Project ----- Week06
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Problem 1

Create call and put options with:

Current Stock Price: $165
Current Date: 02/25/2022
Options Expiration Date: 03/18/2022
Risk Free Rate: 0.25%
Continuously Compounding Coupon: 0.53%

Calculate Time to maturity: (using calendar days)

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Time to maturity is:0.058Years
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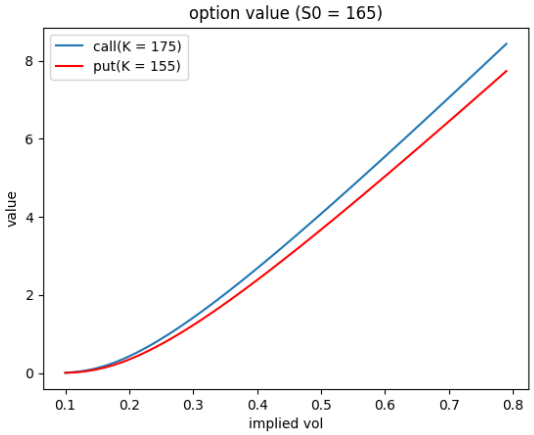
Changing the volatility and calculate the option value with the BS function.

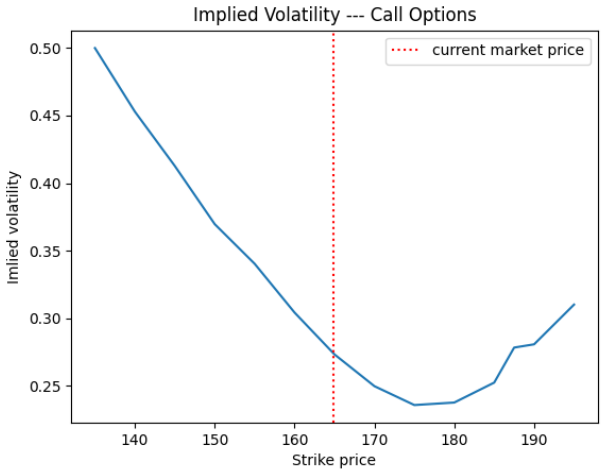


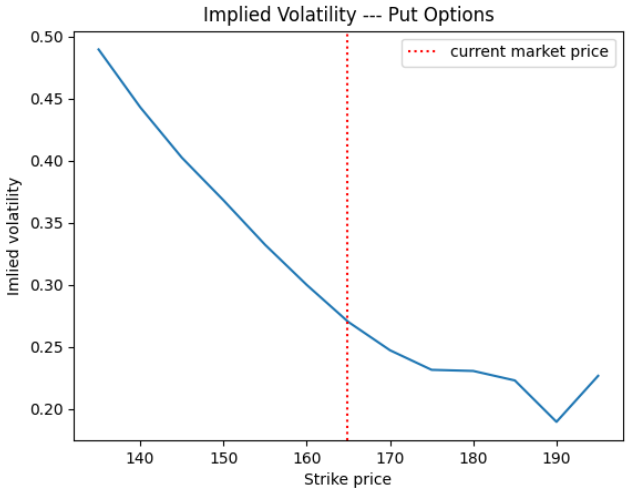
Both the value of call option and the value of put option will increase as the volatility gets higher.

We can get implied volatility from observed market price of the options. When the demand is larger than supply, the prices of the options will go higher, which will make the implied volatility higher. If the opposite happens (demand is lower and having excess supply), the price will be lower and implied volatility will be lower.

Problem 2

Calculate implied volatilities for each option contract based on their current quoted price. Plot them against the strike price.



