PROJECT----WEEK04

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Problem 1

① Classical Brownian Motion:

$$P_t = P_{t-1} + r_t$$

$$E(P_t) = P_{t-1} + E(r_t) \qquad r_t \sim N(0, \sigma^2)$$

$$= P_{t-1}$$

$$Var(P_t) = Var(r_t)$$

$$= \sigma^2$$

② Arithmetic Return

$$P_t = P_{t-1}(1 + r_t)$$

$$E(P_t) = P_{t-1} + P_{t-1} E(r_t) \qquad r_t \sim N(0, \sigma^2)$$

$$= P_{t-1}$$

$$Var(P_t) = P_{t-1}^2 \, Var(r_t)$$

$$= P_{t-1}^2 \sigma^2$$

③ Log Return / Geometric Brownian Motion

$$P_t = P_{t-1} e^{r_t}$$

$$E(P_t) = P_{t-1} E(e^{r_t}) \qquad \ln(e^{r_t}) \sim N(0, \sigma^2)$$

$$= e^{\frac{\sigma^2}{2}} P_{t-1} > P_{t-1} \qquad E(e^{r_t}) = e^{\frac{\sigma^2}{2}}$$

$$(\sigma^2 > 0) \qquad Var(e^{r_t}) = (e^{\sigma^2} - 1) e^{\sigma^2}$$

$$Var(P_t) = P_{t-1}^2 \, Var(e^{r_t})$$

$$= P_{t-1}^2 (e^{\sigma^2} - 1) e^{\sigma^2}$$

```
rt ~ N( 0, 1 )
Pt-1 =  1
Classical Brownian Method:
 E(P) =  1.0035256192471083 Expected(Pt-1): 1
Var(P) =  0.9942590689266244 Expected(σ^2): 1
Arithmetic Return:
 E(P) =  1.0035256192471083 Expected(Pt-1): 1
Var(P) =  0.9942590689266244 Expected(Pt-1^2*σ^2): 1
Geometic Brownian Motion:
 E(P) =  1.6502507854553548 Expected(e^(σ^2/2) * Pt-1): 1.6487212707001282
Var(P) =  4.626707012527816 Expected(Pt-1^2 * (e^(σ^2)-1) ) * e^(σ^2)): 4.670774270471604
```

```
rt ~ N( 0, 1 )
Pt-1 =  3
Classical Brownian Method:
 E(P) =  3.0055686519128573 Expected(Pt-1): 3
Var(P) =  0.9940276415693544 Expected(σ^2): 1
Arithmetic Return:
 E(P) =  3.016765955738573 Expected(Pt-1): 3
Var(P) =  8.945526765809502 Expected(Pt-1^2*σ^2): 9
Geometic Brownian Motion:
 E(P) =  4.96597377171533 Expected(e^(σ^2/2) * Pt-1): 4.946163812100385
Var(P) =  41.90967028298915 Expected(Pt-1^2 * (e^(σ^2)-1) ) * e^(σ^2)): 42.036968434244436
```

