Université Libre de Bruxelles - Vrije Universiteit van Brussel
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MATH-H401
Modulation and Coding : Project

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Section:
Electrical Engineering

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Contents

1 Introduction

In the context of the first master year of the electrical engineering major of the-Bruface program and the project sessions of the Modulation and Coding course, the group was asked to design and simulate the DVB-S2 communication chain. The work was divided in three main parts, corresponding to the functionality of a typical modem :

- A simulation of the optimal communication chain over the ideal channel.
- A simulation of the LDPC channel encoder and decoder.
- A simulation of the time/frequency synchronisation algorithms.

These parts has defined the structure of this report presenting simulations and results.

2 Communication chain over the ideal channel

In this first section, a communication chain over an ideal channel has been simulated. The purpose of this communication chain is to transfer a generated bit message from a transmitter to a receiver.
But the transmission is done with complex symbols. The transmitter and receiver will thus need to apply modulation and demodulation in order to transfer the bit message. Different types of modulation exists, and for this work, four different modulations have been simulated and compared : BPSK,QPSK,16QAM,64QAM.
As the message is considered to be transferred from transmitter to receiver via an ideal channel, it has been simulated by adding AWGN to the transmitted signal.
To maximise the Signal to Noise Ratio, a matched filter has been applied.
The Figure 1 illustrates the block diagram of the communication chain simulation (the coder/decoder shown on the image are going to be implemented and explained in the next section).

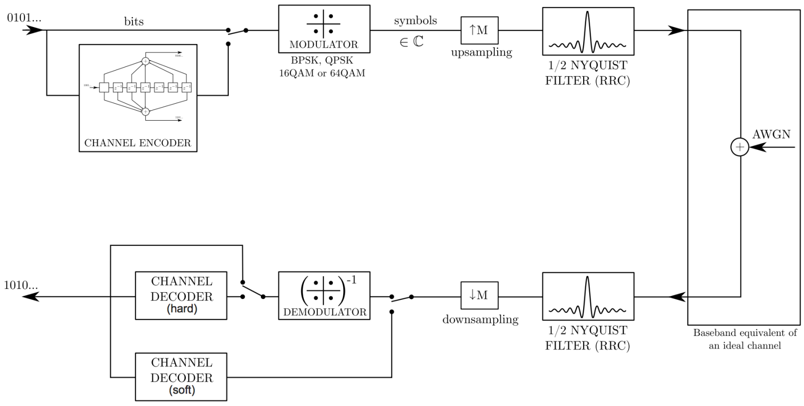

Figure 1: Block scheme of the ideal channel

2.1 Simulation Results

2.1.1 The halfroot Nyquist filter

The raised-cosine filter (RC) limits the bandwidth occupation as shown in Figure 2. In order to maximize the SNR, the RC is split into a pair of matched halfroot Nyquis filter (RRC) at both of the TX side and RX side. The overall RC filter satisfies the Nyquist ISI criterion and introduces the cancellation of the inter-symbol interference (ISI). As shown in Figure 3, when a symbol is sampled, all the other symbols are suppressed by convolving a sinc function.

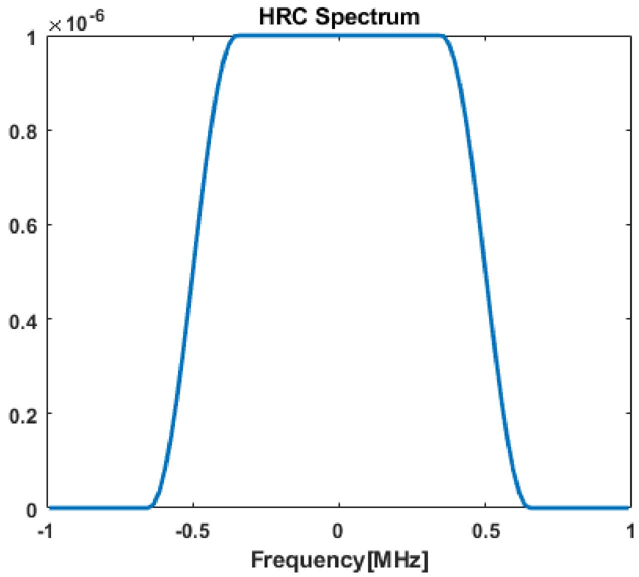

Figure 2: Illustration of the lowpass filter bandwith

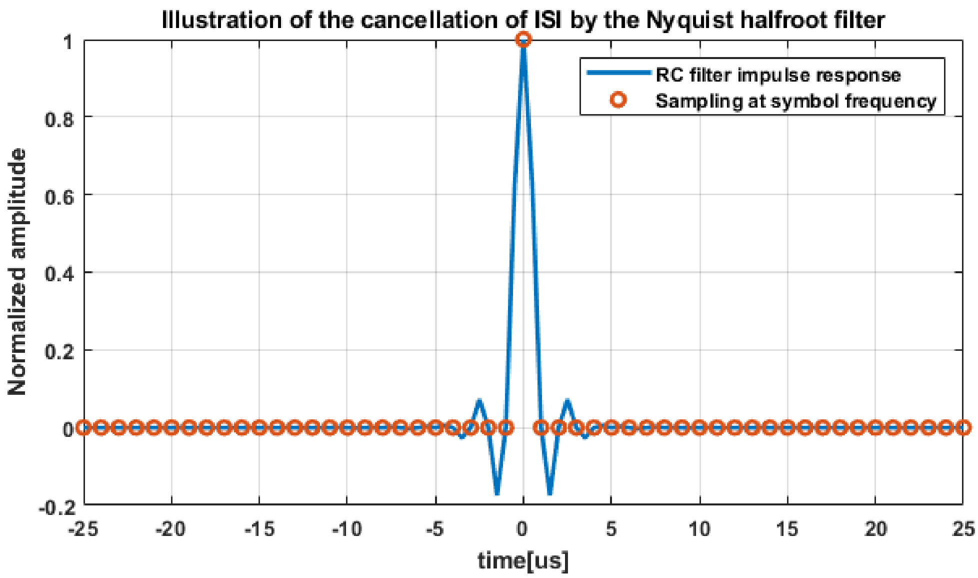

Figure 3: Illustration of the cancellation of ISI by the Nyquist root raised cosine filter at the symbol sampling frequency

