Credit Valuation Adjustment and Funding Valuation Adjustment

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Summary

- Credit Valuation Adjustment (CVA) Definition
- Funding Valuation Adjustment (FVA) Definition
- CVA and FVA Calculation: Credit Exposure Approach
- CVA and FVA Calculation: Least Square Monte Carlo Approach
- Master Agreement
- CSA Agreement
- Risk Neutral Simulation
- Credit Exposure Approach Implementation
- Least Square Monte Carlo Approach Implementation

CVA Definition

- CVA is defined as the difference between the risk-free portfolio value and the true/risky portfolio value
- CVA is the market price of counterparty credit risk
- In practice, CVA should be computed at portfolio level. That m.eans calculation should take Master agreement and CSA agreement into account.

FVA Definition

- FVA is introduced to capture the incremental costs of funding uncollateralized derivatives.
- FVA is the difference between the rate paid for the collateral to the bank's treasury and rate paid by the clearinghouse.
- FVA can be thought of as a hedging cost or benefit arising from the mismatch between an uncollateralized derivative and a collateralized hedge in the interdealer market.
- FVA should be also calculated at portfolio level.

CVA Calculation: Credit Exposure Approach

- Model

$$CVA = (1 - R)\int_0^T EE^*(t)dPD(0,t)$$

where $EE^*(t)$ is the discounted risk-neutral expected credit exposure; R is the recovery rate and PD is the risk neutral probability of default.

- Pros
  - Simple and intuitive
  - Make best reuse of the existing counterparty credit risk system
  - Relatively easy to implement
- Cons
  - Theoretically unsound
  - Inaccurate

CVA Calculation: Least Square Monte Carlo Approach

- Model

$$CVA = V_f(t) - V_r(t)$$

where $V_r(t) = E[Y(t,T)X_T] = E\left[D(t,T)\left(1 - 1_{X_T \geq 0}q(1-R)\right)\right]$ is the risky/true value;

$V_f(t) = E[D(t,T)X_T]$ is the risk-free value;

$D(t,T)$ is the risk-free discount factor;

q is the risk neutral survival probability.

- Introduced by Xiao(1) and then Lee(2)

1. Xiao, T., "An accurate solution for credit value adjustment (CVA) and wrong way risk," Journal of Fixed Income, 25(1), 84-95, 2015.

2. Lee, D., "Pricing financial derivatives subject to counterparty credit risk and credit value adjustment," http://www.finpricing.com/lib/derivativeCVA.pdf

CVA Calculation: Least Square Monte Carlo Approach (Cont’d)

- Pros
  - Theoretically sound: can be rigorously proved.
  - Accurate valuation
  - Valuation is performed by Longstaff-Schwartz least squares Monte Carlo approach.
- Cons
  - Calculation procedure is different from credit exposure computation.
  - Hardly reuse the existing credit exposure system.

Master Agreement

- Master agreement is a document agreed between two parties, which applies to all transactions between them.
- Close out and netting agreement is part of the Master Agreement.
- If two trades can be netted, the credit exposure is

$$E(t) = max(V_1(t) + V_2(t), 0)$$

- If two trade cannot be netted (called non-netting), the credit exposure is

$$E(t) = max(V_1(t), 0) + max(V_2(t), 0)$$

CSA Agreement

- Credit Support Annex (CSA) or Margin Agreement or Collateral Agreement is a legal document that regulates collateral posting.
- Trades under a CSA should be also under a netting agreement, but not vice verse.
- It defines a variety of terms related to collateral posting.
  - Threshold
  - Minimum transfer amount (MTA)
  - Independent amount (or initial margin or haircut)

Risk Neutral Simulation: Interest Rate and FX

- Recommended 1-factor model: Hull-White

$$dr_t = (\theta_t - \alpha r_t)dt + \sigma_t dW_t$$

- Recommended multi-factor model: 2-factor Hull-White or Libor Market Model (LMM)
- All curve simulations should be brought into a common measure.
  - Simulate interest rate curves in different currencies.
  - Change measure from the risk neutral measure of a quoted currency to the risk neutral measure of the base currency.
- Forward FX rate can be derived using interest rate parity

$$F = S_0 exp(r_s - r_q)t$$

Risk Neutral Simulation: Equity Price

- Geometric Brownian Motion (GBM)

$$\frac{dr}{r} = \mu dt + \sigma dw$$

- Pros
  - Simple
  - Non-negative stock price
- Cons
  - Simulated values could be extremely large for a longer horizon.

