Puttable Bond and Vaulation

Puttable Bond

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Puttable Bond Definition

- A puttable bond is a bond in which the investor has the right to sell the bond back to the issuer at specified times (puttable dates) for a specified price (put price).
- At each puttable date prior to the bond maturity, the investor may sell the bond back to its issuer and get the investment money back.
- The underlying bonds can be fixed rate bonds or floating rate bonds.
- A puttable bond can therefore be considered a vanilla underlying bond with an embedded Bermudan style option.
- Puttable bonds protect investors. Therefore, a puttable bond normally pay the investor a lower coupon than a non-callable bond.

Puttable bond

Advantages of Puttable Bond

- Although a puttable bond is a lower income to the investor and an uncertainty to the issuer comparing to a regular bond, it is actually quite attractive to both issuers and investors.
- For investors, puttable bonds allow them to reduce interest costs at a future date should rate increase.
- For issuers, puttable bonds allow them to pay a lower interest rate of return until the bonds are sold back.
- If interest rates have increased since the issuer first issues the bond, the investor is like to put its current bond and reinvest it at a higher coupon.

Puttable Bond

Puttable Bond Payoffs

- At the bond maturity *T*, the payoff of a Puttable bond is given by

$$V_p(t) = \begin{cases} F + C & if\ not\ ptted \\ \max(P_p, F + C) & if\ putted \end{cases}$$

where *F* – the principal or face value; *C* – the coupon; P_p – the call price; *min* (*x*, *y*) – the minimum of x and y

- The payoff of the Puttable bond at any call date T_i can be expressed as

$$V_p(T_i) = \begin{cases} \overline{V}_{T_i} & if\ not\ putted \\ \max\left(P_p, \overline{V}_{T_i}\right) & if\ putted \end{cases}$$

where $\overline{V}_{T_i}$ – continuation value at T_i

Puttable Bond

Model Selection Criteria

- Given the valuation complexity of puttable bonds, there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numerical solution to price them numerically.
- The selection of interest rate term structure models
  - Popular interest rate term structure models:

    Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).
  - HJM and LMM are too complex.
  - Hull-White is inaccurate for computing sensitivities.
  - Therefore, we choose either LGM or QGM.

Model Selection Criteria (Cont)

- The selection of numeric approaches
  - After selecting a term structure model, we need to choose a numerical approach to approximate the underlying stochastic process of the model.
  - Commonly used numeric approaches are tree, partial differential equation (PDE), lattice and Monte Carlo simulation.
  - Tree and Monte Carlo are notorious for inaccuracy on sensitivity calculation.
  - Therefore, we choose either PDE or lattice.
- Our decision is to use LGM plus lattice.

Puttable Bond

LGM Model

- The dynamics

$$dX(t) = \alpha(t)dW$$

where X is the single state variable and W is the Wiener process.

- The numeraire is given by

$$N(t,X) = \left(H(t)X + 0.5H^2(t)\zeta(t)\right)/D(t)$$

- The zero coupon bond price is

$$B(t,X;T) = D(T)exp\left(-H(t)X - 0.5H^2(t)\zeta(t)\right)$$

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LGM Assumption

- The LGM model is mathematically equivalent to the Hull-White model but offers
  - Significant improvement of stability and accuracy for calibration.
  - Significant improvement of stability and accuracy for sensitivity calculation.
- The state variable is normally distributed under the appropriate measure.
- The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfected correlated.

LGM calibration

- Match today's curve

  At time t=0, X(0)=0 and H(0)=0. Thus Z(0,0;T)=D(T). In other words, the LGM automatically fits today's discount curve.

- Select a group of market swaptions.
- Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.

Valuation Implementation

- Calibrate the LGM model.
- Create the lattice based on the LGM: the grid range should cover at least 3 standard deviations.
- Calculate the payoff of the puttable bond at each final note.
- Conduct backward induction process iteratively rolling back from final dates until reaching the valuation date.
- Compare exercise values with intrinsic values at each exercise date.
- The value at the valuation date is the price of the puttable bond.

Puttable Bond

A real world example

Bond specification		Puttable schedule	
Buy Sell	Buy	Put Price	Notification Date
Calendar	NYC	100	1/26/2015
Coupon Type	Fixed	100	7/25/2018
Currency	USD		
First Coupon Date	7/30/2013		
Interest Accrual Date	1/30/2013		
Issue Date	1/30/2013		
Last Coupon Date	1/30/2018		
Maturity Date	7/30/2018		
Settlement Lag	1		
Face Value	100		
Pay Receive	Receive		
Day Count	dc30360		
Payment Frequency	6		
Coupon	0.01		

Thanks!

You can find more details at
https://finpricing.com/lib/IrCurveIntroduction.html