ECE 231A HW8 Shengze Ye 205418959

1. Unusual multiple access channel.

$\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{Y} = \{0,1\}$

if $(x_1, x_2) = (0,0)$ then $Y=0$ $(x_1, x_2) = (0,1)$ then $Y=1$

if $(x_1, x_2) = (1,0)$ then $Y=1$ if $(x_1, x_2) = (1,1)$

$Y=0$ with $p=\frac{1}{2}$, $Y=1$ with $p=\frac{1}{2}$

(a) Show that rate pair $(1,0)$ and $(0,1)$ are achievable.

① for pair $(1,0)$

If we evaluate the achievable region for distribution $p(x_1)p(x_2)$ with

$p(x_1=0) = p(x_1=1) = \frac{1}{2}$, $p(x_2=0)=1$, $p(x_2=1)=0$

$\rightarrow I(X_1;Y|X_2) = p(x_2=0)\, I(X_1;Y|x_2=0) + p(x_2=1)\, I(X;Y|x_2=1)$

$= I(X_1;Y|x_2=0)\cdot 1 = H(Y|X_2) = 1$

$\rightarrow I(X_2;Y|X_1) = p(x_1=0)\, I(X_2;Y|x_1=0) + p(x_1=1)\, I(X_2;Y|x_1=1)$

$= \frac{1}{2} I(X_2;Y|x_1=0) + \frac{1}{2} I(X_2;Y|x_1=1)$

$= \frac{1}{2} H(Y|x_1=0, x_2=0) + \frac{1}{2} H(Y|x_1=1, x_2=0)$

$= 0+0 = 0$

$\rightarrow I(X_1, X_2; Y) = H(Y) - H(Y|X_1, X_2)$ since $p(x_2=0)=1$

$p(Y=0) = p(x_2=0)\cdot p(x_1=0) + p(x_2=1)\cdot p(x_1=1)\cdot\frac{1}{2}$

$= 1\cdot\frac{1}{2} + 0 = \frac{1}{2}$

$\therefore p(Y=1) = 1 - \frac{1}{2} = \frac{1}{2}$

$H(Y) = H(\frac{1}{2}) = 1$

$H(Y|X_1, X_2) = 0$ since $X_1 = 1$ and $X_2 = 1$ cannot be achieved

$\therefore I(X_1, X_2; Y) = 1 - 0 = 1$

$\therefore$ we have $R_1 = 1 \le I(X_1; Y|X_2) = 1$

$R_2 = 0 \le I(X_2; Y|X_1) = 0$

$R_1 + R_2 = 1 \le I(X_1, X_2; Y) = 1$

$\therefore (R_1, R_2) = (1, 0)$ is achievable.

② for pair (0.1)

similar to ①, we have the probability distribution

$p(X_1=0) = 1 \quad p(X_1=1) = 0 \quad p(X_2=0) = p(X_2=1) = \frac{1}{2}$

$\therefore I(X_1; Y|X_2) = 0$

$I(X_2; Y|X_1) = 0$

$I(X_1, X_2; Y) = 1$

$(R_1, R_2) = (0, 1)$ is achievable

(b) show that for any non-degenerate distribution

$p(x_1)p(x_2)$, we have $I(X_1, X_2; Y) < 1$

let $p_1 = p(X_1 = 1) \quad p_2 = p(X_2 = 1)$

By definition of non-degenerate $p_1 \ne 0, 1$, $p_2 \ne 0, 1$

$p(Y=0) = (1-p_1)(1-p_2) + \frac{1}{2} p_1 \cdot p_2$

$p(Y=1) = p_1(1-p_2) + p_2(1-p_1) + \frac{1}{2} p_1 p_2$

$I(X_1, X_2 ; Y) = H(Y) - H(Y|X_1, X_2)$

$= H\left((1-p_1)(1-p_2) + \frac{1}{2} p_1 \cdot p_2\right) - p_1 p_2 \cdot H(Y|X_1=1, X_2=1)$

$= H\left((1-p_1)(1-p_2) + \frac{1}{2} p_1 \cdot p_2\right) - p_1 p_2$

$\leq 1 - p_1 p_2$

$< 1 \qquad \because p_1 p_2 > 0$

$\therefore$ for any non-degenerate distribution, we have $I(X_1, X_2 ; Y) < 1$

(c) Argue that there are points in the capacity region of this multiple access channel that can only be achieved by time sharing

The degenerate distribution have either R_1 or R_2 equal to 0. Plus the result we get from (b)

we know that $R_1 + R_2 \leq I(X_1, X_2 ; Y) < 1$

$\therefore$ For example rate pair $(R_1, R_2) = (\frac{1}{2}, \frac{1}{2})$ could never be achieved.

However, the rate pair $(\frac{1}{2}, \frac{1}{2})$ could be achieved by timesharing between the $(R_1, R_2) = (1, 0)$ and $(R_1, R_2) = (0, 1)$

$(R_1, R_2) = (\frac{1}{2}, \frac{1}{2})$ lies in the convex hull of the union of the achievable regions, but not the union itself. So the operation of taking the convex hull has strictly increased the capacity region for this multiple access channel.

2. Consider the two-user multiplication channel $Y = X_1 \times X_2$, with the following alphabets $X_1 = \{0, 1\}$ $X_2 = \{1, 2, 3\}$

(a) For a fixed value of $p = P(X_1 = 1)$, find the set of equations that describe the associated "pentagon". What distribution on X_2 maximizes the area of this pentagon?

if $X_1 = 0$ $\quad Y = X_2 \cdot 0 = 0$

if $X_1 = 1$ $\quad Y = X_2 \cdot 1 = X_2$

$$
\begin{aligned}
I(X_1; Y|X_2) &= H(X_1|X_2) - H(X_1|X_2, Y) \\
&= H(X_1|X_2) - 0 = H(X_1) \\
&= H(p)
\end{aligned}
$$

$$
\begin{aligned}
I(X_2; Y|X_1) &= p(X_1=0)\, I(X_2; Y|X_1=0) + p(X_1=1)\, I(X_2; Y|X_1=1) \\
&= 0 + p \cdot I(X_2; Y|X_1=1) = p \cdot H(X_2) \\
&\leq p \cdot \log 3
\end{aligned}
$$

