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The Profitability of Pairs Trading Strategies: An Empirical Comparison between Distance and Copula Approach

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1 Introduction

If stock markets are weakly efficient as described by Fama (1970), then no statistically and economically significant excess returns should be obtainable by the information contained in past stock prices. However, it has been well documented that disarmingly simple trading strategies, so-called pairs trading strategies, can achieve statistically and economically significant excess returns using past stock prices (Do & Faff, 2010, 2012; Gatev et al., 2006; Rad et al., 2016). The question is whether these simple pairs trading strategies are still profitable and how more sophisticated strategies perform compared to them.

The idea of pairs trading is in theory simple: find a pair of stocks that share some pattern in prices, e.g., that did move closely together. If they diverge from this pattern, buy long the stock that is relatively undervalued and sell short the stock that is relatively overvalued. Once the stocks converge back to their historical pattern, i.e., equilibrium pricing, close the positions to realize a return. While in theory simple, the techniques to identify pairs in a first step, and generate trading signals in a second step can become quite sophisticated.

The most simple pairs trading strategy, the distance method, derives trading signals from the spread between normalised prices. If the spread is too large in absolute value, we simultaneously enter a long and short position and hope that the spread returns to zero to make a profit. A more sophisticated strategy is the copula method. Copulas enable us to model marginal and joint dependence structures independently of each other. This is useful, as equities are shown to exhibit asymmetry in their joint dependence (Patton, 2004) and thus require a more flexible framework to model the dependence structure between two stocks (Rad et al., 2016).

In an extensive study, Rad et al. (2016) analyse the profitability of pairs trading strategies on the entire US equity market from 1962 to 2014. In particular, they examine the distance, cointegration and copula method and find that all three strategies remain profitable even after transaction costs. However, they also find that trading opportunities and profitability of the distance and cointegration method are declining, whereas the copula method is relatively stable in both dimensions.

The findings of Rad et al. (2016) and the theoretical foundation of copulas make it worthwhile to further assess the suitability of copulas for pairs trading. In this study, we examine the profitability of the copula method on stocks that are part of the S&P 500, a stock index which contains the largest US stocks in terms of market capitalisation. As a benchmark, we study the well-documented distance method. We find that, similar to Rad et al. (2016) the distance and copula method generate statistically significant monthly excess returns of 0.57% and 0.68% before transaction costs for the top 20 pairs respectively.

The remainder of this is study is structured as follows. Section 2 provides an overview of recent pairs trading literature, with respect to the distance and copula method. Section 3 provides a formal introduction into the most important aspects of copulas in light of their application to pairs trading. Section 4 provides a detailed description of the distance and copula methodology. Section 5 provides both the empirical findings of this study and a discussion. Section 6 concludes with potential issues that require further investigation of researchers.

2 Literature Overview

Gatev et al. (2006) assess the profitability of the simplest pairs trading strategy, namely the distance method, from 1962 until 2002 for the entire US stock market. They find that this strategy yields an average monthly excess return of 1.308% for the distance method's top 5 pairs and 1.436% for the distance method's top 20 pairs. Additionally, they find that pairs trading profitability was particularly strong in the 1970s and 1980s, however, this trend is declining since the 1990s. The authors suggest that this decline is due to an increase in arbitrage activity which aims at exploiting the profitability previously observed. Do and Faff (2010, 2012) examine the robustness of the distance method's profitability with respect to commissions, market impact and short selling fees. They confirm the finding of Gatev et al. (2006) of strong profitability during the 1970s and 1980s, but also the decline in profitability in subsequent years. Additionally, they find that pairs trading performs particularly well during the Dot-com (2000-2002) and Global Financial Crisis (2007-2009) periods. The authors conclude that on average, the baseline distance method as implemented in Gatev et al. (2006) has lost its profitability, after controlling for costs and accounting for systematic risks. Pairs trading remains only slightly profitable for a few adjusted versions of the baseline distance method.

Liew and Wu (2013) not only analyse the distance method but also the copula method on a pre-selected, highly correlated pair of stocks from the health care sector. In particular, they make use of, what Krauss and Stübinger (2017) call, the return-based copula method, where conditional probabilities of relative mispricing are calculated every day during the trading period. If the conditional probabilities of relative mispricing are high/low enough a long short position is entered. Krauss and Stübinger (2017) point out that this method comes with some flaws. First, pair selection is not copula based, it is based on a co-movement criterion. Second, the copula-based trading signals only depend on the most recent returns realized of both stocks in the pair. Thus, the time structure is lost. Besides these conceptional issues, there is no large-scale study of the return-based copula method which tests the robustness of this method.

Xie et al. (2016) also apply the distance and copula method. They assess 89 stocks from the US utility sector from 2003 until 2012. Contrary to Liew and Wu (2013) their copula approach is, what Krauss and Stübinger (2017) call, the level-based copula method. Instead of using the daily obtained conditional probabilities in isolation, they construct relative mispricing indices, which are essentially the cumulative deviations from equilibrium pricing. They are thus able to retain the time structure of the pairs relative (mis-)pricing. They find significant average monthly excess returns for the copula method which are higher than the distance method's returns.

Rad et al. (2016) provide a recent and extensive study of pairs trading profitability. They analyse the entire US stock market from 1962 until 2014. In particular, they test the distance, cointegration and copula method. Their findings concerning the distance method are in line with Do and Faff (2010, 2012) and Gatev et al. (2006). Similar to Xie et al. (2016) they also make use of the level-based copula method, i.e., their trading decisions follow relative mispricing indices. They find that all three strategies are significantly profitable, even after accounting for transaction costs. The distance method is the most profitable strategy with an average monthly excess return of 0.91% before transaction costs, followed by the cointegration and copula methods which have an average monthly excess return of 0.85% and 0.43% respectively.

The mentioned studies are all based on the pair selection process using the SSD between normalised prices during the formation period. Krauss and Stübinger (2017) suggest another procedure. Instead of using a co-movement measure, they extend the formation period, which is typically set to 12 months, to 60 months. During this 60 months formation period they use 12-month estimation periods, where Student-t copulas are fitted for all potential pair combinations. The estimation periods are followed by 1-month pseudo trading periods. They start a new estimation period every month, which results in 48 different portfolios during the formation period. After the 60-month formation period, they enter a 12-month trading period in which they trade the top k pairs, $k \in \{5, 10, 20\}$, which are the ones that performed best during the entire formation period. During the trading period, they distinguish between mean reversion and momentum pairs. The former refers to pairs which converge back to their equilibrium pricing level within the trading period. The latter consequently refers to pairs which do not converge back. The authors already distinguish between these types of pairs during the trading process to apply different trading rules, in particular stop-loss rules, which aim at preventing high losses due to non-convergence. They do this for stocks that are part of the S&P 100 stock index from 1990 until 2014 and find that their approach is significantly profitable for the top k mean-reversion and top k momentum pairs. For the top 20 mean-reversion and momentum pairs, they find average monthly excess returns of 0.45% and 0.55% respectively.

