



Financial Sensitivity

Sensitivity

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Financial Sensitivity Definition

- ◆ Financial sensitivity is the measure of the value reaction of a financial instrument to changes in underlying factors.
- ◆ The value of a financial instrument is impacted by many factors, such as interest rate, stock price, implied volatility, time, etc.
- ◆ Financial sensitivities are also called Greeks, such as Delta, Gamma, Vega and Theta.
- ◆ Financial sensitivities are risk measures that are more important than fair values.
- ◆ They are vital for risk management: isolating risk, hedging risk, explaining profit and loss, etc.

Delta Definition

- ◆ Delta is a first-order Greek that measures the value change of a financial instrument with respect to changes in the underlying asset price.

- ◆ Interest rate Delta:

$$IrDelta = \frac{\partial V}{\partial r} = \frac{V(r + 0.0001) - V(r)}{0.0001}$$

where $V(r)$ is the instrument value and r is the underlying interest rate.

- ◆ PV01, or dollar duration, is analogous to interest rate Delta but has the change value of a one-dollar annuity given by

$$PV01 = V(r + 0.0001) - V(r)$$

Delta Definition (Cont)

- ◆ Credit Delta applicable to fixed income and credit product is given by

$$\text{CreditDelta} = \frac{\partial V}{\partial c} = \frac{V(c + 0.0001) - V(c)}{0.0001}$$

where c is the underlying credit spread.

- ◆ CR01 is analogous to credit Delta but has the change value of a one-dollar annuity given by

$$PV01 = V(r + 0.0001) - V(r)$$

- ◆ Equity/FX/Commodity Delta

$$\text{Delta} = \frac{\partial V}{\partial S} = \frac{V(1.01S) - V(S)}{0.01 * S}$$

where S is the underlying equity price or FX rate or commodity price

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Vega Definition

- ◆ Vega is a first-order Greek that measures the value change of a financial instrument with respect to changes in the underlying implied volatility.

$$Vega = \frac{\partial V}{\partial \sigma} = \frac{V(\sigma + \Delta\sigma) - V(\sigma)}{\Delta\sigma}$$

where σ is the implied volatility.

- ◆ Only non-linear products, such as options, have Vegas.

Gamma Definition

- ◆ Gamma is a second order Greek that measures the value change of a financial instrument with respect to changes in the underlying price.

$$Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{V(S + 0.5 * \Delta S) + V(S - 0.5 * \Delta S) - 2V(S)}{\Delta S^2}$$

Theta Definition

- ◆ Theta is a first order Greek that measures the value change of a financial instrument with respect to time.

$$Theta = \frac{\partial V}{\partial t} = \frac{V(t + \Delta t) - V(t)}{\Delta t}$$

Curvature Definition

- ◆ Curvature is a new risk measure for options introduced by Basel FRTB.
- ◆ It is a risk measure that captures the incremental risk not captured by the delta risk of price changes in the value of an option.

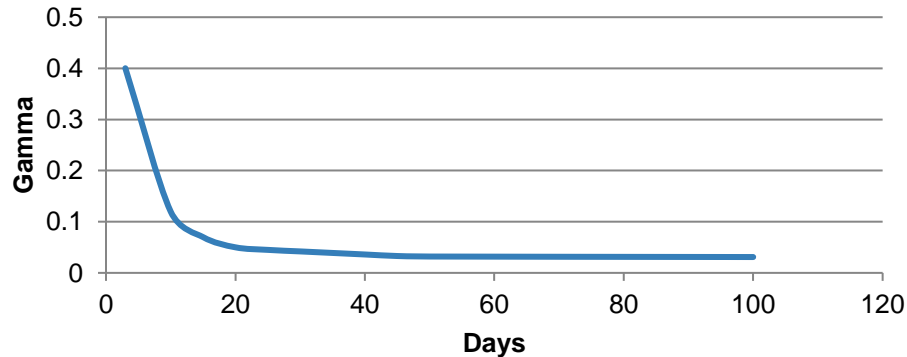
$$Curvature = \min\{V(S + \Delta W) - V(S) - \Delta W * Delta, V(S - \Delta W) - V(S) - \Delta W * Delta\}$$

where ΔW is the risk weight.