when X_2 follows a uniform distribution, the area of the pentagon is the largest

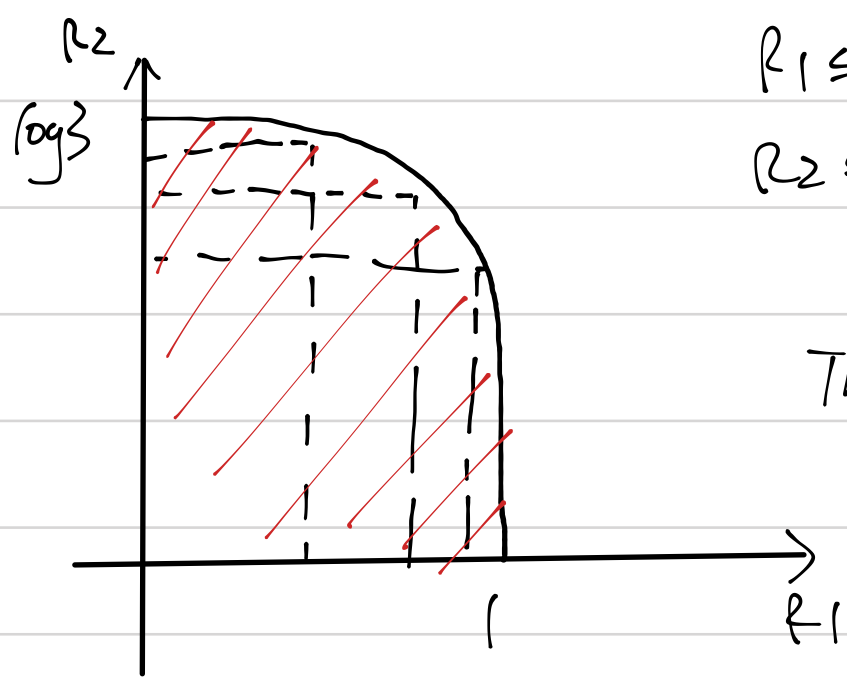
$$I(X_1, X_2; Y) = I(X_1; Y) + I(X_2; Y | X_1)$$
$$= H(p) + p \cdot H(X_2)$$
$$\le H(p) + p \log 3 = I(X_1; Y | X_2) + I(X_2; Y | X_1)$$

$$I(X_1; Y | X_2) + I(X_2; Y | X_1) = I(X_1, X_2; Y)$$

The shape of "pentagons" is actually rectangal

$\therefore$ The rate region is $R_1 \le H(p)$
$R_2 \le p \log 3$

(b) Find the achievable region of this multiple access channel.



$R_1 \le H(p)$
$R_2 \le p \log 3$

p varies from 0 to 1

The region in red is the achievable region

(c) In this case, does the convex hull operation increase the achievable region beyond the union of all pentagons?

No. Because All the rate pair (R_1, R_2) could be achieved under some probability distribution, as the "pentagon" is actually a rectangle.

3. Multiple Access for a Modulo Additional channel
$Y = X_1 + X_2$ with $X_1 = \{0,1\}$, $X_2 = \{0,1,2,3\}$.
Define $p = p(X_1 = 1)$ Note that addition is modulo-4 so that, $2+3=1$

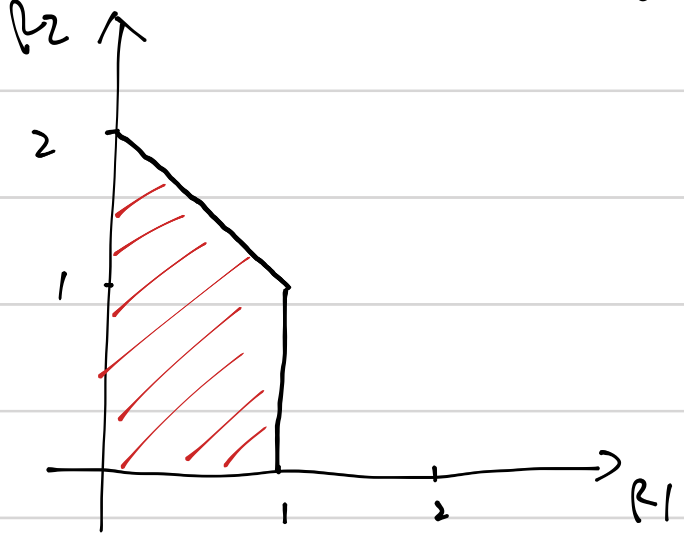
Find and draw the achievable rate region for this multiple access channel.

$$I(X_1;Y|X_2) = H(Y|X_2) - H(Y|X_1,X_2)$$
$$= H(Y|X_2) = H(X_1) \leq 1$$
$$I(X_2;Y|X_1) = H(Y|X_1) - H(Y|X_1,X_2)$$
$$= H(Y|X_1) = H(X_2) \leq 2$$
$$I(X_1,X_2;Y) = H(Y) - H(Y|X_1,X_2) = H(Y) \leq 2$$

$\therefore R_1 \leq 1 \quad R_2 \leq 2, \quad R_1 + R_2 \leq 2$

The upper bound is achieved when X_1, X_2 follows a uniform distribution, when X_1, X_2 is uniform distribution, Y is also a uniform distribution

The achievable rate region is:



The red part is achievable rate region

4. Multiple Access for Binary Adder Channel

Consider the two-user adder channel $Y = X_1 + X_2$ with

$\mathcal{X}_1 = \{0,1\}$ $\quad$ $\mathcal{X}_2 = \{0,1\}$

$p_1 = P(X_1 = 1)$ $\quad$ $p_2 = P(X_2 = 1)$

(a) For fixed, specified p_1 and p_2, find the upper bounds on R_1 and R_2 used for constructing a pentagon of the MAC capacity region.

$$
\begin{aligned}
I(X_1; Y|X_2) &= H(Y|X_2) - H(Y|X_1, X_2) \\
&= H(Y|X_2) \\
&= H(Y|X_2=1)\, p(X_2=1) + H(Y|X_2=0)\, p(X_2=0) \\
&= H(p_1) \cdot p_2 + H(p_1) \cdot (1-p_2) \\
&= H(p_1)
\end{aligned}
$$

$$I(X_2;Y|X_1) = H(Y|X_1) - H(Y|X_1,X_2)$$
$$= H(Y|X_1)$$
$$= H(Y|X_1=0)\,p(X_1=0) + H(Y|X_1=1)\,p(X_1=1)$$
$$= H(p_2)\cdot(1-p_1) + H(p_2)\cdot p_1$$
$$= H(p_2)$$

$$I(X_1,X_2;Y) = H(Y) - H(Y|X_1,X_2)$$
$$= H(Y)$$

$$p(Y=2) = p_1p_2$$
$$p(Y=1) = (1-p_1)p_2 + p_1(1-p_2)$$
$$P(Y=0) = (1-p_1)(1-p_2)$$
$$\therefore H(Y) = H(p_1p_2,\ (1-p_1)(1-p_2),\ (1-p_1)p_2 + p_1(1-p_2))$$

$$\therefore\ R_1 \le H(p_1)$$
$$R_2 \le H(p_2)$$
$$R_1 + R_2 \le H(Y) = H(p_1p_2,\ (1-p_1)(1-p_2),\ (1-p_1)p_2 + p_1(1-p_2))$$

(b) use the grouping axiom to show that
$$H(p_1) + H(p_2) = H(p_1(1-p_2),\ p_1p_2,\ (1-p_1)(1-p_2),\ (1-p_1)p_2)$$

$$H(p_1(1-p_2),\ p_1p_2,\ (1-p_1)(1-p_2),\ (1-p_1)p_2)$$
$$= H(p_1,\ (1-p_1)(1-p_2),\ (1-p_1)p_2) + p_1\cdot H(p_2)$$
$$= H(p_1,\ 1-p_1-p_2+p_1p_2+p_2-p_1p_2) + (1-p_1)H\left(\frac{(1-p_1)(1-p_2)}{1-p_1},\ p_2\right)$$

