Derivative and Collateral Introduction

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ABSTRACT

This paper presents a model for pricing derivatives subject to collateralization. We assess the joint effect of derivative valuation and collateralization on asset price and risk. Because implied swap premium spreads take into account both counterparty risk and collateralization, we assign the model-implied spreads as the explanatory variable and the market spreads as the response variable. The new adjusted R^2 is 0.9906, suggesting that counterparty risk and collateralization together have high explanatory power on premium spreads. The finding leads to practical implications, such as collateralization modeling allows forecasting credit spread.

Key words: derivative pricing, collateralization, credit risk swap premium spread, CVA, VaR.

Collateralization is a critical component of the plumbing of the financial system. The effect of collateralization on valuation and risk is an understudied area.

Due to the complexity of collateralization, the literature seems to turn away from direct and detailed modeling. For example, Johannes and Sundaresan [2007], and Fuijii and Takahahsi [2012] model collateralization via a cost-of-collateral instantaneous rate. Piterbarg [2010] regards collateral as a regular asset and uses the replication approach to price collateralized derivatives.

Contrary to previous studies, we present a model that characterizes a collateral process directly based on the fundamental principal and legal structure of CSA. The model is devised that allows for collateralization adhering to bankruptcy laws. Our model is very useful for valuing off-the-run or outstanding derivatives.

Interest rate swaps collectively account for two-thirds of all outstanding derivatives. An ISDA mid-market swap rate is based on a mid-day polling. The adjustment or swap premium is determined by many factors, such as credit risk, liquidity risk, funding cost, operational cost and expected profit, etc.

Unlike generic mid-market swap rates, swap premia are determined in a competitive market according to the basic principles of supply and demand. A swap client first contacts a number of swap dealers for a quotation and then chooses the most competitive one. If a premium is too low, the dealer may lose money. If a premium is too high, the dealer may lose the competitive advantage.

Unfortunately, we do not know the detailed allocation of a swap premium, i.e., what percentage of the adjustment is charged for each factor. Thus, a direct empirical assessment of the impact of collateralization on swap rate is impossible.

To circumvent this difficulty, this article uses an indirect empirical approach. We reasonably believe that if two contracts are identical except counterparties, the swap premium spread should reflect counterparty credit risk only, as all other risks/costs are identical.

Empirically, we obtain a unique proprietary dataset from an investment bank. We use these data and a statistical measurement R^2 to examine whether credit risk and collateralization, alone or in combination, are sufficient to explain market swap premium spreads. We first study the marginal impact of credit risk. The estimation result shows that the adjusted R^2 is 0.7472, implying that approximately 75% of market spreads can be explained by counterparty credit risk. In other words, counterparty risk alone can provide a good but not overwhelming prediction on spreads.

We then assess the joint effect. Because implied or model-generated spreads take into account both counterparty risk and collateralization, we assign the model-implied spreads as the explanatory variable and the market spreads as the response variable. The new adjusted R^2 is 0.9906, suggesting that counterparty risk and collateralization together have high explanatory power on premium spreads. The finding leads to practical implications, such as collateralization modeling allows forecasting credit spread.

Second, how does collateralization affect counterparty credit risk? Credit value adjustment (CVA) is the most prominent measurement in counterparty credit risk. We find that the CVA of a collateralized counterparty portfolio is always smaller than the one of the same portfolio without collateralization. We also find that credit risk is negatively correlated with collateralization as an increase in collateralization causes a decrease in credit risk. The empirical tests corroborate our theoretical conclusions that collateralization can reduce CVA charges and mitigate counterparty risk.

Finally, how do collateralization and credit risk, either alone or in combination, impact market risk? How do they interact with each other? We find that there is a positive correlation between market risk and credit risk as VaR increases after considering counterparty credit risk. We also find that collateralization and market risk have a negative correlation, i.e., collateral posting can actually reduce VaR.

Pricing Collateralized Financial Derivatives

A CSA is a legal document that regulates collateral posting. It specifies a variety of terms including threshold, independent amount, and minimum transfer amount (MTA). A threshold is the unsecured credit exposure that a party is willing to bear. A MTA is used to avoid the workload associated with a frequent transfer of insignificant collateral amounts. An independent amount plays the same role as an initial margin or haircut. We define the effective collateral threshold as the threshold plus the MTA.

