony. 1989. Investigation and implementation of mine dewatering systems. World Gold '89 Conference, Singapore.

Bear, J. 1979. *Hydraulics of Groundwater.* New York: McGraw-Hill Inc.

Carslaw, H.S., and J.C. Jaeger. 1959. *Conduction of Heat in Solids.* Oxford, England: Oxford University Press.

Goodman, R.E., D.G. Moye, A. Van Schalkwyk, and I. Janvandel. 1965. Ground water inflows during tunnel driving. *Engineering Geology* 2, 39-56.

Hanna, T.M., A.A. Elfadil, and L.C. Atkinson. 1994. Use of an analytical solution for preliminary estimates of ground water inflow to a pit. *Mining Engineering* 46, no. 2: 149-152.

McWhorter, D.B., and D.K. Sunada. 1977. *Ground Water Hydrology and Hydraulics.* Highlands Ranch, Colorado: Water Resources Publications.

Naugle, G.D., and L.C. Atkinson. 1993. Estimating the rate of post-mining filling of pit lakes. *Mining Engineering* 45, no. 4: 402-404.

Singh, R.N., and A.S. Atkins. 1984. Application of analytical solutions to simulate some mine inflow problems in underground coal mines. *International Journal of Mine Water* 3, no. 4: 1-27.

Verma, R.D., and W. Brutsaert. 1970. Unconfined aquifer seepage by capillary flow theory. *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers* 96 (HY6), 1331-1344.

Verma, R.D., and W. Brutsaert. 1971. Unsteady free-surface groundwater seepage. *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers* 97 (HY8), 1213-1229.

Appendix: Derivation of the Zone 1 Analytical Solution

The differential equation for steady-state, axially symmetric, horizontal, unconfined flow is given by McWhorter and Sunada (1977):

$$Q_1(r) = 2\pi K_{h1} r h \frac{dh}{dr} \tag{A1}$$

where r is horizontal radial distance, Q_1 is the flow rate across the cylindrical surface of radius r, h is the saturated thickness above the base of the aquifer, and K_{h1} is the horizontal hydraulic conductivity of the aquifer materials. If it is assumed that all flow is derived from uniform recharge within the radius of influence (r_o), then the flow rate at radius r is also given by

$$Q_1(r) = W\pi (r_o^2 - r^2) \tag{A2}$$

where W is the distributed recharge flux. Substituting Equation A2 into Equation A1 and integrating leads to

$$\frac{W}{2K_{h1}} \int_{r_p}^{r} \left(\frac{r_o^2}{r} - r\right) dr = \int_{h_p}^{h} h\, dh \tag{A3}$$

where r_p is the radial distance from the center of the pit to the pit wall and h_p is the saturated thickness at the pit wall. Carrying out the integration gives

$$h(r) = \sqrt{h_p^2 + \frac{W}{K_{h1}}\left[r_o^2 \ln\left(\frac{r}{r_p}\right) - \frac{(r^2 - r_p^2)}{2}\right]} \tag{A4}$$

Hydraulic head is equal to

$$H_1(r) = H_o - h_o + h(r) \tag{A5}$$

where H_1 is the elevation (hydraulic head) of the current water table and H_o is the elevation of the initial static water table. Substitution of Equation A4 into Equation A5 leads to

$$H_1(r) = H_o - h_o + \sqrt{h_p^2 + \frac{W}{K_{h1}}\left[r_o^2 \ln\left(\frac{r}{r_p}\right) - \frac{(r^2 - r_p^2)}{2}\right]} \tag{A6}$$

which is Equation 3 of the main text. Evaluating Equation A4 at $r = r_o$ and requiring that the initial saturated thickness (h_o) occurs at the radius of influence (r_o) gives

$$h_o = \sqrt{h_p^2 + \frac{W}{K_{h1}}\left[r_o^2 \ln\left(\frac{r_o}{r_p}\right) - \frac{(r_o^2 - r_p^2)}{2}\right]} \tag{A7}$$

which is Equation 1 of the main text. Evaluating Equation A2 at $r = r_p$ gives the flow rate through the pit walls:

$$Q_1 = W\pi (r_o^2 - r_p^2) \tag{A8}$$

which is Equation 2 of the main text.