$+ p_1 H(p_2)$

$= H(p_1) + (1-p_1) H(p_2) + p_1 H(p_2)$

$= H(p_1) + H(p_2)$

$\therefore H(p_1) + H(p_2) = H(p_1(1-p_2), p_1 p_2, (1-p_1)(1-p_2), (1-p_1)p_2)$

(c) For fixed, specified p_1 and p_2 find the upper bound on the sum $R_1 + R_2$ used for constructing a pentagon.

$$R_1 + R_2 \le I(X_1, X_2; Y) = H(Y)$$

$$H(Y) = H(p_1 p_2, (1-p_1)(1-p_2), p_1(1-p_2) + p_2(1-p_1))$$

we know that

$$H(p_1) + H(p_2) = H(p_1(1-p_2), p_1 p_2, (1-p_1)(1-p_2), (1-p_1)p_2)$$

$$= H(p_1 p_2, (1-p_1)(1-p_2), p_1(1-p_2) + p_2(1-p_1))$$

$$+ (p_1 + p_2 - 2p_1 p_2) H\left(\frac{p_1 - p_1 p_2}{p_1 + p_2 - 2p_1 p_2}, \frac{p_2 - p_1 p_2}{p_1 + p_2 - 2p_1 p_2}\right)$$

$$\therefore H(p_1) + H(p_2) = H(Y) + (p_1 + p_2 - 2p_1 p_2) H\left(\frac{p_1 - p_1 p_2}{p_1 + p_2 - 2p_1 p_2}, \frac{p_2 - p_1 p_2}{p_1 + p_2 - 2p_1 p_2}\right)$$

$$\therefore R_1 + R_2 \le H(p_1) + H(p_2) - \alpha H(q)$$

$$\alpha = p_1 + p_2 - 2p_1 p_2 \quad q = \frac{p_1 - p_1 p_2}{p_1 + p_2 - 2p_1 p_2} \text{ or } \frac{p_2 - p_1 p_2}{p_1 + p_2 - 2p_1 p_2}$$

(d) sketch the pentagon for $p_1 = p_2 = 0.5$ and for one other choice of (p_1, p_2). Does the second pentagon lie inside the first pentagon?

for $p_1 = p_2 = 0.5$

$H(p_1) = 1 \quad H(p_2) = 1$

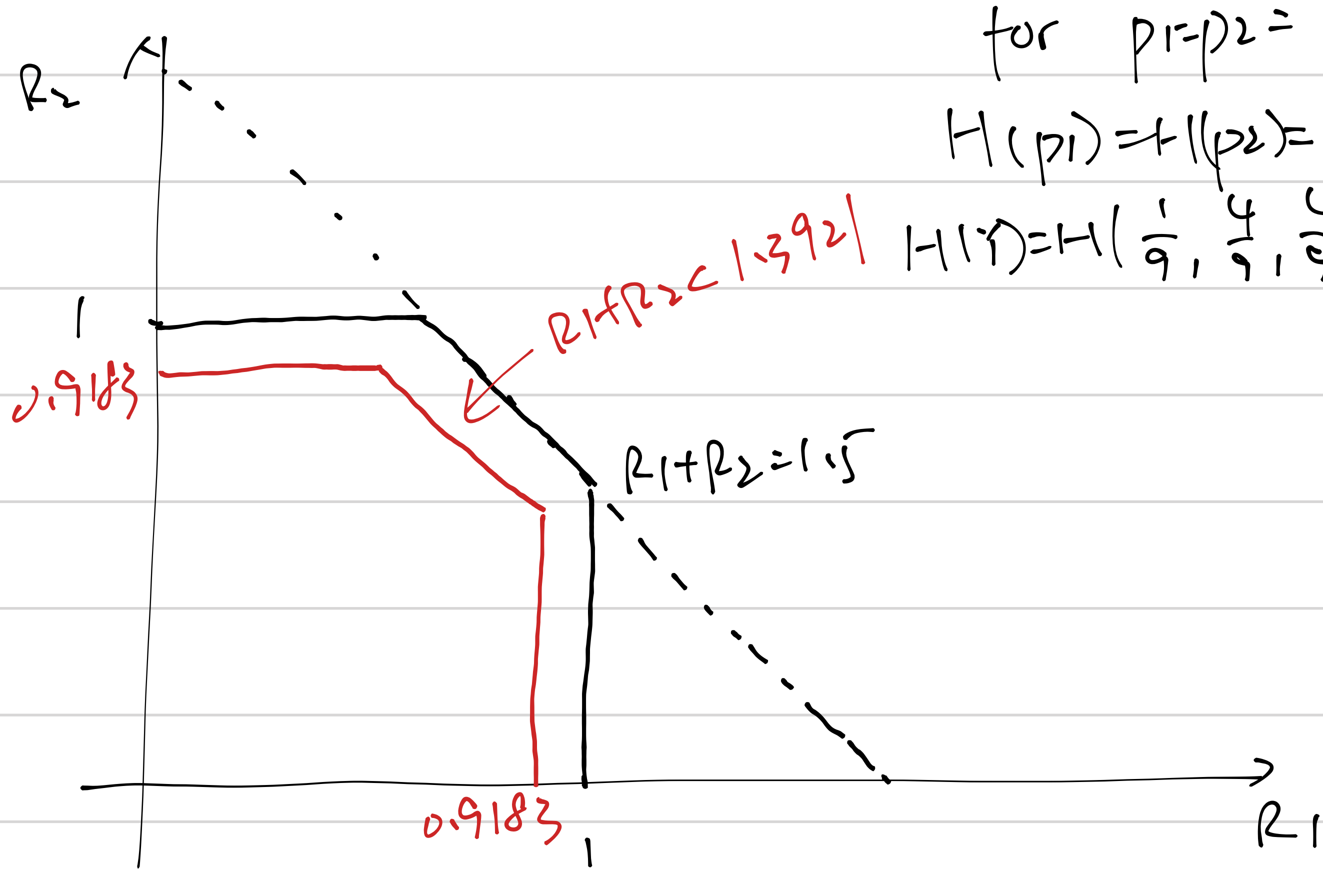
$H(Y) = H(\frac{1}{2}, \frac{1}{4}, \frac{1}{2}) = 1.5$

$\therefore R_1 \le 1 \quad R_2 \le 1 \quad R_1 + R_2 \le 1.5$

for $p_1 = p_2 = \frac{1}{3}$

$H(p_1) = H(p_2) = 0.9183$

$H(Y) = H(\frac{1}{9}, \frac{4}{9}, \frac{4}{9}) = 1.3921$



$I(X_1; Y | X_2) = H(p_1) \le 1$

$I(X_2; Y | X_1) = H(p_2) \le 1$

$I(X_1, X_2; Y) = H(Y) \le 1.5$

The second pentagon lies inside the first pentagon

5. TDMA vs. CDMA

Consider the two-user ($m=2$) multiple access Gaussian channel with users both constrained to have $P=3N$ where the receiver sees $Y = X_1 + X_2 + Z$ and $Z \sim N(0,N)$