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Option Sensitivity Pattern

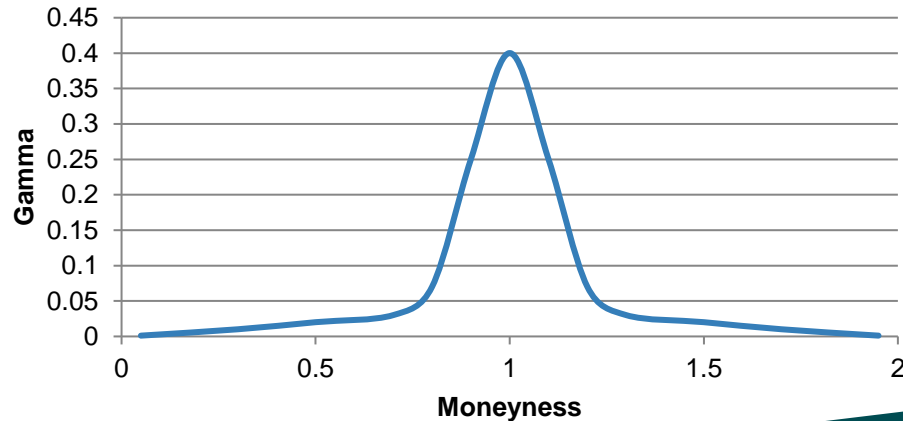
- ◆ Sensitivity behaviors are critical for managing risk.
- ◆ Gamma
 - ◆ Gamma behavior in relation to time to maturity shown below.
 - ◆ Gamma has a greater effect on shorter dated options.



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Option Sensitivity Pattern (Cont)

- ◆ Gamma behavior in relation to moneyness shown below.
- ◆ Gamma has the greatest impact on at-the-money options.

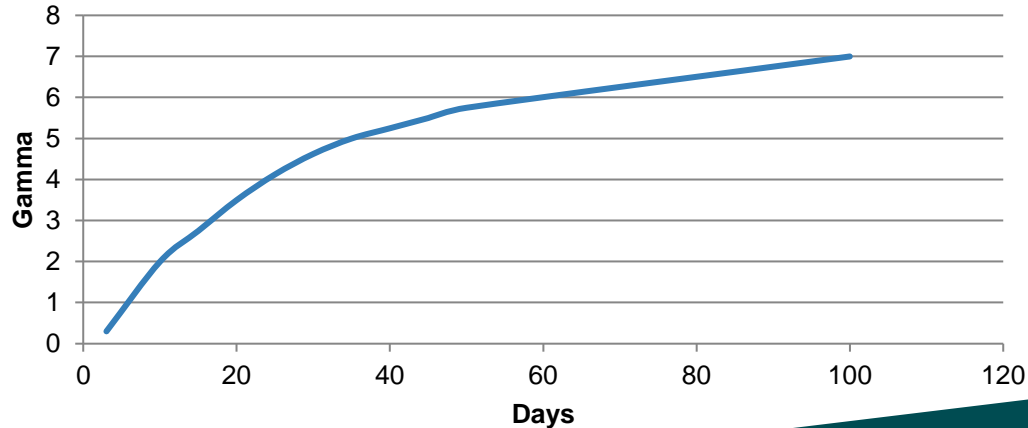


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Option Sensitivity Pattern (Cont)

