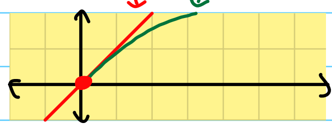
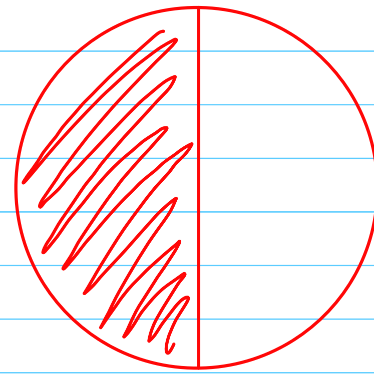
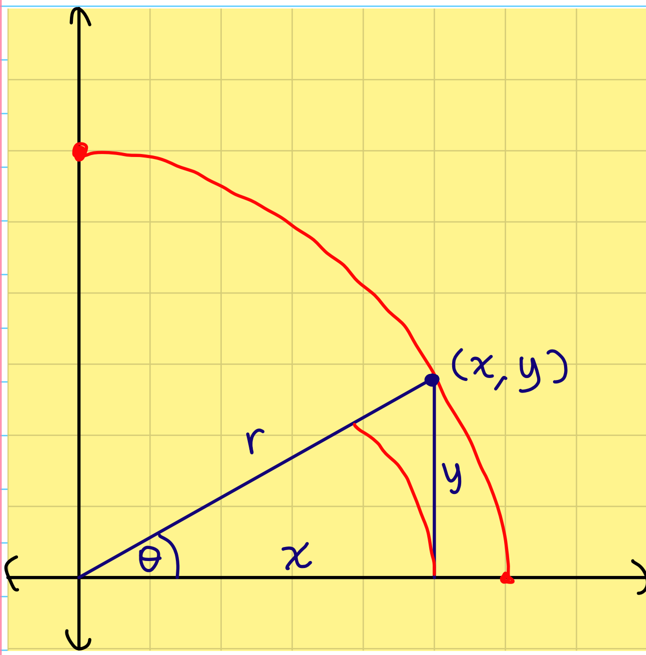
$$\lim_{x\to 0} \frac{\sin x}{x}$$



$= 1$

PROVE





A of half circle $= \frac{1}{2}\pi r^2$

$\frac{\pi}{2\pi}$

A of small — A of triangle — A of big

$$\frac{\theta}{2\pi}\pi x^2 \le \frac{1}{2}xy \le \frac{\theta}{2\pi}\pi r^2$$

$$\theta x^2 \le xy \le \theta r^2$$

$$\theta(r\cos\theta)^2 \le r\cos\theta \, r\sin\theta \le \theta r^2$$

$\sin\theta = \frac{y}{r}$ $\quad$ $\cos\theta = \frac{x}{r}$

$y = r\sin\theta$ $\quad$ $x = r\cos\theta$

$$\frac{\theta\cos^2\theta}{\theta\cos\theta} \le \frac{\cos\theta\sin\theta}{\theta\cos\theta} \le \frac{\theta}{\theta\cos\theta}$$

$$\lim_{\theta\to 0}\cos\theta \le \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \le \lim_{\theta\to 0}\frac{1}{\cos\theta}$$

$$1 \le \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \le 1$$

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

OCT 31

$$\lim_{x\to0} \frac{\cos x - 1}{x}$$

$\sin^2 x + \cos^2 x = 1$ pythagorean identity

$\sin^2 x = 1 - \cos^2 x$

$$= \lim_{x\to0} \frac{1-\cos x}{x} \cdot \frac{1+\cos x}{1+\cos x}$$

$$= \lim_{x\to0} \frac{\sin^2 x}{x(1+\cos x)}$$

$$\frac{a^2}{bc} = \frac{a}{b} \times \frac{a}{c}$$

$$= \lim_{x\to0} \frac{\sin x}{x} \times \frac{\sin x}{1+\cos x}$$

$$= 1 \times \frac{0}{1+1}$$

$$= 0$$

$$\boxed{\lim_{x\to0} \frac{\sin x}{x} = 1} \qquad \boxed{\lim_{x\to0} \frac{\cos x - 1}{x} = 0}$$

