Interest Rate Cancelable Swap Valuation and Risk

FinPricing

Summary

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- Bermudan Swaption Payoffs
- Valuation Model Selection Criteria
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- LGM Assumption
- LGM calibration
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Cancelable Swap Definition

- A cancelable swap gives the holder the right but not the obligation to cancel the swap at predetermined dates prior to maturity.
- It can be decomposed into a vanilla swap and a Bermudan swaption.

$$PV_{CancellablePayerSwap} = PV_{PayerSwap} - PV_{ReceiverBermudanSwaption}$$

$$PV_{CancellableReceiverSwap} = PV_{ReceiverSwap} - PV_{PayerBermudanSwaption}$$

- A vanilla swap is well understood. Hence we focus on Bermudan swaption for the rest of this presentation.
- A Bermudan swaption gives the holder the right but not the obligation to enter an interest rate swap at predefined dates.

Bermudan Swaption Payoffs

- At the maturity T, the payoff of a Bermudan swaption is given by

$$Payoff(T) = \max(0, V_{swap}(T))$$

where $V_{swap}(T)$ is the value of the underlying swap at T.

- At any exercise date T_i, the payoff of the Bermudan swaption is given by

$$Payoff(T_i) = max\left(V_{swap}(T_i), I(T_i)\right)$$

where $V_{swap}(T_i)$ is the exercise value of the Bermudan swap and $I(T_i)$ is the intrinsic value.

Model Selection Criteria

- Given the complexity of Bermudan swaption valuation, there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numeric solution to price Bermudan swaptions numerically.
- The selection of interest rate term structure models
  - Popular interest rate term structure models:

    Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).
  - HJM and LMM are too complex.
  - Hull-White is inaccurate for computing sensitivities.
  - Therefore, we choose either LGM or QGM.

Cancelable Swap

Model Selection Criteria (Cont)

- The selection of numeric approaches
  - After selecting a term structure model, we need to choose a numeric approach to approximate the underlying stochastic process of the model.
  - Commonly used numeric approaches are tree, partial differential equation (PDE), lattice and Monte Carlo simulation.
  - Tree and Monte Carlo are notorious for inaccuracy on sensitivity calculation.
  - Therefore, we choose either PDE or lattice.
- Our decision is to use LGM plus lattice.

LGM Model

- The dynamics

$$dX(t) = \alpha(t)dW$$

where X is the single state variable and W is the Wiener process.

- The numeraire is given by

$$N(t,X) = \left(H(t)X + 0.5H^2(t)\zeta(t)\right)/D(t)$$

- The zero coupon bond price is

$$B(t,X;T) = D(T)exp\left(-H(t)X - 0.5H^2(t)\zeta(t)\right)$$

LGM Assumption

- The LGM model is mathematically equivalent to the Hull-White model but offers
  - Significant improvement of stability and accuracy for calibration.
  - Significant improvement of stability and accuracy for sensitivity calculation.
- The state variable is normally distributed under the appropriate measure.
- The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfected correlated.

LGM calibration

- Match today's curve

  At time $t=0$, $X(0)=0$ and $H(0)=0$. Thus $Z(0,0;T)=D(T)$. In other words, the LGM automatically fits today's discount curve.

- Select a group of market swaptions.
- Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.

Cancelable Swap

Valuation Implementation

- Calibrate the LGM model.
- Create the lattice based on the LGM: the grid range should cover at least 3 standard deviations.
- Calculate the underlying swap value at each final note.
- Conduct backward induction process iteratively rolling back from final dates until reaching the valuation date and also Compare exercise values with intrinsic values at each exercise date.
- The value at the valuation date is the price of the Bermudan swaption.
- The final value of the cancelable swap is given by

$$PV_{CancellablePayerSwap} = PV_{PayerSwap} - PV_{ReceiverBermudanSwaption}$$
$$PV_{CancellableReceiverSwap} = PV_{ReceiverSwap} - PV_{PayerBermudanSwaption}$$

Cancelable Swap

cancelable swap definition		
Counterparty	xxx	
Buy or sell	Buy	
Payer or receiver	Payer	
Currency	USD	
Settlement	Physical	
Trade date	9/12/2012	
Underlying swap definition	**Leg 1**	**Leg2**
Day Count	dcAct360	dcAct360
Leg Type	Fixed	Float
Notional	250000	250000
Payment Frequency	1	1
Pay Receive	Receive	Pay
Start Date	9/14/2012	9/14/2012
End Date	9/14/2022	9/14/2022
Fix rate	0.0398	NA
Index Type	NA	LIBOR
Index Tenor	NA	1M
Index Day Count	NA	dcAct360
Exercise Schedules		
Exercise Type	Notification Date	Settlement Date
Call	1/12/2017	1/14/2017
Call	1/10/2018	1/14/2018

A real world example

Thanks!

You can find more details at
https://finpricing.com/lib/IrCurveIntroduction.html