EE 231A Information Theory
Lecture 17
Broadcast Channels

A. Introduction to Broadcast Channels
B. Achievable rates on the broadcast channel
C. Degraded broadcast channels
D. Binary Symmetric Degraded Broadcast Channel
E. Gaussian Degraded Broadcast Channel
F. Coding Theorem for degraded broadcast channels

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Part 17A:

Introduction to Broadcast Channels

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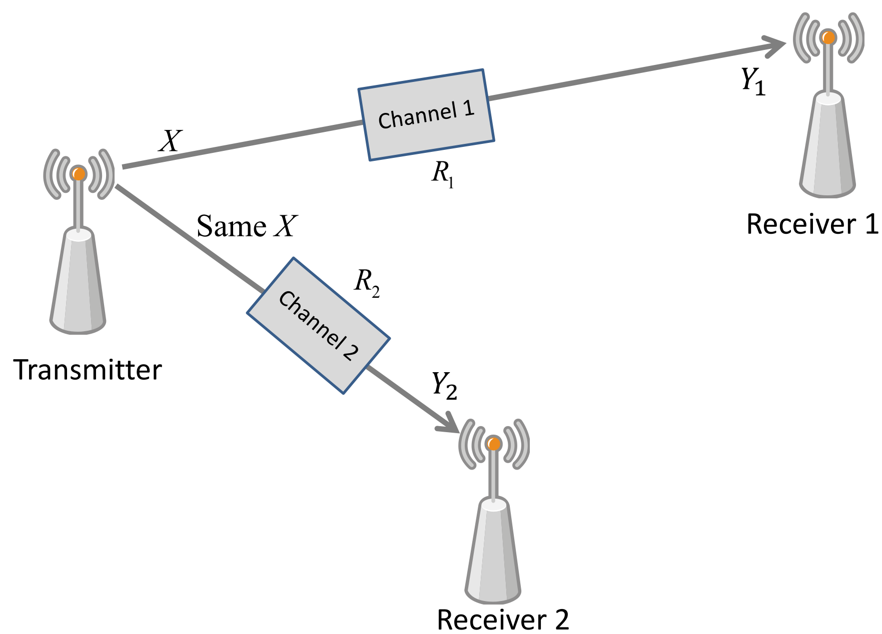
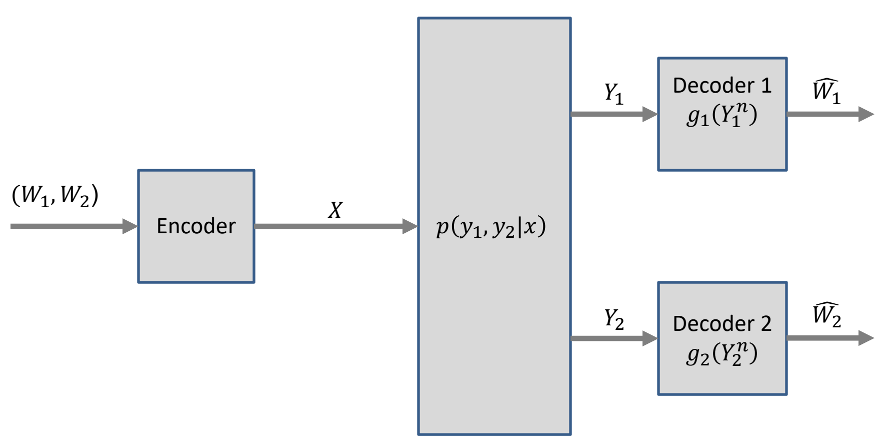
Broadcast Channel Illustration



Broadcast Channel Diagram



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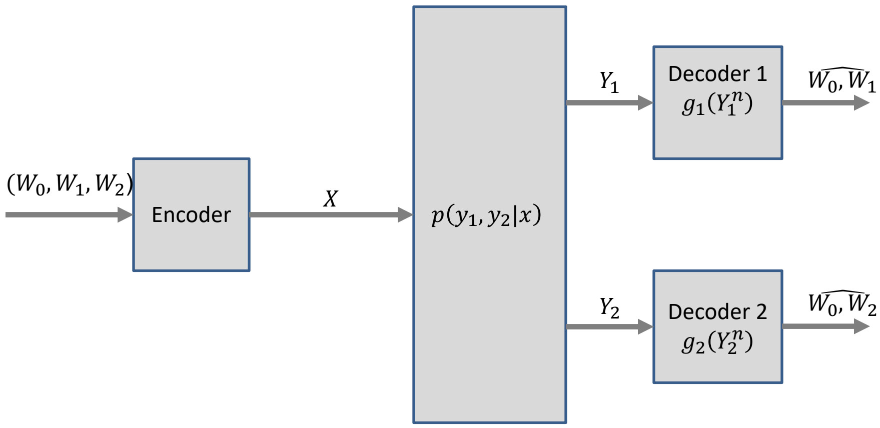
Common information



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Television and Lectures

- Standard Television Broadcast: W_0 only.
- HDTV and standard TV simultaneously transmitted: Standard TV is W_0. Extra info for HDTV is W_1.
- Lecturer in a classroom $H(W_1) > \mathrm{H}(W_2)$, $H(W_2|W_1) \neq 0$. W_1 and W_2 are both non-zero entropy, but W_0 is hopefully significant.