test it with different value for sigma and Pt-1

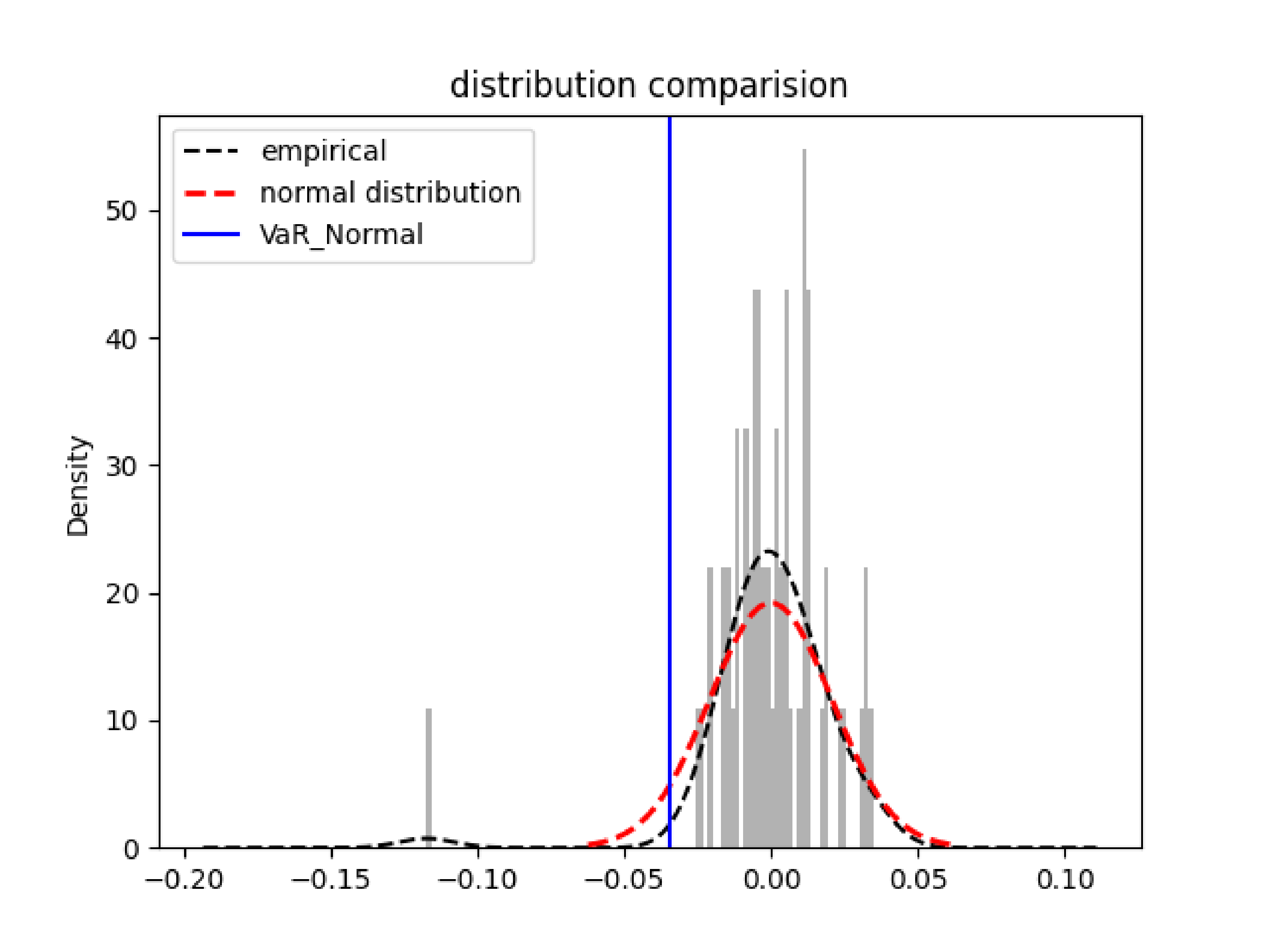
```
rt ~ N( 0, 1 )
Pt-1 =  -1
Classical Brownian Method:
 E(P) =  -0.9948295922477342 Expected(Pt-1): -1
Var(P) =  1.0045670281323267 Expected(σ^2): 1
Arithmetic Return:
 E(P) =  -1.005150407752266 Expected(Pt-1): -1
Var(P) =  1.0045672345486363 Expected(Pt-1^2*σ^2): 1
Geometic Brownian Motion:
 E(P) =  -1.6591232757206302 Expected(e^(σ^2/2) * Pt-1): -1.6487212707001282
Var(P) =  4.6532396964212595 Expected(Pt-1^2 * (e^(σ^2)-1) ) * e^(σ^2)): 4.670774270471604
```