There are three types of collateralization: partial, over or full. A positive effective threshold corresponds to partial-collateralization where the posting of collateral is less than the MTM value. A negative effective threshold represents over-collateralization where the posting of collateral is greater than the MTM value. A zero-value effective threshold equates with full-collateralization where the posting of collateral is equal to the MTM value.

Since the only reason for taking collateral is to reduce/eliminate credit risk, collateral analysis should be closely related to credit risk modeling. There are two primary types of models that attempt to describe default processes in the literature: structural models and reduced-form models.

The choice of modeling assumptions for collateralization should be based on the legal structure of CSA. According to the Bankruptcy Law, if the demand for default payment exceeds the collateral value, the balance of the demand will be treated as an unsecured claim and subject to its pro rate distribution under the Bankruptcy Code's priority scheme (see Garlson [1992], Routh and Douglas [2005], and Edwards and Morrison [2005]).

We choose interest rate swaps for our empirical study. Ultimately, it is the objective of this subsection to test if counterparty credit risk and collateralization are sufficient to explain market swap premium spreads. We choose a statistical measurement R^2 to determine how much market spreads can be interpreted by model-implied spreads that take counterparty risk and collateralization into account.

Due to a close relationship, any statistical software that performs linear regression analysis will outputs R^2 value. Thus, conveniently we report R^2 together with other regression results that may provide additional statistical and financial insights.

Swap rate is the fixed rate that sets the market value of a swap at initiation to zero. ISDAFIX provides average mid-market swap rates based on a mid-day polling from a panel of dealers. In practice, the mid-market swap rates are generally not the actual swap rates transacted with counterparties, but are instead the benchmarks against which the actual swap rates are set. A swap dealer that arranges a contract and provides liquidity to the market involves costs.

Unlike the generic benchmark swap rates, swap premia are determined according to the basic principles of supply and demand. The swap market is highly competitive. In a competitive market, prices are determined by the impersonal forces of demand and supply, but not by the manipulations of powerful buyers or sellers.

Prior research has primarily focused on the generic mid-market swap rates and results appear puzzling. Sorensen and Bollier [1994] believe that swap spreads are partially determined by counterparty default risk. Whereas Duffie and Huang [1996], Minton [1997] and Grinblatt [2001] find weak or no evidence of the impact of counterparty credit risk on swap spreads. Collin-Dufresne and Solnik [2001] and He [2001] further argue that many credit enhancement devices, e.g., collateralization, have essentially rendered swap contracts risk-free. Meanwhile, Duffie and Singleton [1999], and Liu, Longstaff and Mandell [2006] conclude that both credit and liquidity risks have an impact on swap spreads.

In contrast to previous research, this subsection mainly studies swap adjustments/premia related to credit risk and collateralization. It empirically measures the effect of collateralization on pricing and compares it with model-implied prices.

A swap premium is supposed to cover the expected profit and all the expenses, including the cost of bearing unsecured credit risk. Unfortunately, however, we do not know what percentage of the market swap premium is allocated to the unsecured credit risk, which makes a direct verification impossible.

To circumvent this difficulty, we design an indirect verification process in which we select some CSA swap pairs such that the two contracts in each pair have exactly the same terms and conditions but are traded with different counterparties under different collateral agreements

We obtain a unique proprietary dataset from FinPricing (FinPricing 2017). The dataset contains derivative contract data, counterparty data (including collateral agreements, recovery rates, etc), and market data. The trading dates are from May 6, 2005 to May 11, 2012. We find a total of 1002 swap pairs in the dataset, where the two contracts in each pair have the same terms and conditions but are traded with different CSA counterparties.

By accounting for both credit risk and collateralization, we calculate the model-implied swap rates as 0.048780 and 0.048790 shown in Exhibit 5. Consequently, the model-implied swap premia are 0.09 bps and 0.19 bps. The results imply that only a small portion of a swap premium is attributed to unsecured credit risk. Intuitively the small impact of credit risk and collateral posting on a swap premium is mainly due to 1) only the swap coupons rather than the national amount are exposed to counterparty risk; 2) the initial swap rate sets the contract value close to zero and there is only 50% chance to develop counterpart risk; 3) collateralization mitigates credit risk. This would certainly not be the case for other derivatives. The result is in line with the findings of Duffie and Huang [1996], Duffie and Singleton [1999], and Minton [1997].

