

```

1 import torch
2 import time
3 import os
4 from typing import List, Callable
5 import torch.nn.functional as F
6 import torch.distributed as dist
7 import torch.distributed.fsdp
8 from execute_util import text, image, link, system_text
9 from torch_util import get_device
10 from lecture_util import article_link
11 from lecture_08_utils import spawn, int_divide, summarize_tensor, get_init_params, render_duration
12

```

```

13 def main():

```

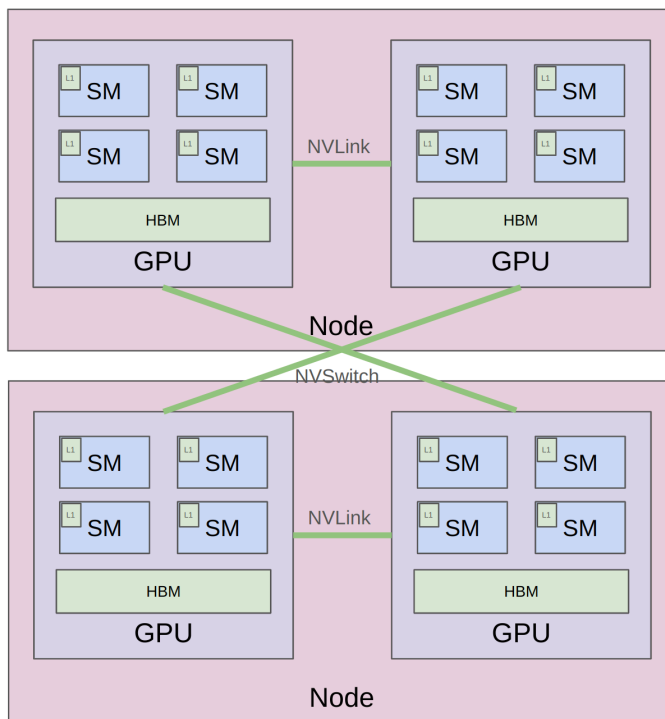
14 Last week: parallelism within a single GPU

15 This week: parallelism across multiple GPUs

```

16

```



```

17

```

18 In both cases, **compute** (arithmetic logic units) is far from inputs/outputs (**data**).

19 Unifying theme: orchestrate computation to avoid data transfer bottlenecks

```

20

```

21 Last week: reduce memory accesses via fusion/tiling

22 This week: reduce communication across GPUs/nodes via replication/sharding

```

23

```

24 Generalized hierarchy (from small/fast to big/slow):

- 25 • Single node, single GPU: L1 cache / shared memory
- 26 • Single node, single GPU: HBM
- 27 • Single node, multi-GPU: NVLink
- 28 • Multi-node, multi-GPU: NVSwitch

```

29

```

30 This lecture: concretize the concepts from last lecture in code

```

31

```

32 [\[stdout for this lecture\]](#)

Part 1: building blocks of distributed communication/computation

```
collective_operations()  # Conceptual programming interface
torch_distributed()      # How this is implemented in NCCL/PyTorch
benchmarking()          # Measure actual NCCL bandwidth
```

Part 2: distributed training

Walk through bare-bones implementations of each strategy on deep MLPs.

Recall that MLPs are the compute bottleneck in Transformers, so this is representative.

```
data_parallelism()      # Cut up along the batch dimension
tensor_parallelism()    # Cut up along the width dimension
pipeline_parallelism()  # Cut up along the depth dimension
```

What's missing?

- More general models (with attention, etc.)
- More communication/computation overlap
- This requires more complex code with more bookkeeping
- Jax/TPUs: just define the model, the sharding strategy, and the Jax compiler handles the rest [\[levanter\]](#)
- But we're doing PyTorch so you can see how one builds up from the primitives

Summary

- Many ways to parallelize: data (batch), tensor/expert (width), pipeline (depth), sequence (length)
- Can **re-compute** or store in **memory** or store in another GPU's memory and **communicate**
- Hardware is getting faster, but will always want bigger models, so will have this hierarchical structure

```
def collective_operations():
```