3 Copula Preliminaries

When encountering multivariate distributions one might be interested in two things: how do the marginal distributions of the multivariate distribution behave individually and how do they interact with each other? These questions can be examined using copulas. The idea of copulas is to link marginal distribution functions to their joint distribution function (Rad et al., 2016). In the context of pairs trading this can be useful to identify pairs, but also to generate trading signals for a given set of pairs. The focus of this study lies in the generation of trading signals using copulas and the conditional probability functions that can be derived from them.

3.1 Copula Concept

A formal introduction to copulas can be found in Nelsen (2007). However, due to the focus of this study on the copula application to pairs trading, we adopt definitions and notation from McNeil et al. (2015), who provide besides a formal, also a practical introduction to copulas in the context of risk management.

Definition 1 (Copula)**.** A d-dimensional copula is a distribution function on $[0,1]^d$ with standard uniform marginal distributions.

Denote the multivariate distribution functions that are copulas as $C(\mathbf{u}) = C(u_1, ..., u_d)$. Then C is a mapping of the form $C : [0,1]^d \to [0,1]$, hence a mapping of the unit hypercube into the unit interval (McNeil et al., 2015). To qualify as a copula, C has to fulfil three properties:

1. $C(u_1, ..., u_d) = 0$ if $u_i = 0$ for any i
2. $C(1, ..., 1, u_i, 1, ..., 1) = u_i$ for all $i \in \{1, ..., d\}$, $u_i \in [0,1]$
3. For all $(a_1, ..., a_d), (b_1, ..., b_d) \in [0,1]^d$ with $a_i \leq b_i$, we have

$$\sum_{i_1=1}^{2} \cdots \sum_{i_d=1}^{2} (-1)^{i_1+\cdots+i_d} C(u_{1i_1}, ..., u_{di_d}) \geq 0,$$

where $u_{j1} = a_j$ and $u_{j2} = b_j$ for all $j \in \{1, ..., d\}$.

Copulas establish a functional relationship between multivariate distribution functions and their corresponding marginal distributions (Krauss & Stübinger, 2017). Sklar's theorem (Sklar, 1959) states that if F is a joint distribution function with marginal distributions $F_1, ..., F_d$, then, there is a d-copula C, such that, for all $(x_1, ..., x_d) \in \mathbb{R}^d$:

$$F(x_1, ..., x_d) = C(F_1(x_1), ..., F_d(x_d)) \tag{1}$$

If F is absolutely continuous and $F_1, ..., F_d$ are strictly increasing continuous, the joint probability density function f can be written as:

$$f(x_1, ..., x_d) = \left(\prod_{k=1}^{d} f_k(x_k)\right) \times c(F_1(x_1), ..., F_d(x_d)), \tag{2}$$

where the copula density function c is obtained taking the partial derivative of C, d times with respect to each marginal:

$$c(u_1, ..., u_d) = \frac{\partial^d C(u_1, ..., u_d)}{\partial u_1 ... \partial u_d} \tag{3}$$

Equation (2) and (3) are fundamental to understand. By decomposing the multivariate distribution into the marginal probability density functions, and the copula density function we can capture the characteristics of the marginal distributions as well as the dependence characteristics of them (Rad et al., 2016). Thus no assumption on the joint behaviour of the marginal distributions is required since the marginal behaviour is modelled separately from the joint structure. Hence copulas allow for higher flexibility in modelling multivariate distributions.

3.2 Copulas in Pairs Trading

Let X_1 and X_2 be two random variables, e.g., the daily return distributions of two stocks for a given period, with probability functions $F_1(X_1)$ and $F_2(X_2)$. By feeding the random variables into their distribution function we transform the random variables into uniform random variables, i.e, $U_i = F(X_i)$ where $U_i \sim$

U(0, 1) for $i = 1, 2$ (Rad et al., 2016). Define $C(u_1, u_2) = \mathrm{P}(U_1 \leq u_1, U_2 \leq u_2)$, their copula function. By taking the partial derivative of the copula function, we obtain the conditional distribution function (Aas et al., 2009):

$$h_1(u_1|u_2) = \mathrm{P}(U_1 \leq u_1|U_2 = u_2) = \frac{\partial C(u_1, u_2)}{\partial u_2} \tag{4}$$

$$h_2(u_1|u_2) = \mathrm{P}(U_2 \leq u_2|U_1 = u_1) = \frac{\partial C(u_1, u_2)}{\partial u_1} \tag{5}$$

The conditional distribution functions h_1 and h_2 allow us to estimate the probability outcomes if one random variable is smaller than a certain value, given the other random variable has a specific value (Rad et al., 2016). Using the conditional distribution functions for pairs trading allows us to calculate the conditional probability of a stock increasing or decreasing in price, given the current realization of the other stock's price. This is done every day in the trading period.

There are two methods to derive trading signals from the conditional probabilities. The return-based method uses the most recent realized return of both stocks in a given pair to derive trading signals. Krauss and Stübinger (2017) and Liew and Wu (2013) define upper and lower bound values for h_1 and h_2 for which they start trading. In particular, if h_1 happens to be smaller than 0.05, while h_2 happens to be larger than 0.95 on a given day, we are sufficiently confident in the mispricing and conclude that stock 2 is overvalued relative to stock 1 and simultaneously open a long short position.[1] Similar, if h_1 happens to be larger than 0.95, while h_2 happens to be smaller than 0.05 on a given day, we conclude that stock 1 is overvalued relative to stock 2 and we simultaneously open a long short position. Once the conditional probabilities return to 0.5, i.e., equilibrium pricing, we close both positions. The alternative method is based on relative mispricing over time using cumulative mispricing indices (Rad et al., 2016; Xie et al., 2016). A detailed discussion of this method follows in Section 4.4, as this is the copula method used in this study.