2.1.2 Comparing modulations types

Different kinds of modulation methods can be used to convert bits into analog symbols. The Quadrature Amplitude Modulation (QAM) will modulate two quadra-

ture carriers thus modulating amplitude and phase. The M in the MQAM refers to 2^M bit per symbols.
The Bit Error Rate represents the rate of bit error in a transmitted message in function of the SNR of the signal, we thus want a low BER. In order to assure the same bit error rate, when number of bits per symbol is larger, SNR should be larger, as shown in Figure 4. It is also interesting to note that the BER curves of BPSK and 4QAM(QPSK) coincide with each other. This shows that the performances of modulating two quadrature carriers is the same as modulating one carrier.

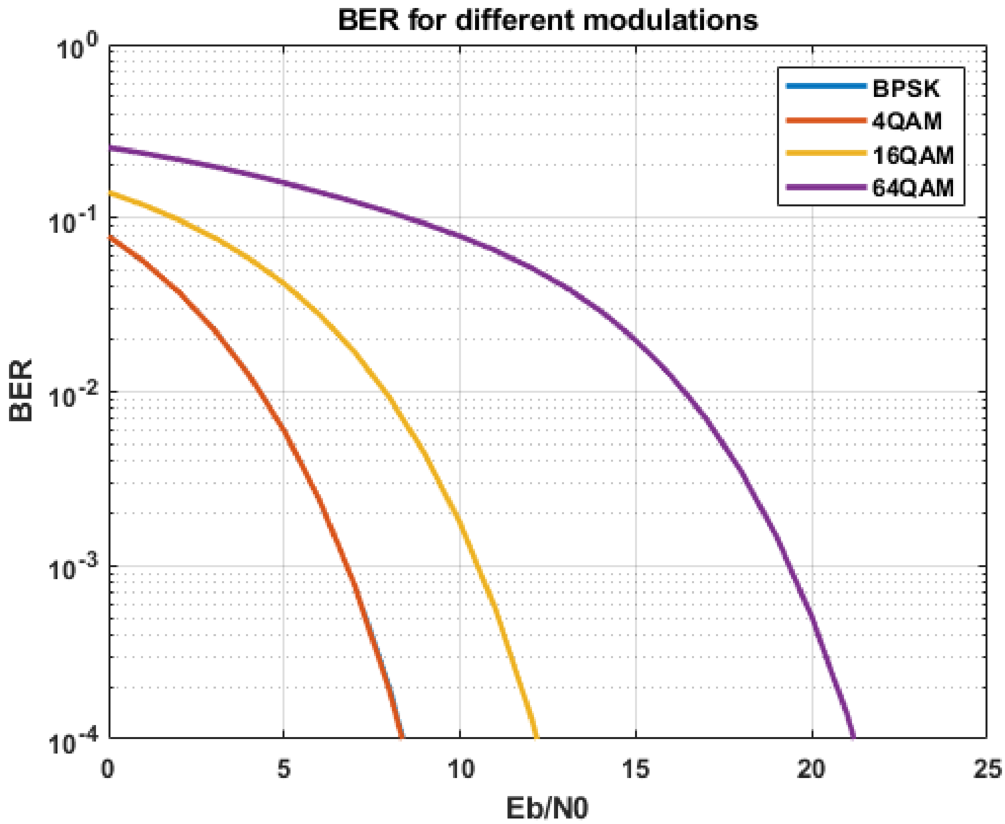

Figure 4: BER over SNR for different types of modulations

2.2 Questions answers

2.2.1 Simulation

- **It is proposed to use the baseband equivalent model of the AWGN channel. Would it be possible to live with a bandpass implemen-**

tation of the system?
The advantages of simulating in baseband are:

  - Reducing significantly the necessary signal sampling rate (computational complexity).
  - Developing modulation/demodulation techniques independently of carrier frequency (support system flexibility)

In our case, the required sampling rate is extremely high, thus it is possible but unrealistic to implement the bandpass model in practice while baseband equivalent model is always preferred due to the advantages above.

- **How do you choose the sample rate in Matlab?**
The sampling rate must be at least twice as high as the symbol frequency in order to respect the Nyquist criterion.

- **How do you make sure you simulate the desired E_b/N_0 ratio?**
First we calculate the bit energy. With the bit energy and the E_bN_0 we predefined, we can derive the corresponding noise power. The procedures are as follows:

$$SignalEnergy = \int signal^2 dt \tag{1}$$

$$E_b = \frac{SignalEnergy}{NbEncodedBit} \tag{2}$$

In order to get E_b in passband signal we should calculate $E_b = \frac{E_b}{2}$. Then we can obtain $N_0 = \frac{E_b}{E_bN_0}$. In the end we have $NoisePower = 2N_0Fs$.
The noise vector is given by:

$$k = \sqrt{\frac{NoisePower}{2}}(N(0,1) + jN(0,1)) \tag{3}$$

We have the square root here because we need the baseband representation of the noise. The noise power is divided by 2 in order to have equal distribution on the real and imaginary part.

- **How do you choose the number of transmitted data packets and their length ?**
We should send enough amount of bits to compute the bit error rate in an

accurate way, assuring that the the BER doesn't have any significant changes between two simulations. In our case the minimum BER is 10^{-4}, and we send 10^6 bits.

2.2.2 Communication system

- **Determine the supported (uncoded) bit rate as a function of the physical bandwidth**
  The bit rate (R) is the number of bits that are conveyed per second. Thus, R can be expressed as the product of numbers of bits per symbol (Nbps) and the numbers of symbol per seconds (symbol frequency) :
  $$R = Nbps * F_{symbol} = log_2(M) * F_{symbol}$$
  Where F_{symbol} is the bandwidth and the number of bit per symbol is given by $log_2(M)$.