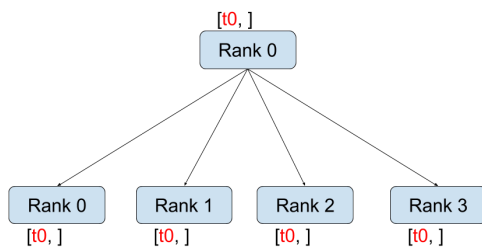
Collective operations are the conceptual primitives used for distributed programming [\[article\]](#)

- Collective means that you specify communication pattern across many (e.g., 256) nodes.
- These are classic in the parallel programming literature from the 1980s.
- Better/faster abstraction than managing point-to-point communication yourself.

Terminology:

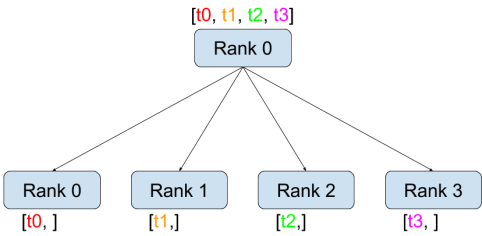
- **World size**: number of devices (e.g., 4)
- **Rank**: a device (e.g., 0, 1, 2, 3)

Broadcast



71

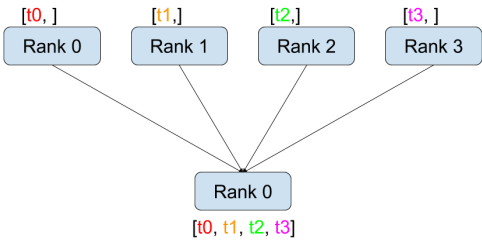
Scatter



72

73

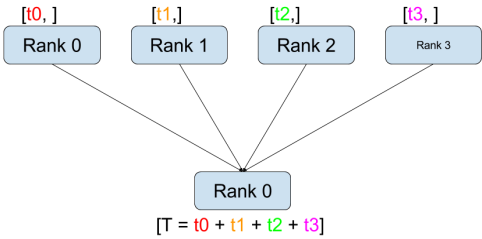
Gather



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75

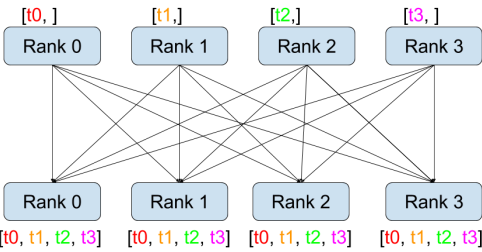
Reduce



76

77

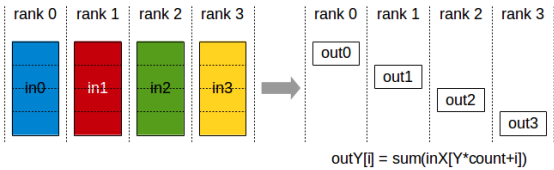
All-gather



78

79

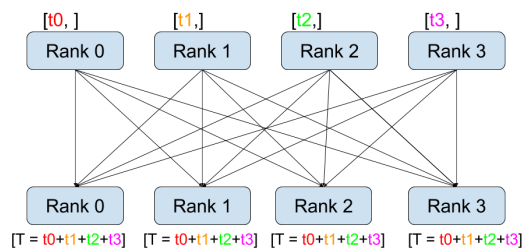
Reduce-scatter



80

81

All-reduce = reduce-scatter + all-gather



82

83 Way to remember the terminology:

- 84 • Reduce: performs some associative/commutative operation (sum, min, max)
- 85 • Broadcast/scatter is inverse of gather
- 86 • All: means destination is all devices

