



# Bond Future Option Valuation Guide



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## Bond Future Option

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## Bond Future Option

### Bond Future Option Introduction

- A bond future option is an option contract that gives the holder the right but not the obligation to buy or sell a bond future at a predetermined price.
- The writer/seller receives a premium from the buyer for undertaking this obligation.
- Options are leveraged instruments that allow the owner to control a large amount of the underlying asset with a smaller amount of money.
- Bond future options offer significant advantages for reducing costs, enhancing returns and managing risk.
- Bond future options could be European style or American style.





## Bond Future Option

### The Use of Bond Future Options

- Bond futures options are also exchange traded derivatives on treasury instruments.
- Bond future options provide market participants with the ability to adjust their interest rate exposures.
- A bond future option is also a good tool for hedging, income enhancement, duration adjustments, interest rate speculation and spread trading.
- Investors use bond future options to hedge an existing portfolio against adverse interest rate movements or enhance the long-term performance.
- Arbitrageurs profit from the price difference between the spot bonds and the bond futures.
- Speculators use bond future option in the hope of making a profit on short-term movements in prices.



## Bond Future Option

### Valuation: European Style

- The present value of a call bond future option is represented as:

$$PV(0) = N[F_T \Phi(d_1) - K \Phi(d_2)] D_T$$

- The present value of a put bond future option is represented as:

$$PV(0) = N[K \Phi(-d_2) - F_T \Phi(-d_1)] D_T$$

where

- $d_{1,2} = [\ln(F_T/K) \pm \sigma^2 T/2] / \sigma \sqrt{T}$
- $F_T = [(P - C_\Sigma) \exp(r_T T) - A] / CF$  the forward clean price of the delivered bond (CTD) at *time 0*.
- $C_\Sigma = \sum_{t_i \leq T} C \exp(-r_i t_i)$  the summed present value of all coupons of the underlying bond between 0 and *T*.
- $K$  the strike.



## Bond Future Option

### Valuation: European Style (Cont)

- $N$  the notional.
- $T$  the option maturity date.
- $D_T$  the discount factor.
- $CF$  the conversion factor for a bond to deliver in a bond futures contract.
- $A$  the accrual interest before  $T$ .
- $P$  the bond dirty price at 0.
- $r_T$  the continuously compounded interest rate between  $t$  and  $T$ .
- $\sigma = \alpha Dy\sigma_y/CF$  the volatility of forward bond price.



## Bond Future Option

### Valuation (Cont)

- $\sigma_y$  the forward yield volatility of the CTD bond of the underlying futures. We use the swaption volatility
- $\alpha$  the implied volatility scaling factor.
- $y$  the forward yield that can be solved by

$$P - C_{\Sigma} = \sum_{T \leq t_i \leq T_B} C e^{-y t_i}$$

- $T_B$  the maturity of the underlying CTD bond
- $D = \frac{\sum_{T \leq t_i \leq T_B} t_i C e^{-y t_i}}{\sum_{T \leq t_i \leq T_B} C e^{-y t_i}}$  the forward modified duration of the CTD bond of the underlying futures



## Bond Future Option

### Valuation: American Style

- We use the Cox-Ross-Rubinstein (CRR) binomial tree to price American bond future option.
- Build forward bond price tree.

$$F_0 = [(P - C_{\Sigma}) \exp(r_T T) - A] / CF$$

$$F_j^u = F_j e^{\sigma \sqrt{\Delta t}} \text{ with probability } p = \frac{1 - e^{-\sigma \sqrt{\Delta t}}}{e^{\sigma \sqrt{\Delta t}} - e^{-\sigma \sqrt{\Delta t}}} \text{ where } \Delta t = T/m$$

$$F_j^d = F_j e^{-\sigma \sqrt{\Delta t}} \text{ with probability } 1-p \text{ where } j = 1, \dots, m$$

$$\sigma = \alpha D y \sigma_y / CF \text{ is the volatility described above}$$

- After constructing the tree, valuation is performed backward until the valuation date. The option value at node 0 is the present value of the bond future option.





# Bond Future Option

## Practical Guide

- First compute the CTD forward bond price first.
- Then determine the volatility of the forward bond price.
- After that, call Black formula for pricing European bond future options.
- Or build binomial tree to value American bond future options.



# Bond Future Option

## A Real World Example

Option Specification		Future Specification	
Buy Sell	Buy	Contract Size	33333.33
Call Put	Call	Conversion Factor	0.8851
Currency	USD	First Delivery Date	6/1/2017
Option Maturity Date	5/26/2017	Last Delivery Date	6/30/2017
Option Expiry Date	5/26/2017	Future Ticker	USM17
Strike	151	Future Ticker Size	32
Option Ticker	USM17C 151	Future Ticker Value	31.25
Settlement Amount	-173875	Number of Contract	52
Settlement Date	2/9/2017	Quote Price	3.34375
		Trade Date	2/9/2017
		Future Maturity Date	6/21/2017
		Underlying Bond Type	UST
		Underlying Bond Coupon	0.05
		Underlying Bond Maturity Date	5/15/2037



# Thank You

You can find more information at

<http://www.finpricing.com/lib/FiBondFutureOption.html>