Repeating this exercise for the remaining pairs, we find that the model-implied spreads fluctuate randomly around the market spreads. We refer to the differences between the model-implied premium spreads and the market quoted premium spreads as the *model-market premium spread differentials*. It shows that the average of the model-market spread differentials is only -0.03 bps, which can be partly attributed to noise. The results indicate prima facie that the model performs quite well. The empirical tests corroborate the theoretical prediction on premium spreads.

According to ISDA Margin Survey (ISDA [2013], 73.7% of OTC derivatives are subject to collateral agreements. For large firms, the figure is 80.7%. Accounting for collateralization has become increasingly important in pricing OTC derivatives. Since the implied spreads generated by our model take into account both credit risk and collateralization, the statistical relationship between the market spreads and the model-implied spreads should refer to the joint effect of counterparty credit risk and collateralization on market spreads. Thus, we present another regression model where the market spreads are regressed on the implied spreads.

The empirical results shed light on the economic and statistical significance of collateralization. The increase in the explanatory power of swap premium spreads bears an interesting finding: It seems that credit risk alone has a modest explanatory power on premium spreads. Only the combination of credit risk and collateralization can sufficiently explain them.

In this subsection, we study how collateralization affects credit risk by measuring CVA changes due to collateral posting. CVA is the market price of counterparty credit risk that has become a central part of counterparty credit risk management.

From the same dataset above, we find that there are a total of 3052 counterparties having live trades as of May 11, 2012. 516 of them have CSA agreements. We randomly select one counterparty portfolio that contains 476 interest rate swaps, 36 interest rate swaptions and 223 interest rate caps/floors. First, we compute the risk-free value $V^F = 2{,}737{,}702$ that is relatively straightforward as the risk-free portfolio value is what trading systems or pricing models normally report.

Second, we assume that there is counterparty credit risk but no collateral agreement. Based on the pricing model proposed by Xiao [2015], we compute the risky value of the portfolio as $V^N = 2{,}688{,}014$ after considering counterparty credit risk. By definition, the CVA without collateralization is equal to $CVA_N = V^F - V^N = 49{,}688.$

Next, we further assume that there is a CSA agreement in which the threshold is 2 million and the MTA is 100,000. The risky value of the portfolio is calculated as $V^C = 2{,}725{,}094$ according to Proposition 2. The CVA with collateralization is given by $CVA_C = V^F - V^C = 12{,}608$.

Full collateralization makes a portfolio appear to be risk-free. An increase in collateral threshold leads to a rise in unsecured credit exposure, and thereby an increase in CVA. In particular, CVA reaches the maximum when the threshold is infinite representing no collateral arrangement.

We extend our analysis to other CSA portfolios. The results hold across different CSA counterparties and collateral agreements. Our findings show that collateral posting can reduce credit risk and CVA. The results also suggest a negative correlation between collateralization and CVA as an increase in collateralization causes a decrease in CVA charge and vice-versa. These findings improve our understanding of the relationship between collateralization and CVA.

We study how collateralization impacts market risk by gauging VaR changes due to collateral arrangements. VaR is the regulatory measurement for assessing market risk. It is defined as the maximum loss likely to be suffered on a portfolio for a given probability defined as a confidence level over a given period time. In its most general form, VaR measures 10-day 99th percentile of potential loss that can be incurred.

There are three commonly used methodologies to calculate VaR – parametric, historical simulation and Monte Carlo simulation. Parametric model estimates VaR directly from the standard deviation of portfolio returns typically assuming returns are normally distributed. Historical simulation calculates VaR from the distribution of actual historical returns. Whilst Monte Carlo simulation computes VaR from a distribution constructed from random outcomes. In this paper, we calculate historical VaR.

For monitoring market risk, many organizations segment portfolios in some manner. They may do so by traders and trading desks. Typically, a market risk portfolio contains derivatives across multiple counterparties, while a counterparty portfolio comprises transactions among different traders and trading desks. We fortuitously select a trading portfolio and then partition it into single-counterparty sub-portfolios. One sub-portfolio contains 172 interest rate swaps, 68 caps and floors, 25 European swaptions and 17 cancelable swaps.

First, we compute the VaR of the sub-portfolio without considering credit risk. The calculated result as of May 11, 2012 is -386,570. Here the VaR is a negative value by definition. However, the industry convention is to report VaR as a positive number that is the amount of money one can lose. Thus, people in the market usually say that the VaR is 386,570 in this case. This convention can be confusing in places. If one says that VaR increases, the numbers actually become smaller or move into the left tail.

