



CLOUD COMPUTING CONCEPTS

with Indranil Gupta (Indy)

GOSSIP

Lecture C

GOSSIP ANALYSIS

PROPERTIES

Claim that the simple Push protocol

- Is lightweight in large groups
- Spreads a multicast quickly
- Is highly fault-tolerant

ANALYSIS

From old mathematical branch of *Epidemiology* [Bailey 75]

- Population of $(n+1)$ individuals mixing homogeneously
- Contact rate between any individual pair is β
- At any time, each individual is either uninfected (numbering x) or infected (numbering y)
- Then, $x_0 = n, y_0 = 1$
and at all times $x + y = n + 1$
- Infected–uninfected contact turns latter infected, and it stays infected

ANALYSIS (CONTD.)

- Continuous time process
- Then

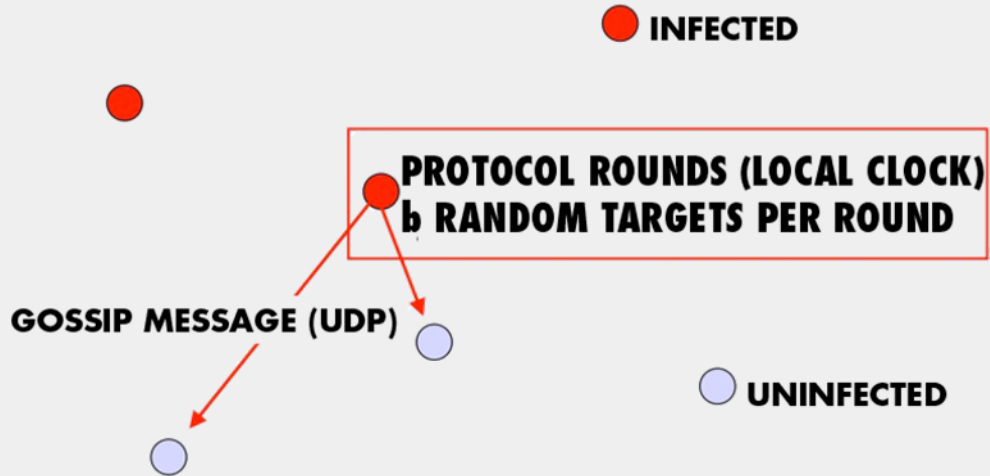
$$\frac{dx}{dt} = -\beta xy \quad (\text{why?})$$

with solution:

$$x = \frac{n(n+1)}{n + e^{\beta(n+1)t}}, \quad y = \frac{(n+1)}{1 + ne^{-\beta(n+1)t}}$$

(can you derive it?)

EPIDEMIC MULTICAST



EPIDEMIC MULTICAST ANALYSIS

$$\beta = \frac{b}{n} \quad (\text{why?})$$

Substituting, at time $t=c\log(n)$, the number of infected is

$$y \approx (n+1) \frac{1}{n^{cb-2}}$$

(correct? can you derive it?)

ANALYSIS (CONTD.)

- Set c, b to be small numbers independent of n
- Within $c \log(n)$ rounds, **[low latency]**
 - all but $\frac{1}{n^{cb-2}}$ number of nodes receive the multicast
[reliability]
- each node has transmitted no more than $c b \log(n)$ gossip messages **[lightweight]**

WHY IS LOG(N) LOW?

- $\text{Log}(N)$ is not constant in theory
- But pragmatically, it is a very slowly growing number
- Base 2
 - $\text{Log}(1000) \sim 10$
 - $\text{Log}(1\text{M}) \sim 20$
 - $\text{Log}(1\text{B}) \sim 30$
 - $\text{Log}(\text{all IPv4 address}) = 32$

FAULT-TOLERANCE

- Packet loss
 - 50% packet loss: analyze with b replaced with $b/2$
 - To achieve same reliability as 0% packet loss, takes twice as many rounds
- Node failure
 - 50% of nodes fail: analyze with n replaced with $n/2$ and b replaced with $b/2$
 - Same as above

FAULT-TOLERANCE

- With failures, is it possible that the epidemic might die out quickly?
 - Possible, but improbable:
 - Once a few nodes are infected, with high probability, the epidemic will not die out
 - So the analysis we saw in the previous slides is actually behavior *with high probability*
- [Galey and Dani 98]
- Think: Why do rumors spread so fast? Why do infectious diseases cascade quickly into epidemics? Why does a virus or worm spread rapidly?