- **Explain the trade-off communication capacity/reliability achieved by varying the constellation size**
  Increasing the constellation size is done by increasing the number of bits transmitted per symbol, thus the constellation size allows to increase the bit rate. However, the minimum euclidean distance between the possible symbols in the constellation decreases, causing the probability of detecting the good symbol to decrease. Thus more energy would be required to achieve a similar BER when the constellation size increases.

- **Why do we choose the halfroot Nyquist filter to shape the complex symbols ?**

  - The overall filter satisfies the Nyquist ISI criterion, which cancels inter symbol interference : the combined transmit and receive filters form a raised-cosine filter which does have zero at the intervals of ± Ts.
  - The matched filter at receiver maximizes the SNR of the output signal.

- **How do we implement the optimal demodulator? Give the optimization criterion**
  The received signal is matched filtered and sampled at symbol rate. Matched filters have the property that they maximize the SNR at $Eb/2N_0$ (optimization criterion).

- **How do we implement the optimal detector? Give the optimization criterion.**

We have seen two optimisation criteria in order to minimize the error probability of the transmitted signal $\underline{s_m}$ based on the observation $\underline{\mathrm{r}}$:

-**The maximum a posteriori criterion (MAP)** will maximize the probability of doing a correct decision.

$$\underline{s_m'}^{MAP} = \max_{\underline{s_m}} p(\underline{s_m}|\underline{r}) \tag{4}$$

By using Bayes'rule, the equation can be rewritten :

$$\underline{s_m'}^{MAP} = \max_{\underline{s_m}} p(\underline{r}|\underline{s_m})p(\underline{s_m}) \tag{5}$$

-**The maximum likelihood criterion(ML)** is equivalent to the MAP when all the M symbol are equally probable thus with p($\underline{s_m}$) constant.

3 Simulation of the LDPC channel encoder and decoder

To help the receiver detect if some errors has slipped in the retrieved message during the communication, can avoid loosing time sending back the message again. That is why a channel encoder with redundant information is added at the transmitter. This will allow a decoder at the receiver's side to correct errors.

3.1 Simulation Results

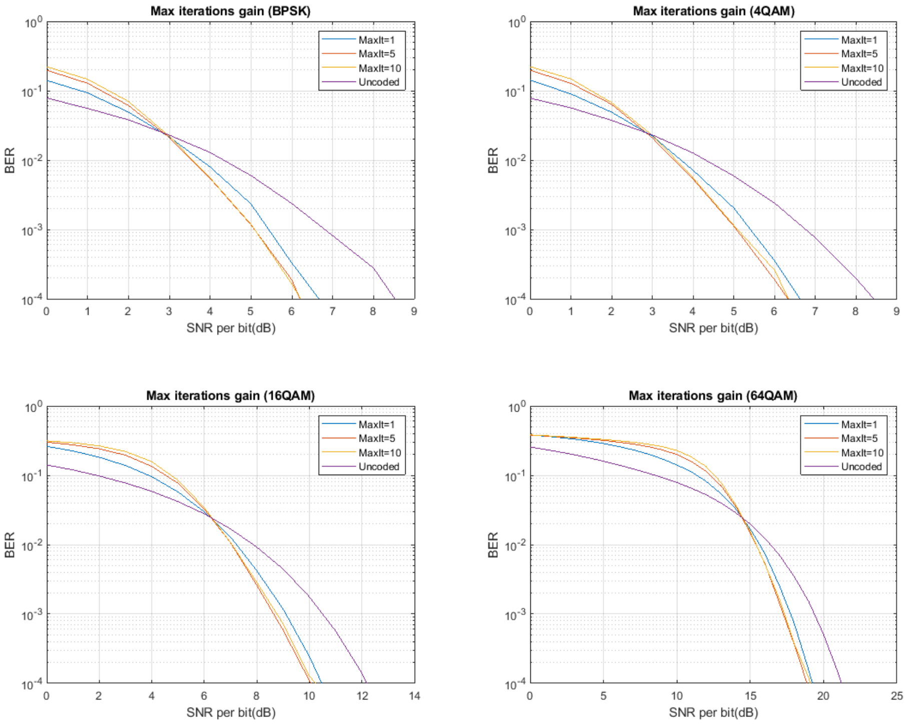

Figure 5: Hard decoding LDPC on BER curves for different modulations

Illustrations of the Hard decoding LDPC effect on BER curves for different modulations is shown in Figure 5, with different max iterations. As shown in the figure, when the max iteration is bigger, the BER drops faster. This is because more corrections are done improving result at a trade off of time calculation. It is also worthy to note that when max iteration increases from 5 to 10, the BER doesn't improve much. This is due to that we have two iteration stop conditions: maximal iteration and syndrome. When the max iteration is big, the syndrome condition dominates.

What's more, when the SNR is bigger than a certain critical point (intersection point of uncoded and encoded), the BER is much lower, and so much better for the encoded message. This is thanks to the forward correction of the message. Of course, this comes with a trade off : the code vector is longer than the message vector with redundant information. It thus requires a higher bandwidth. We can also observe that when the BER is lower than the critical point, the encoded vector has a higher bit error rate. This is due to amplifications of the errors for too small SNR. Indeed there are so much error that the correction of the decoding algorithm gives a worse message code result than encoded one. While comparing among the different modulations, it can be observed that as M increases, the critical point moves to the right, which means that decoding is effective for higher bit energy.