[1]Equilibrium pricing requires $h_1 = h_2 = 0.5$. The interpretation of h_1 and h_2 will be discussed in Section 4.4.

4 Methodology

4.1 Data

The data set of this analysis contains the daily closing prices of stocks included in the S&P 500 index.[2] For simplicity, we do not consider the history of S&P 500 constituents, but rather obtain prices for the stocks that are in the index as of July 28, 2022, from January 2005 until June 2022. In January 2005, only 345 of the S&P 500 constituents were already stock market listed and thus available for trading. However, due to subsequent IPOs, this number increases steadily over time to 473 available stocks in December 2020. Furthermore, we only consider securities of the type common stock.

4.2 Identifying Pairs

In line with the pairs trading literature we identify pairs in a 12-month formation period and trade the nominated pairs in the subsequent 6 months, namely the trading period (Do & Faff, 2010; Gatev et al., 2006; Rad et al., 2016). During the formation period, we calculate the sum of squared difference (SSD) between normalised prices for any possible stock combination. Normalised prices are defined as the cumulative return indices which are rescaled to one in every formation period. To be considered as a pair, we require both stocks to have prices for each day of the formation and trading period. We start a new formation period every month, which results in up to 6 overlapping trading periods each month and a total of 192 periods. The 6 overlapping periods can be interpreted as six individual portfolios, which are active for trading during a given month. For a given number of stocks N, the number of pairs to check is equal to $(N[N-1]/2)$. In the first period, there are 345 stocks available, resulting in 59,340 pairs to check. Over time the number of available stocks increases until it reaches 473 in the final period, which starts in December 2020. This results in 109,278 pairs to check in the final period. In total, we end up with 15,924,065 pairs to evaluate during the

[2] S&P 500 constituents and the daily stock prices have been obtained from Bloomberg Terminal at the Leibniz Institute for Financial Research SAFE data room on July 28, 2022.

entire observation period. Once we obtained the SSD for each pair, we nominate the 20 pairs with the lowest SSD in each formation period for trading.

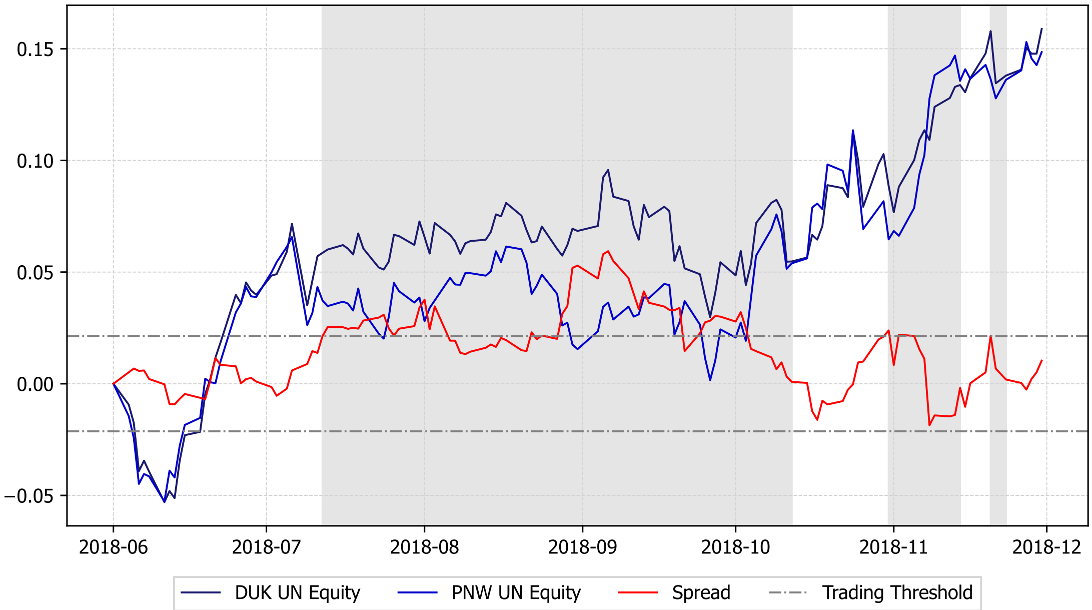
4.3 Distance Method

The distance method is by far the simplest pairs trading strategy. Similar to Gatev et al. (2006) we nominate the 20 pairs that have the lowest SSD in normalised prices during their respective formation period for trading. We also obtain the standard deviation of the daily difference between the stock prices, henceforth "spread", during the aforementioned formation period, which serves as a trading threshold in the trading period. During the trading period, we monitor the spread of each nominated pair daily. If the spread is larger than two historical standard deviations in absolute value (obtained during the preceding formation period), we simultaneously open a long and short position. If the spread increases and exceeds two historical standard deviations, we say that stock 1 is overvalued, relative to stock 2 and therefore enter a short position in stock 1 and a long position in stock 2. If the spread falls below two negative historical standard deviations the opposite is true. Stock 2 is said to be overvalued relative to stock 1 and we enter a stock 2 short and a stock 1 long position. Once the spread converges back to zero, we close both positions and continue to monitor the pair for further trading opportunities during the remainder of the trading period.

Figure 1 demonstrates the described trading rule for two stocks from the utility sector. At the beginning of the trading period, both stocks tend to move close together and the spread fluctuates around zero. During June and July 2018 both stocks increase in price, however, the price of DUK (stock 1) stays at a higher price whereas the price of PNW (stock 2) drops. Therefore, according to the trading rule, DUK is overvalued relative to PNW; thus we buy long PNW and sell short DUK. Around mid of October 2018, the spread between the two stocks decreases and converges (shortly) back to zero. We gain on both positions and end up with a return of 2.0648%. Later that period two additional opportunities arise, which we also exploit.

Figure 1: Distance Method Trading Rule

This figure shows the cumulative return indices for Duke Energy Corp (DUK) and Pinnacle West Capital Corp (PNW), two companies from the utility sector. Furthermore, the spread is calculated for each day and the trading threshold (two historical standard deviations from the preceding formation period) is graphed. The shaded areas correspond to periods where long and short positions were active.



Note that by the end of the period, all positions are closed by natural convergence of the spread to zero. This is not always the case. If the spread does not converge to zero, and eventually diverges even further, both positions will be closed on the last day of the trading period, irrespective of whether the stocks converged back or not. This can potentially cause large losses and reduce the profitability of the distance method severely. A detailed discussion on the convergence/non-convergence of pairs and the effect on the overall profitability of pairs trading strategies follows in Section 5.2.