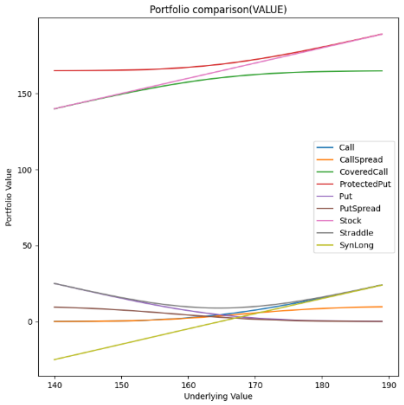


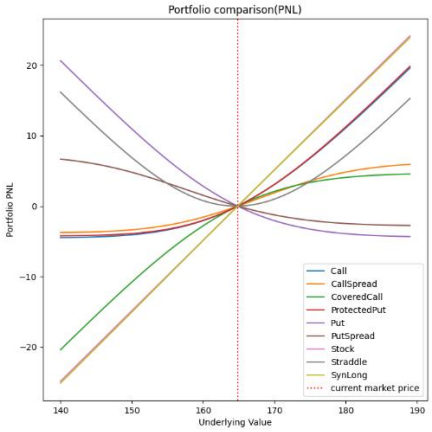
The "smile" shaped curve can be observed when plotting implied volatility against strike price. The intrinsic and theoretical price of an option is calculated with the BS function using an assumed volatility, while the implied volatility is derived from using the function backwardly based on empirical market price of options. As a result, the shape of the curve indicate that out-of-the-money and in-the-money options are priced higher by the market, or BS model is undervaluing these options. (Not sure why the lowest implied volatility does not occur exactly when the option is "at-the money" but a little in-the-money for calls and out-of-the-money for puts.)

In addition to the "smile", the shape of curves is not symmetrical, more inclined to be a "smirk" (reverse skew), showing that options with a lower strike price (out-of-the-money calls and in-the-money puts) are priced relatively higher. One rationale for observing this characteristic is that investors are currently more concerned about the risk of the stock price going down, thus buying protective option contracts for those potential circumstances, creating more demands for low strike price, pushing the market price up, showing a higher implied volatility in our calculation.

Problem 3

First calculate the implied volatility based on real market price. Use this volatility to get option value when changing the value of the underlying stock. Plot the portfolio's value against underlying stock price.





As volatility is derived from the current prices of option contracts, all portfolios will have a value of zero when the underlying value is at the current price.

Observations from the graphs:

1. The "ProtectedPut" portfolio and the "Call" portfolio behave the same when asset price moves. They almost have the same payoff across circumstances and have the approximately the same value now, which verified the no-arbitrage theory.

Portfolio	Type	Underlying	Holding	OptionType	ExpirationDate	Strike	CurrentPrice
Call	Option	AAPL	1	Call	3/18/2022	165.0	4.50
ProtectedPut	Stock	AAPL	1	NaN	NaN	NaN	164.85
ProtectedPut	Option	AAPL	1	Put	3/18/2022	165.0	4.40

This shows that with the same strike price, a long position on the underlying asset and a long position on a put can replicate a long position on a call option, which will be hedged against losses when the stock price decreases and gain when price raises.

2. The "SynLong" portfolio behaves the same as "Stock". They almost have the same payoff across circumstances and have the approximately the same value now (almost zero), which verified the no-arbitrage theory.

Portfolio	Type	Underlying	Holding	OptionType	ExpirationDate	Strike	CurrentPrice
SynLong	Option	AAPL	1	Call	3/18/2022	165.0	4.5
SynLong	Option	AAPL	-1	Put	3/18/2022	165.0	4.4

Longing a call and shorting a put (at the same strike) will replicate a long position on the underlying asset.

3. Comparing to "Call" and "Put", the "CallSpread" and "PutSpread" portfolios are hedged against value changes when the stock price moves toward both directions.

Portfolio	Type	Underlying	Holding	OptionType	ExpirationDate	Strike	CurrentPrice
CallSpread	Option	AAPL	1	Call	3/18/2022	165.0	4.50
CallSpread	Option	AAPL	-1	Call	3/18/2022	175.0	0.72
PutSpread	Option	AAPL	1	Put	3/18/2022	165.0	4.40
PutSpread	Option	AAPL	-1	Put	3/18/2022	155.0	1.60

4. "CoveredCall" behaves like a short position on a put option, having limited gain in increasing asset price and infinite loss when the opposite happens.
5. The "Staddle" portfolio can be seen as a long position on the volatility. It will gain when the price has a bigger move, but paying the option premiums when the price is more stable.

