Supplementary Appendix
Currency Areas and Voluntary Transfers

Derivation of equations (15) and (16) from Section 6.1

In the absence of transfer, the consumption and labour in state s in Home country, when the money supply is $M_{0,s}$, are given by

$$L_s = \frac{\vartheta M_{0,s}}{W}, \quad \text{and} \quad X_s = \frac{\xi}{\mu}\frac{\vartheta M_{0,s}}{AB_s W}$$

where

$$B_s \equiv \left(\tfrac{1}{2}b_s + \tfrac{1}{2}b_s^* \epsilon_s^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

and (note that ϵ is the relative wage and is different from the exchange rate ε)

$$\epsilon_s \equiv \left(\frac{W^*}{W}\right)\varepsilon_s = \left(\frac{b_s}{b_s^*}\right)^{-\frac{1}{\sigma}}\left(\frac{W}{W^*}\right)^{-\frac{1}{\sigma}}\left(\frac{M_{0,s}}{M_{0,s}^*}\right)^{\frac{1}{\sigma}}.$$

Central banks choose a contingent money supply simultaneously and independently. They choose money supplies as Stackelberg leaders in the wage setting process. With two states and $\gamma = 1$, the Home equilibrium wage is given by

$$W = \left(\kappa_0 \frac{1}{2}\vartheta^2\left(M_{0,G}^2 + M_{0,B}^2\right)\right)^{\frac{1}{2}}.$$

The objective function of the Home central bank is to maximize the expected utility of Home households. With $\gamma = 1$, this is equivalent (ignoring constants) to maximizing

$$\log\left(M_{0,G}\right) - \log\left(B_G\right) + \log\left(M_{0,B}\right) - \log\left(B_B\right) - 2\log\left(W\right).$$

The first-order condition for $M_{0,G}$ is

$$\begin{aligned}&\left(\frac{\epsilon_G}{B_G}\frac{\partial B_G}{\partial \epsilon_G}\right)\left(\frac{M_G}{\epsilon_G}\frac{\partial \epsilon_G}{\partial M_G} + \left(\frac{W}{\epsilon_G}\frac{\partial \epsilon_G}{\partial W}\right)\left(\frac{M_G}{W}\frac{\partial W}{\partial M_G}\right)\right)\\ &+\left(\frac{\epsilon_B}{B_B}\frac{\partial B_B}{\partial \epsilon_B}\right)\left(\frac{W}{\epsilon_B}\frac{\partial \epsilon_B}{\partial W}\right)\left(\frac{M_G}{W}\frac{\partial W}{\partial M_G}\right) = 1 - 2\left(\frac{M_G}{W}\frac{\partial W}{\partial M_G}\right).\end{aligned}$$

Similarly, the first-order condition for $M_{0,B}$ is

$$\begin{aligned}&\left(\frac{\epsilon_B}{B_B}\frac{\partial B_B}{\partial \epsilon_B}\right)\left(\frac{M_B}{\epsilon_B}\frac{\partial \epsilon_B}{\partial M_B} + \left(\frac{W}{\epsilon_B}\frac{\partial \epsilon_B}{\partial W}\right)\left(\frac{M_B}{W}\frac{\partial W}{\partial M_B}\right)\right)\\ &+\left(\frac{\epsilon_G}{B_G}\frac{\partial B_G}{\partial \epsilon_G}\right)\left(\frac{W}{\epsilon_G}\frac{\partial \epsilon_G}{\partial W}\right)\left(\frac{M_B}{W}\frac{\partial W}{\partial M_B}\right) = 1 - 2\left(\frac{M_B}{W}\frac{\partial W}{\partial M_B}\right).\end{aligned}$$

We have

$$\frac{\epsilon_G}{B_G}\frac{\partial B_G}{\partial \epsilon_G} = \frac{\epsilon_G \frac{M_{0,G}}{W}}{\frac{M^*_{0,G}}{W^*} + \epsilon_G \frac{M^*_{0,G}}{W^*}} = \frac{1}{1 + \left(\frac{b_G}{b^*_G}\right)^{\frac{1}{\sigma}} \left(\frac{\frac{M_{0,G}}{W}}{\frac{M^*_{0,G}}{W^*}}\right)^{\frac{\sigma-1}{\sigma}}},$$