87

88

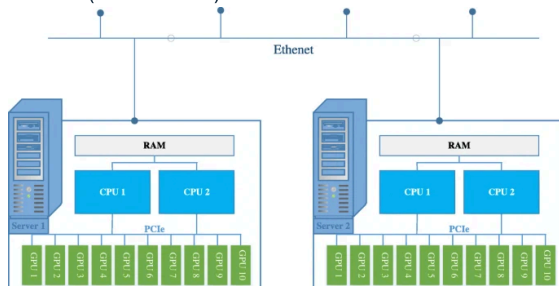
89 `def torch_distributed():`

90

Hardware

91 Classic (in the home):

92

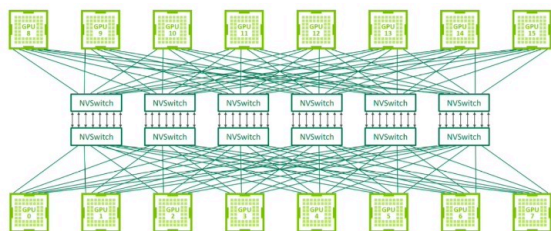


- 93 • GPUs on same node communicate via a PCI(e) bus (v7.0, 16 lanes => 242 GB/s) [\[article\]](#)
- 94 • GPUs on different nodes communicate via Ethernet (~200 MB/s)

95

96 Modern (in the data center):

97



- 98 • Within a node: NVLink connects GPUs directly, bypass CPU
- 99 • Across nodes: NVSwitch connects GPUs directly, bypass Ethernet

100

101 Each H100 has 18 NVLink 4.0 links, for a total of 900GB/s [\[article\]](#)

102 In comparison, memory bandwidth for HBM is 3.9 TB/s [\[article\]](#)

103

104 Let's check what our hardware setup is. [\[article\]](#)

105 `if torch.cuda.is_available():`

106 `os.system("nvidia-smi topo -m")`

107 Note GPUs are connected via NV18, also connected to NICs (for PCIe)

108

109

NVIDIA Collective Communication Library (NCCL)

110 NCCL translates collective operations into low-level packets that are sent between GPUs. [\[talk\]](#)

- 111 • Detects topology of hardware (e.g., number of nodes, switches, NVLink/PCIe)

- Optimizes the path between GPUs
- Launches CUDA kernels to send/receive data

PyTorch distributed library (torch.distributed)

[\[Documentation\]](#)

- Provides clean interface for collective operations (e.g., `all_gather_into_tensor`)
- Supports multiple backends for different hardware: gloo (CPU), nccl (GPU)
- Also supports higher-level algorithms (e.g., `FullyShardedDataParallel`) [not used in this course]

