May 14 Integration by parts

$$\frac{d}{dx}uv = du \cdot v + u \cdot dv$$

$$u \cdot dv = \frac{d}{dx}uv - du \cdot v$$

$$\int u\,dv = \int \frac{d}{dx}uv - \int v\,du$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$\int x \sin x\,dx$ $\quad u = x \quad dv = \sin x\,dx$

$du = dx \quad v = -\cos x$

$= -x\cos x - \int -\cos x\,dx$

$= -x\cos x + \int \cos x\,dx$

$= -x\cos x + \sin x + C$

$\int xe^x\,dx$ $\quad u = x \quad dv = e^x\,dx$

$du = dx \quad v = e^x$

$= xe^x - \int e^x\,dx$

$= xe^x - e^x + c$

$\int \sec^3 x\,dx$ $\quad u = \sec x \quad dv = \sec^2 x\,dx$

$du = \sec x \tan x\,dx \quad v = \tan x$

$= \sec x \tan x - \int \tan^2 x \sec x\,dx$

$= \sec x \tan x - \int \sec x(\sec^2 x - 1)\,dx$

$= \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$

$\int x \sec x \tan x\,dx$

$u = x \quad dv = \sec x \tan x\,dx$

$du = dx \quad v = \sec x$

$= x \sec x - \int \sec x\,dx$

$= x \sec x - \ln|\sec x + \tan x| + C$

$$\int \sec^5 x \, dx$$

$u = \sec^3 x \qquad dv = \sec^2 x \, dx$

$du = 3\sec^3 x \tan x \, dx \qquad v = \tan x$

$$= \sec^3 x \tan x - 3\int \sec^3 x \tan^2 x \, dx$$

$$= \sec^3 x \tan x - 3\int \sec^3 x (\sec^2 x - 1) \, dx$$

$$= \sec^3 x \tan x - 3\int \sec^5 x \, dx + 3\int \sec^3 x \, dx$$

$$= \frac{1}{4}\left(\sec^3 x \tan x + 3\int \sec^3 x \, dx \right)$$

$u = \sec x \qquad dv = \sec^2 x \, dx$

$du = \sec x \tan x \, dx \qquad v = \tan x$

$$= \frac{1}{4}\sec^3 x \tan x + \frac{3}{4}\left(\frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| \right) + C$$

$$= \frac{1}{4}\sec^3 x \tan x + \frac{3}{8}\sec x \tan x + \frac{3}{8}\ln|\sec x + \tan x| + C$$

$$\int x^2 e^x \, dx$$

$u = x^2 \qquad dv = e^x \, dx$

$du = 2x \, dx \qquad v = e^x$

$$= x^2 e^x - 2\int x e^x \, dx$$

$u = x \qquad dv = e^x \, dx$

$du = dx \qquad v = e^x$

$$= x^2 e^x - 2x e^x + 2\int e^x \, dx$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$\int x^2 \sin x \, dx$

$u = x^2 \qquad dv = \sin x \, dx$

$du = 2x \, dx \qquad v = -\cos x$

$= -x^2 \cos x - \int -2x \cos x \, dx$

$= -x^2 \cos x + 2\int x \cos x \, dx$

$u = x \qquad dv = \cos x \, dx$

$du = dx \qquad v = \sin x$

$= -x^2 \cos x + 2x \sin x - 2\int \sin x \, dx$

$= -x^2 \cos x + 2x \sin x + 2\cos x + c$

$\int e^x \sin x \, dx$

$u = \sin x \qquad dv = e^x \, dx$

$du = \cos x \, dx \qquad v = e^x$

$= e^x \sin x - \int e^x \cos x \, dx$

$u = \cos x \qquad dv = e^x \, dx$

$du = -\sin x \, dx \qquad v = e^x$

$= e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$

$= \frac{1}{2} e^x \sin x - \frac{1}{2} e^x \cos x + c$

$\int e^{\sin x} \sin 2x \, dx$

$= 2\int e^{\sin x} \sin x \cos x \, dx$

$m = \sin x$

$dm = \cos x \, dx$

$= 2\int e^m m\,dm$

$u = m \qquad dv = e^m dm$

$du = dm \qquad v = e^m$

$= 2me^m - 2\int e^m dm$

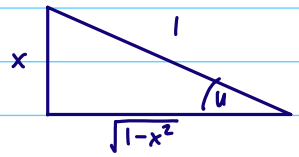
$= 2me^m - 2e^m + c$

$= 2e^{\sin x}\sin x - 2e^{\sin x} + c$

$\int \arcsin x\,dx$

$u = \arcsin x \qquad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}dx \qquad v = x$



$= x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}dx$

$= x\arcsin x + \sqrt{1-x^2} + c$