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Part 17B:

Achievable Rates on Broadcast Channels

Definition of achievable rate pair

- A rate pair (R_1, R_2) is said to be achievable for the broadcast channel if there exists a sequence of $\big((2^{nR_1}, 2^{nR_2}), n\big)$ codes with $P_e^{(n)} \to 0$.
- Where

$$P_e^{(n)} = P(g_1(Y_1^n) \neq W_1 or g_2(Y_2^n) \neq W_2)$$

Definition of Capacity Region

- The capacity region of the broadcast cahnnel is the closure of the set of achievable rate pairs, or more generally tuples.
- The capacity region of the broadcast channel depends only on the conditional marginal distributions $p(y_1|x)$ and $p(y_2|x)$.

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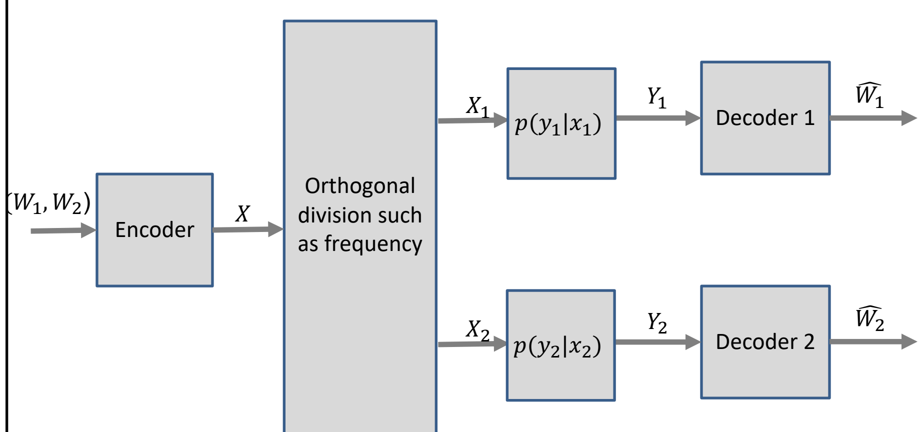
Orthogonal Broadcast Channel



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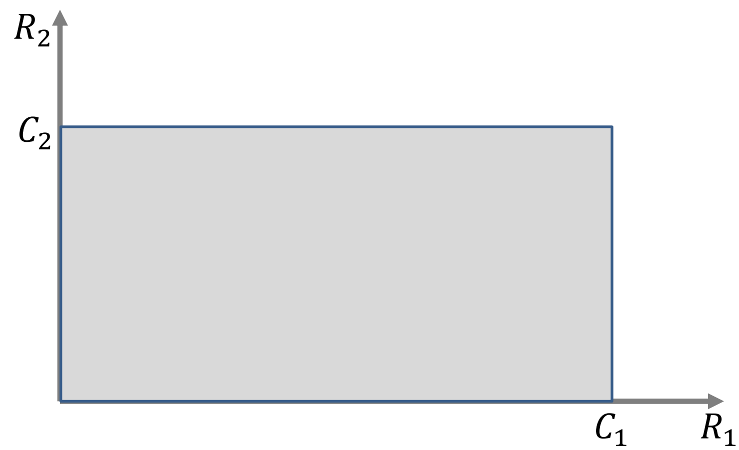
Orthogonal Broadcast Channel Rate Region (assuming independent power constraints)



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Achievable triple for common information

- A rate pair (R_0, R_1, R_2) is said to be achievable for the broadcast channel with common information if there exists a sequence of $\left((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n\right)$ codes with $P_e^{(n)} \to 0$.

- Where
- $P_e^{(n)} = P(g_1(Y_1^n) \neq W_0, W_1 \text{ or } g_2(Y_2^n) \neq W_0, W_2)$

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Part 17C:

Degraded Broadcast Channels

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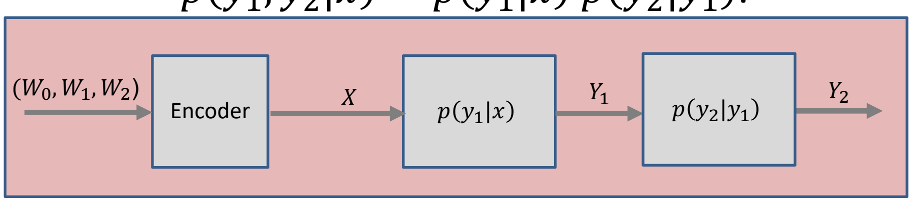
Physically Degraded Broadcast Channels

- A broadcast channel is said to be *physically* degraded if

$$p(y_1, y_2|x) = p(y_1|x)\, p(y_2|y_1).$$



- A broadcast channel is said to be *stochastically* degraded if for some distribution $\acute{p}(y_2|y_1)$

$$p(y_2|x) = \sum_{y_1} p(y_1|x)\acute{p}(y_2|y_1).$$

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Degraded Broadcast Channel Capacity Region

- The capacity region for sending independent information over the degraded broadcast channel $X \to Y_1 \to Y_2$ is the convex hull of the closure of all (R_1, R_2) satisfying

$$R_2 \le I(U; Y_2)$$
$$R_1 \le I(X; Y_1|U)$$

For some joint distribution $p(u)p(x|u)p(y_1, y_2|x)$
Where $|\mathcal{U}| \le \min\{|\mathcal{X}|, |\mathcal{Y}_1|, |\mathcal{Y}_2|\}$.