Let's walk through some examples.

```
spawn(collective_operations_main, world_size=4)
```

```
def collective_operations_main(rank: int, world_size: int):
    """This function is running asynchronously for each process (rank = 0, ..., world_size - 1)."""
    setup(rank, world_size)

    # All-reduce
    dist.barrier() # Waits for all processes to get to this point (in this case, for print statements)

    tensor = torch.tensor([0., 1, 2, 3], device=get_device(rank)) + rank # Both input and output

    print(f"Rank {rank} [before all-reduce]: {tensor}", flush=True)
    dist.all_reduce(tensor=tensor, op=dist.ReduceOp.SUM, async_op=False) # Modifies tensor in place
    print(f"Rank {rank} [after all-reduce]: {tensor}", flush=True)

    # Reduce-scatter
    dist.barrier()

    input = torch.arange(world_size, dtype=torch.float32, device=get_device(rank)) + rank # Input
    output = torch.empty(1, device=get_device(rank)) # Allocate output

    print(f"Rank {rank} [before reduce-scatter]: input = {input}, output = {output}", flush=True)
    dist.reduce_scatter_tensor(output=output, input=input, op=dist.ReduceOp.SUM, async_op=False)
    print(f"Rank {rank} [after reduce-scatter]: input = {input}, output = {output}", flush=True)

    # All-gather
    dist.barrier()

    input = output # Input is the output of reduce-scatter
    output = torch.empty(world_size, device=get_device(rank)) # Allocate output

    print(f"Rank {rank} [before all-gather]: input = {input}, output = {output}", flush=True)
    dist.all_gather_into_tensor(output_tensor=output, input_tensor=input, async_op=False)
    print(f"Rank {rank} [after all-gather]: input = {input}, output = {output}", flush=True)

    Indeed, all-reduce = reduce-scatter + all-gather!

    cleanup()

def benchmarking():
    Let's see how fast communication happens (restrict to one node).

    # All-reduce
```

```

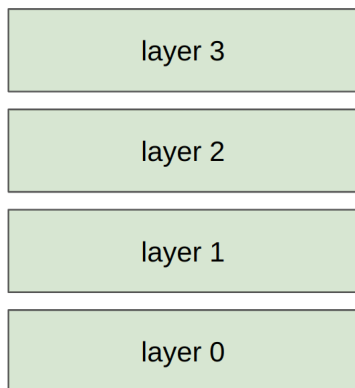
168     spawn(all_reduce, world_size=4, num_elements=100 * 1024**2)
169
170     # Reduce-scatter
171     spawn(reduce_scatter, world_size=4, num_elements=100 * 1024**2)
172
173     # References
174     How to reason about operations
175     Sample code
176
177
178 def all_reduce(rank: int, world_size: int, num_elements: int):
179     setup(rank, world_size)
180
181     # Create tensor
182     tensor = torch.randn(num_elements, device=get_device(rank))
183
184     # Warmup
185     dist.all_reduce(tensor=tensor, op=dist.ReduceOp.SUM, async_op=False)
186     if torch.cuda.is_available():
187         torch.cuda.synchronize() # Wait for CUDA kernels to finish
188         dist.barrier()           # Wait for all the processes to get here
189
190     # Perform all-reduce
191     start_time = time.time()
192     dist.all_reduce(tensor=tensor, op=dist.ReduceOp.SUM, async_op=False)
193     if torch.cuda.is_available():
194         torch.cuda.synchronize() # Wait for CUDA kernels to finish
195         dist.barrier()           # Wait for all the processes to get here
196     end_time = time.time()
197
198     duration = end_time - start_time
199     print(f"[all_reduce] Rank {rank}: all_reduce(world_size={world_size}, num_elements={num_elements}) took
{render_duration(duration)}", flush=True)
200
201     # Measure the effective bandwidth
202     dist.barrier()
203     size_bytes = tensor.element_size() * tensor.numel()
204     sent_bytes = size_bytes * 2 * (world_size - 1) # 2x because send input and receive output
205     total_duration = world_size * duration
206     bandwidth = sent_bytes / total_duration
207     print(f"[all_reduce] Rank {rank}: all_reduce measured bandwidth = {round(bandwidth / 1024**3)} GB/s", flush=True)
208
209     cleanup()
210
211
212 def reduce_scatter(rank: int, world_size: int, num_elements: int):
213     setup(rank, world_size)
214
215     # Create input and outputs
216     input = torch.randn(world_size, num_elements, device=get_device(rank)) # Each rank has a matrix
217     output = torch.empty(num_elements, device=get_device(rank))
218
219     # Warmup
220     dist.reduce_scatter_tensor(output=output, input=input, op=dist.ReduceOp.SUM, async_op=False)
221     if torch.cuda.is_available():
222         torch.cuda.synchronize() # Wait for CUDA kernels to finish
223         dist.barrier()           # Wait for all the processes to get here
224

```

```

225     # Perform reduce-scatter
226     start_time = time.time()
227     dist.reduce_scatter_tensor(output=output, input=input, op=dist.ReduceOp.SUM, async_op=False)
228     if torch.cuda.is_available():
229         torch.cuda.synchronize() # Wait for CUDA kernels to finish
230         dist.barrier()           # Wait for all the processes to get here
231     end_time = time.time()
232
233     duration = end_time - start_time
234     print(f"[reduce_scatter] Rank {rank}: reduce_scatter(world_size={world_size}, num_elements={num_elements}) took
{render_duration(duration)}", flush=True)
235
236     # Measure the effective bandwidth
237     dist.barrier()
238     data_bytes = output.element_size() * output.numel() # How much data in the output
239     sent_bytes = data_bytes * (world_size - 1) # How much needs to be sent (no 2x here)
240     total_duration = world_size * duration # Total time for transmission
241     bandwidth = sent_bytes / total_duration
242     print(f"[reduce_scatter] Rank {rank}: reduce_scatter measured bandwidth = {round(bandwidth / 1024**3)} GB/s",
flush=True)
243
244     cleanup()
245
246
247 def data_parallelism():
248