4.4 Copula Approach

Similar to the distance method we nominate the 20 pairs with the lowest SSD during the formation period for trading in each period. The copula approach to pairs trading is more involved than the distance method. Fitting pairs to copulas is a two-step process. First, we must find the marginal distributions that describe the daily returns during the formation period of each nominated pair's stocks best. Similar to Rad et al. (2016), we allow marginal distributions to be chosen from Normal, Student-t, Generalized Logistic, and Generalized Extreme Value distributions and ultimately choose the one that has the lowest Aikake information criterion (AIC). By feeding the daily returns into the cumulative distribution function of the marginal distribution that fits best, we obtain uniform marginals. Second, we use the uniform marginals of the formation period to find the copula that models the pairs dependence structure best. Again, similar to Rad et al. (2016), we allow for Student-t, Clayton, Gumbel (where for the Clayton and Gumbel copula rotated versions can be chosen). The copula parameters are estimated via maximum likelihood and we ultimately choose the copula that features the lowest AIC. Table 1 provides an overview of the frequencies with which marginal distributions and copulas have been selected.

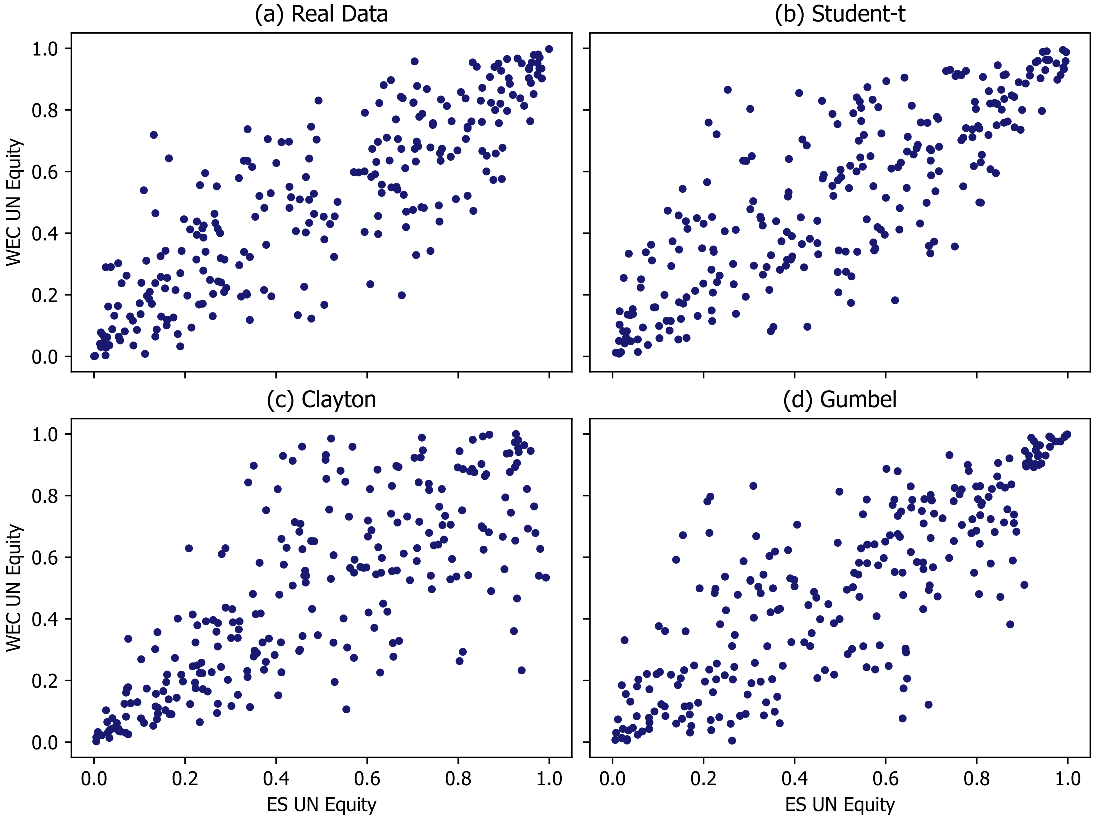
Table 1: Share of Selected Marginal Distributions and Copulas

		Marginal Distribution		
Marginal	Student-t	Gen. Logistic	Normal	Gen. Extreme Value
% of Stocks	51.70	40.87	7.24	0.19
		Copula		
Copula	Student-t	Gumbel	Rotated Gumbel	Clayton
% of Pairs	75.36	3.44	19.74	1.46

Figure 2 illustrates the marginal daily return distribution of two stocks from the utility sector and different fitted copulas. In particular, Figure 2(a) contains the realized uniform marginal daily returns of the formation period. We immediately notice the high dependence in both tails. This is captured best by the Student-t copula, Figure 2(b). Clayton and Gumbel copula exhibit high dependence in only one of their tails. Having obtained the copula that provides a parsimonious fit for each nominated pair allows us to turn to the trading period. Figure 3 illustrates the uniform daily returns realized during the subsequent trading period and simulated data, based on the Student-t copula, parameterised during the formation period. The relationship captured during the formation period still holds, thus the Student-t copula continues to provide a parsimonious fit.