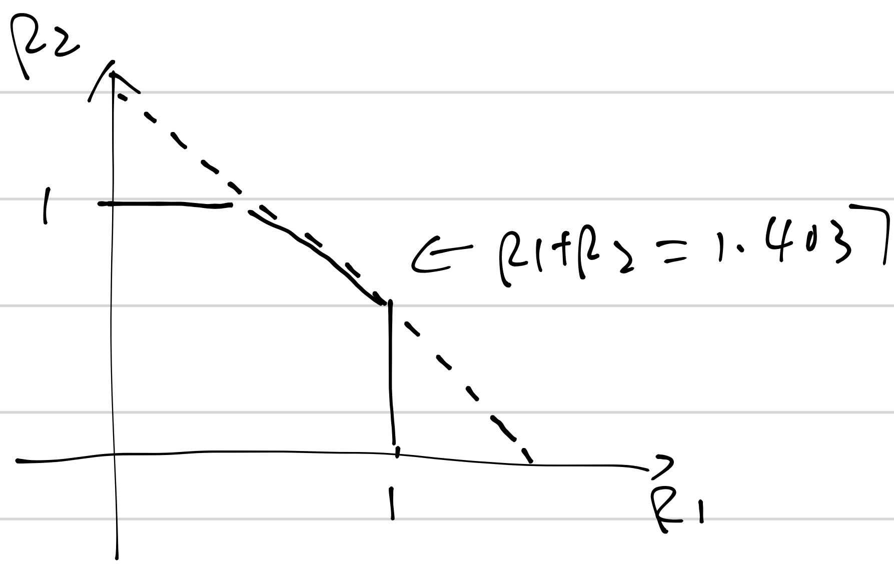
(a) Plot the region of achievable rate for this multiple access channel.

$R_1 < \frac{1}{2}\log(1+P/N) = \frac{1}{2}\log 4 = 1$

$R_2 < \frac{1}{2}\log(1+P/N) = \frac{1}{2}\log 4 = 1$

$R_1+R_2 < \frac{1}{2}\log(1+2P/N) = \frac{1}{2}\log(1+6) = \frac{1}{2}\log 7 = 1.4037$

$\therefore$



(b) Consider a time-sharing arrangement, λ of time user one sends on the channel with $P = 3N/\lambda$, and the other $1-\lambda$ of time, user two sends on the channel with $P = 3N/(1-\lambda)$, show that both users obey their power constraints and plot the curve of rate pairs achieved by this arrangement.

For user one:

$$P = \frac{3N}{\lambda} \cdot \lambda + 0 \cdot (1-\lambda) = 3N$$

For user two:

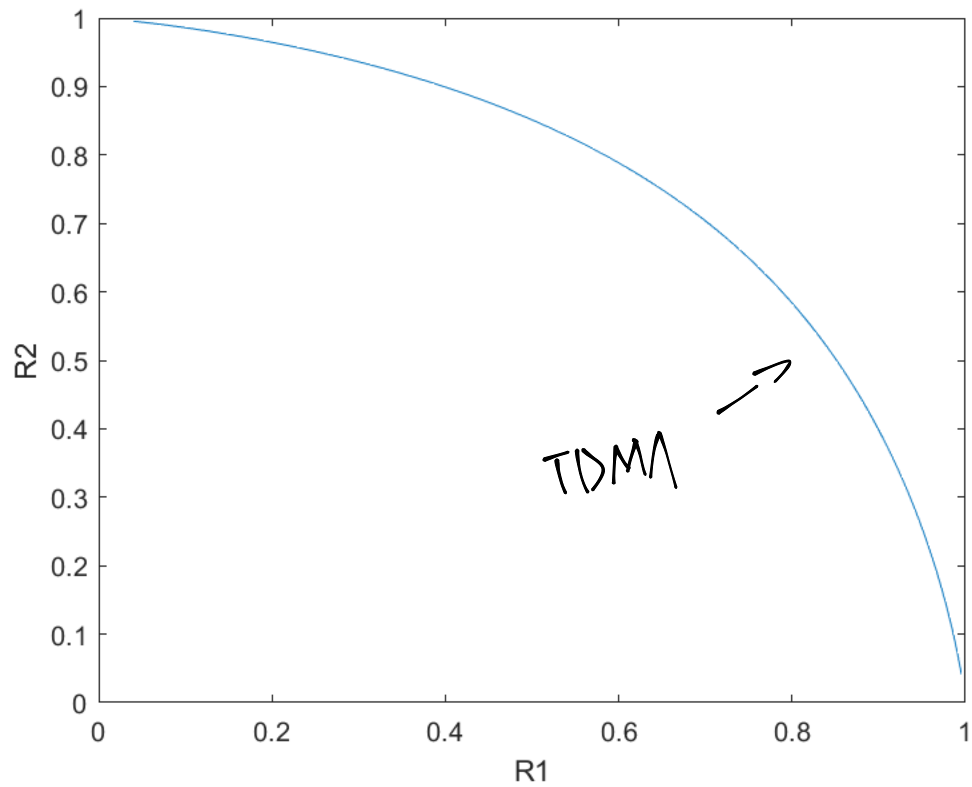
$$P = \frac{3N}{(1-\lambda)} \cdot (1-\lambda) + 0 \cdot \lambda = 3N$$

$\therefore$ Both user obey the power constraint.

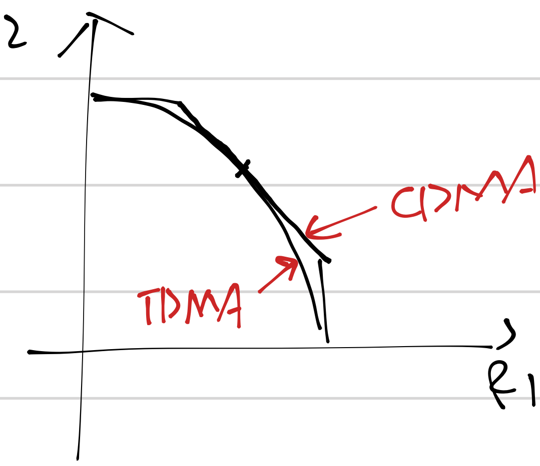
with TDMA

$$R_1 < \lambda \cdot \frac{1}{2} \log\left(1 + \frac{3N}{\lambda N}\right) = \frac{1}{2}\lambda \log\left(1 + \frac{3}{\lambda}\right)$$

$$R_2 < (1-\lambda)\frac{1}{2}\log\left(1 + \cancel{\frac{3N}{(1-\lambda)N}}\right) = \frac{1}{2}(1-\lambda)\log\left(1+\frac{3}{1-\lambda}\right)$$



The curve of rate pairs is shown in the figure above combined with CDMA, the curve should be



(c) when, if ever, does time-sharing achieve optimal pair (R_1, R_2)

we want R_1+R_2 to be maximized

$$R_1+R_2 = \frac{1}{2}\lambda \cdot \log\left(1+\frac{3}{\lambda}\right) + \frac{1}{2}(1-\lambda)\log\left(1+\frac{3}{1-\lambda}\right)$$

optimal pair is achieved when $\lambda = \frac{1}{2}$

in this case $R_1+R_2 = \frac{1}{2}\log 7$, $R_1 = R_2 = 0.7018$

(d) compair CDMA and TDMA

I think CDMA is a little bit better than TDMA for cellular telephone network.