Risk Neutral Simulation: Commodity Price

- Simulate commodity spot, future and forward prices as well as pipeline spreads
- Two factor model

$$\log(S_t) = q_t + \mathcal{X}_t + \mathcal{Y}_t$$
$$d\mathcal{X}_t = (\alpha_1 - \gamma_1 \mathcal{X}_t)dt + \sigma_1 dW_t^1$$
$$d\mathcal{Y}_t = (\alpha_2 - \gamma_2 \mathcal{Y}_t)dt + \sigma_2 dW_t^2$$
$$dW_t^1 dW_t^2 = \rho dt$$

where S_t is the spot price or spread; q_t is the deterministic function; $\mathcal{X}_t$ is the short term deviation and $\mathcal{Y}_t$ is the long term equilibrium level

- This model leads to a closed form solution of forward prices and thus forward term structure.

Risk Neutral Simulation: Volatility

- In the risk neutral world, the volatility is embedded in the price simulation.
- Thus, there is no need to simulate implied volatilities.

Credit Exposure Approach Implementation

- Obtain the risk-free value $V_f(t)$ of a counterparty portfolio that should be reported by trading systems.
- The solution is based on the existing credit exposure framework.
- Switch simulation from the real-world measure to the risk neutral measure.
- Calculate discounted risk-neutral credit exposures (EEs) and take master agreement and CSA into account.
- One can directly compute CVA using the following formula

$$CVA = (1 - R) \sum_{k=1}^{N} [PD(t_k) - PD(t_{k-1})] EE^*(t)$$

Credit Exposure Approach Implementation (Cont'd)

- Or one can compute the risky value $V_r(t)$ of the portfolio via discounting positive EEs by counterparty's CDS spread + risk-free interest rate as the positive EEs bearing counterparty risk and negative EEs by the bank's own CDS spread + risk-free interest rate as the negative EEs bearing the bank's credit risk.

$$CVA = V_f(t) - V_r(t)$$

- Furthermore, you can compute the funding value $V_F(t)$ of the portfolio via discounting positive EEs by counterparty's bond spread + risk-free interest rate and negative EEs by the bank's own bond spread + risk-free interest rate.

$$FVA = V_f(t) - V_F(t) - CVA = V_r - V_F$$

Least Square Monte Carlo Approach Implementation

- Obtain the risk-free value $V_f(t)$ of a counterparty portfolio that should be reported by trading systems.
- Simulate market risk factors in the risk-neutral measure.
- Generate payoffs for all trades based on Monte Carlo simulation.
- Aggregate payoffs based on the Master agreement and CSA.
- Compute the risky value $V_r(t)$ of the portfolio using Longstaff-Schwartz approach.

LSMC Approach Implementation (Cont'd)

- Positive cash flows should be discounted by counterparty's CDS spread + risk-free interest rate while negative cash flows should be discounted by the bank's own CDS spread + risk-free interest rate.
- $CVA = V_f(t) - V_r(t)$
- Moreover, you can compute the funding value $V_F(t)$ of the portfolio using Longstaff-Schwartz approach as well
- Positive cash flows should be discounted by counterparty's bond spread + risk-free interest rate while negative cash flows should be discounted by the bank's own bond spread + risk-free interest rate.
- $FVA = V_f(t) - V_F(t) - CVA = V_r - V_F$

Thanks!

You can find more details at
http://www.finpricing.com/lib/cvaFva.pdf