$$\frac{\epsilon_B}{B_B}\frac{\partial B_B}{\partial \epsilon_B} = \frac{\epsilon_B \frac{M^*_{0,B}}{W^*}}{\frac{M_{0,B}}{W} + \epsilon_B \frac{M^*_{0,B}}{W^*}} = \frac{1}{1 + \left(\frac{b_B}{b^*_B}\right)^{\frac{1}{\sigma}} \left(\frac{\frac{M_{0,B}}{W}}{\frac{M^*_{0,B}}{W^*}}\right)^{\frac{\sigma-1}{\sigma}}},$$

$$\frac{M_{0,G}}{\epsilon_G}\frac{\partial \epsilon_G}{\partial M_{0,G}} = \frac{1}{\sigma}, \quad \frac{M_{0,B}}{\epsilon_B}\frac{\partial \epsilon_B}{\partial M_{0,B}} = \frac{1}{\sigma}, \quad \frac{W}{\epsilon_G}\frac{\partial \epsilon_G}{\partial W} = -\frac{1}{\sigma}, \quad \frac{W}{\epsilon_B}\frac{\partial \epsilon_B}{\partial W} = -\frac{1}{\sigma},$$

$$\frac{M_{0,G}}{W}\frac{\partial W}{\partial M_{0,G}} = \frac{\frac{M^2_{0,G}}{M^2_{0,B}}}{1 + \frac{M^2_{0,G}}{M^2_{0,B}}}, \quad \frac{M_{0,B}}{W}\frac{\partial W}{\partial M_{0,B}} = \frac{1}{1 + \frac{M^2_{0,G}}{M^2_{0,B}}}.$$

Hence, the first-order condition for $M_{0,G}$ satisfies

$$\left(\frac{1}{1 + \left(\frac{b_G}{b^*_G}\right)^{\frac{1}{\sigma}} \left(\frac{\frac{M_{0,G}}{W}}{\frac{M^*_{0,G}}{W^*}}\right)^{\frac{\sigma-1}{\sigma}}}\right)\left(\frac{1}{1 + \frac{M^2_{0,G}}{M^2_{0,B}}}\right) - \left(\frac{1}{1 + \left(\frac{b_B}{b^*_B}\right)^{\frac{1}{\sigma}} \left(\frac{\frac{M_{0,B}}{W}}{\frac{M^*_{0,B}}{W^*}}\right)^{\frac{\sigma-1}{\sigma}}}\right)\left(\frac{\frac{M^2_{0,G}}{M^2_{0,B}}}{1 + \frac{M^2_{0,G}}{M^2_{0,B}}}\right) = \sigma\left(\frac{1 - \frac{M^2_{0,G}}{M^2_{0,B}}}{1 + \frac{M^2_{0,G}}{M^2_{0,B}}}\right). \tag{S.1}$$

The first-order condition for $M_{0,B}$ is identical because money is neutral and only the ratio $M_{0,G}/M_{0,B}$ matters. With symmetry between countries, $b^*_G = b_B$, $b^*_B = b_B$ and the symmetric Nash equilibrium has $M_{0,G}/M_{0,B} = M^*_{0,B}/M^*_{0,G}$ and $W = W^*$. Substituting these conditions into equation (S.1) gives equation (15) in the text.

Letting $\varrho = M_{0,G}/M_{0,B}$ and recalling that $b_G/b_B = z^{1-\sigma}$, equation (15) can be written as

$$1 - \varrho^2 z^{\frac{1-\sigma}{\sigma}} \varrho^{\frac{\sigma-1}{\sigma}} = \sigma(1-\varrho^2)\left(1 + z^{\frac{1-\sigma}{\sigma}} \varrho^{\frac{\sigma-1}{\sigma}}\right). \tag{S.2}$$

Denote the solution as $\varrho(z)$ and note that $\varrho(1) = 1$. Taking a linear approximation of both sides of equation (15) about $z = 1$ and solving gives

$$\varrho'(1) = \frac{\sigma - 1}{4\sigma^2 - 3\sigma + 1}.$$

Hence, the linear approximation for the solution of $\varrho(z)$ about $z = 1$ is

$$\varrho(z) = \varrho(1) + \varrho'(1)(1-z) = 1 + \left(\frac{\sigma-1}{4\sigma^2 - 3\sigma + 1}\right)(1-z),$$

which corresponds to equation (16) in the text.