Portfolio	Type	Underlying	Holding	OptionType	ExpirationDate	Strike	CurrentPrice
Straddle	Option	AAPL	1	Call	3/18/2022	165.0	4.5
Straddle	Option	AAPL	1	Put	3/18/2022	165.0	4.4

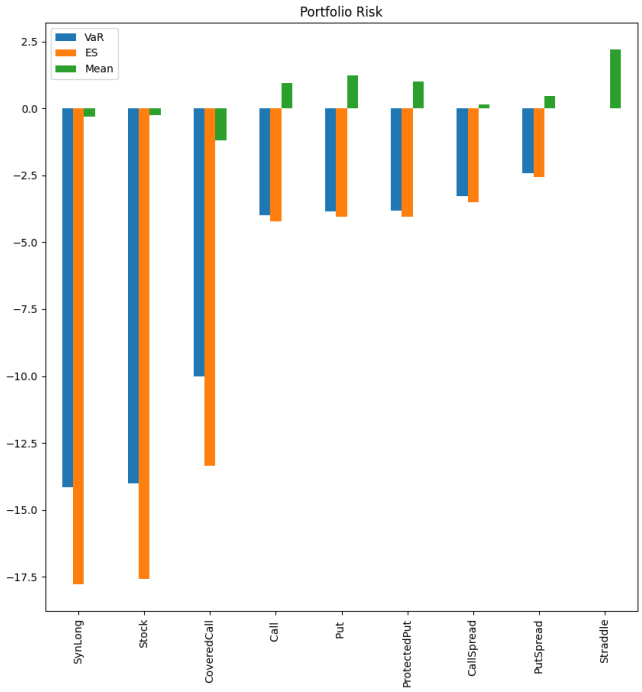
Simulating 10 days forward returns assuming normal distribution. Get the simulated price from the return series and look at values changes of the different portfolios.

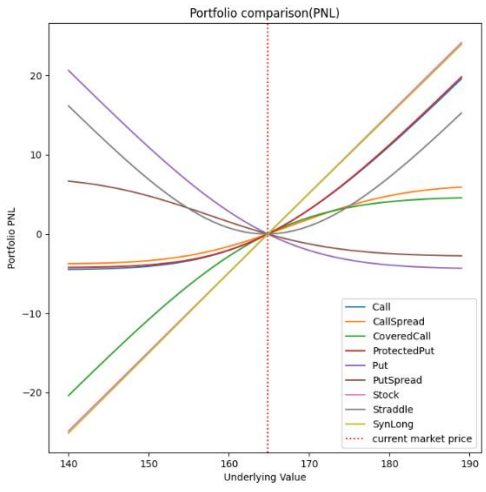
Portfolio Risk

	VaR	ES	Mean
SynLong	-14.162529	-17.782067	-0.293449
Stock	-14.005476	-17.585555	-0.241394
CoveredCall	-10.017347	-13.354718	-1.199110
Call	-3.988129	-4.230837	0.957716
Put	-3.848169	-4.054137	1.251165
ProtectedPut	-3.831075	-4.034325	1.009771
CallSpread	-3.280661	-3.515448	0.151209
PutSpread	-2.423064	-2.564451	0.454832
Straddle	0.007194	0.001442	2.208881

Portfolio Risk (%)

	VaR(%)	ES(%)	Mean(%)
SynLong	-14162.529398	-17782.066588	-2.934490
Call	-88.627050	-94.020688	0.212831
Put	-87.460364	-92.141574	0.284362
CallSpread	-86.789979	-93.001273	0.040002
PutSpread	-86.537990	-91.587525	0.162440
Stock	-8.495891	-10.667610	-0.001464
CoveredCall	-6.247172	-8.328475	-0.007478
ProtectedPut	-2.263561	-2.383650	0.005966
Straddle	0.080836	0.016206	0.248194





From the graph previously plotted, a larger downside risk can be expected for portfolios that has a larger range of value change when stock price changes. This is verified by the result from the VaR and ES calculations.

“SynLong” and “Stock” shows similarly large VaR and ES. Though their mean returns to not vary much from other portfolios, they have relatively large risks.

“CoveredCall” has limited gain and infinite potential loss, but the absolute value of loss is partially compensated by the income of option premium, so it has smaller VaR and ES comparing to the previous two.

“Call”, “Put”, “ProtectedPut” have similar shapes, showing similar VaR and ES.

“CallSpresad” and “PutSpread” are headed from both directions of underlying value change, their VaR and ES are smaller while mean return is also smaller.

“Straddle” will result in a gain when the underlying price moves toward any direction. It can be seen that its simulation has near zero (also positive) VaR and ES, and highest mean.