Anti-Derivatives find a function from its slope

$\int x \, dx$

↳ what to take derivative of to get x

$\frac{d}{dx} x^2 = 2x$ multiply by $\frac{1}{2}$ to get rid of 2

$\frac{d}{dx} \frac{1}{2}x^2 = x$

$\int x \, dx = \frac{1}{2}x^2 + C$

always add a constant
since $\frac{d}{dx} C = 0$

$\int (2x-1)^3 \, dx$

$\frac{d}{dx} (2x-1)^4 = 4(2x-1)^3 \cdot \frac{d}{dx}(2x-1)$

$= 8(2x-1)^3$

$\frac{d}{dx} \frac{1}{8}(2x-1)^4 = (2x-1)^3$

$\int (2x-1)^3 \, dx = \frac{1}{8}(2x-1)^4 + C$

$\int e^{2x} \, dx$

$\frac{d}{dx} e^{2x} = e^{2x} \cdot 2$

$\frac{d}{dx} \frac{1}{2}e^{2x} = e^{2x}$

$\int e^{2x} = \frac{1}{2}e^{2x} + C$

$$\int 2^{3x-1}\,dx$$

$$\frac{d}{dx}\,2^{3x-1} = 2^{3x-1}(\ln 2)(3)$$

$$\frac{d}{dx}\,\frac{1}{3\ln 2}\,2^{3x-1} = 2^{3x-1}$$

$$= \int 2^{3x-1} = \frac{1}{3\ln 2}\,2^{3x-1} + c$$

$$\int 3xe^{x^2}\,dx$$

$$\frac{d}{dx}\,e^{x^2} = e^{x^2}\cdot 2x$$

$$= \frac{3}{2}e^{x^2} + C$$

$$\int \cos x\,\sin^4 x\,dx$$

$$\frac{d}{dx}\,\sin^5 x = 5\sin^4 x\cdot\cos x$$

$$= \frac{1}{5}\sin^5 x + c$$

$\int \sin x \cos x \, dx$

$\frac{d}{dx} \sin^2 x = 2 \sin x \cos x$

$\frac{d}{dx} \cos^2 x = -2 \cos x \sin x$

$= \frac{1}{2} \sin^2 x + c$

$= -\frac{1}{2} \cos^2 x + c$

$\int \tan^3 x \sec^2 x \, dx$

$\frac{d}{dx} \tan^4 x = 4 \tan^3 x \sec^2 x$

$= \frac{1}{4} \tan^4 x + c$

$\int x \sqrt{1-x^2} \, dx$

$\frac{d}{dx} (1-x^2)^{\frac{3}{2}}$

$= \frac{3}{2} (1-x^2)^{\frac{1}{2}} (-2x)$

$= -\frac{1}{3} (1-x^2)^{\frac{3}{2}} + c$

OR

let $m = 1-x^2$

$dm = -2x \, dx$

$-\frac{1}{2} dm = x dx$

$\int x \sqrt{1-x^2} \, dx$

$= \int \sqrt{m} \left(-\frac{1}{2}\right) dm$

$\frac{d}{dx} m^{\frac{3}{2}} = \frac{3}{2} \sqrt{m}$

$= -\frac{1}{3} m^{\frac{3}{2}} + c$

$= -\frac{1}{3} (1-x^2)^{\frac{3}{2}} + c$

$$\int \frac{x+1}{\sqrt{x}}\,dx$$

$$= \int \left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right) dx$$

$$= \int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}x^{\frac{3}{2}} = \frac{3}{2}\sqrt{x}$$

$$= \frac{2}{3}x^{\frac{3}{2}} + 2\sqrt{x} + C$$

$$= \frac{2}{3}\sqrt{x^3} + 2\sqrt{x} + C$$

$$\int \frac{x}{\sqrt{x+1}}\,dx$$

let $m = x+1$, $x = m-1$

$dm = dx$

$$\int \frac{x}{\sqrt{x+1}}\,dx$$

$$= \int \frac{m-1}{\sqrt{m}}\,dm$$

$$= \int \left(\sqrt{m} - \frac{1}{\sqrt{m}}\right) dm$$

$$= \frac{2}{3}m^{\frac{3}{2}} - 2m^{\frac{1}{2}} + C$$

$$= \frac{2}{3}(x+1)^{\frac{3}{2}} - 2(x+1)^{\frac{1}{2}} + C$$

Apr 2

$$\int \frac{(x-1)^2}{x^2}\,dx$$

$$= \int \frac{x^2-2x+1}{x^2}\,dx$$

$$= \int \frac{x^2}{x^2}\,dx - \int \frac{2x}{x^2}\,dx + \int \frac{1}{x^2}\,dx$$

$$= \int dx - \int 2x^{-1}\,dx + \int x^{-2}\,dx$$

$$= x - 2\ln|x| - x^{-1} + c$$

$$\int \frac{x^2}{(x-1)^2}\,dx$$

let $m = x-1$ $\quad x = m+1$

$dm = dx$

$$= \int \frac{x^2}{(x-1)^2}\,dx$$

$$= \int \frac{(m+1)^2}{m^2}\,dm$$

$$= \int \frac{m^2}{m^2}\,dm + \int \frac{2m}{m^2}\,dm + \int \frac{1}{m^2}\,dm$$

$$= \int dm + \int 2m^{-1}\,dm + \int m^{-2}\,dm$$

$$= m + 2\ln|m| - m^{-1} + c$$

$$= x-1 + 2\ln|x-1| - (x-1)^{-1} + c$$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$\frac{d}{dx}\sqrt{1-x^2} = \frac{-x}{\sqrt{1-x^2}}$$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= -\sqrt{1-x^2} + c$$

