

OCT 16

Using the limit definition, deriving:

$$\frac{d}{dx} x^n = nx^{n-1} \cdot \frac{d}{dx} x$$

$$\begin{aligned}\frac{d}{dx} x^7 &= 7x^6 \cdot \frac{d}{dx} x \\ &= 7x^6\end{aligned}$$

$$\begin{aligned}\frac{d}{dx} (2x-1) &= \frac{d}{dx} 2x - \frac{d}{dx} 1 \\ &= 2-0 \\ &= 2\end{aligned}$$

$$\begin{aligned}\frac{d}{dx} (2x-1)^3 &\xleftarrow{\text{DR}} \frac{d}{dx} (2x-1)^3 = 3(2x-1)^2 \cdot \frac{d}{dx} (2x-1) \\ &= \frac{d}{dx} (8x^3 - 12x^2 + 6x - 1) &= 6(2x-1)^2 \# \\ &= \frac{d}{dx} 8x^3 - \frac{d}{dx} 12x^2 + \frac{d}{dx} 6x - \frac{d}{dx} 1 \\ &= 24x^2 - 24x + 6 - 0 \\ &= 6(4x^2 - 4x + 1) \\ &= 6(2x-1)^2 \#\end{aligned}$$

$$\begin{aligned}\frac{d}{dx} \sqrt{x - \sqrt{1-x}} \\ &= \frac{d}{dx} (x - (1-x)^{\frac{1}{2}})^{\frac{1}{2}} \\ &= \frac{1}{2} (x - (1-x)^{\frac{1}{2}})^{-\frac{1}{2}} \cdot \frac{d}{dx} (x - (1-x)^{\frac{1}{2}}) \\ &= \frac{1}{2} (x - (1-x)^{\frac{1}{2}})^{-\frac{1}{2}} \cdot \left(\frac{d}{dx} x - \frac{d}{dx} (1-x)^{\frac{1}{2}} \right) \\ &= \frac{1}{2} (x - (1-x)^{\frac{1}{2}})^{-\frac{1}{2}} \cdot \left(1 - \frac{1}{2} (1-x)^{-\frac{1}{2}} \cdot \frac{d}{dx} (1-x) \right) \\ &= \frac{1}{2} (x - (1-x)^{\frac{1}{2}})^{-\frac{1}{2}} \cdot \left(1 - \frac{1}{2} (1-x)^{-\frac{1}{2}} \cdot \left(\frac{d}{dx} 1 - \frac{d}{dx} x \right) \right) \\ &= \frac{1}{2} (x - (1-x)^{\frac{1}{2}})^{-\frac{1}{2}} \cdot \left(1 - \frac{1}{2} (1-x)^{-\frac{1}{2}} \cdot \left(\frac{d}{dx} 1 - \frac{d}{dx} x \right) \right)\end{aligned}$$

$$= \frac{1}{2} (x - (1-x)^{\frac{1}{2}})^{-\frac{1}{2}} \cdot \left(1 - \frac{1}{2} (1-x)^{-\frac{1}{2}} \cdot (-1) \right)$$

$$= \frac{1}{2} (x - (1-x)^{\frac{1}{2}})^{-\frac{1}{2}} \cdot \left(1 + \frac{1}{2} (1-x)^{-\frac{1}{2}} \right) \quad \#$$

$$\frac{d}{dx} \frac{8}{(x^2 - \sqrt{x})^3}$$

$$= \frac{d}{dx} 8 (x^2 - \sqrt{x})^{-3}$$

$$= -24 (x^2 - \sqrt{x})^{-4} \cdot \frac{d}{dx} (x^2 - x^{\frac{1}{2}})$$

$$= -24 (x^2 - \sqrt{x})^{-4} \cdot \left(\frac{d}{dx} x^2 - \frac{d}{dx} x^{\frac{1}{2}} \right)$$

$$= -24 (x^2 - \sqrt{x})^{-4} \cdot \left(2x - \frac{1}{2} x^{-\frac{1}{2}} \right) \quad \#$$