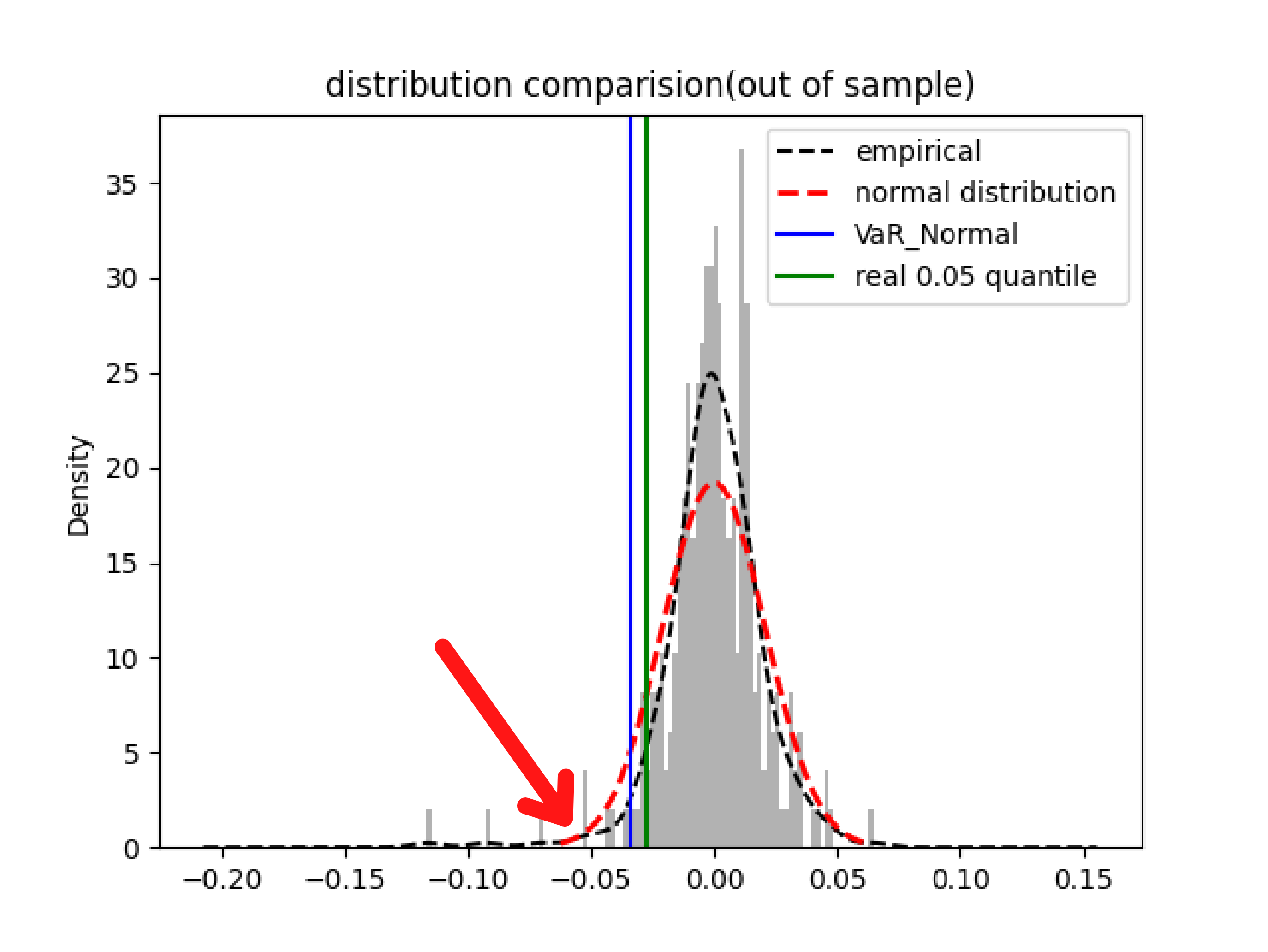
```
rt ~ N( 0, 2 )
Pt-1 =  1
Classical Brownian Method:
 E(P) =  0.9987160968410883 Expected(Pt-1): 1
Var(P) =  2.0030020362142107 Expected(σ^2): 2
Arithmetic Return:
 E(P) =  0.9987160968410883 Expected(Pt-1): 1
Var(P) =  2.0030020362142107 Expected(Pt-1^2*σ^2): 2
Geometic Brownian Motion:
 E(P) =  2.712746459474568 Expected(e^(σ^2/2) * Pt-1): 2.718281828459045
Var(P) =  48.70272583230862 Expected(Pt-1^2 * (e^(σ^2)-1) ) * e^(σ^2)): 47.20909393421359
```