OR

$$= \int \frac{x}{\sqrt{m}} \cdot \frac{-dm}{2x}$$

$$= -\frac{1}{2}\int m^{-\frac{1}{2}}\,dm$$

$$= -\frac{1}{2}\, 2\sqrt{m} + c$$

$$= -\sqrt{m} + c$$

$$= -\sqrt{1-x^2} + c$$

$m = 1-x^2$

$dm = -2x\,dx$

$dx = \frac{-dm}{2x}$

$$\int \frac{x^2}{\sqrt{1-x^3}}\,dx$$

$$\frac{d}{dx}\sqrt{1-x^3} = -\frac{3x^2}{2\sqrt{1-x^3}}$$

$$\int \frac{x^2}{\sqrt{1-x^3}}\,dx$$

$$= -\frac{2\sqrt{1-x^3}}{3} + c$$

$$\int \frac{dx}{\sqrt{1-x^2}}$$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1-\sin^2\theta$

$$= \int \frac{\cos\theta\,d\theta}{\cos\theta}$$

let $x = \sin\theta$

$dx = \cos\theta\,d\theta$

$$= \int d\theta$$

$$= \theta + c$$

$= \arcsin x + c$

$= \sin^{-1} x + c$

Apr 4

$$\int \frac{x^3}{\sqrt{1-x^2}}\, dx$$

let $m = 1-x^2$ $\quad x^2 = 1-m$

$dm = -2x\, dx$

$$= \int \frac{x^2 \cdot x}{\sqrt{m}} \cdot \frac{-dm}{2x}$$

$$= \frac{1}{2} \int \frac{m-1}{\sqrt{m}}\, dm$$

$$= \frac{1}{2}\left(\int \frac{m}{\sqrt{m}}\, dm - \int \frac{1}{\sqrt{m}}\, dm \right)$$

$$= \frac{1}{2}\left(\int m^{\frac{1}{2}}\, dm - m^{-\frac{1}{2}}\, dm \right)$$

$$= \frac{1}{2}\left(\frac{2}{3} m^{\frac{3}{2}} - 2m^{\frac{1}{2}} + c \right)$$

$$= \frac{1}{3} m^{\frac{3}{2}} - m^{\frac{1}{2}} + c$$

$$= \frac{1}{3}(1-x^2)^{\frac{3}{2}} - \sqrt{1-x^2} + c$$

$$\int \frac{x^2}{\sqrt{1-x^2}}\, dx$$

let $x = \sin\theta$

$dx = \cos\theta\, d\theta$

$$= \int \frac{\sin^2\theta}{\cos\theta} \cos\theta\, d\theta$$

$$= \int \sin^2\theta\, d\theta$$

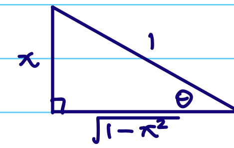
$$\cos 2\theta = \left\{ \begin{array}{l} \cos^2\theta - \sin^2\theta \\ 1 - 2\sin^2\theta \\ 2\cos^2\theta - 1 \end{array} \right. \Big\} \Rightarrow \sin^2\theta = \tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta$$

$$= \int \left(\frac{1}{2} - \frac{1}{2}\cos 2\theta \right) d\theta$$

$$\frac{d}{dx} \cos 2\theta = 2\sin 2\theta$$

$$= \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + c$$

SINCE $x = \sin\theta \Rightarrow$



$$= \frac{1}{2}\arcsin x - \frac{1}{4}(2)(x)(\sqrt{1-x^2}) + c$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= \frac{\arcsin x}{2} - \frac{x\sqrt{1-x^2}}{2} + c$$