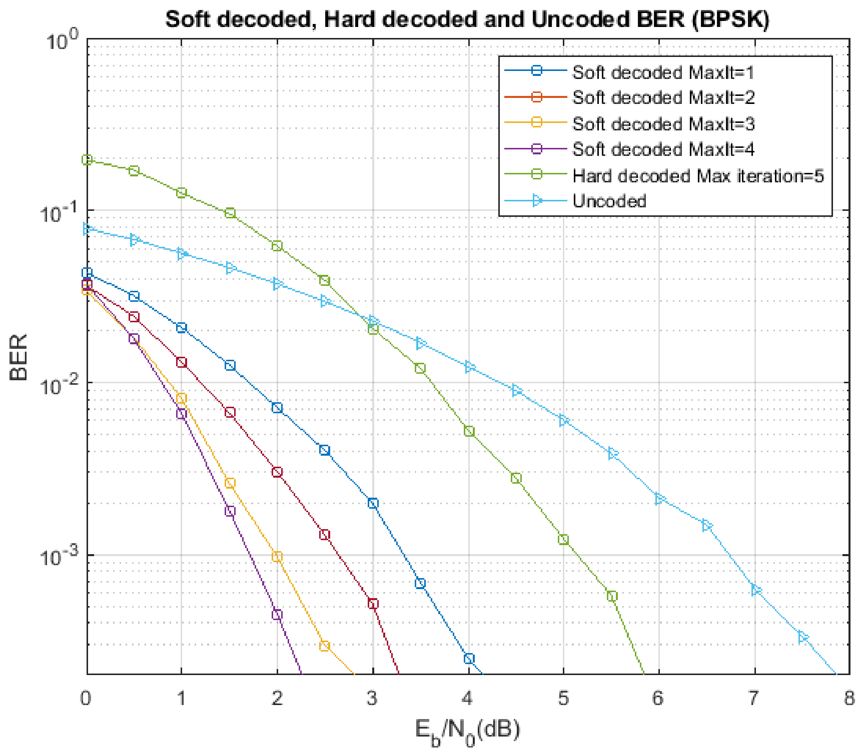

Figure 6: Channel coding gain achieved with hard and soft decoding

On Figure 6, we can see that the soft decoding performs better than the hard decoding. To the contrary of hard decoding, the BER of soft decoding stays lower than the uncoded one for low SNR. As explained above, for higher max iteration, the BER improves.

3.2 Questions answers

3.2.1 Regarding the simulation:

- **When building the new BER curves, do you consider the uncoded or coded bit energy on the x-axis?**

In our simulation, the coded bit energy was considered because we add AWGN noise to the coded bits. However both of them are feasible.

- **How do you limit the number of decoder iterations?**
For hard decoding, iterations stop when the syndrome is zero or when it reaches maximum number of iteration we defined.
For soft decoding, iterations stop when there is no more detected errors or when the maximum number of iterations is reached.

- **Why is it much simpler to implement the soft decoder for BPSK or QPSK than for 16- QAM or 64-QAM?**
For BPSK and QPSK, we only need to check the real and imaginary part to make the decision. BPSK has one bit per symbol so that the probability of the received bit to be one or zero is simply given by the real part of the received symbol. For QPSK, which can be seen two orthogonal QBSK, the probability that the first bit is a one or zero is given by the real part of the received symbol and the probability that the second bit is one or zero is given by its imaginary part.While for 16-QAM and 64-QAM, we need to calculate the euclidean distance.

3.2.2 Regarding the communication system

- **Demonstrate analytically that the parity check matrix is easily deduced from the generator matrix when the code is systematic.**
A code is systematic when the mapping is such that the latter half part of the code vector coincides with the message vector. The generator matrix is defined as follow:

$$G = [P|I_K] \tag{6}$$

The parity-check matrix H needs to respect the following condition:

$$G.H^T = 0 \tag{7}$$

Thus, the parity-check matrix is chosen as follow

$$H = [I_{N-K}|P^T] \tag{8}$$

since the condition above is well respected when considering a modulo-2 addition. Indeed :

$$[P|I_K].[I_{N-K}|P^T]^T = P \oplus P = \bar{0} \tag{9}$$

Note that all the letter written in upper case are matrices.

- **Explain why we can apply linear combinations on the rows of the parity check matrix to produce an equivalent systematic code.**
  The parity check matrix has the property to span the vector subspace complementary to the codewords subspace, which is spanned by the generator matrix. So the linear combinations of the base vectors that form the columns of parity check matrix H will span the same subspace, and thus produces a code that will be equivalent.

- **Why is it especially important to have a sparse parity check matrix (even more important than having a sparse generator matrix)?**
  Having a sparse parity check matrix is important to reduce the computation time of the decoding. This is because the complexity of the decoder will stay low since it reduces the number of messages and responses (connections) between variable nodes and check nodes. Indeed, every check node will send a message to a low number of variable nodes (few ones in every row) and every variable node will intervene in a low number of variable nodes (few ones in every column). Because of the noise that is introduce after coding, the complexity of decoding is much bigger than the coding. Having a sparse H is thus more important than having a sparse G to reduce the decoding time.

- **Explain why the check nodes only use the information received from the other variable nodes when they reply to a variable node.**
  The check nodes only use the information retrieved from the other variable in order to be stochastically independent. This one will assume the other nodes as correct and the node it is responding to as indeterminate.

4 Simulation of the time/frequency synchronisation algorithms

In this last part, the synchronisation problems existing between the transmitter and the receiver as well as the solutions and algorithm implemented in order to compensate these errors are going to be presented.

The synchronisation error is a small difference in the time clock of the transmitter and receiver due to the limited accuracy of their cristal and also due to the random carrier phase and time shift between both sides.

These errors are characterised by the Carrier frequency Offset (CFO) and the phase offset, referred as $\Delta\omega$ and ϕ_0. These errors are also characterised by the Sampling clock Offset (SCO) and the time shift, referred as δ and t_0. These errors can impact the communication chain by adding a rotation in the constellation and by causing Inter symbol Interference(ISI)

In order to avoid degradation due to these errors, an evaluation of their impact will first be made to fix some specifications, then a structure will be added to compensate them assuring the desired specifications.