Figure 2: Copula Estimation of Uniform Daily Returns

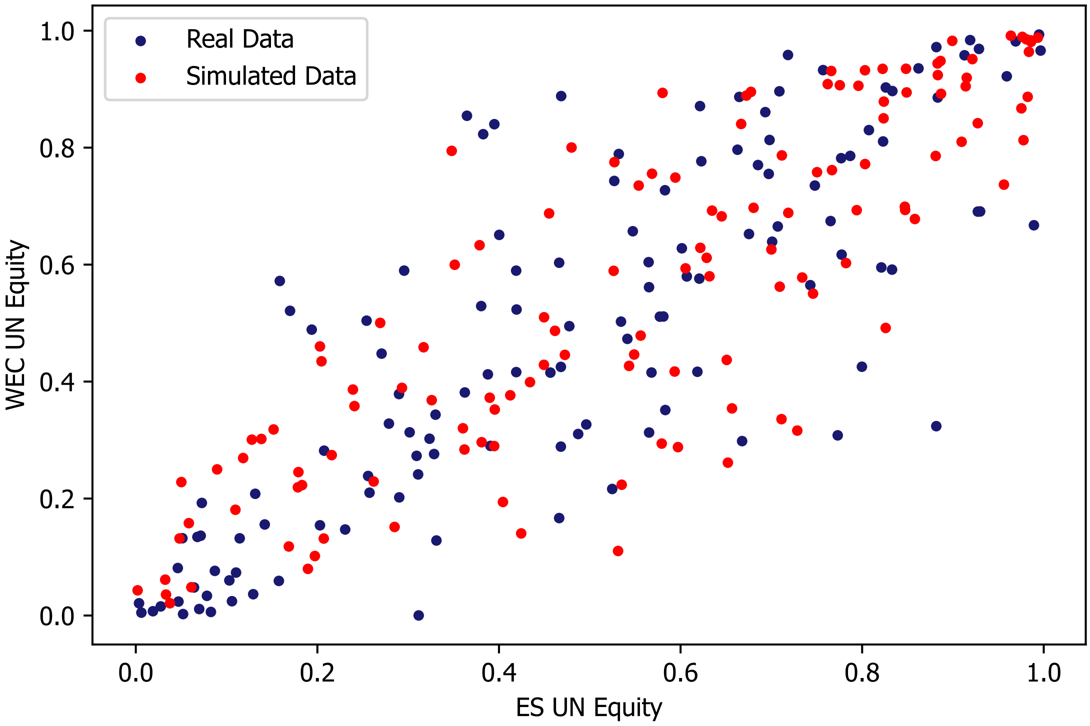
This figure illustrates the copula estimation for Eversource Energy (ES) and WEC Energy Group Inc (WEC), two companies from the utility sector during the formation period which started in April 2013. Figure 2(a) contains the realized uniform marginal daily returns. Figures 2(b)-(d) contain fitted copulas.



During the trading period, we monitor each pair daily. We calculate the conditional probabilities, h_1 and h_2, as defined in Equations (4) and (5). Rad et al. (2016) provide an intuitive interpretation for the conditional probabilities: if h_1 is equal to 0.5, the probability of stock 1's price to fall below its current realization, given the price of stock 2, is 50%. This interpretation is analogously valid for h_2. Consequently, a value of h_1 above 0.5 implies that the probability for stock 1 to fall below its current realization is higher than the probability for stock 1 to increase in price (Rad et al., 2016). Similarly, a value of h_1 below 0.5 predicts an increase in price to be more probable than a decrease. Again, interpretations hold analogously for h_2. To monitor the development of the relative over-/underpricing of stocks 1 and 2, we first introduce two mispriced indices (Rad et al., 2016; Xie

Figure 3: Copula Predictions for the Trading Period

This figure contains the uniform realized daily returns and the student-t copula predictions for the trading period following the formation period illustrated in Figure 2.



et al., 2016) which are calculated for each day of the trading period:

$$m_{1,t} = h_1(u_1|u_2) - 0.5 = \mathrm{P}(U_1 \leq u_1|U_2 = u_2) - 0.5 \tag{6}$$

$$m_{2,t} = h_2(u_2|u_1) - 0.5 = \mathrm{P}(U_2 \leq u_2|U_1 = u_1) - 0.5 \tag{7}$$

Secondly, we calculate the cumulative mispriced indices for each day of the respective trading period:

$$M_{1,t} = M_{1,t-1} + m_{1,t} \tag{8}$$

$$M_{2,t} = M_{2,t-1} + m_{2,t} \tag{9}$$

If M_1 is positive, while M_2 turns out to be negative, stock 1 is said to be overvalued relative to stock 2. Similarly, if M_2 is negative and M_1 is positive, stock 2 is overvalued. In line with Xie et al. (2016), we open a long and short position if the cumulative daily deviations from equilibrium pricing are sufficiently large enough. That is the case if one of the cumulative mispriced indices greater than 0.5 while the other one is smaller than -0.5. Once the indices return to zero, the positions are closed and we continue to evaluate the pair for further trading opportunities.

4.5 Performance Evaluation

The performance evaluation of this study follows Gatev et al. (2006), who provide two types of excess returns which are commonly used in the pairs trading literature. Determining the profitability of pairs trading is a nontrivial issue. This is because pairs can open and converge multiple times during the trading period. In particular, if a pair opens and converges during the trading period, a positive cash flow results. If a pair opens and does not converge, the position is closed on the final day of the trading period; either a positive or negative cash flow will occur. Hence, the cash flows of a pairs trading strategy are a set of randomly distributed positive cash flows throughout the trading period and a set of cash flows that can be either positive or negative at the end of the period (Gatev et al., 2006). Per assumption, we invest one dollar long and short for a given pair. Therefore the payoffs are interpreted as excess returns. The excess return of a given pair is equal to the reinvested payoffs during the trading interval, where the long-short portfolio positions are marked-to-market daily. In particular, we obtain the daily return of pair P on day t as a value-weighted return:

$$r_{P,t} = \frac{\sum_{i \in P} w_{i,t} r_{i,t}}{\sum_{i \in P} w_{i,t}} \tag{10}$$

$$w_{i,t} = w_{i,t-1}(1 + r_{i,t-1}) = (1 + r_{i,1}) \cdots (1 + r_{i,t-1})$$

where $r_{i,t}$ denotes the return of stock i of pair P on day t and $w_{i,t}$ denotes a corresponding weight. To obtain monthly returns, daily returns are compounded (Gatev et al., 2006).

Similar to Gatev et al. (2006) and Rad et al. (2016) we calculate two types of excess returns, namely the return on employed capital and the return on committed capital. Return on employed capital of a given portfolio in month m is defined as the sum of payoffs of the portfolio's pairs divided by the number of pairs that opened for trading during the respective trading period. Return on committed capital in month m is defined as the sum of payoffs of the portfolios pairs divided by the number of nominated pairs to trade during the respective trading period. In our specification this number is equal to 20 for a given portfolio, regardless of whether the pairs actually have been traded or not. Ultimately, the monthly excess return is calculated as the equally-weighted average return of the 6 portfolios that are active for trading in a given month. Comparing these two measures of excess returns, the return on committed capital is more conservative, as it takes into account the opportunity costs of capital (Rad et al., 2016).

