Q1. Product Reviews 1a) Target (continuous) Future sales volume (units) over a fixed horizon (e.g., next 30 days). 1b) Linear model with rating + judgment words Let Score be the numeric rating. Let freq_good, freq_bad, freq_doesnt_work be normalized word frequencies (e.g., per 100 tokens). Model: SalesNext30d = b0 + b1*Score + b2*freq_good + b3*freq_bad+ b4*freq_doesnt_work+ error 1c) Reducing features - LASSO (L1) regularization to shrink unhelpful coefficients to zero. - Dimensionality reduction (e.g., PCA on text features). - Aggregate to a single sentiment score instead of multiple word counts. 1d) Mixed scales (1–5 vs 1–10) Normalize ratings to a common scale before modeling (e.g., convert 1–10 to 1–5 by dividing by 2, or standardize to z-scores). Optionally add an indicator feature for original scale (e.g., is_scale_10) to allow different intercepts/slopes if needed. Without normalization, the Score coefficient is not comparable across scales. Q2. Fruit dataset 2a) Categorical variables type, color, size 2b) Total variables after one-hot coding - If you one-hot encode all levels (no drop): 7 dummy vars + price = 8 total. - If you use drop-first (to avoid multicollinearity with an intercept): 4 dummy vars + price = 5 total. (Counting only the encoded features: 7 vs 4.) Q3. Small linear regression Given: House 1: Size=1400, Beds=3, Price=245 House 2: Size=1600, Beds=3, Price=312 House 3: Size=1700, Beds=4, Price=279 (Price in

$$1000s)Model: Price = \beta0 + \beta1 \cdot Size + \beta2 \cdot Bedrooms(We'llscaleSizeby100foreasierarithmetic: Size100 = Size/100)3a)FeaturematrixAA$$

$$= [[1,14,3],[1,16,3],[1,17,4]]3b)Targetvectoryy = [245,312,279]^T3c)Normalequation\beta = (A^\top A)^{-1}A^\top ygives\beta = [\beta0,\beta1,\beta2]$$

$$= [-24.5,33.5,-66.5](IfyoudoNOTscaleSize: \beta = [-24.5,0.335,-66.5])3d)PredictforSize = 1500sqft(Size100 = 15),Bedrooms = 3: Pric\hat{e} = -24.5 + 33$$

$$= 278.5(in$$

1000s) ⇒ Predicted price ≈ $278,500 Q4. When is ($A^TA$) invertible? Claim. For A ∈ ℝ^{n×p}, (A^TA) is invertible ⇔ the columns of A are linearly independent. Proof. (⇒) Suppose A^TA is invertible. If A's columns were dependent, there would exist x ≠ 0 with Ax = 0 (i.e., a nontrivial vector in the null space of A). Then A^TA x = A^T(Ax) = A^T·0 = 0, so x is in the null space of A^TA. But an invertible matrix has only the zero vector in its null space. Contradiction. Hence A's columns are linearly independent. (⇐) Suppose A's columns are linearly independent. Then rank(A) = p (full column rank), so for any x ≠ 0 we have Ax ≠ 0 and x^TA^TA x = $\|Ax\|^2$ > 0. Thus A^TA is symmetric positive definite and therefore invertible. (Equivalently, by the rank identity rank(A^TA) = rank(A) = p; since A^TA is p×p with rank p, it is invertible.)

In [2]:

```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_california_housing
from sklearn.linear_model import LinearRegression

california = fetch_california_housing(as_frame=True)
X_df = california.data
y = california.target

print("5a) Feature names:", list(X_df.columns))

print("\n5b) Rows 10–15 for HouseAge, AveRooms, Population:")
print(X_df.loc[10:15, ['HouseAge','AveRooms','Population']])

# 5c) Simple linear regression: MEDV ~ AveOccup
X = X_df[['AveOccup']].values
n = len(y)
split = (2*n)//3
X_train, X_test = X[:split], X[split:]
y_train, y_test = y.values[:split], y.values[split:]

model = LinearRegression()
model.fit(X_train, y_train)

print("\n5d) R^2 (train):", model.score(X_train, y_train))
print("5d) R^2 (test):", model.score(X_test, y_test))
print("Intercept:", model.intercept_)
print("Slope for AveOccup:", model.coef_[0])

plt.figure()
sample = np.arange(0, n, 50)
plt.scatter(X_df['AveOccup'].iloc[sample], y.iloc[sample], s=8)
xs = np.linspace(X.min(), X.max(), 200).reshape(-1,1)
ys = model.predict(xs)
plt.plot(xs, ys, linewidth=2)
plt.xlabel("AveOccup")
plt.ylabel("MedHouseValue ($100k)")
plt.title("Simple Linear Regression: MedHouseValue ~ AveOccup")
plt.show()
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5a) Feature names: ['MedInc', 'HouseAge', 'AveRooms', 'AveBedrms', 'Population', 'AveOccup', 'Latitude', 'Longitude']

5b) Rows 10–15 for HouseAge, AveRooms, Population:
    HouseAge  AveRooms  Population
10      52.0  5.477612       910.0
11      52.0  4.772480      1504.0
12      52.0  5.322650      1098.0
13      52.0  4.000000       345.0
14      52.0  4.262903      1212.0
15      50.0  4.242424       697.0

5d) R^2 (train): 0.0023374510886858824
5d) R^2 (test): -0.03700899339889241
Intercept: 2.038064223107151
Slope for AveOccup: -0.009751795964041849
```

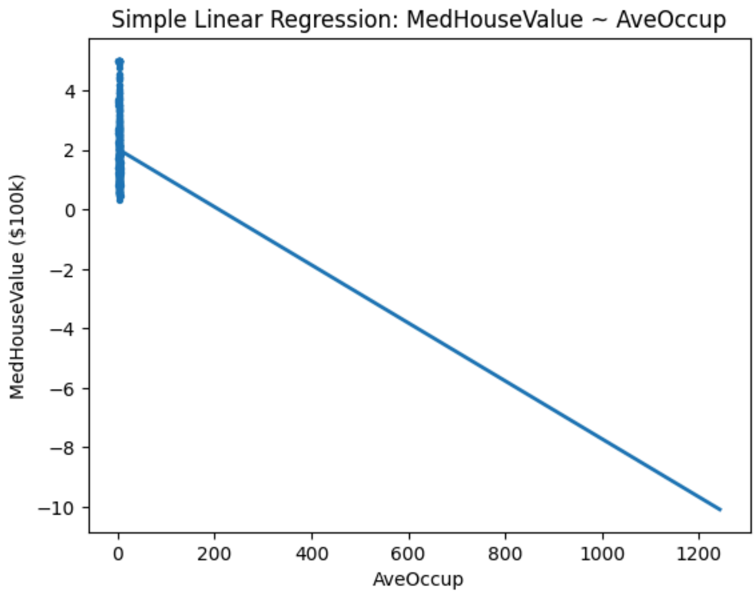

5e) R^2: fraction of target variance explained; expect low with one predictor. Big train–test gap ⇒ overfitting; both low ⇒ weak model.Q6: 6a) $b \notin col(A)$ 6b) $b \in col(A)$ and $rank(A)=n$ 6c) $b \in col(A)$ and $rank(A)<n$