$$\frac{d}{dx} \sin x = \lim_{h\to0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to0} \frac{\sin(x+h) - \sin(x)}{h}$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$= \lim_{h\to0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to0} \frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h}$$

$$= \lim_{h\to0} \sin x \left(\cancelto{0}{\frac{\cos h - 1}{h}}\right) + \cos x \left(\cancelto{1}{\frac{\sin h}{h}}\right)$$

$$\frac{d}{dx} \sin x = \cos x$$ ‼

$$\boxed{\frac{d}{dx} \sin x = \cos x}$$

$$\frac{d}{dx}\cos x = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h}$$

$\cos(A+B) = \cos A\cos B + \sin A\sin B$

$$= \lim_{h\to 0}\frac{\cos x\cosh - \sin x\sinh - \cos x}{h}$$

$$= \lim_{h\to 0}\frac{\cos x\cosh - \cos x}{h} - \frac{\sin x\sinh}{h}$$

$$= \lim_{h\to 0}\cos x\left(\cancelto{0}{\frac{\cosh - 1}{h}}\right) - \sin x\left(\cancelto{1}{\frac{\sinh}{h}}\right)$$

[redacted]

$$\frac{d}{dx}\cos x = -\sin x \quad \#$$

$$\boxed{\frac{d}{dx}\cos x = -\sin x}$$

$$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x}$$

$$= \frac{\frac{d}{dx}\sin x\cdot\cos x - \sin x\cdot\frac{d}{dx}\cos x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \left(\frac{1}{\cos x}\right)^2$$

$\cos x = \frac{1}{\sec x}$

$\sin x = \frac{1}{\csc x}$

$\tan x = \frac{1}{\cot x}$

$$\frac{d}{dx}\tan x = \sec^2 x \quad \#$$

$$\frac{d}{dx}\sec x$$

$$= \frac{d}{dx}\frac{1}{\cos x}$$

$$= -1(\cos x)^{-2}\cdot\frac{d}{dx}\cos x$$

$$\frac{d}{dx}\csc x$$

$$= \frac{d}{dx}\frac{1}{\sin x}$$

$$= -1(\sin x)^{-2}\cdot\frac{d}{dx}\sin x$$

$$\frac{d}{dx}\cot x$$

$$= \frac{d}{dx}\frac{\cos x}{\sin x}$$

$$= \frac{\frac{d}{dx}\cos x\cdot\sin x - \cos x\cdot\frac{d}{dx}\sin x}{\sin^2 x}$$

$$= \frac{-1(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \tan x \cdot \sec x$$

$$= \frac{-1(\cos x)}{\sin^2 x}$$

$$= \frac{-\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$= -\cot x \cdot \csc x$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= -\csc^2 x$$

Derivatives of
Cs → cot, cos, csc
are NEGATIVE

$$\frac{d}{dx} \sin^3(\cos x)$$

$$= \frac{d}{dx} (\sin(\cos x))^3$$

$$= 3(\sin(\cos x))^2 \cdot \frac{d}{dx} \sin(\cos x)$$

$$= 3(\sin(\cos x))^2 (\cos(\cos x))(-\sin x)$$

$$= 3\sin^2(\cos x)(\cos(\cos x))(-\sin x) \quad \#$$

$$\frac{d}{dx} \tan^2 \sqrt{x}$$

$$= \frac{d}{dx} (\tan \sqrt{x})^2$$

$$= 2(\tan \sqrt{x}) \cdot \frac{d}{dx}(\tan \sqrt{x})$$

$$= 2(\tan \sqrt{x})(\sec^2 \sqrt{x})\left(\frac{d}{dx} \sqrt{x}\right)$$

$= 2(\tan\sqrt{x})(\sec^2\sqrt{x})(\frac{1}{2}x^{-\frac{1}{2}})$ #

$\frac{d}{dx} \sec^4(\csc^3(\sin x))$

$= 4(\sec^3(\csc^3(\sin x))) \cdot \frac{d}{dx}\sec(\csc^3(\sin x))$

$= 4(\sec^3(\csc^3(\sin x))) \cdot \tan(\csc^3(\sin x)) \cdot \sec(\csc^3(\sin x)) \cdot \frac{d}{dx}(\csc^3(\sin x))$