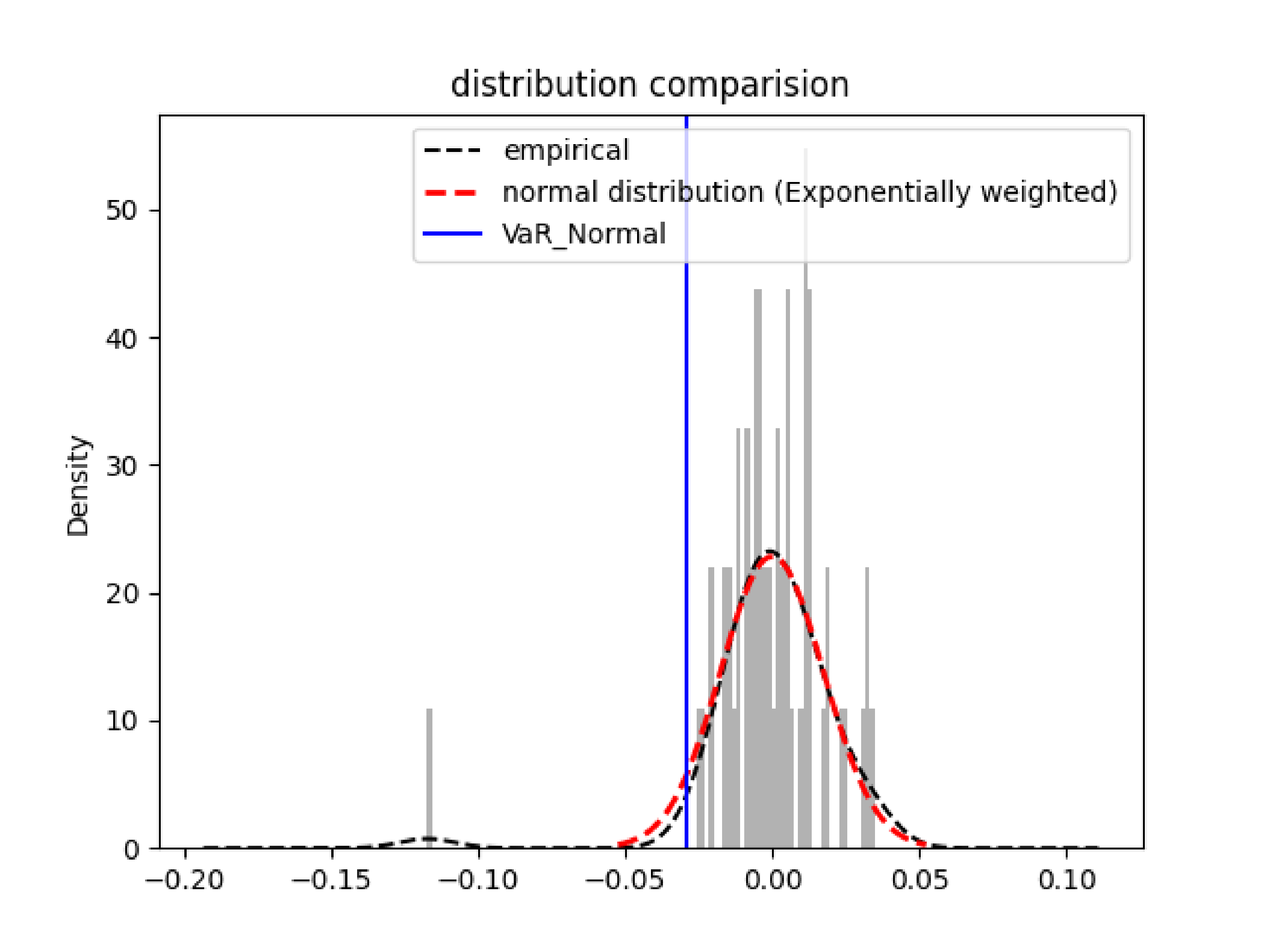
Problem 2

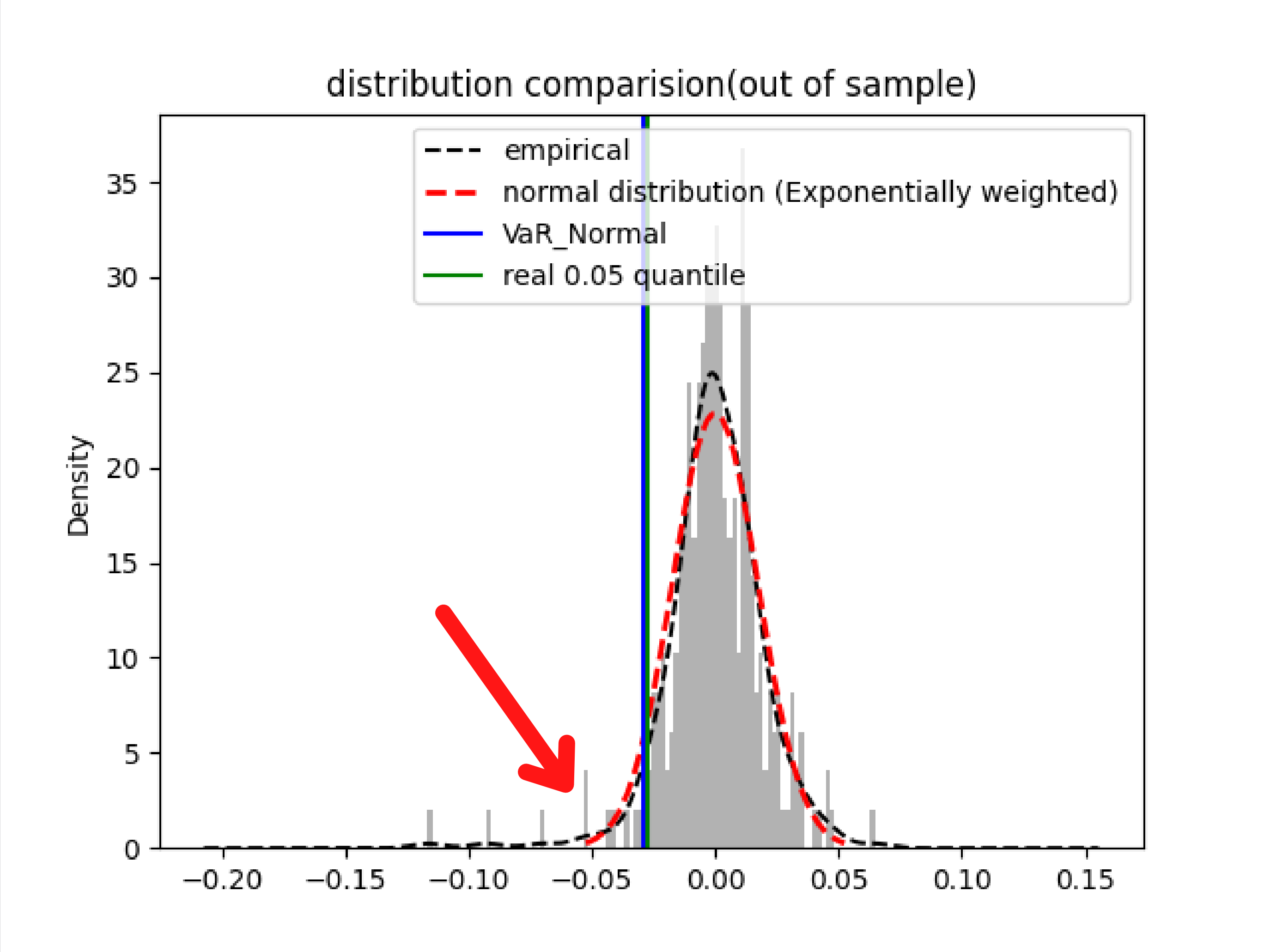
Calculate VaR
1. Using a normal distribution.
2. Using a normal distribution with an Exponentially Weighted variance (λ = 0. 94)
3. Using a MLE fitted T distribution.
4. Using a Historic Simulation.