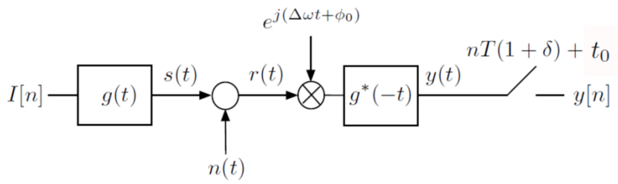

Figure 7: Synchronisation errors in communication chain

4.1 Simulation Results

The BER degradation for increasing values of the CFO

The impact of phase shift can be seen intuitively from the constellation as shown in Figure 8 . The noise has spread the received symbol around the theoretical symbols. As the phase shift is introduced, the noisy received symbols are rotated.

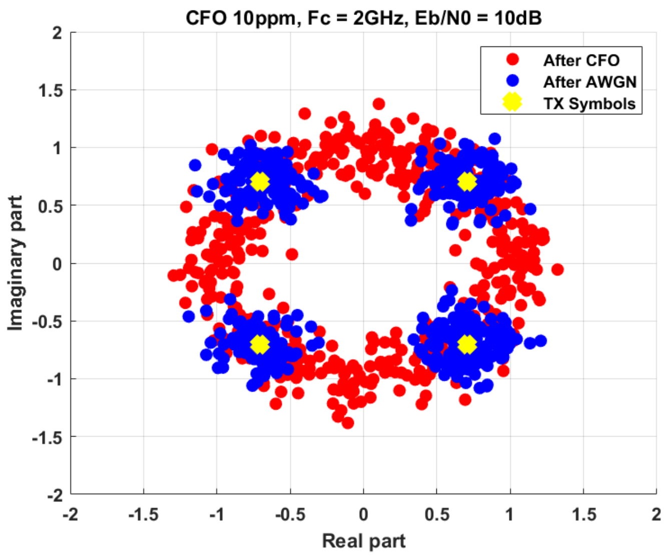

Figure 8: Constellation at different stages in the communication

Effect of the CFO-caused ISI and time shift on the BER

The Figure 9 illustrates the BER degradation due to the ISI only after doing the

perfect compensation. When observing horizontally, we can say that as CFO gets bigger and the ISI increases higher E_b/N_0 is needed for the desired BER. On the other hand, less CFO leads a smaller BER when the SNR is fixed. In our case, the ISI due to the 20ppm CFO has little impact on BER performances.
The Figure 10 displays a BER degradation due to the time shift t_0. As time shift increases, the BER gets worse as expected. In our case, when time shift reduces to 0.02T, the impact could be neglected.

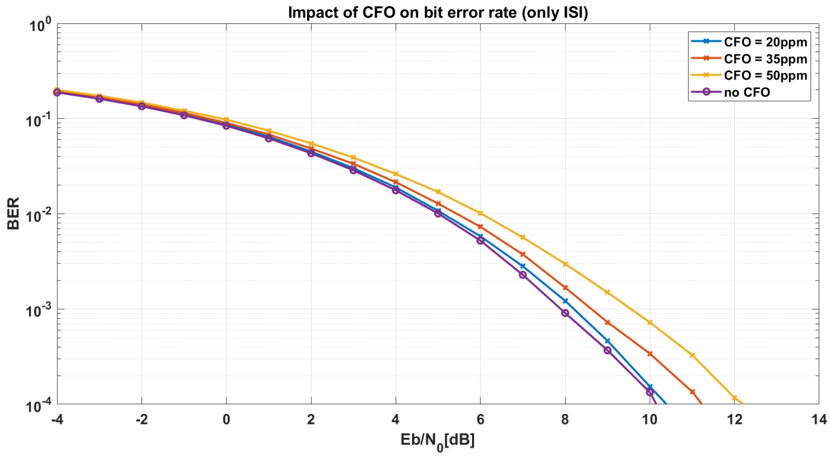

Figure 9: Impact of CFO on bit error rate(only ISI)

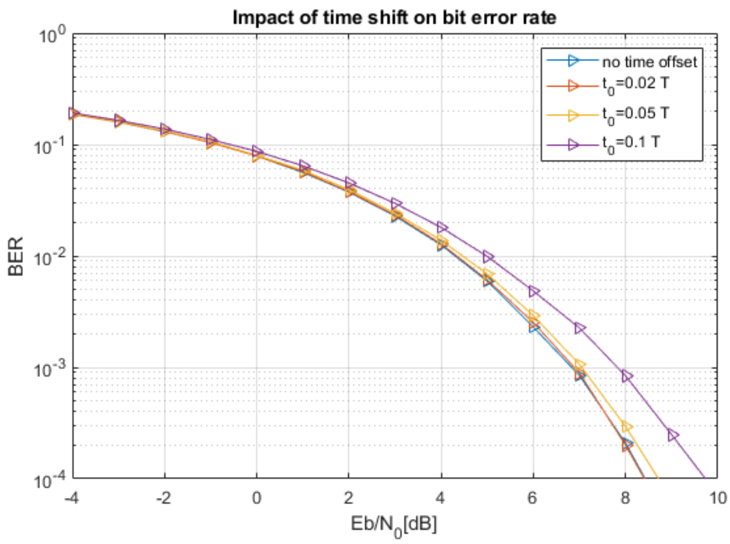

Figure 10: Impact of time shift on Bit Error Rate

Sample time shift error correction using the Gardner Algorithm
The Figure11 shows the correction of Gardner algorithm to time shift error. It is obvious that after the Gardner algorithm is employed, the BER improves over the E_b/N_0.