$= 4(\sec^3(\csc^3(\sin x))) \cdot \tan(\csc^3(\sin x)) \cdot \sec(\csc^3(\sin x))(3\csc^2(\sin x)) \cdot \frac{d}{dx}\sin x$

$= 4(\sec^3(\csc^3(\sin x))) \cdot \tan(\csc^3(\sin x)) \cdot \sec(\csc^3(\sin x)) \cdot 3\csc^2(\sin x) \cdot \cos x$ #

$\frac{d}{dx} \cos^4(\sin^3(\csc^2 x))$

$= 4\cos^3(\sin^3(\csc^2 x)) \cdot \frac{d}{dx}\cos(\sin^3(\csc^2 x))$

$= 4\cos^3(\sin^3(\csc^2 x))(-\sin(\sin^3(\csc^2 x))) \cdot \frac{d}{dx}(\sin^3(\csc^2 x))$

$= 4\cos^3(\sin^3(\csc^2 x))(-\sin(\sin^3(\csc^2 x)))(3(\sin^2(\csc^2 x))) \cdot \frac{d}{dx}(\sin(\csc^2 x))$

$= 4\cos^3(\sin^3(\csc^2 x))(-\sin(\sin^3(\csc^2 x)))(3(\sin^2(\csc^2 x)))(\cos(\csc^2 x)) \cdot \frac{d}{dx}\csc^2 x$

$= 4\cos^3(\sin^3(\csc^2 x))(-\sin(\sin^3(\csc^2 x)))(3(\sin^2(\csc^2 x)))(\cos(\csc^2 x))(2\csc x) \cdot \frac{d}{dx}\csc x$

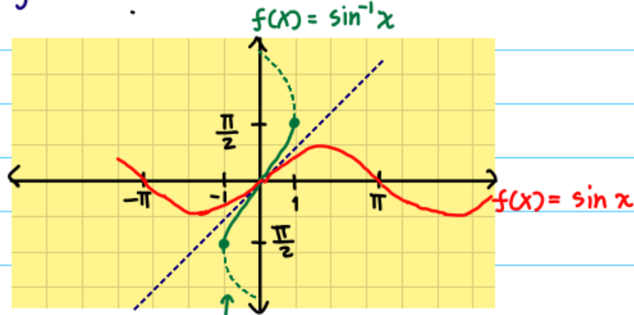
$= 4\cos^3(\sin^3(\csc^2 x))(-\sin(\sin^3(\csc^2 x)))(3(\sin^2(\csc^2 x)))(\cos(\csc^2 x))(2\csc x)(-\cot x\csc x)$

#

[redacted]