A main issue for TDMA is time synchronization we need to make sure every user shares the same timing. otherwise the users cannot normally recieve the information due to the chaos and confusion of the time slot.

6. Noiseless Multiple Access channel

Consider the two-user multiple access channel with no noise so that $Y = f(X_1, X_2)$ and f is a deterministic function

Show that each pentagon includes the constraint $R_1 + R_2 < H(Y)$

$$R_1 + R_2 < I(X_1, X_2; Y) = H(Y) - H(Y|X_1, X_2)$$

since $Y = f(X_1, X_2)$ is a deterministic function we know Y as long as we know X_1 and X_2

$\therefore H(Y|X_1, X_2) = 0$

$$R_1 + R_2 < I(X_1, X_2; Y) = H(Y)$$

$\therefore$ each pentagon includes the constraint $R_1 + R_2 < H(Y)$

7. Slepian-Wolf. for deterministically related sources

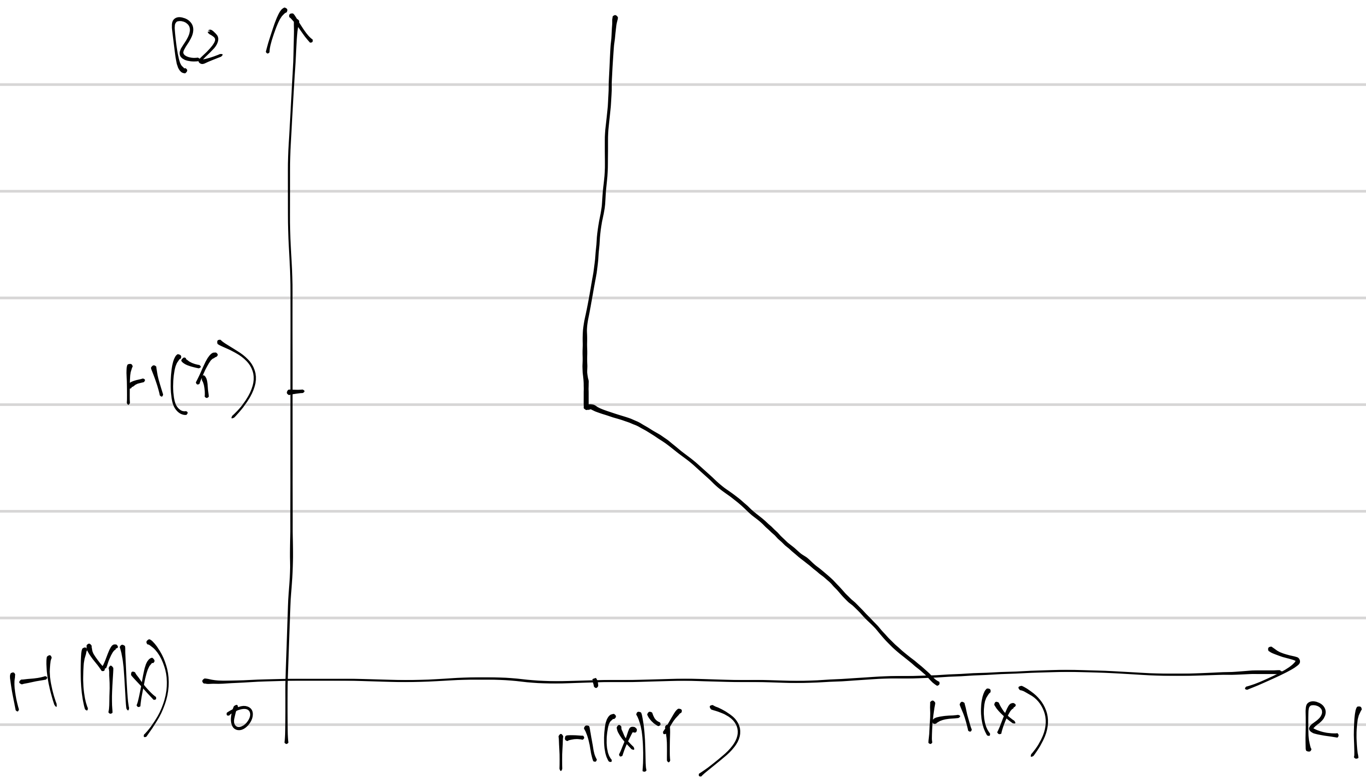
Find and sketch the Slepian-Wolf rate region for the simultaneous data compression of (X, Y), where $y = f(x)$ is some deterministic function of x

$$H(X, Y) = H(X) + H(Y|X)$$

$\because$ $y = f(x)$ is a deterministic function

$\therefore H(X,Y) = H(X) + H(Y|X) = H(X)$

and $H(X|Y) \geq 0$ $\quad H(Y|X) = 0$



f. Slepian - wolf.

let X_i iid. Bernoulli (p). let Z_i iid $\sim$ Bernoulli (r), and let Z be independent of X. Finally, let $Y = X \oplus Z$, (mod 2 addition). let X be described at rat R_1 and Y be R_2, what region of rates allows recovery of X, Y with probability of error tending to zero?