$$\int \frac{dx}{\sqrt{4-x^2}}$$

let $x = 2\sin\theta$ $\quad\quad$ $\sin\theta = \frac{x}{2}$

$dx = 2\cos\theta \, d\theta$

$$= \int \frac{2\cos\theta \, d\theta}{\sqrt{4-4\sin^2\theta}}$$

$$= \int \frac{2\cos\theta \, d\theta}{\sqrt{4(1-\sin^2\theta)}}$$

$$= \int \frac{2\cos\theta \, d\theta}{2\cos\theta}$$

$$= \int d\theta$$

$$= \theta + c$$

$$= \arcsin\frac{x}{2} + c$$

$$\int \frac{dx}{\sqrt{9-4x^2}}$$

let $x = \frac{3}{2}\sin\theta$

$$= \int \frac{\frac{3}{2}\cos\theta \, d\theta}{\sqrt{9-9\sin^2\theta}}$$

$dx = \frac{3}{2}\cos\theta \, d\theta$

$$= \int \frac{\frac{3}{2}\cos\theta \, d\theta}{3\cos\theta}$$

$$= \frac{1}{2} \int d\theta$$

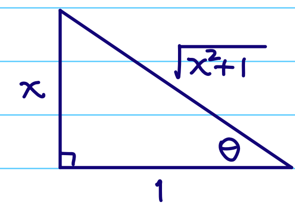
$$= \frac{1}{2} \theta + c$$

$$= \frac{1}{2} \arcsin \frac{2x}{3} + c$$

$$\int \frac{dx}{\sqrt{1+x^2}}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$



Let $x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

$$= \int \frac{\sec^2\theta \, d\theta}{\sqrt{1+\tan^2\theta}}$$

$$= \int \frac{\sec^2\theta \, d\theta}{\sec\theta}$$

$$= \int \sec\theta \, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + c$$

$$= \ln\left|\sqrt{x^2+1} + x\right| + c$$

$$\frac{d}{dx} \ln|\sec x + \tan x|$$

$$= \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$$

$$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$$

$$= \sec x$$

Apr 8 $\int \frac{dx}{x^2+2x}$

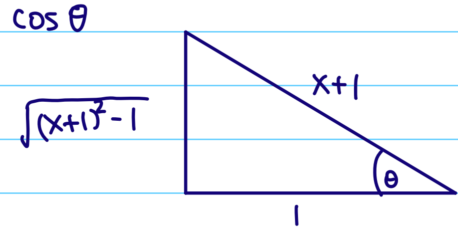
$= \int \frac{dx}{(x+1)^2-1}$

$\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

let $x+1 = \sec\theta$ $\quad dx = \sec\theta\tan\theta\, d\theta$

$\frac{1}{x+1} = \cos\theta$



$= \int \frac{dx}{\sec^2\theta - 1}$

$= \int \frac{\sec\theta\tan\theta\, d\theta}{\tan^2\theta}$

$= \int \frac{\sec\theta}{\tan\theta}\, d\theta$

$= \int \csc\theta\, d\theta$

$= \ln|\csc\theta - \cot\theta| + c$

$= \ln\left|\frac{x+1}{\sqrt{(x+1)^2-1}} - \frac{1}{\sqrt{(x+1)^2-1}}\right| + c$

$= \ln\left|\frac{x}{\sqrt{x^2+2x}}\right| + c$

$= \ln|x| - \frac{1}{2}\ln|x^2+2x| + c$

$= \ln|x| - \frac{1}{2}\left(\ln|x| + \ln|x+2|\right) + c$

$= \frac{1}{2}\ln|x| - \frac{1}{2}\ln|x+2| + c$

$$\boxed{\begin{aligned} &\frac{d}{dx} \ln|\csc x - \cot x| \\ &= \frac{-\cot x\csc x + \csc^2 x}{\csc x - \cot x} \\ &= \csc x \end{aligned}}$$

OR

$$\int \frac{dx}{x^2+2x}$$

$$= \int \frac{dx}{x(x+2)}$$

$$= \int \left(\frac{a}{x} + \frac{b}{x+2}\right) dx$$

$$= \int \left(\frac{1}{2}\cdot\frac{1}{x} - \frac{1}{2}\cdot\frac{1}{x+2}\right) dx$$

$$= \frac{1}{2}\int \left(\frac{1}{x} - \frac{1}{x+2}\right) dx$$

$$= \frac{1}{2}\ln|x| - \frac{1}{2}\ln|x+2| + C$$

$$\frac{1}{x(x+2)} = \frac{a}{x} + \frac{b}{x+2}$$

$$\frac{1}{x(x+2)} = \frac{a(x+2)+bx}{x(x+2)}$$

$$\boxed{1} + \boxed{0x} = \underline{2a} + \underline{(a+b)x}$$

$1 = 2a$ AND $0x = (a+b)x$

$a = \frac{1}{2}$ $\quad$ $b = -\frac{1}{2}$

$$\int \frac{dx}{x^2+2x+10}$$

$$= \int \frac{dx}{(x+1)^2+9}$$

$\tan^2\theta + 1 = \sec^2\theta$

$(3\tan\theta)^2 + 9 = (3\sec\theta)^2$

Let $x+1 = 3\tan\theta$ $\quad$ $dx = 3\sec^2\theta \, d\theta$

$\frac{x+1}{3} = \tan\theta$

$$= \int \frac{\sec^2\theta \, d\theta}{(3\tan\theta)^2+9}$$

$\tan^{-1}\left(\frac{x+1}{3}\right) = \theta$

$$= \int \frac{\sec^2\theta \, d\theta}{3\sec^2\theta}$$

$$= \frac{1}{3}\int d\theta$$

$$= \frac{1}{3}\theta + C \quad = \frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right) + C$$