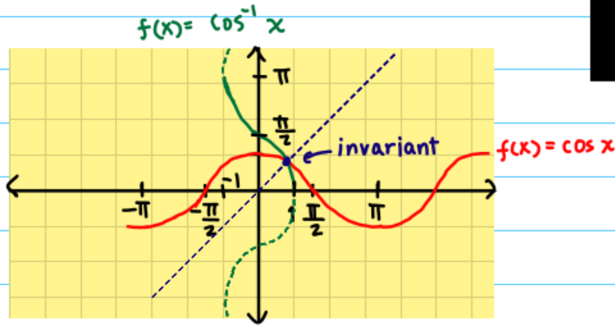
NOV 8

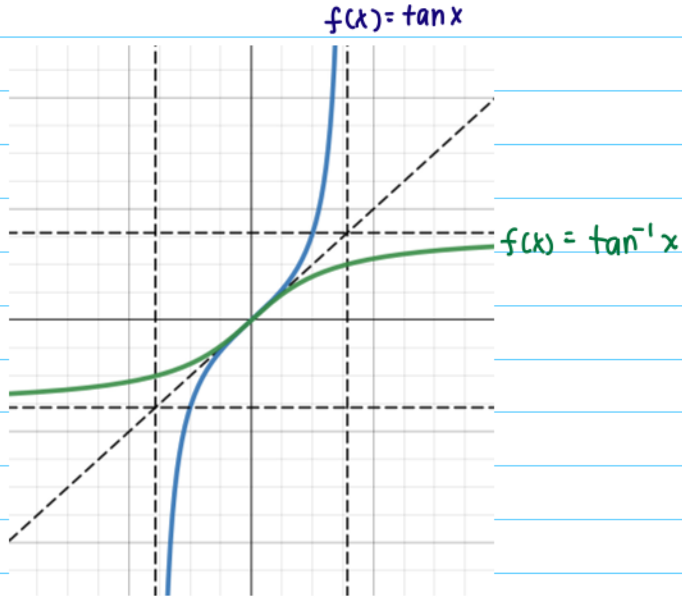
$y = \sin x$



not the dotted part because it will not pass the vertical line test

[redacted]





Implicit Differentiation

$$y = x^2$$
$$\frac{d}{dx} y = \frac{d}{dx} x^2$$
$$\frac{dy}{dx} = 2x \cdot \frac{d}{dx} x$$
$$\frac{dy}{dx} = 2x$$

$$y^2 - y + x^2 = 1$$
$$\frac{d}{dx} y^2 - \frac{d}{dx} y + \frac{d}{dx} x^2 = \frac{d}{dx} 1$$
$$2y\left(\frac{dy}{dx}\right) - \frac{dy}{dx} + 2x = 0$$
$$\left(\frac{dy}{dx}\right)(2y-1) = -2x$$
$$\frac{dy}{dx} = \frac{-2x}{2y-1}$$

NOV 10

$$y = x^2 - 2x + 1$$

$$\frac{d}{dt}(y) = \frac{d}{dt}(x^2 - 2x + 1)$$ ← take derivative with respect to time

$$\frac{dy}{dt} = 2x \frac{dx}{dt} - 2\frac{dx}{dt}$$

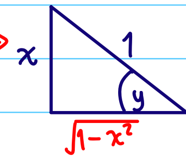
$$4y^2 - 2xy + x^2 = 5$$

$$\frac{d}{dt} 4y^2 - \frac{d}{dt} 2xy + \frac{d}{dt} x^2 = \frac{d}{dt} 5$$

$$8y \cdot \frac{dy}{dt} - \left(2\frac{dx}{dt} \cdot y + 2x \cdot \frac{dy}{dt}\right) + 2x \cdot \frac{dx}{dt} = 0$$

to find: $\frac{d}{dx} \arcsin x$

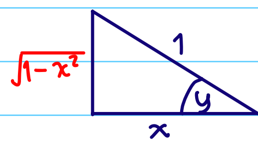
$$y = \arcsin x$$
$$\sin y = \sin(\arcsin x)$$
$$\sin y = x$$
$$\cos y \cdot \frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = \frac{1}{\cos y}$$
$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$



THEN $\boxed{\cos y = \frac{\sqrt{1-x^2}}{1}}$

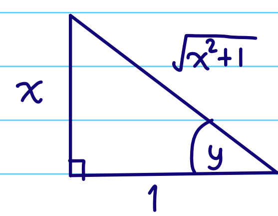
$\frac{d}{dx} \arccos x$

$$y = \arccos x$$
$$\cos y = x$$
$$-\sin y \cdot \frac{d}{dx} y = 1$$
$$\frac{dy}{dx} = \frac{1}{-\sin y}$$
$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$$



