DEC 4

$$\lim_{x\to 0} \frac{2^x - 1}{x} = \ln 2 \quad (2<e \text{ then } \ln 2 < 1)$$

$$\lim_{x\to 0} \frac{3^x - 1}{x} = \ln 3 \quad (3>e \text{ then } \ln 3 > 1)$$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = \ln e = 1$$

$$\begin{aligned}
\frac{d}{dx} b^x &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{b^{x+h}-b^x}{h} \\
&= \lim_{h\to 0} \frac{b^x \cdot b^h - b^x}{h} \\
&= \lim_{h\to 0} \frac{b^x(b^h-1)}{h} \\
&= \lim_{h\to 0} (b^x) \cdot \lim_{h\to 0}\left(\frac{b^h-1}{h}\right)
\end{aligned}$$

(chain rule)

$$\frac{d}{dx} b^x = b^x \cdot \ln b \cdot \frac{d}{dx} b$$

$$\begin{aligned}
\frac{d}{dx} e^x &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{e^{x+h}-e^x}{h} \\
&= \lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h} \\
&= \lim_{h\to 0} \frac{e^x(e^h-1)}{h} \\
&= \lim_{h\to 0} (e^x) \cdot \lim_{h\to 0}\left(\frac{e^h-1}{h}\right) \\
&= e^x \cdot 1
\end{aligned}$$

$$\frac{d}{dx} e^x = e^x$$

$$\boxed{\frac{d}{dx} b^a = b^a \cdot \ln b \cdot \frac{d}{dx} a}$$

where $b \neq 1$ and $b>0$ (straight line) and a is a function of x → negative base to a power is undefined $(-2)^{3.1} = ?$

$$\frac{d}{dx} e^{\sin 2x} = (e^{\sin 2x}) \cdot (\ln e) \cdot \left(\frac{d}{dx} \sin 2x\right)$$

$$= e^{\sin 2x} \cdot \cos 2x \cdot 2$$

$$\frac{d}{dx} \sin 3x \cdot 3e^{\sin 3x}$$

$$= \cos 3x \cdot 3 \cdot 3e^{\sin 3x} + \sin 3x \cdot 3 \cdot e^{\sin 3x} \cdot \cos 3x \cdot 3$$

$$\frac{d}{dx} 3^{x\sin x} \cdot \sin^2(3x)$$

$$= 3^{x\sin x} \cdot \ln 3 \cdot (\sin x + x\cos x) \sin^2(3x) + 3^{x\sin x} \cdot 2\sin(3x)\cos(3x) \cdot 3$$

DEC 8

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} \log x = \frac{1}{x \ln 10}$$

$$\boxed{\frac{d}{dx} \ln \text{☺} = \frac{1}{\text{☺}} \frac{d}{dx} \text{☺}}$$

$$\boxed{\frac{d}{dx} \log_b \text{☺} = \frac{1}{\text{☺} \ln b} \frac{d}{dx} \text{☺}}$$

$$\frac{d}{dx} \ln|\sec x + \tan x|$$

$$= \frac{1}{\sec x + \tan x} \cdot \frac{d}{dx}(\sec x + \tan x)$$

$$= \frac{1}{\sec x + \tan x} \cdot (\tan x \cdot \sec x + \sec^2 x)$$

$$= \frac{\tan x \cdot \sec x + \sec^2 x}{\sec x + \tan x}$$

$$= \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}$$

$$= \sec x \quad \#$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\frac{d}{dx}\ln|\csc x - \cot x|$$

$$= \frac{1}{\csc x - \cot x} \cdot \frac{d}{dx}(\csc x - \cot x)$$

$$= \frac{1}{\csc x - \cot x} \cdot (-\cot x \cdot \csc x + \csc^2 x)$$

$$= \frac{\csc x\,(\csc x - \cot x)}{\csc x - \cot x}$$

$$= \csc x \quad \#$$

$$\frac{d}{dx}\, x\ln|x|$$

$$= \ln|x| + x \cdot \frac{1}{x}$$

$$= \ln|x| + 1 \quad \#$$

$$\frac{d}{dx}(x\ln|x| - x)$$

$$= \ln|x| + x \cdot \frac{1}{x} - 1$$

$$= \ln|x|$$

$$\frac{d}{dx}\ln|x\sin x|$$

$$= \frac{1}{x\sin x} \cdot \frac{d}{dx}\, x\sin x$$

$$= \frac{1}{x\sin x} \cdot (\sin x + x\cos x)$$

$$= \frac{\sin x + x\cos x}{x\sin x}$$

$$= x^{-1} + \cot x \quad \#$$

$$\frac{d}{dx} \log \left| 2^x \cdot \ln|\sec x| \right|$$

$$= \frac{\frac{d}{dx} 2^x \cdot \ln|\sec x|}{2^x \cdot \ln|\sec x| \cdot \ln 10}$$

$$= \frac{2^x \cdot \ln 2 \cdot \ln|\sec x| + 2^x \cdot \frac{1}{\cancel{\sec x}} \cdot \tan x \cancel{\sec x}}{2^x \cdot \ln|\sec x| \cdot \ln 10}$$

$$= \frac{2^x (\ln 2 \cdot \ln|\sec x| + \tan x)}{2^x \cdot \ln|\sec x| \cdot \ln 10}$$

$$= \frac{\ln 2 \cdot \ln|\sec x| + \tan x}{\ln|\sec x| \cdot \ln 10}$$

$$= \frac{\ln 2}{\ln 10} + \frac{\tan x}{\ln|\sec x| \cdot \ln 10}$$

$$= \log_{10} 2 + \frac{\tan x}{\ln|\sec x| \cdot \ln 10} \qquad \boxed{\frac{\log_m a}{\log_m b} = \log_b a}$$

$$= \log 2 + \frac{\tan x}{\ln|\sec x| \cdot \ln 10} \quad \#$$

$$y = x^x$$

$$\ln y = \ln x^x$$

$$\ln y = x \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \ln x + x \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y \ln x + y$$

$$\frac{dy}{dx} = x^x \cdot \ln x + x^x$$

$$\frac{dy}{dx} = x^x (\ln x + 1)$$