$p(X=1) = p \quad p(X=0) = 1-p \qquad p(Z=1) = r \quad p(Z=0) = 1-r$

$Y = X \oplus Z$

$P(Y=1) = p(1-r) + r \cdot (1-p)$

$P(Y=0) = pr + (1-p)(1-r)$

$H(X) = H(p) \quad H(Y) = H(p(1-r)+r(1-p))$

$H(X,Y) = H(X,Z) = H(X) + H(Z)$

$= H(p) + H(r)$

$\therefore H(Y|X) = H(p)+H(r) - H(p) = H(r)$

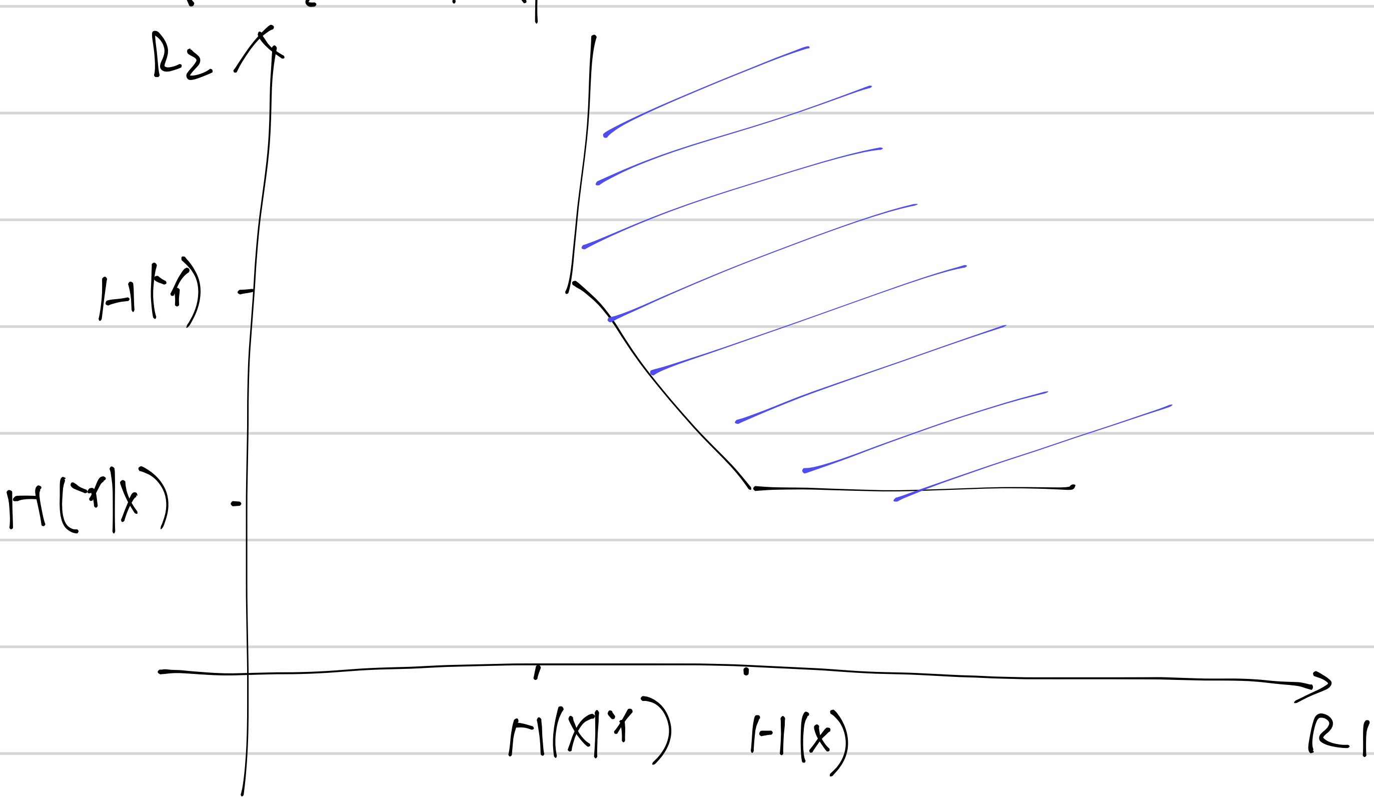
$H(X|Y) = H(p)+H(r) - H(p(1-r)+r(1-p))$

The region is :

$R_1 > H(p)$

$R_2 > H(p(1-r)+r(1-p))$

$R_1+R_2 > H(p)+H(r)$



9. Slepian-Wolf for multiplication

Let X_1 be a binary random variable with

$p(x_1=1)=p_1$, z be uniformly distributed over the four integers $\{1,2,3,4\}$, $X_2=X_1 \times z$

compute the Slepian-Wolf region of rate pairs that allow the recovery of X_1 and X_2. Plot your region for $p_1=0.5$