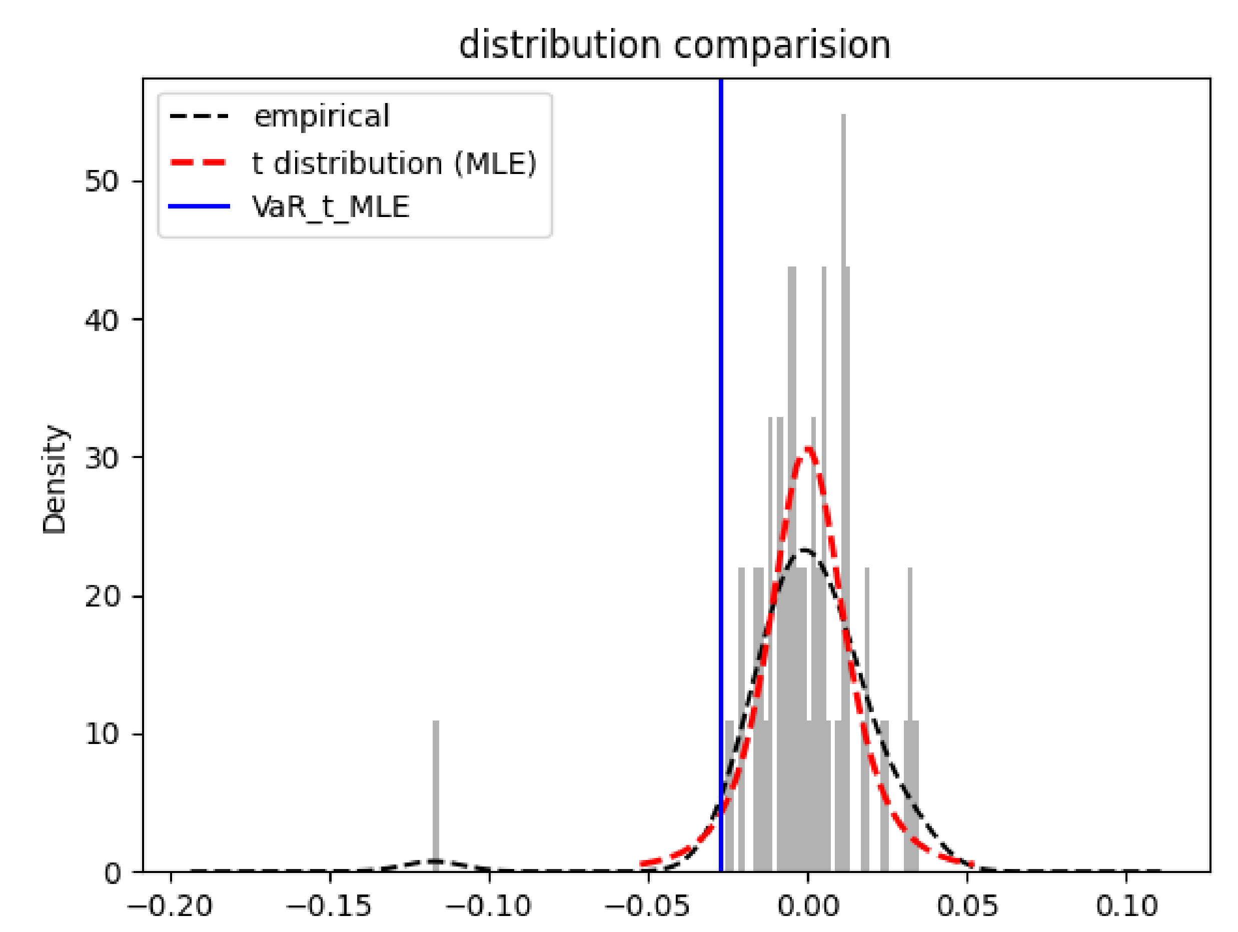
```
VaR calculations:
   Normal Distribution: 3.41% in dollars: 1.902
Exponentially Weighted: 2.87% in dollars: 1.600
         MLE fitted T: 2.73% in dollars: 1.519
   Historic Simulation: 2.50% in dollars: 1.391
```