```



```

249     Sharding strategy: each rank gets a slice of the data
250
251     data = generate_sample_data()
252     spawn(data_parallelism_main, world_size=4, data=data, num_layers=4, num_steps=1)
253
254     Notes:
255     • Losses are different across ranks (computed on local data)
256     • Gradients are all-reduced to be the same across ranks
257     • Therefore, parameters remain the same across ranks
258
259
260 def generate_sample_data():
261     batch_size = 128
262     num_dim = 1024
263     data = torch.randn(batch_size, num_dim)

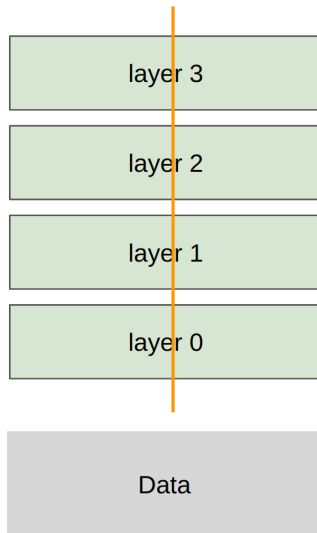
```

```

264     return data
265
266
267 def data_parallelism_main(rank: int, world_size: int, data: torch.Tensor, num_layers: int, num_steps: int):
268     setup(rank, world_size)
269
270     # Get the slice of data for this rank (in practice, each rank should load only its own data)
271     batch_size = data.size(0) # @inspect batch_size
272     num_dim = data.size(1) # @inspect num_dim
273     local_batch_size = int_divide(batch_size, world_size) # @inspect local_batch_size
274     start_index = rank * local_batch_size # @inspect start_index
275     end_index = start_index + local_batch_size # @inspect end_index
276     data = data[start_index:end_index].to(get_device(rank))
277
278     # Create MLP parameters params[0], ..., params[num_layers - 1] (each rank has all parameters)
279     params = [get_init_params(num_dim, num_dim, rank) for i in range(num_layers)]
280     optimizer = torch.optim.AdamW(params, lr=1e-3) # Each rank has own optimizer state
281
282     for step in range(num_steps):
283         # Forward pass
284         x = data
285         for param in params:
286             x = x @ param
287             x = F.gelu(x)
288         loss = x.square().mean() # Loss function is average squared magnitude
289
290         # Backward pass
291         loss.backward()
292
293         # Sync gradients across workers (only difference between standard training and DDP)
294         for param in params:
295             dist.all_reduce(tensor=param.grad, op=dist.ReduceOp.AVG, async_op=False)
296
297         # Update parameters
298         optimizer.step()
299
300         print(f"[data_parallelism] Rank {rank}: step = {step}, loss = {loss.item()}, params =
{[summarize_tensor(params[i]) for i in range(num_layers)]}", flush=True)
301
302     cleanup()
303
304
305 def tensor_parallelism():

```


306



307 Sharding strategy: each rank gets part of each layer, transfer all data/activations

308

309 data = generate_sample_data()

310 spawn(tensor_parallelism_main, world_size=4, data=data, num_layers=4)

311

312

313 def tensor_parallelism_main(rank: int, world_size: int, data: torch.Tensor, num_layers: int):

314 setup(rank, world_size)

315

316 data = data.to(get_device(rank))

317 batch_size = data.size(0) # @inspect batch_size

318 num_dim = data.size(1) # @inspect num_dim

319 local_num_dim = int_divide(num_dim, world_size) # Shard `num_dim` @inspect local_num_dim

320

321 # Create model (each rank gets 1/world_size of the parameters)

322 params = [get_init_params(num_dim, local_num_dim, rank) for i in range(num_layers)]

323

324 # Forward pass

325 x = data

326 for i in range(num_layers):

327 # Compute activations (batch_size x local_num_dim)

328 x = x @ params[i] # Note: this is only on a slice of the parameters

329 x = F.gelu(x)