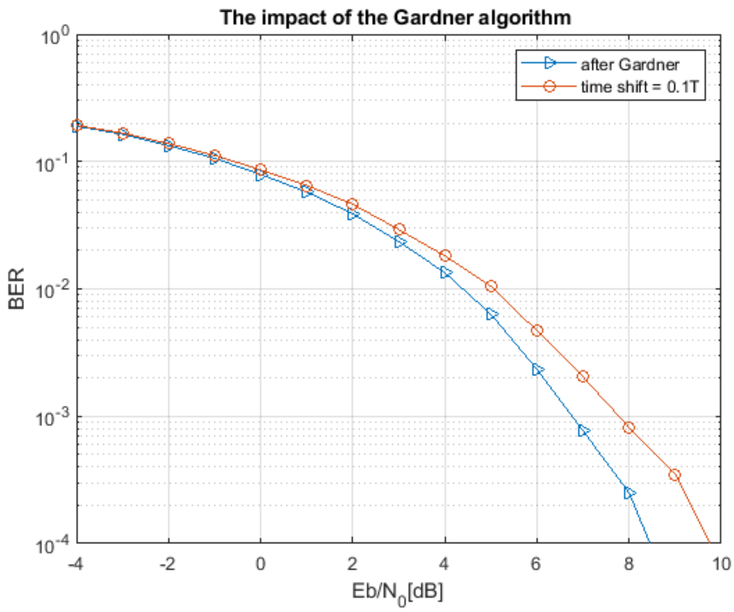

Figure 11: Impact of Gardner algorithm on time shift error

The convergence of Gardner for different error weight
The Gardner algorithm, compensating the sampling time errors, is the first algorithm that has been implemented because of it's robustness to CFO (explained later). The Figure 12 illustrates the convergence of the algorithm for different values of the error weight. As we can see from the12, smaller K returns us a slower but better convergence while the bigger K converges faster but it is less reliable. The robustness of Gardner to CFO is displayed in the Figure.13. The Gardner is applied to 50ppm CFO and error doesn't improve. This proves the robustness.

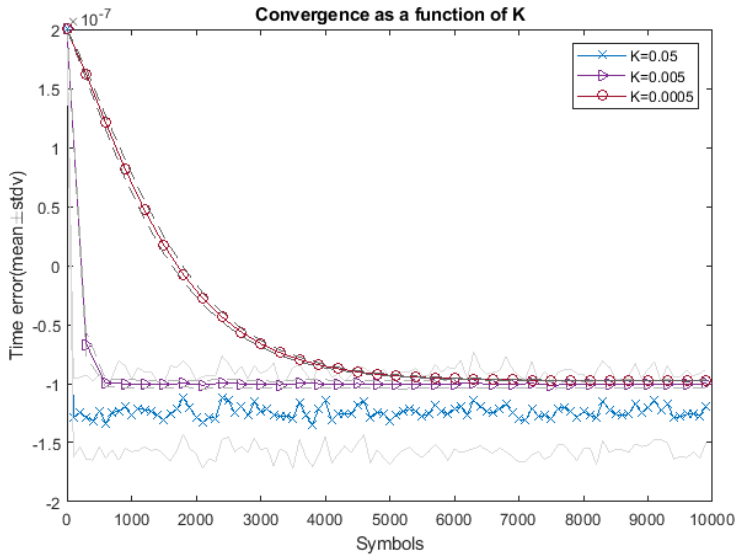

Figure 12: The convergence of Gardner for different error weight

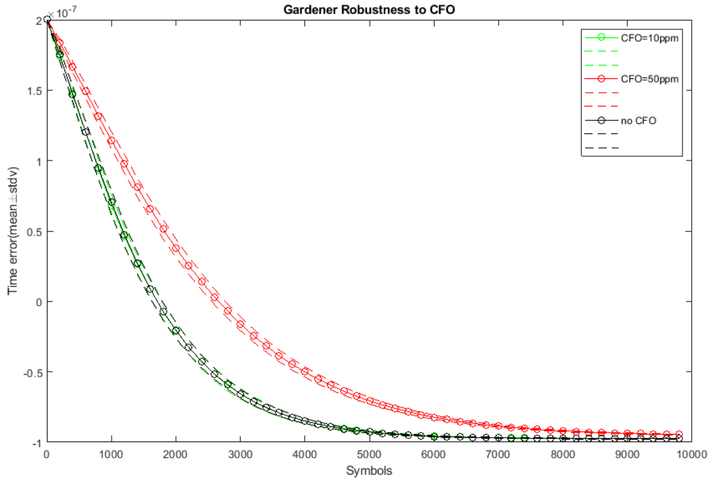

Figure 13: Garderner Robustness to CFO

Time of Arrival and CFO Estimation Using Frame Acquisition

After we corrected the sampling time shift, a pilot (a known random sequence) is sent between frames of data to estimate the Time of Arrival (ToA) and the CFO. By changing the length of the pilot N and the size of the averaging windows K, the dependency of estimation of ToA and CFO on these parameters can be highlighted.