$\frac{d}{dx} \arctan x$

$$y = \arctan x$$
$$\tan y = x$$
$$\sec^2 y \cdot \frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$
$$\frac{dy}{dx} = \frac{1}{x^2+1}$$



$$\sec y = \frac{\sqrt{x^2+1}}{1}$$
$$\sec^2 y = x^2+1$$

$\frac{d}{dx} \arcsin(2\sqrt{x})$

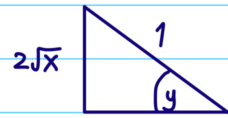
$y = \arcsin(2\sqrt{x})$

$\sin y = \frac{1}{\sqrt{x}}$

$\cos y \cdot \frac{dy}{dx} = \frac{1}{\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{x}\ \cos y}$

$\frac{dy}{dx} = \frac{1}{\sqrt{x}\sqrt{1-4x}}$



$\boxed{\frac{d}{dx}\arcsin \text{☺} = \frac{1}{\sqrt{1-\text{☺}^2}} \cdot \frac{d}{dx}\text{☺}}$

$\frac{d}{dx}\, x^2 \arccos x$

$= 2x \cdot \arccos x + x^2 \cdot \frac{-1}{\sqrt{1-x^2}}$

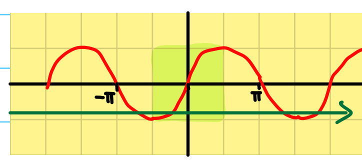
$= 2x \cdot \arccos x - \frac{x^2}{\sqrt{1-x^2}}$

NOV 15

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\sin\theta = -\frac{\sqrt{3}}{2}$

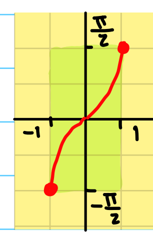
$\theta = -\frac{\pi}{3}$



hits ∞ amount of points

BUT

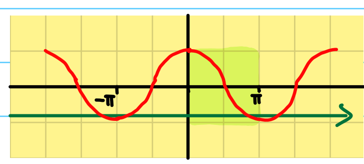
restriction only Q1, 4

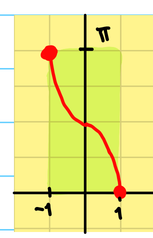


$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\cos\theta = \frac{-\sqrt{3}}{2}$

$\theta = \frac{5\pi}{6}$





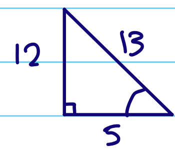
NOV 17

$\sin\left(\tan^{-1}\left(\frac{12}{5}\right)\right)$

$\theta = \tan^{-1}\left(\frac{12}{5}\right)$

$\tan\theta = \frac{12}{5}$

$\sin\theta = \frac{12}{13}$



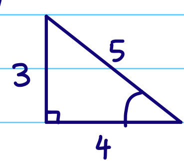
Pythagorean Triplets

3	4	5
5	12	13
8	15	17
7	24	25

$\cos\left(2\sin^{-1}\left(\frac{3}{5}\right)\right)$

$\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$\sin\theta = \frac{3}{5}$



$\boxed{\cos 2\theta = 1 - 2\sin^2\theta}$ cosine identity

$\cos 2\theta = 1 - 2\left(\frac{3}{5}\right)^2$

$= \frac{25}{25} - \frac{18}{25}$

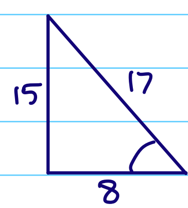
$= \frac{7}{25}$

$\tan\left(3\cos^{-1}\left(\frac{8}{17}\right)\right)$

$\theta = \cos^{-1}\left(\frac{8}{17}\right)$

$\cos\theta = \frac{8}{17}$

$\tan 3\theta$



$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(2A) = \tan(A+A)$

$= \frac{2\tan A}{1-\tan^2 A}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{15}{8}\right)}{1-\left(\frac{15}{8}\right)^2} \cdot \frac{8^2}{8^2}$