330

331 # Allocate memory for activations (world_size x batch_size x local_num_dim)

332 activations = [torch.empty(batch_size, local_num_dim, device=get_device(rank)) for _ in range(world_size)]

333

334 # Send activations via all gather

335 dist.all_gather(tensor_list=activations, tensor=x, async_op=False)

336

337 # Concatenate them to get batch_size x num_dim

338 x = torch.cat(activations, dim=1)

339

340 print(f"[tensor_parallelism] Rank {rank}: forward pass produced activations {summarize_tensor(x)}", flush=True)

341

342 # Backward pass: homework exercise

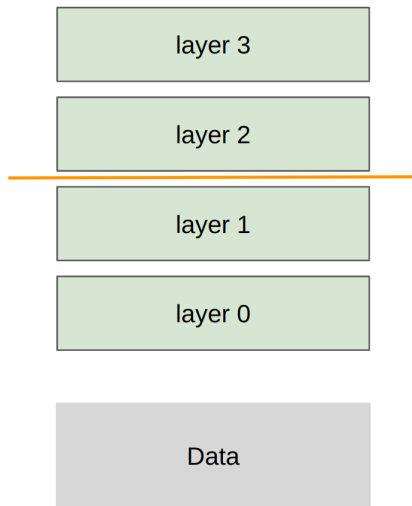
343

344 cleanup()

345

346

347 def pipeline_parallelism():



Sharding strategy: each rank gets subset of layers, transfer all data/activations

```

349
350
351 data = generate_sample_data()
352 spawn(pipeline_parallelism_main, world_size=2, data=data, num_layers=4, num_micro_batches=4)
353
354
355 def pipeline_parallelism_main(rank: int, world_size: int, data: torch.Tensor, num_layers: int, num_micro_batches: int):
356     setup(rank, world_size)
357
358     # Use all the data
359     data = data.to(get_device(rank))
360     batch_size = data.size(0) # @inspect batch_size
361     num_dim = data.size(1) # @inspect num_dim
362
363     # Split up layers
364     local_num_layers = int_divide(num_layers, world_size) # @inspect local_num_layers
365
366     # Each rank gets a subset of layers
367     local_params = [get_init_params(num_dim, num_dim, rank) for i in range(local_num_layers)]
368
369     # Forward pass
370
371     # Break up into micro batches to minimize the bubble
372     micro_batch_size = int_divide(batch_size, num_micro_batches) # @inspect micro_batch_size
373     if rank == 0:
374         # The data
375         micro_batches = data.chunk(chunks=num_micro_batches, dim=0)
376     else:
377         # Allocate memory for activations
378         micro_batches = [torch.empty(micro_batch_size, num_dim, device=get_device(rank)) for _ in
379 range(num_micro_batches)]
380
381     for x in micro_batches:
382         # Get activations from previous rank
383         if rank - 1 >= 0:
384             dist.recv(tensor=x, src=rank - 1)
385
386         # Compute layers assigned to this rank
387         for param in local_params:
388             x = x @ param
389             x = F.gelu(x)

```

```

390         # Send to the next rank
391         if rank + 1 < world_size:
392             print(f"[pipeline_parallelism] Rank {rank}: sending {summarize_tensor(x)} to rank {rank + 1}", flush=True)
393             dist.send(tensor=x, dst=rank + 1)
394
395     Not handled: overlapping communication/computation to eliminate pipeline bubbles
396
397     # Backward pass: homework exercise
398
399     cleanup()
400
401     #####
402
403     def setup(rank: int, world_size: int):
404         # Specify where master lives (rank 0), used to coordinate (actual data goes through NCCL)
405         os.environ["MASTER_ADDR"] = "localhost"
406         os.environ["MASTER_PORT"] = "15623"
407
408         if torch.cuda.is_available():
409             dist.init_process_group("nccl", rank=rank, world_size=world_size)
410         else:
411             dist.init_process_group("gloo", rank=rank, world_size=world_size)
412
413
414     def cleanup():
415         torch.distributed.destroy_process_group()
416
417
418     if __name__ == "__main__":
419         main()

```