$H(x_1)=H(p_1)$

$X_2=X_1\times z \quad \therefore p(X_2=0)=1-p_1$

$p(X_2=1)=\frac{1}{4}\cdot p_1=p(X_2=2)=p(X_2=3)=p(X_2=4)$

$R_1>H(x_1|X_2)=0$

$R_2>H(X_2|X_1)=H(X_2|X_1=0)\,p(X_1=0)+H(X_2|X_1=1)\,p(X_1=1)$

$=0+H(z)\cdot p_1=\log 4\cdot p_1$

$=2p_1$

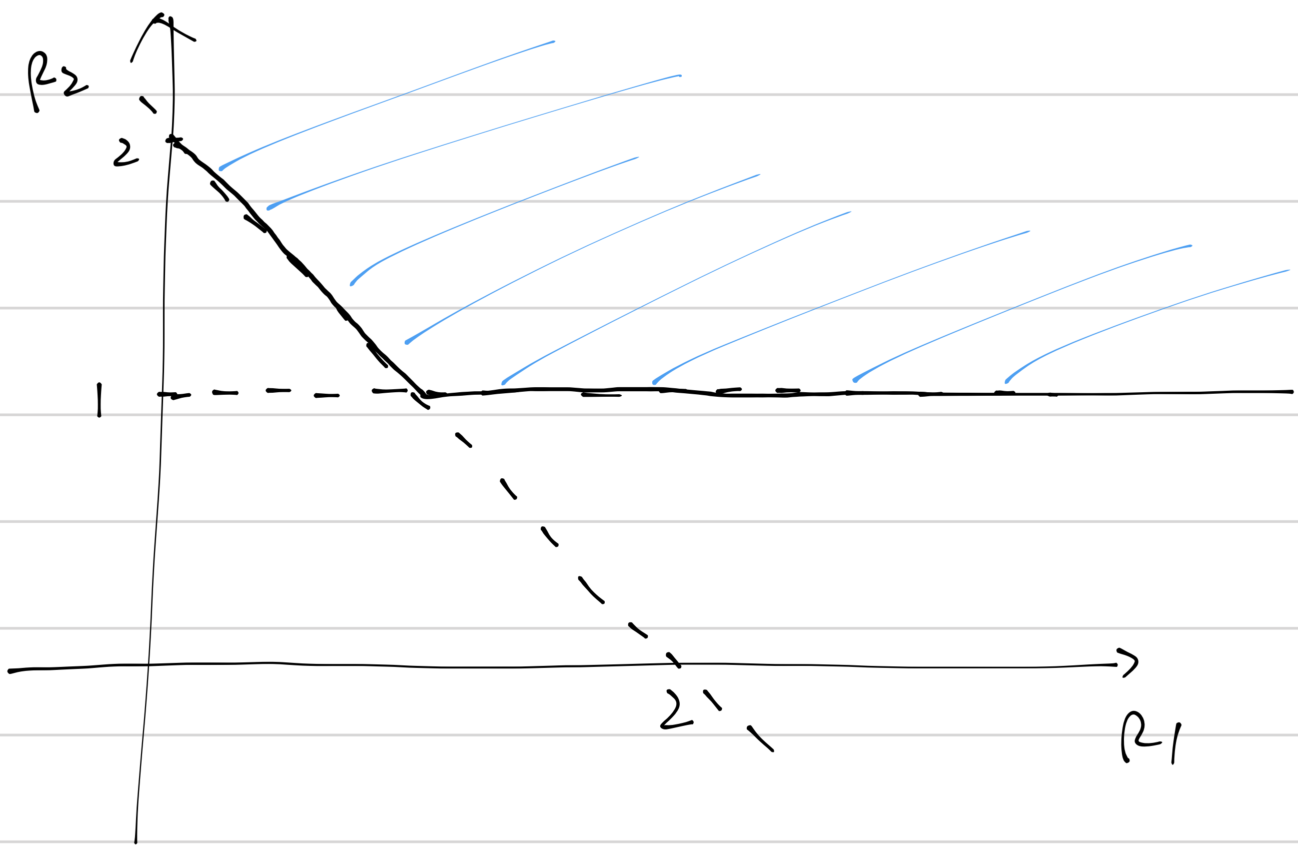
$R_1+R_2>H(X_1,X_2)=H(X_1)+H(X_2|X_1)=$

$=H(p_1)+2p_1$

when $p_1=0.5$

$H(X_1|X_2)=0 \qquad H(X_2|X_1)=1$

$H(X_1,X_2)=H(\frac{1}{2})+2\cdot 0.5=2$



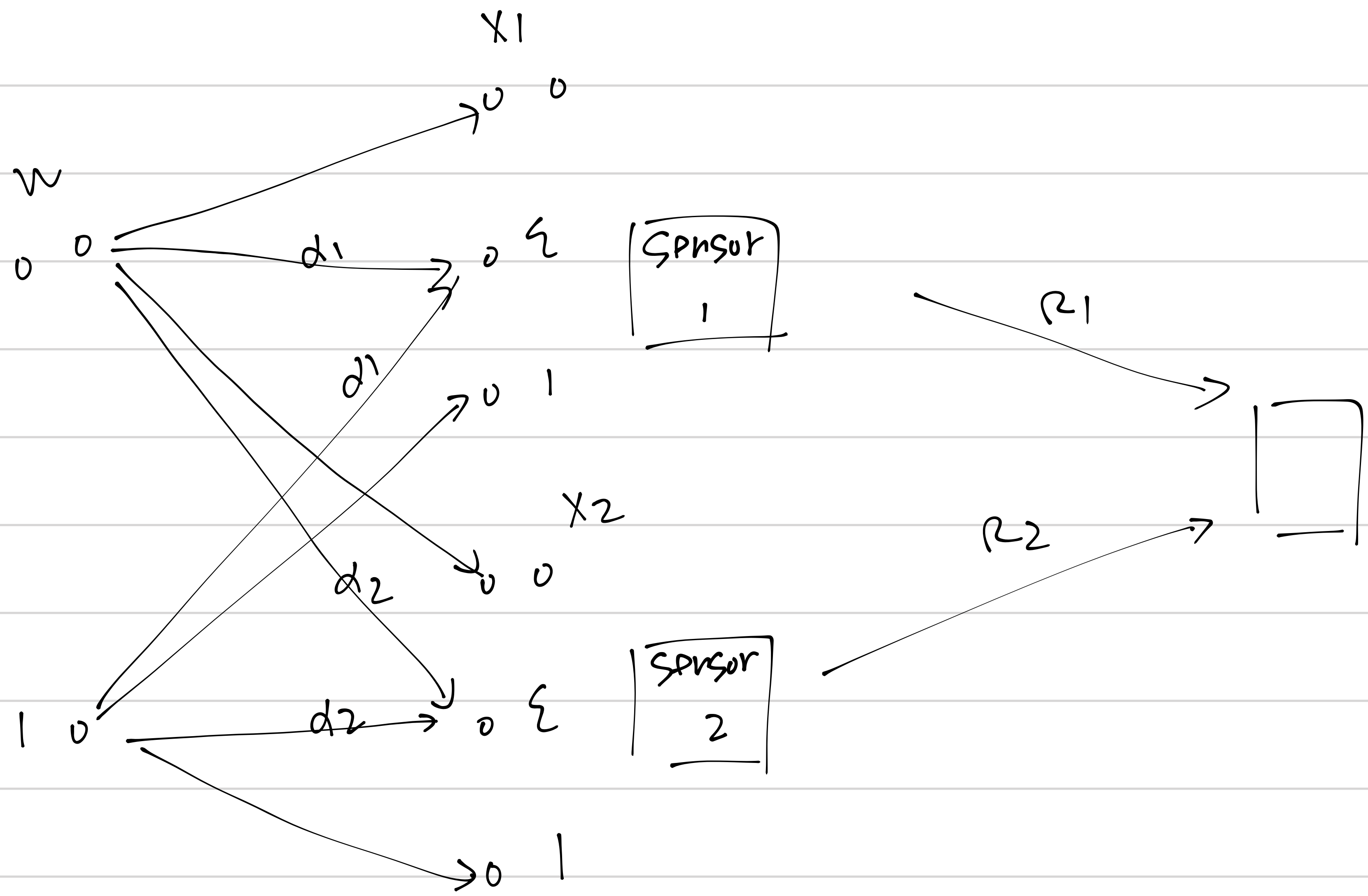
The region is marked by blue lines.

10. Slepian-wolf for distributed sensors with erasures.

Two sensor nodes each observe the same bit $w \in \{0,1\}$. W is Bernoulli with $P(W=1) = p$

X_i, the observation of the ith sensor node $i \in \{1,2\}$ is described by a binary erasure channel with erasure probability α_i.

Find AND DRAW the Slepian-wolf region of rate pairs that will accomplish this.



For sensor 1

$p(x_1=0)=(1-d_1)\cdot(1-p)$

$p(x_1=\varepsilon)=d_1$

$p(x_1=1)=(1-d_1)\cdot p$

For sensor 2

$p(x_2=0)=(1-d_2)(1-p)$

$p(x_2=\varepsilon)=d_2$

$p(x_2=1)=(1-d_2)\cdot p$

$$H(X_2|X_1)=H(X_2|X_1=0)\,p(X_1=0)+H(X_2|X_1=\varepsilon)\,p(X_1=\varepsilon)$$
$$+H(X_2|X_1=1)\,p(X_1=1)$$

if $X_1=0$ $\rightarrow$ X_2 could only be 0 or erasure

if $X_1=1$ $\rightarrow$ X_2 could only be 1 or erasure

since sensor 1 and 2 are observing the same source

$\therefore H(X_2|X_1=0) = H(\alpha_2)$

$H(X_2|X_1=1) = H(\alpha_2)$

$H(X_1) = H(\alpha_1) + (1-\alpha_1)H(p)$

$H(X_2) = H(\alpha_2) + (1-\alpha_2)H(p)$

$$H(X_2|X_1=\varepsilon) = H((1-\alpha_2)(1-p), \alpha_2, (1-\alpha_2)\cdot p)$$
$$= H(\alpha_2) + (1-\alpha_2)H(p)$$

$$\therefore H(X_2|X_1) = (1-\alpha_1)(1-p)H(\alpha_2)$$
$$+ \alpha_1 \cdot [H(\alpha_2) + (1-\alpha_2)H(p)]$$
$$+ (1-\alpha_1)\cdot p \cdot H(\alpha_2)$$
$$= H(\alpha_2) + \alpha_1(1-\alpha_2)H(p)$$

by symmetry

$$H(X_1|X_2) = (1-\alpha_2)(1-p)H(\alpha_1) + \alpha_2[H(\alpha_1) + (1-\alpha_1)H(p)]$$
$$+ (1-\alpha_2)\cdot p \cdot H(\alpha_1)$$
$$= H(\alpha_1) + \alpha_2(1-\alpha_1)H(p)$$

$$H(X_1, X_2) = H(X_2|X_1) + H(X_1)$$
$$= H(\alpha_2) + \alpha_1(1-\alpha_2)H(p) + H(\alpha_1) + (1-\alpha_1)H(p)$$
$$= H(\alpha_1) + H(\alpha_2) + (1-\alpha_1\alpha_2)H(p)$$