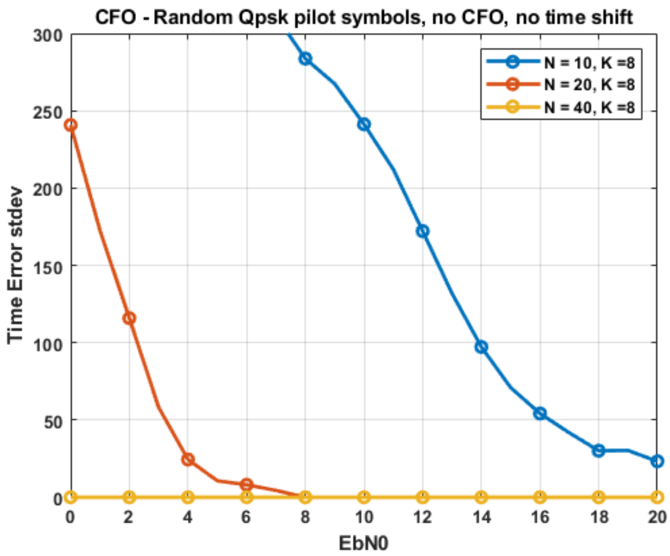

Figure 14: Random QPSK pilot symbols, no CFO, no time shift

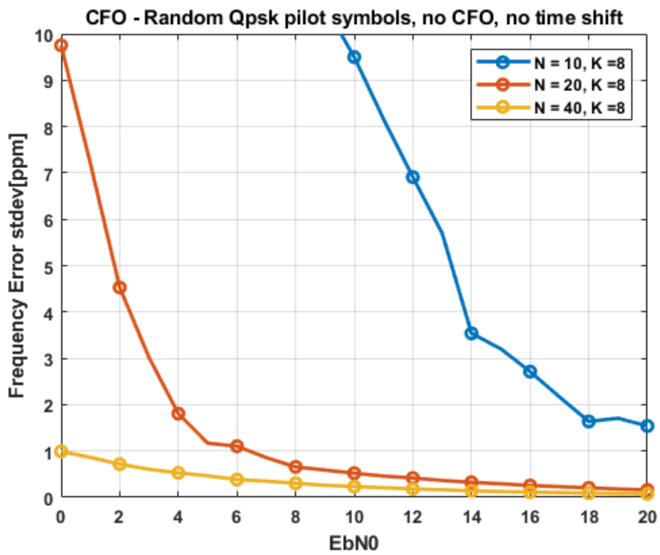

Figure 15: Random QPSK pilot symbols, no CFO, no time shift

We can see from Fig.14 and Fig.15 that the time error stdv and frequency error stdv decreases as Eb/N0 increases, which indicates that the pilots can be detected more accurately when we have less noise. Also as N increases, we have smaller time error stdv, which leads to better estimation of CFO. At N = 40 and K = 8, the pilot is always detected correctly with a freq error less than 1 ppm.

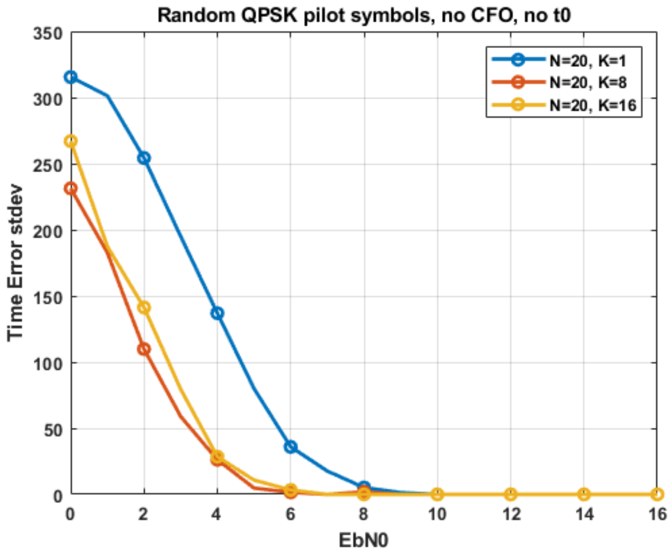

Figure 16: Random QPSK pilot symbols, no CFO, no time shift

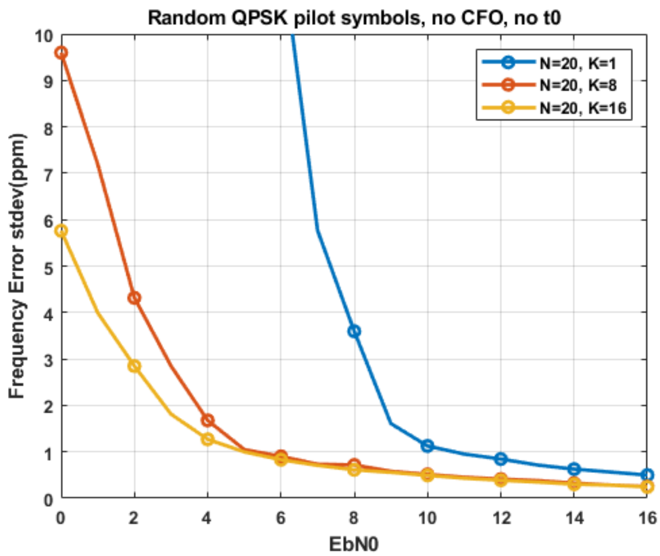

Figure 17: Random QPSK pilot symbols, no CFO, no time shift

As shown in Fig.16 and Fig.17, when $K \geq 8$, we have better frame synchronization and the frequency error stdv is less than 2 ppm for $Eb/N0 \geq 4dB$.

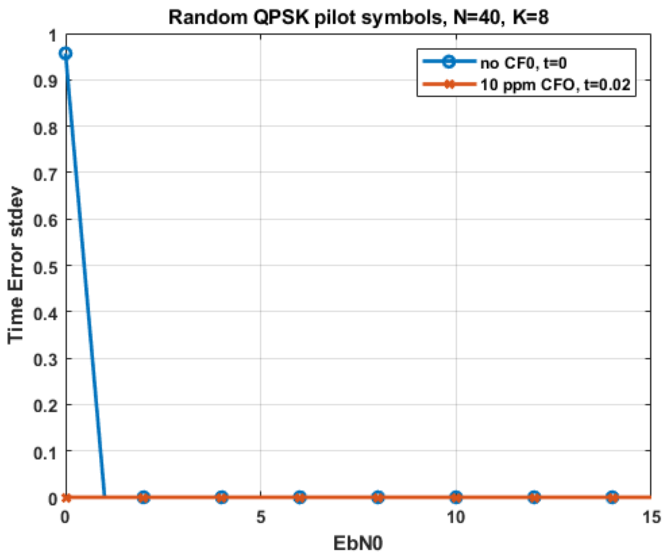

Figure 18: Random QPSK pilot symbols, no CFO, no time shift

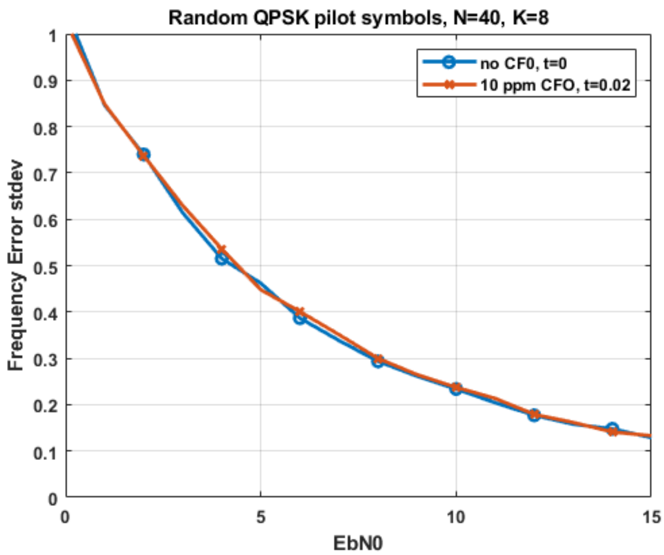

Figure 19: Random QPSK pilot symbols, no CFO, no time shift

We can see from Fig.18 and Fig.19 that the robustness of the frame acquisition to CFO is quite good as it has nearly no impact on the estimation.