◆ Vega

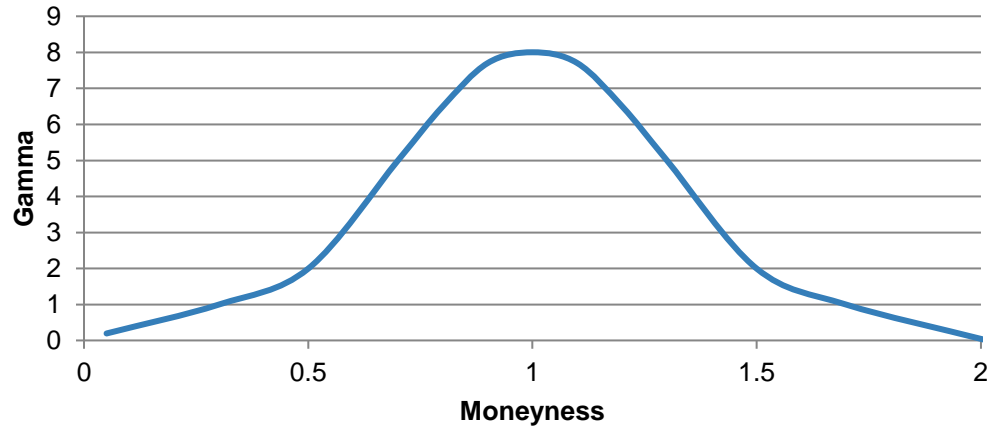
- ◆ Vega behavior in relation to time to maturity shown below.
- ◆ Vega has a greater effect on longer dated options.



Sensitivity

Option Sensitivity Pattern (Cont)

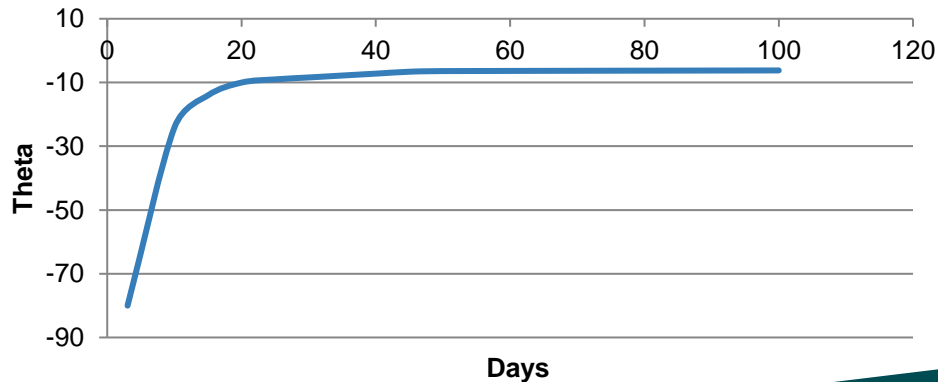
- ◆ Vega behavior in relation to moneyness shown below.
- ◆ Vega has the greatest impact on at-the-money options.



Sensitivity

Option Sensitivity Pattern (Cont)

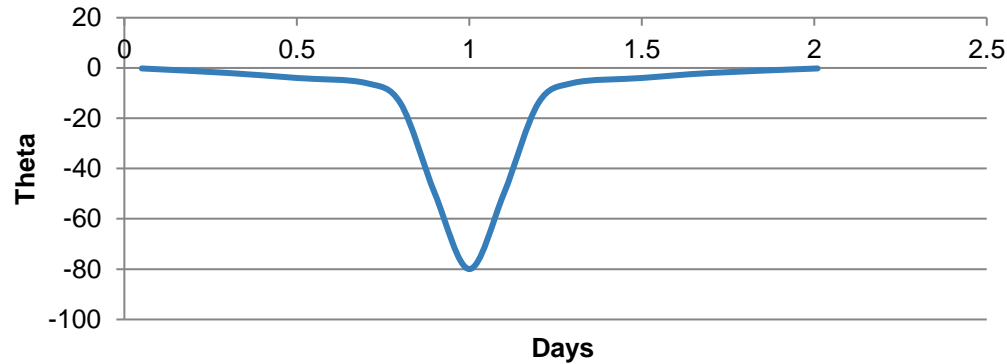
- ◆ Theta or time decay
 - ◆ Theta is normally negative except some deeply in-the-money deals.
 - ◆ Theta behavior in relation to time to maturity shown below.
 - ◆ Theta has a greater effect on shorter dated options.



