
# Interest Rate Cancelable Swap Valuation and Risk

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# Summary

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- Bermudan Swaption Payoffs
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# Cancelable Swap Definition

- A cancelable swap gives the holder the right but not the obligation to cancel the swap at predetermined dates prior to maturity.
- It can be decomposed into a vanilla swap and a Bermudan swaption.

$$PV_{CancellablePayerSwap} = PV_{PayerSwap} - PV_{ReceiverBermudanSwaption}$$
$$PV_{CancellableReceiverSwap} = PV_{ReceiverSwap} - PV_{PayerBermudanSwaption}$$

- A vanilla swap is well understood. Hence we focus on Bermudan swaption for the rest of this presentation.
- A Bermudan swaption gives the holder the right but not the obligation to enter an interest rate swap at predefined dates.

# Bermudan Swaption Payoffs

- At the maturity T, the payoff of a Bermudan swaption is given by

$$Payoff(T) = \max(0, V_{swap}(T))$$

where $V_{swap}(T)$ is the value of the underlying swap at T.

- At any exercise date $T_i$, the payoff of the Bermudan swaption is given by

$$Payoff(T_i) = max\big(V_{swap}(T_i), I(T_i)\big)$$

where $V_{swap}(T_i)$ is the exercise value of the Bermudan swap and $I(T_i)$ is the intrinsic value.

# Model Selection Criteria

- Given the complexity of Bermudan swaption valuation, there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numeric solution to price Bermudan swaptions numerically.
- The selection of interest rate term structure models
  - Popular interest rate term structure models:

    Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).
  - HJM and LMM are too complex.
  - Hull-White is inaccurate for computing sensitivities.
  - Therefore, we choose either LGM or QGM.

# Model Selection Criteria (Cont)

- The selection of numeric approaches
  - After selecting a term structure model, we need to choose a numeric approach to approximate the underlying stochastic process of the model.
  - Commonly used numeric approaches are tree, partial differential equation (PDE), lattice and Monte Carlo simulation.
  - Tree and Monte Carlo are notorious for inaccuracy on sensitivity calculation.
  - Therefore, we choose either PDE or lattice.
- Our decision is to use LGM plus lattice.

# LGM Model

- The dynamics

$$dX(t) = \alpha(t)dW$$

where X is the single state variable and W is the Wiener process.

- The numeraire is given by

$$N(t,X) = \left(H(t)X + 0.5H^2(t)\zeta(t)\right)/D(t)$$

- The zero coupon bond price is

$$B(t,X;T) = D(T)exp\left(-H(t)X - 0.5H^2(t)\zeta(t)\right)$$

# LGM Assumption

- The LGM model is mathematically equivalent to the Hull-White model but offers
  - Significant improvement of stability and accuracy for calibration.
  - Significant improvement of stability and accuracy for sensitivity calculation.
- The state variable is normally distributed under the appropriate measure.
- The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfected correlated.

# LGM calibration

- Match today's curve

  At time t=0, X(0)=0 and H(0)=0. Thus Z(0,0;T)=D(T). In other words, the LGM automatically fits today's discount curve.

- Select a group of market swaptions.
- Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.

# Cancelable Swap

## Valuation Implementation

- Calibrate the LGM model.
- Create the lattice based on the LGM: the grid range should cover at least 3 standard deviations.
- Calculate the underlying swap value at each final note.
- Conduct backward induction process iteratively rolling back from final dates until reaching the valuation date and also Compare exercise values with intrinsic values at each exercise date.
- The value at the valuation date is the price of the Bermudan swaption.
- The final value of the cancelable swap is given by

$$PV_{CancellablePayerSwap} = PV_{PayerSwap} - PV_{ReceiverBermudanSwaption}$$
$$PV_{CancellableReceiverSwap} = PV_{ReceiverSwap} - PV_{PayerBermudanSwaption}$$

# Cancelable Swap

## A real world example

| cancelable swap definition | | |
|---|---|---|
| Counterparty | xxx | |
| Buy or sell | Buy | |
| Payer or receiver | Payer | |
| Currency | USD | |
| Settlement | Physical | |
| Trade date | 9/12/2012 | |
| **Underlying swap definition** | **Leg 1** | **Leg2** |
| Day Count | dcAct360 | dcAct360 |
| Leg Type | Fixed | Float |
| Notional | 250000 | 250000 |
| Payment Frequency | 1 | 1 |
| Pay Receive | Receive | Pay |
| Start Date | 9/14/2012 | 9/14/2012 |
| End Date | 9/14/2022 | 9/14/2022 |
| Fix rate | 0.0398 | NA |
| Index Type | NA | LIBOR |
| Index Tenor | NA | 1M |
| Index Day Count | NA | dcAct360 |
| **Exercise Schedules** | | |
| Exercise Type | Notification Date | Settlement Date |
| Call | 1/12/2017 | 1/14/2017 |
| Call | 1/10/2018 | 1/14/2018 |

# Thanks!



You can find more details at
http://www.finpricing.com/lib/IrCancelableSwap.html