Whenever we enter a trade, we sell one stock short. In practice, the sale of this stock would provide us with the necessary capital to buy the other one long in equal amounts. Therefore, this strategy is self-financing. However, especially by short-selling stocks, fees arise. For simplicity, we abstract from such fees and other costs. As mentioned in Section 2, Do and Faff (2012) provide a study of pairs trading profitability considering costs inherent to these strategies. The findings of Do and Faff (2012) have been adopted in Rad et al. (2016). They make the assumption, that by screening out stocks that have a low dollar value and market capitalization, i.e., are illiquid and more costly to trade, the remaining stocks are cheap to sell short. Given that our stock sample consists of S&P 500 stocks, which are regarded as very liquid, the assumption of relatively low short-selling costs is thus reasonable for our study as well.

5 Results

5.1 Performance Statistics

Table 2 provides average monthly excess returns for both strategies. We differentiate between the top 5 and top 20 pairs in terms of the SSD criterion for trading nomination.

Table 2: Average Monthly Excess Returns of Top 5 and Top 20 Pairs
This table contains average monthly excess returns for the distance and copula methods' top 5 and top 20 pairs before transaction costs.

	Top 5		Top 20	
	Distance	Copula	Distance	Copula
Panel A: Return on Employed Capital				
Mean	0.0069***	0.0074***	0.0057***	0.0068***
t-stat	3.1030	2.7764	2.8881	2.8576
Std	0.0314	0.0373	0.0278	0.0334
Sharpe Ratio	0.2211	0.1978	0.2058	0.2036
Skewness	−0.6603	−0.6797	−1.1000	−1.0015
Kurtosis	1.4210	1.5332	2.8108	2.5776
Panel B: Return on Committed Capital				
Mean	0.0043***	0.0070***	0.0038***	0.0063***
t-stat	2.5277	2.7480	2.3975	2.7903
Std	0.0238	0.0358	0.0224	0.0319
Sharpe Ratio	0.1801	0.1958	0.1708	0.1988
Skewness	−0.9362	−0.6531	−1.5087	−1.0414
Kurtosis	2.8988	1.6590	5.2777	2.8847

*** indicates significance at the 1% level.

For both Panel A and Panel B, the average monthly excess return is higher for the top 5 pairs than for the top 20 pairs which implies a higher suitability of pairs with a low SSD for pairs trading. The distance methods average monthly excess return for the top 20 pairs is with 0.57% lower than what Rad et al. (2016) find (0.91%). This is in line with the declining trend of the distance methods' profitability and the fact that we consider large-cap stocks, which are very liquid and receive a lot of attention from investors and analysts. Thus (equilibrium) pricing should be very efficient which leaves little to no arbitrage opportunities.

Furthermore, Rad et al. (2016) include the 1970s and 1980s, which were very profitable for the distance method. This might render their average monthly excess return higher. Compared to Rad et al. (2016) not only the average monthly excess return is lower, but also the standard deviation is higher, which in turn results in a lower sharpe ratio of 0.0278 for the top 20 pairs of the distance method. In our study, the performance of the copula method is superior to the distance method with an average monthly excess return of 0.68%. Interestingly, the drop in average monthly excess return in moving from the top 5 to the top 20 pairs is smaller for the copula method. The distance method seems to be more sensitive to pair quality in terms of the SSD criterion for trading nomination. Furthermore, while for the distance method the sharpe ratio decreases in moving from the top 5 to the top 20 pairs, for the copula method it increases.

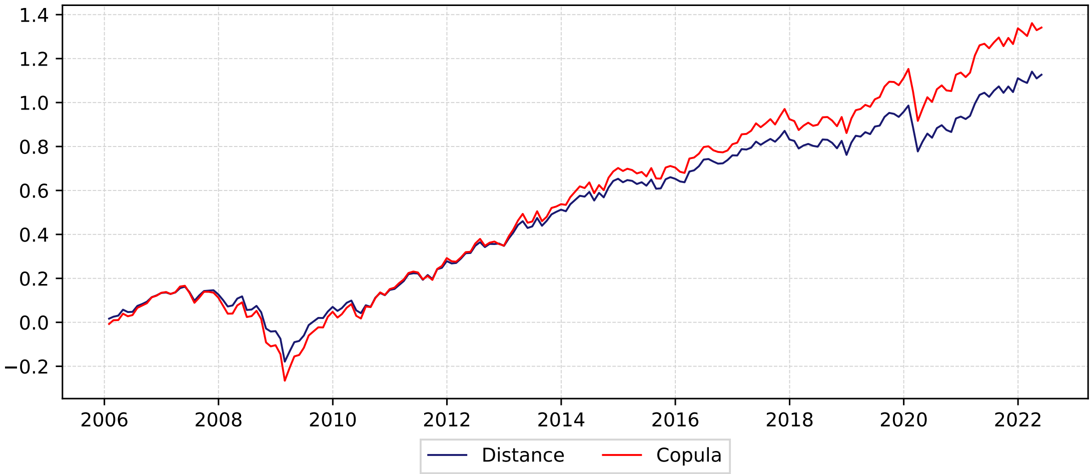
The difference between the average monthly excess return on employed capital and committed capital is larger for the distance method than for the copula method. This results from the fact that for the copula method we observe relatively early divergence during the trading period, usually in the first month. Furthermore, we observe very little convergence later on in the trading period. Thus, by definition return on employed and committed capital are very similar for the copula method. This is not the case for the distance method, where the difference between the return on employed capital and return on committed capital is larger because fewer pairs open for trading during a given trading period.

Figure 4 provides the cumulative monthly excess return on employed capital for the distance and copula method. We can see a steady and similar increase in cumulative excess return for both strategies, although the copula method is slightly more profitable in terms of excess return. Furthermore, we immediately recognize two bumps in the graph, namely during the Global Financial Crisis (2007-2009) and the COVID-19 Pandemic (2019-2022). A detailed discussion of the strategies' performance during severe exogenous shocks to global financial markets will follow in Section 5.3.