For many reasons, both historical and practical, market and credit risk have often been treated as if they are unrelated source of risk: the risk types have been measured separately, managed separately, and economic capital against each risk type has been assessed separately.

However, market risk and credit risk actually reinforce each other. Default-induced credit losses can be driven by market price changes. At the same time, the changes in prices depend on the rating migration or increase in the default perception of the firm. The Basel Committee on Banking Supervision (see BCBS [2009]) also finds that those banks that were more severely affected in the global financial crisis measured their market and credit risk separately, whereas banks that used an integrated approach to market and credit risk measurement were much less impacted. Measuring market risk without considering credit risk may mask the significant impact of credit risk and often leads to underestimation of risk.

Therefore, we further calculate VaR by taking credit risk into account. Let us first assume that there is no CSA agreement. Each trade is valued by the risky model developed by Xiao [2015]. The VaR with credit risk is computed as -418,948. It can be seen that VaR has increased from 386,570 to 418,948 after accounting for credit risk. The above empirical results show that market fluctuation has a larger impact on VaR when credit risk is taken into account. Our research highlights the linkage between market and credit risk.

Our findings suggest that there are important interactions between market and credit risk. We find a positive correlation between market and credit risk as they increase or decrease together. We also find that collateralization is actually negatively correlated with market risk, i.e., an increase in collateralization causes a drop in market risk. Said differently, collateral posting can reduce market risk. Our research contributes to the understanding of the interaction between market and credit risk.

Conclusion

We present a new model for pricing collateralized financial contracts based on the fundamental principal and legal structure of CSA. The model can back out market prices. This is very useful for pricing outstanding collateralized derivatives.

Empirically, we use a unique proprietary dataset to measure the effect of collateralization on pricing and compare it with model-implied prices. The empirical results show that the model-implied prices are quite close to the market-quoted prices, suggesting that the model is fairly accurate on pricing collateralized derivatives.

We find strong evidence that counterparty credit risk alone plays a significant but not overwhelming role in determining credit-related spreads. Only the joint effect of collateralization and credit risk has high explanatory power on unsecured credit costs.

We also find evidence that there is a strong linkage between market and credit risk. Our research results suggest that banks and regulators need to think about an integrated framework to capture material interactions of these two types of risk. This requires all profits and losses are gauged in a consistent way across risk types as they tend to be driven by the same economic factors. Our finding leads to an improved understanding of the interaction between market and credit risk and how this interaction is related to risk measurement and management.

References

Edwards, F. and Morrison, E. “Derivatives and the bankruptcy code: why the special treatment?” Yale Journal on Regulation, 22 (2005), pp. 101-133.

Feldhutter, P. and Lando, D. “Decomposing swap spreads. Journal of Financial Economics,” 88 (2008), pp. 375-405.

FinPricing, 2017, Valuation service, https://finpricing.com/lib/EqBarrier.html

Fuijii, M. and Takahahsi, A. “Collateralized credit default swaps and default dependence: Implications for the central counterparties.” Journal of Credit Risk, 8(2012), pp. 1-10.

Garlson, D. “Secured creditors and the eely character of bankruptcy valuations.” 41 American University Law Review, 63 (1991-1992), pp. 63-106.

Grinblatt, M., “An analytic solution for interest rate swap spreads.” Review of International Finance, 2 (2001), pp. 113-149.

Johannes, M. and Sundaresan, S. “The impact of collateralization on swap rates.” Journal of Finance, 62 (2007), pp. 383-410.

Liu, J., Longstaff, F. and Mandell, R. “The Market Price of Risk in Interest Rate Swaps: The Roles of Default and Liquidity Risks,” The Journal of Business, 79 (2006), pp. 2337-2360.

Minton, B. “An Empirical Examination of Basic Valuation Models for Plain Vanilla U.S. Interest Rate Swaps.” Journal of Financial Economics, 44 (1997), pp. 251-277.

Moody’s Investor’s Service. “Historical default rates of corporate bond issuers, 1920-99.” 2000

Piterbarg, V. “Funding beyond discounting: collateral agreements and derivatives pricing.” Risk Magazine, 2010 (2010), pp. 97-102.

Routh, R. and Douglas, M. “Default interest payable to oversecured creditor subject to reasonableness limitation.” ABF Journal, Vol. 4, No. 3 (2005), pp. 1-10.