$$\int \frac{x-1}{x^2-x-2}\,dx$$

$$\frac{A}{x-2} + \frac{B}{x+1}$$

$$A(x+1) + B(x-2) = x-1$$

$$Ax + A + Bx - 2B = x-1$$

$$(A+B)x + (A-2B) = x-1$$

$$= \int \frac{x-1}{(x-2)(x+1)}\,dx$$

$$\left.\begin{aligned} A+B &= 1 \\ A-2B &= -1 \end{aligned}\right\} \quad A = \frac{1}{3}, \quad B = \frac{2}{3}$$

$$= \frac{1}{3}\int \frac{dx}{x-2} + \frac{2}{3}\int \frac{dx}{x+1}$$

$$= \frac{1}{3}\ln|x-2| + \frac{2}{3}\ln|x+1| + c$$

clean up

$$= \frac{1}{3}\left(\ln|x-2| + 2\ln|x+1|\right) + c$$

$$= \frac{1}{3}\left(\ln|x-2| + \ln|x+1|^2\right) + c$$

$$= \frac{1}{3}\ln\left|(x-2)(x+1)^2\right| + c$$

$$\int \frac{x^3+x+1}{x^4+x^3}\,dx$$

$$= \int \frac{x^3+x+1}{x^3(x+1)}\,dx$$

$$= \int \frac{x^3}{x^3(x+1)}\,dx + \int \frac{x+1}{x^3(x+1)}\,dx$$

$$= \int \frac{dx}{x+1} + \int \frac{dx}{x^3}$$

$$= \ln|x+1| - \frac{1}{2x^2} + c$$

$$\int \frac{x-1}{x^3+x^2}\,dx \qquad \underbrace{\frac{A}{x} + \frac{B}{x^2}} + \frac{C}{x+1} = \frac{x-1}{x^2(x+1)}$$

raise to a power
start from degree 1

$$= \int \frac{x-1}{x^2(x+1)}\,dx$$

$$Ax(x+1) + B(x+1) + Cx^2 = x-1$$

$$Ax^2 + Ax + Bx + B + Cx^2 = x-1$$

$$(A+C)x^2 + (A+B)x + B = x-1$$

$$A+C=0 \qquad C=-2$$

$$A+B=1 \qquad A=2$$

$$B=-1$$

$$= \int \frac{2}{x}\,dx - \int \frac{dx}{x^2} - \int \frac{2}{x+1}\,dx$$

$$= 2\ln|x| + \frac{1}{x} - 2\ln|x+1| + C$$

clean up

$$= 2\ln\left|\frac{x}{x+1}\right| + \frac{1}{x} + C$$

$$= \ln\left(\frac{x}{x+1}\right)^2 + \frac{1}{x} + C$$

numerator can be degree 1

denominator is degree 2

$$\int \frac{x^2+x+2}{(x^2+1)(x-1)}\,dx \qquad \frac{Ax+B}{x^2+1} + \frac{C}{x-1} = \frac{x^2+x+2}{(x^2+1)(x-1)}$$

$$(Ax+B)(x-1) + C(x^2+1) = x^2+x+2$$

$$Ax^2 - B + Bx - Ax + Cx^2 + C = x^2+x+2$$

$$(A+C)x^2 + (-A+B)x + (-B+C) = x^2+x+2$$

$$\left.\begin{aligned} A+C &= 1 \\ -A+B &= 1 \\ -B+C &= 2 \end{aligned}\right\} \quad \begin{aligned} A &= -1 \\ B &= 0 \\ C &= 2 \end{aligned}$$

$$= -\int \frac{x}{x^2+1}\,dx + \int \frac{2}{x-1}\,dx$$

$$= -\frac{1}{2}\ln|x^2+1| + 2\ln|x-1| + c$$

$\sin^2 x + \cos^2 x = 1$

$\cot x = \sqrt{\csc^2 x - 1}$

[redacted]

solve $0 \leqslant x < 2\pi$

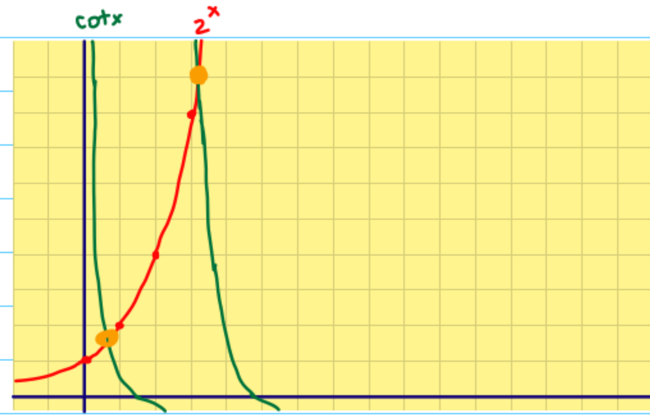
$2^x = \cot x$

$x = \log_2(\cot x)$

$x = \log_2 \cos x - \log_2 \sin x$

$2^x = \cot x$

$x\ln 2 = \ln\cos x - \ln\sin x$



$\ln 2^x = \ln\cot x$

$0 = \ln\cot x - \ln 2^x$

$0 = \ln\left(\frac{\cot x}{2^x}\right)$

$2^x = \cot x$

$2^x = \sqrt{\csc^2 x - 1}$

$x\ln 2 = \ln\sqrt{\csc^2 x - 1}$

$x\ln 2 = \frac{1}{2}\ln(\csc^2 x - 1)$

$x\ln 2 = \frac{1}{2}\ln\left(\frac{1-\sin^2 x}{\sin^2 x}\right)$

$2\ln 2 = \frac{\ln(1-\sin^2 x) - \ln(\sin^2 x)}{x}$

$2\ln 2 = \frac{\ln((1-\sin x)(1+\sin x)) - 2\ln(\sin x)}{x}$

$$2\ln 2 = \frac{\ln(1+\sin x) + \ln(1-\sin x) - 2\ln(\sin x)}{x}$$