



# Interest Rate Bermudan Swaption Valuation and Risk

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# Bermudan Swaption

## Summary

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# Bermudan Swaption

## Bermudan Swaption Definition

- ◆ An interest rate Bermudan swaption is an option on an interest rate swap with predefined exercise schedules.
- ◆ A Bermudan swaption gives the holder the right but not the obligation to enter an interest rate swap at predefined dates.
- ◆ Bermudan swaptions give the holders some flexibility to enter swaps.
- ◆ A comparison of European, American and Bermudan swaptions
  - ◆ European swaption has only one exercise date at the maturity.
  - ◆ American swaption has multiple exercise dates (daily)
  - ◆ Bermudan swaption has multiple exercise dates (but not daily): such as quarterly, monthly, etc.

# Bermudan Swaption

## Bermudan Swaption Payoffs

- ◆ At the maturity  $T$ , the payoff of a Bermudan swaption is given by

$$Payoff(T) = \max(0, V_{swap}(T))$$

where  $V_{swap}(T)$  is the value of the underlying swap at  $T$ .

- ◆ At any exercise date  $T_i$ , the payoff of the Bermudan swaption is given by

$$Payoff(T_i) = \max(V_{swap}(T_i), I(T_i))$$

where  $V_{swap}(T_i)$  is the exercise value of the Bermudan swap and  $I(T_i)$  is the intrinsic value.

# Bermudan Swaption

## Model Selection Criteria

- ◆ Given the complexity of Bermudan swaption valuation, there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numeric solution to price Bermudan swaptions numerically.
- ◆ The selection of interest rate term structure models
  - ◆ Popular interest rate term structure models:  
Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).
  - ◆ HJM and LMM are too complex.
  - ◆ Hull-White is inaccurate for computing sensitivities.
  - ◆ Therefore, we choose either LGM or QGM.

## Model Selection Criteria (Cont)

- ◆ The selection of numeric approaches
  - ◆ After selecting a term structure model, we need to choose a numeric approach to approximate the underlying stochastic process of the model.
  - ◆ Commonly used numeric approaches are tree, partial differential equation (PDE), lattice and Monte Carlo simulation.
  - ◆ Tree and Monte Carlo are notorious for inaccuracy on sensitivity calculation.
  - ◆ Therefore, we choose either PDE or lattice.
- ◆ Our decision is to use LGM plus lattice.

# Bermudan Swaption

## LGM Model

- ◆ The dynamics

$$dX(t) = \alpha(t)dW$$

where  $X$  is the single state variable and  $W$  is the Wiener process.

- ◆ The numeraire is given by

$$N(t, X) = (H(t)X + 0.5H^2(t)\zeta(t))/D(t)$$

- ◆ The zero coupon bond price is

$$B(t, X; T) = D(T)\exp(-H(t)X - 0.5H^2(t)\zeta(t))$$

# Bermudan Swaption

## LGM Assumption

- ◆ The LGM model is mathematically equivalent to the Hull-White model but offers
  - ◆ Significant improvement of stability and accuracy for calibration.
  - ◆ Significant improvement of stability and accuracy for sensitivity calculation.
- ◆ The state variable is normally distributed under the appropriate measure.
- ◆ The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfectly correlated.



# Bermudan Swaption

## LGM calibration

- ◆ Match today's curve

At time  $t=0$ ,  $X(0)=0$  and  $H(0)=0$ . Thus  $Z(0,0;T)=D(T)$ . In other words, the LGM automatically fits today's discount curve.

- ◆ Select a group of market swaptions.

- ◆ Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.

# Bermudan Swaption

## Valuation Implementation

- ◆ Calibrate the LGM model.
- ◆ Create the lattice based on the LGM: the grid range should cover at least 3 standard deviations.
- ◆ Calculate the underlying swap value at each final note.
- ◆ Conduct backward induction process iteratively rolling back from final dates until reaching the valuation date.
- ◆ Compare exercise values with intrinsic values at each exercise date.
- ◆ The value at the valuation date is the price of the Bermudan swaption.

# Bermudan Swaption

A real world example

Swaption definition		
Counterparty	xxx	
Buy or sell	Sell	
Payer or receiver	Receiver	
Currency	USD	
Settlement	Cash	
Trade date	9/12/2012	
Underlying swap definition	Leg 1	Leg2
Day Count	dcAct360	dcAct360
Leg Type	Fixed	Float
Notional	250000	250000
Payment Frequency	1	1
Pay Receive	Receive	Pay
Start Date	9/14/2012	9/14/2012
End Date	9/14/2022	9/14/2022
Fix rate	0.0398	NA
Index Type	NA	LIBOR
Index Tenor	NA	1M
Index Day Count	NA	dcAct360
Exercise Schedules		
Exercise Type	Notification Date	Settlement Date
Call	1/12/2017	1/14/2017
Call	1/10/2018	1/14/2018



# Thanks!



You can find more details at  
<http://www.finpricing.com/lib/lrBermudan.html>