Sensitivity

Option Sensitivity Pattern (Cont)

- ◆ Theta behavior in relation to moneyness shown below.
- ◆ Theta has the biggest impact on at-the-money options.



Sensitivity Hedging

- ◆ The objective of hedging is to have a lower price volatility that eliminates both downside risk (loss) and upside profit.
- ◆ Hedging is a double-edged sword.
- ◆ The profit of a broker or an investment bank comes from spread rather than market movement. Thus it is better to hedge all risks.
- ◆ Delta is normally hedged.
- ◆ Vega can be hedged by using options.
- ◆ Gamma is hardly hedged in real world.

Sensitivity Profit & Loss (P&L)

- ◆ Hypothetic P&L is the P&L that is purely driven by market movement.
- ◆ Hypothetic P&L is calculated by revaluing a position held at the end of the previous day using the market data at the end of the current day, i.e.,

$$\text{HypotheticalP\&L} = V(t-1, P_{t-1}, M_t) - V(t-1, P_{t-1}, M_{t-1})$$

where $t-1$ is yesterday; t is today; P_{t-1} is the position at yesterday; M_{t-1} is yesterday's market and M_t is today's market.

- ◆ Sensitivity P&L is the sum of Delta P&L, Vega P&L and Gamma P&L.
- ◆ Unexplained P&L = HypotheticalP&L – SensitivityP&L.

Sensitivity Profit & Loss (Cont)

- ◆ Delta P&L:

$$DeltaP\&L = Delta * (S_t - S_{t-1})$$

where S_t is today's underlying price and S_{t-1} is yesterday's underlying price.

- ◆ Vega P&L:

$$VegaP\&L = Vega * (\sigma_t - \sigma_{t-1})$$

where σ_t is today's implied volatility and σ_{t-1} is yesterday's implied volatility.

- ◆ Gamma P&L:

$$GammaP\&L = 0.5 * Gamma * (S_t - S_{t-1})^2$$

Backbone Adjustment

- ◆ Backbone adjustment is an advanced topic in sensitivity P&L.
- ◆ It can be best explained mathematically.
- ◆ Assume the value of an option is a function of the underlying price S and implied volatility σ , i.e., $V = F(S, \sigma)$.
- ◆ If the implied volatility is a function of the ATM volatility and strike (sticky strike assumption), i.e., $\sigma = \sigma_A + f(K)$, the first order approximation of the option value is

$$\Delta V = \frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial \sigma_A} d\sigma_A = \text{DeltaP\&L} + \text{VegaP\&L}$$

$$\text{where } \text{DeltaP\&L} = \frac{\partial F}{\partial S} dS \text{ and } \text{VegaP\&L} = \frac{\partial F}{\partial \sigma_A} d\sigma_A$$

Backbone Adjustment (Cont)

- ◆ If the implied volatility is a function of the ATM volatility and moneyness K/S (sticky moneyness or stricky Delta assumption), i.e., $\sigma = \sigma_A + f(S, K)$, the first order approximation of the option value is

$$\Delta V = \frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial \sigma_A} d\sigma_A + \frac{\partial F}{\partial \sigma} \frac{\partial \sigma}{\partial S} dS = \text{DeltaP\&L} + \text{VegaP\&L}$$

$$\text{where } \text{DeltaP\&L} = \left(\frac{\partial F}{\partial S} + \frac{\partial F}{\partial \sigma} \frac{\partial \sigma}{\partial S} \right) dS \text{ and } \text{VegaP\&L} = \frac{\partial F}{\partial \sigma_A} d\sigma_A$$

- ◆ Under sticky moneyness/Delta assumption, the DeltaP&L above has one more item, i.e., $\frac{\partial F}{\partial \sigma} \frac{\partial \sigma}{\partial S} dS$ that is the backbone adjustment.



Thanks!



You can find more details at
<https://finpricing.com/faq.html>