Oct 18

$$\frac{d}{dx} \sqrt[3]{x^2 - (1-x)^3 + (\sqrt{x} + 1)^{-2}}$$

$$= \frac{1}{3} (x^2 - (1-x)^3 + (\sqrt{x} + 1)^{-2})^{-\frac{2}{3}} \times \frac{d}{dx} (x^2 - (1-x)^3 + (\sqrt{x} + 1)^{-2})$$

$$= \frac{1}{3} (x^2 - (1-x)^3 + (\sqrt{x} + 1)^{-2})^{-\frac{2}{3}} \times \left(\frac{d}{dx} x^2 - \frac{d}{dx} (1-x)^3 + \frac{d}{dx} (\sqrt{x} + 1)^{-2} \right)$$

$$= \frac{1}{3} (x^2 - (1-x)^3 + (\sqrt{x} + 1)^{-2})^{-\frac{2}{3}} \times \left(2x - 3(1-x)^2 \cdot \frac{d}{dx} (1-x) - 2(\sqrt{x} + 1)^{-3} \cdot \frac{d}{dx} (\sqrt{x} + 1) \right)$$

$$= \frac{1}{3} (x^2 - (1-x)^3 + (\sqrt{x} + 1)^{-2})^{-\frac{2}{3}} \times \left(2x - 3(1-x)^2 \cdot \left(\frac{d}{dx} 1 - \frac{d}{dx} x \right) - 2(\sqrt{x} + 1)^{-3} \cdot \left(\frac{d}{dx} \sqrt{x} + \frac{d}{dx} 1 \right) \right)$$

$$= \frac{1}{3} (x^2 - (1-x)^3 + (\sqrt{x} + 1)^{-2})^{-\frac{2}{3}} \times \left(2x - 3(1-x)^2 \cdot (-1) - 2(\sqrt{x} + 1)^{-3} \cdot \left(\frac{1}{2} x^{-\frac{1}{2}} \right) \right)$$

$$= \frac{1}{3} (x^2 - (1-x)^3 + (\sqrt{x} + 1)^{-2})^{-\frac{2}{3}} \times \left(2x + 3(1-x)^2 - 2(\sqrt{x} + 1)^{-3} \cdot \left(\frac{1}{2} x^{-\frac{1}{2}} \right) \right) \quad \#$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

only if $a, b \geq 0$

$$\frac{d}{dx} \left(x + \left(x^2 - \frac{4}{x^2 + 2x + 1} \right)^{\frac{1}{2}} \right)^5$$

$$= 5 \left(x + \left(x^2 - 4(x+1)^{-2} \right)^{\frac{1}{2}} \right)^4 \cdot \left(\frac{d}{dx} x + \left(x^2 - 4(x+1)^{-2} \right)^{\frac{1}{2}} \right)$$

$$= 5 \left(x + \left(x^2 - 4(x+1)^{-2} \right)^{\frac{1}{2}} \right)^4 \cdot \left(1 + \frac{1}{2} \left(x^2 - 4(x+1)^{-2} \right)^{-\frac{1}{2}} \cdot \frac{d}{dx} \left(x^2 - 4(x+1)^{-2} \right) \right)$$

$$= 5 \left(x + \left(x^2 - 4(x+1)^{-2} \right)^{\frac{1}{2}} \right)^4 \cdot \left(1 + \frac{1}{2} \left(x^2 - 4(x+1)^{-2} \right)^{-\frac{1}{2}} \cdot (2x + 8(x+1)^{-3} \cdot \frac{d}{dx} (x+1)) \right)$$

$$= 5 \left(x + \left(x^2 - 4(x+1)^{-2} \right)^{\frac{1}{2}} \right)^4 \cdot \left(1 + \frac{1}{2} \left(x^2 - 4(x+1)^{-2} \right)^{-\frac{1}{2}} \cdot (2x + 8(x+1)^{-3}) \right) \quad \#$$