Figure 5 provides monthly excess returns from 2006 until 2022. Although the majority of months have a positive excess return, there are some months with ex-

Figure 4: Cumulative Excess Return on Employed Capital

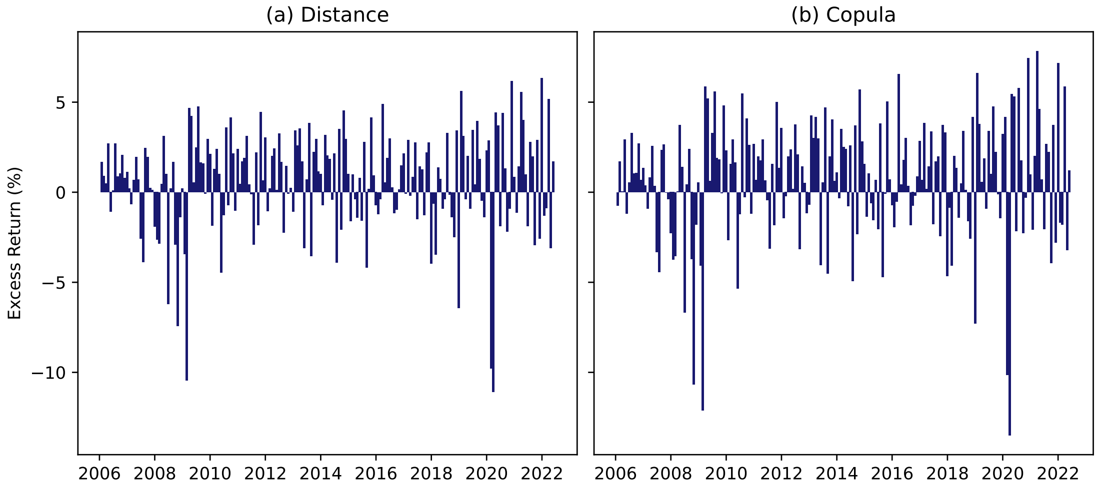
This figure illustrates cumulative excess returns on employed capital for the top 20 pairs.



treme losses, most notably around the time of the Global Financial Crisis and the COVID-19 Pandemic. The discussion of why such events can imply large losses for pairs trading strategies will follow in Section 5.3. However, implementing a simple stop-loss mechanism can enhance the pairs trading profitability.

Figure 5: Monthly Excess Return on Employed Capital

This figure illustrates monthly excess return on employed capital for the top 20 pairs.



5.2 Trading Statistics

Figure 6 provides the return distribution of all trades executed in each of the respective pairs trading strategies. For the distance method, we find similar results as Rad et al. (2016), namely fat left tails, which imply that it is more likely to realize (extreme) negative returns than (extreme) positive returns. This however is not surprising, as the return of the distance method is bound by the relative mispricing we tolerate until we open a long short position by construction (two standard deviations of the spread from the preceding formation period). The copula method does exhibit both extreme negative as well as positive returns because it is not bound in profitability. In theory, this makes the strategy superior to the distance method in terms of return potential.

Figure 6: Return Distribution of Pairs Trading Strategies

This figure contains the trade return distributions of both strategies. Trade returns are defined as the sum of returns from long and short position.

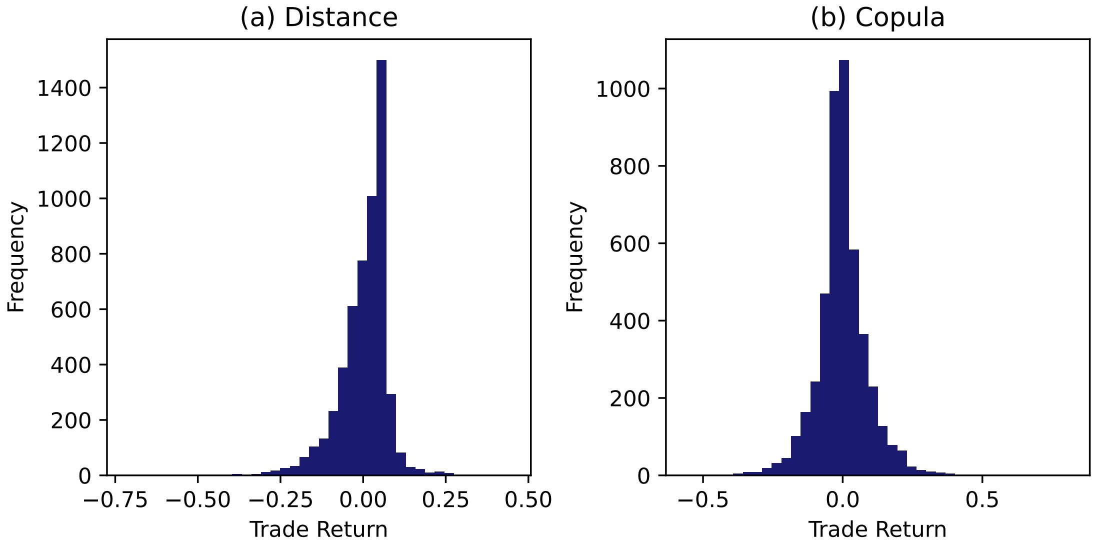


Table 3 provides a detailed summary about the trades executed in each pairs trading strategy. We find that, contrary to Rad et al. (2016), the distance method offers more trading opportunities than the copula method. Additionally, we find a surprisingly low rate of convergence among the distance methods trades. However, the trades that do converge are very profitable on average and overcompensate the negative returns of the unconverged trades. The converged trades also feature a small standard deviation which results in a large sharpe ratio.

For the copula method, we find that, similar to Rad et al. (2016) the convergence rate is lower than in the distance method. Surprisingly though, the average return for the convergent trades is negative, whereas for the non-convergent it is slightly positive. Hence the trades that do not converge are the driver of the copula methods' profitability. This does not necessarily imply that copulas for pairs trading are unsuited. A potential issue of the copula method as implemented in this study is the non-copula-based pair selection process as pointed out by Krauss and Stübinger (2017). A discussion of the pair selection process follows in Section 6.

Table 3: Converged and Unconverged Trade Summary Statistic
Trade type "C" refers to trades which were closed by natural convergence. Trade type "U" refers to trades that did not converge until the end of the trading period and thus were forced to be closed.