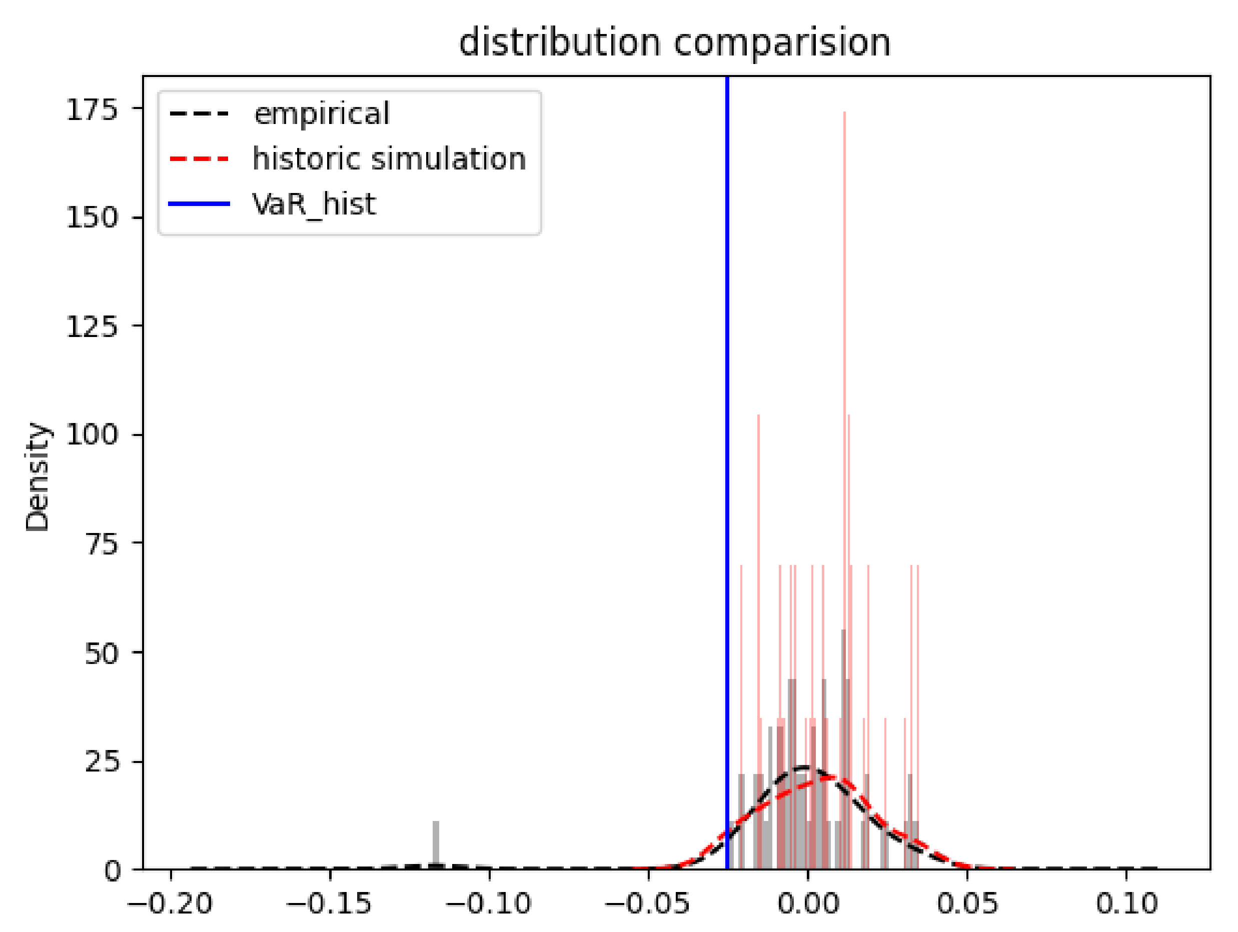

distribution comparision
empirical
normal distribution
VaR_Normal
Density

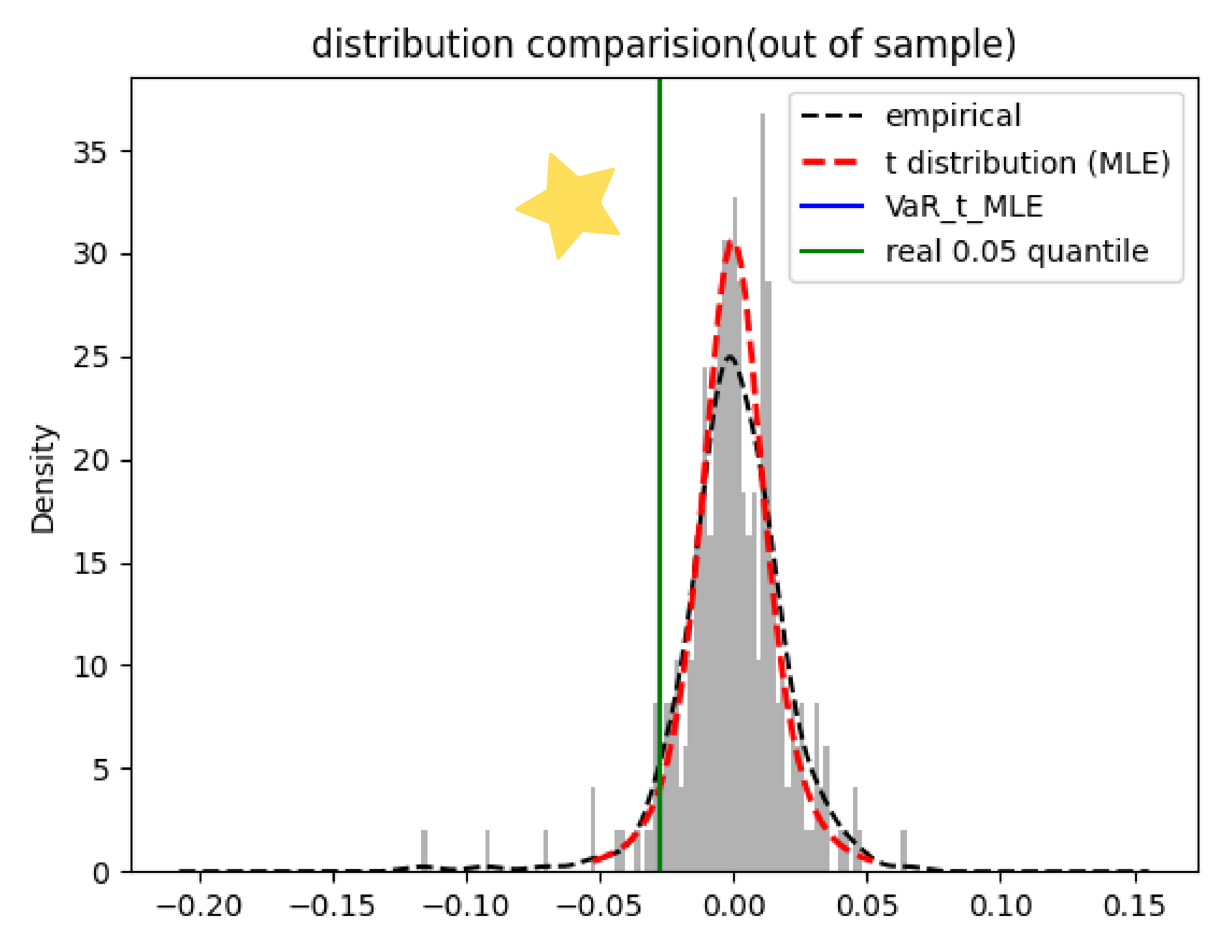

distribution comparision(out of sample)
empirical
normal distribution
VaR_Normal
real 0.05 quantile
Density

