Dividend Risk Model Calibration

Calibration is based on two-year time series of Bloomberg data field BEST_DPS with BEST_FPERIOD_OVERRIDE set to be 1BF, meaning one-year forward rolling window. Some adjustment may be needed to remove data error, such as beginning of year adjustment.

As described in model description and calibration of the model, the daily innovation to expected dividend of an index is simulated from double exponential distribution with specified correlation with all other risk factors. The daily innovation to expected dividend of a single stock is simulated on the fly without correlation with other risk factors.

In theory, index dividend yield can be derived from the corresponding underlying index spot price and index futures price.

For each futures contract with a given delivery date T, we have

$$F_T(t) = E_t^Q\left[S(T)\right]$$

$$= S(t)\cdot \exp\left((r_t - y_t)\cdot(T-t)\right) \qquad (2.1)$$

Where

r_t: libor rate at time t for T-t term, r_t is market observable.

y_t: dividend yield at time t for T-t term

$S(t)$: index price at time t

$F_T(t)$: index T-expiry futures price at time t

We can re-arrange (2.1) to give explicit formula for dividend yield

$$y_t = r_t - \frac{1}{T-t}\ln\left(\frac{F_T(t)}{S(t)}\right) \qquad (2.2)$$

By following the above procedure, we will be able to calculate an annualized dividend yield for SPX index for each futures contract in any business day. Usually the trading on the longer term futures is not as active (liquid) as the front contract so the price on longer term futures may be less reliable as the price of the front contract. For simplicity, we will take the average of the yields calculated based on different futures contract as the dividend yield of the underlying index. This will reduce the term structure of the dividend yield to a scalar valued annualized dividend yield. The rationale is mainly based on practicality of reducing unnecessary risk factors. But it is also based on the fact that the long term yield is very uncertain due to illiquid calibration instruments.

With dividend yield time series, y_t, calculated, daily change in dividend expectation ξ_t can also be calculated from (1.7)

$$\xi_{t+1} = \frac{y_{t+1}}{y_t} \cdot \frac{S_{t+1}}{S_t} - 1 \qquad (2.3)$$

The calculated time series ξ_t can be used to calibrate to model described in (1.10).

However, we found that the day-to-day volatility of expected dividend calculated from index futures is extraordinarily high. For example, the empirical one-day volatility of expected dividend of SPX index is above 40%, which is not reasonable. A careful investigation leads to the following findings.

- Futures market and stock market do not close at the same time, causing unsynchronized close price of futures contract and equity index close points.
- Even using the intraday trading data to get the synchronized closing rate of futures and equity index, a small discrepancy can still cause big error in annualized dividend yield due to short time to maturity, since the most liquid futures contracts are front contract of less than three months to maturity.
- The front contract implied dividend yield is only indicative of the expected dividend amount up to the futures expiry. The uncertainty about dividend amount diminishes as observation time approaches to futures expiry date.
- The futures other than the front contract are illiquid and hence the closing rate is not reliable to be used to back out yield.
- No-arbitrage argument assumes no friction, infinitely liquid market, that is, no transaction cost, bid/ask spread, minimum tick, and margin requirement, market price is not impacted by any order. None of these hold in reality.

We believe that the overly volatile implied dividend from futures market is due to the noise resulting from the above market technicalities. The noise is too large relative to signal that the results are simply not indicative. Therefore, we do not use the futures implied dividend yield approach.

Bloomberg provides projected dividend data for each index at each trading day. The projected dividend amount, according to Bloomberg, has taken into account of information from seven related areas including corporate guidance, historical pattern, and put-call parity. It claims an accuracy rate of over 80% in 2010 and is widely used by most traders as the expected dividend amount for pricing equity derivatives and equity-linked notes. Since the projection is forward looking and incorporate market price of traded options, we deem that this Bloomberg projected dividend amount is a reasonable proxy for expected dividend.

The model parameter is calibrated to fit the empirical variance of ξ_t, one-day relative change of expected dividend.

We can also calculate the correlation between daily return on stock price and relative changes of expected dividend from the above data. Standard correlation technique will be applied to calculate pair-wise correlation.

We will choose one index for each major equity market. Currently we use S&P500, SPTSX Composite, DJ Euro STOXX 50, Nikkei 225 and FTSE100 for stocks traded in USD, CAD, EUR, JPY and GBP, respectively. These dividend risk factors are considered market risk factors and go into global correlation matrix.

The challenge in calibrating daily innovation in dividend expectation of an equity basket is due to lack of data. Most likely there is no data directly available for a customized equity basket underlying a derivative trade. Since dividend yield of the basket is used as pricing input, and delta is calculated with respect to the price of the basket rather than each of the constituent stock, it is not practical to obtain the innovation in dividend of the basket based on that of the constituent stocks. Mapping to an index is considered a reasonable approach. The mapping is based on the market of the underlying basket's constituent stocks. The equity index for that market is mapped as the proxy for the purpose of dividend risk.

In theory, ξ can be calibrated from stock options in a similar way as index ξ is calibrated from index futures. Based on Put-Call parity we can synthetically obtain a forward of the underlying stock. Specifically, for a given maturity, the forward price is

$$F_T(t) = K + e^{r \cdot T} \cdot \left[C\left(t, S(t), K, T\right) - P\left(t, S(t), K, T\right) \right] \qquad (2.4)$$

However, in practice there are many difficulties that hinder the application of (2.4). First, Put-Call parity does not hold for American options – most exchange-traded options are American options. This is evidenced by the two distinct implied volatilities calculated for put and call options at the same strike. Other factors in real trading also cause violation of Put-Call parity [2]. In addition, the exchanged traded stock options for longer maturity are not liquid, thus the data is not reliable.

For the above reasons, we resort to Bloomberg projected dividend amount to calibrate ξ. Our preliminary investigation indicates that the data is available for most of dividend-paying stocks.

The jump frequency and jump size parameters are estimated from the day-to-day change of the projected dividend amount time series. For missing data, we will assume that the projected dividend amount stay the same as the previous day during the period. This will cause a reduced jump frequency and an elevated jump size of daily relative changes of expected dividend. The net effect in terms of extreme P&L scenario tends to lead to fatter tails, hence a conservative estimate of risk.

Specifically, the jump frequency is the percentage of number of occurrence of non-zero day-to-day changes of Bloomberg projected dividend amount in total length of the time series. The jump size parameter is calibrated to fit the empirical variance of non-zero day-to-day relative changes. This is same as in the case of equity index.

A sample calibration on GE stock dividend is given in Appendix D.

Correlation of ξ among different stocks and with equity index is difficult to calibrate. The difficulties lie with the lack of overlapping data of sufficient length of Bloomberg projected dividend amount of different stocks. Since the dividend jumps are usually driven by company specific events, we assume zero correlation among the innovation in dividend expectation of different stocks, or between stock and equity index.

The zero correlation assumption may miss the hedging effect between equity index and equity index ETF, which is treated as single stock in our setting. However, this may only over-estimate the dividend risk. A better gauge of correlation may be developed once better quality data become available.

Reference:

https://finpricing.com/lib/IrBasisCurve.html