Strategy	Total Trades	Trade Type	% of Trades	Mean	Std.	Sharpe Ratio	Skew	Days open		Positive Trades (%)
								Mean	Median	
Distance	5391	C	44.78	0.0516	0.0213	2.4237	1.8280	31.30	25.00	100.00
		U	55.22	−0.0332	0.0797	−0.4168	−0.4168			
Copula	4673	C	20.37	−0.0220	0.0294	−0.7490	−1.2852	30.39	20.00	11.97
		U	79.63	0.0030	0.0997	0.0300	−0.0131			

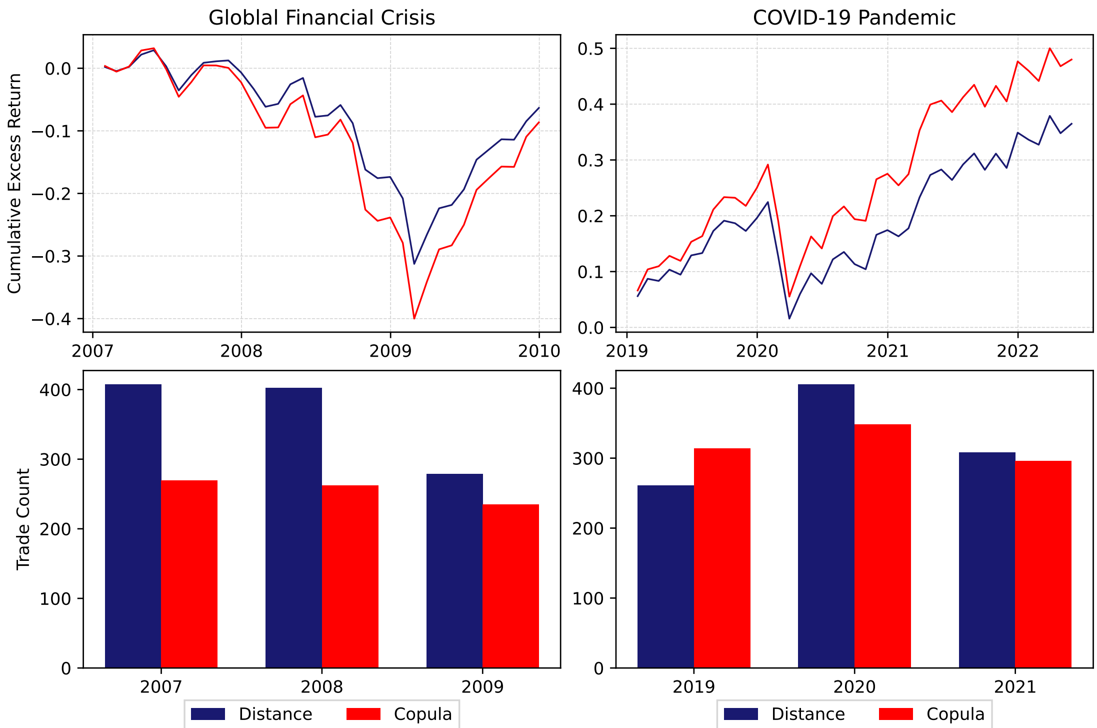
5.3 Sub-Period Analysis

Do and Faff (2010) find that the distance method performed solidly during the Dot-com bear market (2000-2002) and the Global Financial Crisis (2007-2009). This makes it worthwhile to take a closer look at the Global Financial Crisis and the most recent shock to global financial markets, the COVID-19 Pandemic (2019-2022). Figure 7 provides the cumulative excess returns for both strategies and the number of trades executed during the two mentioned time frames. During the financial crisis, the profitability declines for 2 years until the first quarter of 2009, when excess returns begin to increase relatively fast. The profitability of the distance method is slightly higher at the end of the period, although it remains negative. The average monthly excess return during the Global Financial Crisis period is -0.17%. The copula method only achieves an average monthly excess return of -0.24%. For the COVID-19 Pandemic period, we see a very abrupt loss in excess return in the first quarter of 2020, essentially the point in time where the COVID-19 Pandemic hit the global economy. From thereon, excess returns increase. The average monthly excess return from 2019 until 2021 is 0.96% for the distance and 1.3% for the copula method respectively. Trading opportunities during the Global Financial Crisis are relatively stable. Only in

2009, the distance method suffers from a relatively large decrease in pairs traded. Interestingly, during the COVID-19 Pandemic trading opportunities peak in 2020.

These findings potentially have many different causes and require further in-depth analysis to draw robust conclusions. However, it seems plausible that in periods of high disruption such as the COVID-19 Pandemic, uncertainty among investors is rather large. Deriving the fair value of a stock is harder than during non-crisis times because the possible states of the world and thus future cash flows and growth rates of a company are even harder to predict. This might make it less likely for a diverged pair to return back to equilibrium pricing, which in turn implies (high) losses for trades entered prior to the COVID-19 shock to global financial markets.

Figure 7: Excess Returns during Global Financial Crisis & COVID-19 Pandemic
This figure contains cumulative monthly excess returns on employed capital and the number of trades executed for the distance and copula method during the Global Financial Crisis (2007-2009) and the COVID-19 Pandemic (2019-2022).



6 Conclusion

Pairs trading is a methodologically simple trading strategy: buy the stock that is undervalued and sell the one that is overvalued. The fair value of a stock is derived relative to another stock using past prices. In this study, we examined the profitability of the famous distance method and the more sophisticated copula method. We find that both distance and copula methods are statistically significant profitable before transaction costs for S&P 500 stocks from 2005 to 2022. Conversely, we find that the profitability of the copula method is driven by the large share of unconverged trades, which are slightly profitable. The small share of converged trades is not. To conclude whether the copula method does work or not additional research is required.

Given the relatively small stock sample, we are not necessarily able to obtain the ideal pairs to nominate for trading. Rad et al. (2016) for instance use the entire CRSP database. This results in 2,377 stocks and hence a total of 2,823,876 pairs being available for trading in the final period of their study. The distance method seems to have no problem with this since the returns found in this study are similar to the findings of Do and Faff (2010) and Rad et al. (2016). However, the copula method might suffer from the pair quality caused by the small stock sample. To test this hypothesis, a study of the entire US equity market is required.

In this study, we use the SSD between normalised prices to identify pairs. This is in line with the pairs trading literature, though there might be better, more suitable pair identification methods for the copula method, especially in light of trading signals being derived using copulas. Copulas provide a powerful way of decomposing the marginal and joint dependence structure. Using this already in the formation period, as Krauss and Stübinger (2017) did, can generate significant returns. The stock sample used tough is relatively small. Therefore researchers might want to apply their method to the entire US stock market as Rad et al. (2016) did.

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Statutory Declaration

I herewith declare that I have composed the present thesis myself and without use of any other than the cited sources and aids. Sentences or parts of sentences quoted literally are marked as such; other references with regard to the statement and scope are indicated by full details of the publications concerned. The thesis in the same or similar form has not been submitted to any examination body and has not been published. This thesis was not yet, even in part, used in another examination or as a course performance.
Throughout the thesis I am using the pronoun “We”, which is meant as a stylistic device. It does not imply that there are additional authors of this thesis.

Tim Winkler	Date, Place