$R_1 > H(X_1|X_2)$ $\quad R_2 > H(X_2|X_1)$

$R_1 + R_2 > H(X_1, X_2)$

For $\alpha_1 = \alpha_2 = p = 0.5$

$p(X_1=0) = \frac{1}{4}$ $\quad p(X_1=\varepsilon) = \frac{1}{2}$ $\quad p(X_1=1) = \frac{1}{4}$

$p(X_2=0) = \frac{1}{4}$ $\quad p(X_2=\varepsilon) = \frac{1}{2}$ $\quad p(X_2=1) = \frac{1}{4}$

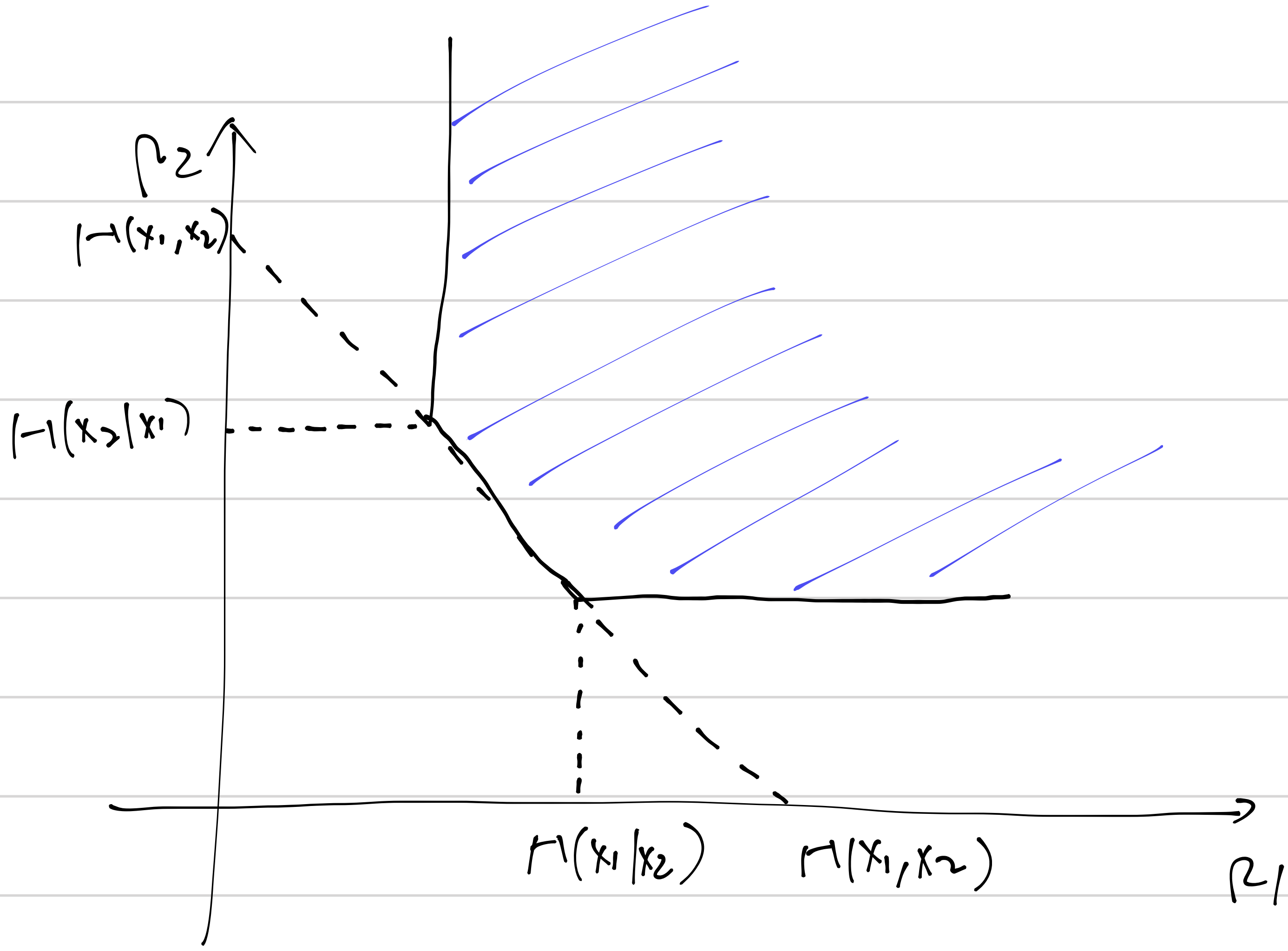
$$H(X_2|X_1) = p(X_1=0)\cdot H(X_2|X_1=0) + p(X_1=1)\, H(X_2|X_1=0)$$
$$+ p(X_1=\varepsilon)\cdot H(X_2|X_1=\varepsilon)$$
$$= \frac{1}{4}\cdot H(\tfrac{1}{2}) + \frac{1}{4}\cdot H(\tfrac{1}{2}) + \frac{1}{2}\cdot H(\tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4})$$
$$= \frac{1}{2}\cdot H(\tfrac{1}{2}) + \frac{1}{2}\cdot\frac{3}{2}$$
$$= \frac{1}{2} H(\tfrac{1}{2}) + \frac{3}{4} = \frac{5}{4}$$

$$H(X_1|X_2) = H(X_2|X_1) = \frac{1}{2} H(\tfrac{1}{2}) + \frac{3}{4} = \frac{5}{4}$$

$$H(X_1, X_2) = H(X_2|X_1) + H(X_1)$$
$$= \frac{1}{2} H(\tfrac{1}{2}) + \frac{3}{4} + H(\tfrac{1}{4}, \tfrac{1}{2}, \tfrac{1}{4})$$
$$= \frac{1}{2} H(\tfrac{1}{2}) + \frac{3}{4} + \frac{3}{2}$$
$$= \frac{1}{2} H(\tfrac{1}{2}) + \frac{9}{4} = \frac{11}{4}$$

$R_1 > H(X_1|X_2)$ $\quad R_1 + R_2 > H(X_1, X_2)$

$R_2 > H(X_2|X_1)$



for $\alpha_1 = \alpha_2 = p = 0.5$

The marked region is the rate pairs that will accomplish this.