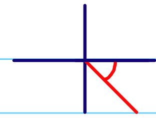
$= \frac{16\cdot 15}{8^2 - 15^2}$

$\tan(2\theta+\theta) = \frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \cdot \tan\theta}$

$\tan\theta = \frac{15}{8}$

$$= \frac{\frac{16\cdot 15}{8^2-15^2} + \frac{15}{8}}{1 - \frac{16\cdot 15}{8^2-15^2}\cdot\frac{15}{8}}$$

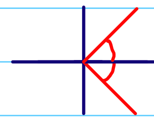
$\cos^{-1}(\cos(-\frac{\pi}{4}))$



$\cos 45^\circ = \frac{1}{\sqrt{2}}$

$\cos -45^\circ = \frac{1}{\sqrt{2}}$

$= \cos^{-1}(\frac{1}{\sqrt{2}}) \longrightarrow$



$= 45^\circ, 315^\circ$

$= \frac{\pi}{4}$

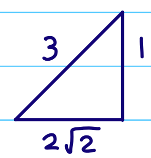
$\sin(\sin^{-1}(\frac{1}{3}) + \sin^{-1}(\frac{2}{3}))$

$\boxed{\sin(A+B) = \sin A \cos B + \cos A \sin B}$

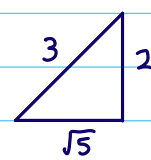
$= \sin(\sin^{-1}(\frac{1}{3}))\cos(\sin^{-1}(\frac{2}{3})) + \cos(\sin^{-1}(\frac{1}{3}))\sin(\sin^{-1}(\frac{2}{3}))$

$= \frac{1}{3}\cos(\sin^{-1}(\frac{2}{3})) + \frac{2}{3}\cos(\sin^{-1}(\frac{1}{3}))$

$= \frac{1}{3} \cdot \frac{\sqrt{5}}{3} + \frac{2\sqrt{2}}{3} \cdot \frac{2}{3}$



$= \frac{\sqrt{5}}{9} + \frac{4\sqrt{2}}{9}$



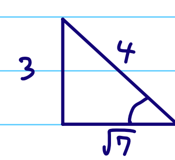
$= \frac{\sqrt{5} + 4\sqrt{2}}{9}$

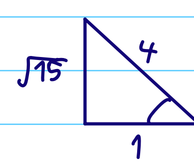
$\cos(\sin^{-1}(\frac{3}{4}) + \cos^{-1}(\frac{1}{4}))$

$\boxed{\cos(A+B) = \cos A \cos B - \sin A \sin B}$

$= \cos(\sin^{-1}(\frac{3}{4}))\cos(\cos^{-1}(\frac{1}{4})) - \sin(\sin^{-1}(\frac{3}{4}))\sin(\cos^{-1}(\frac{1}{4}))$

$= \frac{1}{4}\cos(\sin^{-1}(\frac{3}{4})) - \frac{3}{4}\sin(\cos^{-1}(\frac{1}{4}))$



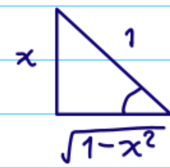


$= \frac{1}{4} \cdot \frac{\sqrt{7}}{4} - \frac{3}{4} \cdot \frac{\sqrt{15}}{4}$

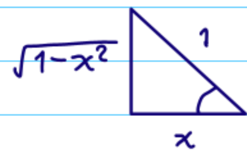
$= \frac{\sqrt{7} - 3\sqrt{15}}{16}$

$\cos(\sin^{-1} x)$

$= \sqrt{1-x^2}$



$\sin(2\cos^{-1}x)$



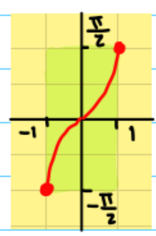
$= \sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\sqrt{1-x^2}\right)(x)$

$= 2x\sqrt{1-x^2}$

$\sin^{-1}(1-x)$



$-1 \leq 1-x \leq 1$

$-2 \leq -x \leq 0$

$0 \leq x \leq 2$

[redacted]