PULL GOSSIP: ANALYSIS

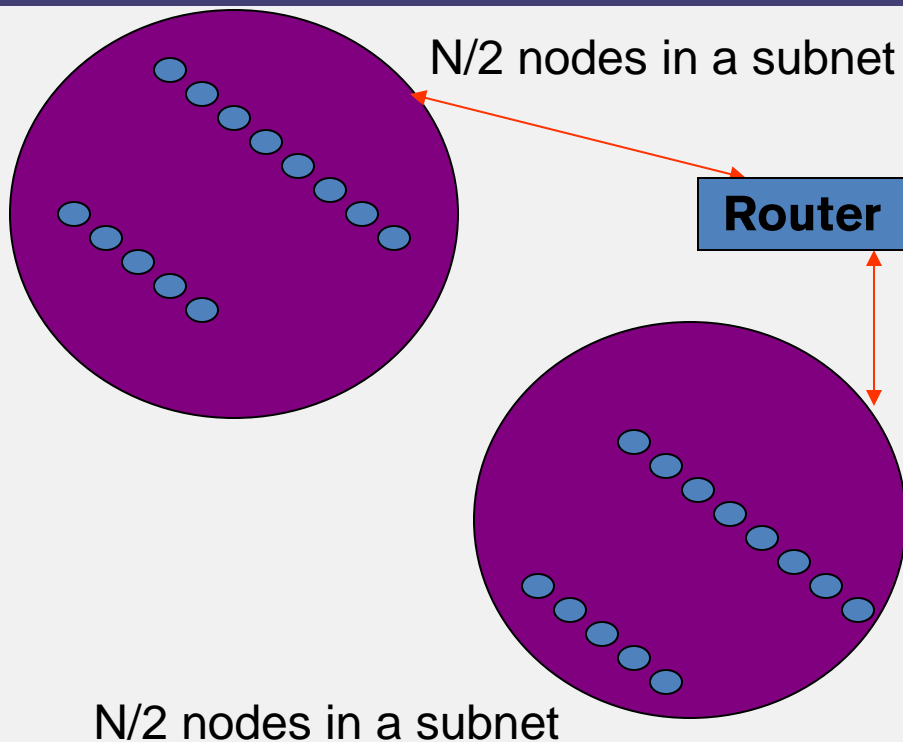
- In all forms of gossip, it takes $O(\log(N))$ rounds before about $N/2$ gets the gossip
 - Why? Because that's the fastest you can spread a message – a spanning tree with **fanout (degree)** of constant degree has $O(\log(N))$ total nodes
- Thereafter, pull gossip is faster than push gossip
- After the i th, round let p_i be the fraction of non-infected processes. Then (k =number of gossip pulls per round per process)

$$p_{i+1} = (p_i)^{k+1}$$

- This is super-exponential
- Second half of pull gossip finishes in time $O(\log(\log(N)))$

TOPOLOGY-AWARE GOSSIP

- Network topology is hierarchical
- Random gossip target selection \Rightarrow core routers face $O(N)$ load (Why?)
- **Fix:** In subnet i , which contains n_i nodes, pick gossip target in your subnet with probability $1/n_i$
- Router load $= O(1)$
- Dissemination time $= O(\log(N))$
- Why?



ANSWER – PUSH ANALYSIS (CONTD.)

Using: $\beta = \frac{b}{n}$

Substituting, at time $t=c\log(n)$

$$\begin{aligned} y &= \frac{n+1}{1 + ne^{-\frac{b}{n}(n+1)c\log(n)}} \approx \frac{n+1}{1 + \frac{1}{n^{cb-1}}} \\ &\approx (n+1)\left(1 - \frac{1}{n^{cb-1}}\right) \\ &\approx (n+1) - \frac{1}{n^{cb-2}} \end{aligned}$$