4.2 Questions answers

- **Derive analytically the baseband model of the channel including the synchronisation errors.**

  $$r(t) = s(t) = \Re\{e_s(t)\}cos(\omega_c t) + \Im\{e_s(t)\}sin(\omega_c t)$$

  $$e_r(t) = r(t)cos((\omega_c + \Delta\omega)t + \phi_0) + r(t)sin((\omega_c + \Delta\omega)t + \phi_0)$$

  The complex received envelop will pass into the root raised cosine filter, acting as a low pass filter. We can thus derive the following expression :
  $y(t) = e_s(t)\exp[j(\Delta\omega t + \phi_0)]$ Where we retrieve the expression of the synchronisations error in the baseband model

- **How do you separate the impact of the carrier phase drift and ISI due to the CFO in your simulation?**
  In order to isolate the impact of ISI we do the perfect compensation by multiplying the expression retrieved after matched filter with the following expression : $exp[-j(\Delta\omega t+\phi_0)]$, thus the phase drift is perfectly compensated.

- **How do you simulate the sampling time shift in practice?**
  We simulate the time shift by first oversampling then shifting the whole symbol stream at different samples lengths. E.g. if the upsampling ratio is 100, we move all the symbols ten samples right, the time shift is 0.1T, where T is the symbol duration before oversampling.

- **How do you select the simulated Eb/N0 ratio?**
  The SNR should be sufficiently high.

- **How do you select the lengths of the pilot and data sequences?**
  The length of the pilot should be sufficiently long to get a correct estimation of ToA and CFO. We can see from the results that the N should be larger then 20. The length of a data sequence should not be too long in order to ensure a correct phase interpolation between two successive pilot sequences for the period of the phase is 2π. But the length of the pilot and the rate of interpolation should not be too high considering the speed of transmission of the signal.

- **In which order are the synchronisation effects estimated and compensated. Why?**

At first, the Gardner algorithm will be applied. It is an acquisition and a tracking algorithm for the sampling time that is robust to CFO. It will thus compensate the time shift t_0 and the sample clock offset (SCO) δ. Secondly, an acquisition of the frame starting time (ToA) , followed by the acquisition of the carrier frequency offset (CFO) are going to be applied. Finally an interpolation of the phase drift will be implemented to compensate the small remaining CFO.

- **Explain intuitively how the error is computed in the Gardner algorithm. Why is the Gardner algorithm robust to CFO?**
As shown in the image, the correction term of estimation ε depends on two adjacent symbols and the midway sample weighted by a factor. The direction of the correction is given by the sign of the midway sample, as described below

$$\hat{\varepsilon}[n+1] \simeq \hat{\varepsilon}[n] + 2\kappa\Re[y_{\hat{\varepsilon}[n]}[n-1/2](y^*_{\hat{\varepsilon}[n]}[n] - y^*_{\hat{\varepsilon}[n-1]}[n-1])] \tag{10}$$

Gardner algorithm is robust to CFO since the $y_{\hat{\varepsilon}[n]}[n-1/2]$ is multiplied with the conjugate of $y_{\hat{\varepsilon}[n]}[n]$, the exponent-term $exp[j(\Delta\omega t + \phi_0)]$ is canceled but not perfectly canceled due to the phase difference between midway point and sample point.

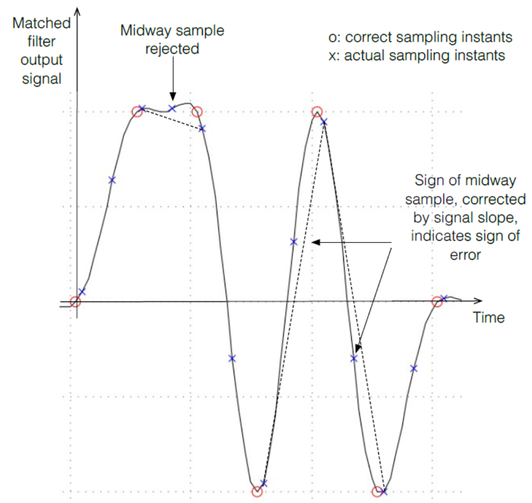

Figure 20: Gardner interpretation. Course note, synchronisation chapter.

- **Explain intuitively why the differential cross-correlation is better suited than the usual cross-correlator? Isn't interesting to start the summation at k = 0 (no time shift)?**

The cross-correlator necessitates a 2D exhaustive search, which is of very high complexity. The differential cross-correlator first estimate the ToA and then the CFO, which lower the complexity of computation. We need the lower complexity of computation so we choose differential cross-correlator.
No. Because the term $D_0[n]$ carry no useful information but only the power of the window.

- **Are the frame and frequency acquisition algorithms optimal? If yes, give the optimisation criterion.**
  No there are not totally optimal due to the residual of the CFO after the Gardner algorithm. But the Gardner algorithm is a non aided data algorithm. By using data aided synchronisation, an optimised estimate of the parameters using the maximum likelihood estimate can be done.

5 Conclusion

This project has introduced the basic for numerical communications. It has presented all the components used to assure a reliable communication between a transmitter and a receiver when AWGN is added to the channel, with a special attention to DVB-S2 communication.

The performances of different types of modulation have been compared, coding and decoding techniques helping improve the communication robustness to noise and algorithms that could compensate synchronisations errors have been introduced.

Moreover, this project was an opportunity to develop teamwork skills with fixed presentations deadlines.