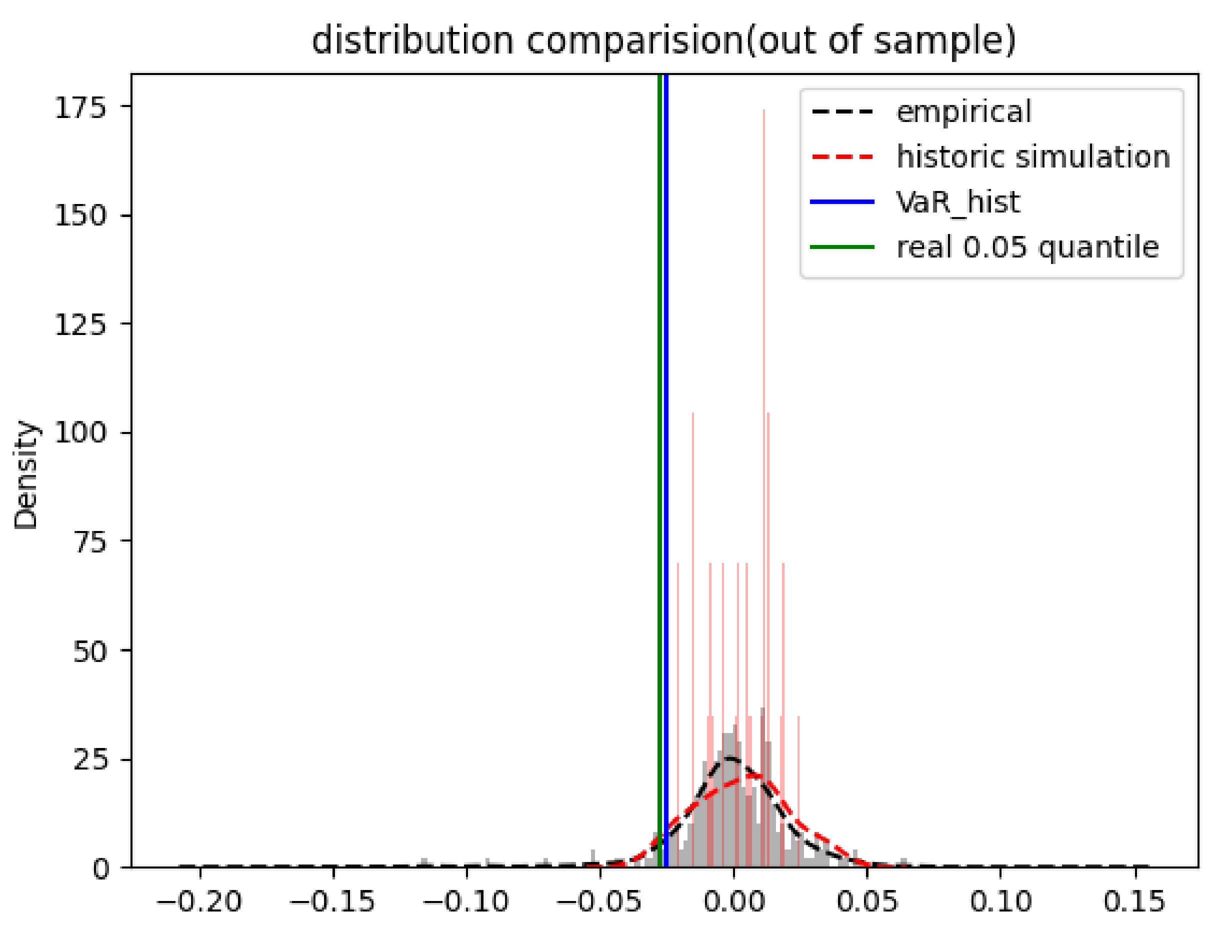

distribution comparision
empirical
normal distribution (Exponentially weighted)
VaR_Normal
Density


distribution comparision(out of sample)
empirical
normal distribution (Exponentially weighted)
VaR_Normal
real 0.05 quantile
Density


distribution comparision
empirical
t distribution (MLE)
VaR_t_MLE
Density


distribution comparision
empirical
historic simulation
VaR_hist
Density


distribution comparision(out of sample)
empirical
t distribution (MLE)
VaR_t_MLE
real 0.05 quantile
Density


distribution comparision(out of sample)
empirical
historic simulation
VaR_hist
real 0.05 quantile
Density

Problem 3

	Pros	Cons
Normal	Not underestimating	Thin-tail
Normal (EW)	Not underestimating, shape fit best from looking at graph	Thin-tail
t-distribution (MLE)	Fat tail, excess kurtosis (suits financial data)	Overestimating kurtosis (more interested in the tail distributions)
Historic simulation	Robust on extreme value	Volatile

```
----Portfolio 1----
VaR[Return] 1.65%
VaR[portfolio Value]: (a loss of)  $ 6002.954453962024

----Portfolio 2----
VaR[Return] 1.46%
VaR[portfolio Value]: (a loss of)  $ 4775.207786430735

----Portfolio 3----
VaR[Return] 1.10%
VaR[portfolio Value]: (a loss of)  $ 3591.438028104183

----Total----
VaR[Return] 1.38%
VaR[portfolio Value]: (a loss of)  $ 14011.428774479446
```

recall for the return of INTC...

```
VaR calculations:
   Normal Distribution: 3.41% in dollars: 1.902
Exponentially Weighted: 2.87% in dollars: 1.600
          MLE fitted T: 2.73% in dollars: 1.519
   Historic Simulation: 2.50% in dollars: 1.391
```

THANKS FOR LISTENING