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Channel Capacity Region with Common Information

- (Theorem 15.6.3) In general, if the rate pair (R_1, R_2) is achievable for a broadcast channel with independent information, the rate triple$(R_0, R_1 - R_0, R_2 - R_0)$ provided that $R_0 \le \min(R_1, R_2)$.

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Channel Capacity Region with Common Information *for a Degraded Channel*

- (Theorem 15.6.4) If the rate pair (R_1, R_2) is achievable for a broadcast channel with independent information, the rate triple$(R_0, R_1, R_2 - R_0)$ provided that $R_0 \leq R_2$.

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Part 17D:

Binary Symmetric Degraded Broadcast Channel

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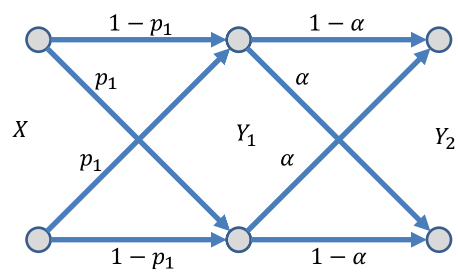
Physically Degraded BSC



$p_2 = p_1(1-\alpha) + (1-p_1)\alpha$

$\alpha = \dfrac{p_2 - p_1}{1 - 2p_1}$

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Achievable Rates for degraded BSC

- Let's use the equations:

$$R_2 \leq I(U;Y_2)$$
$$R_1 \leq I(X;Y_1|U)$$

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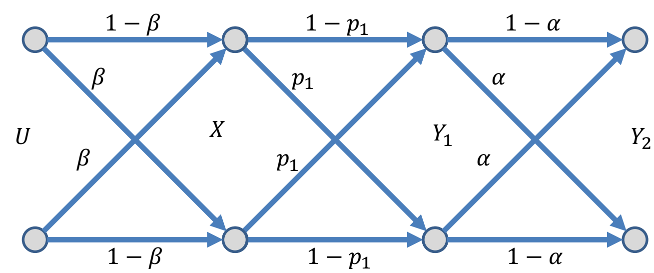
Adding U



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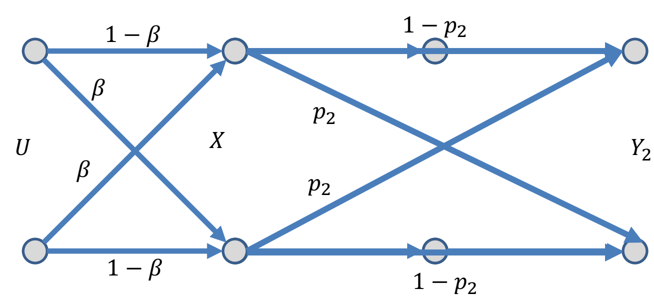
Adding U



$p_{U\to Y_2} = \beta(1-p_2) + (1-\beta)p\_2{=}\beta * p_2$

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Achievable Rates for degraded BSC

- Let’s use the equations:

$$R_2 \leq I(U;Y_2)$$
$$R_1 \leq I(X;Y_1|U)$$

- $R_2 \leq I(U;Y_2) = H(Y_2) - H(Y_2|U) \leq 1 - H(\beta * p_2)$
- $\beta * p_2 = \beta(1-p_2)+(1-\beta)p_2$
- $R_1 \leq I(X;Y_1|U)=H(Y_1|U)- H(Y_1|X,U)$
- $R_1 \leq I(X;Y_1|U)=H(\beta * p_1)- H(Y_1|X)$
- $R_1 \leq I(X;Y_1|U)=H(\beta * p_1)- H(p_1)$

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Achievable Rates for degraded BSC

- So the answer is:

$$R_2 \leq 1 - H(\beta * p_2)$$
$$R_1 \leq H(\beta * p_1)- H(p_1)$$

With $\beta = 0, \beta * p_2 = p_2$ and $\beta * p_1 = p_1$ so that

$$R_2 \leq 1 - H(p_2)$$
$$R_1 \leq 0$$

Using Thm 15.6.4 in this case, for common information we can achieve $(1 - H(p_2), 0,0)$ so that all the information is common information.

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Part 17E:

Gaussian Degraded Broadcast Channel

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Part 17E:

Degraded Broadcast Channel Coding Theorem

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