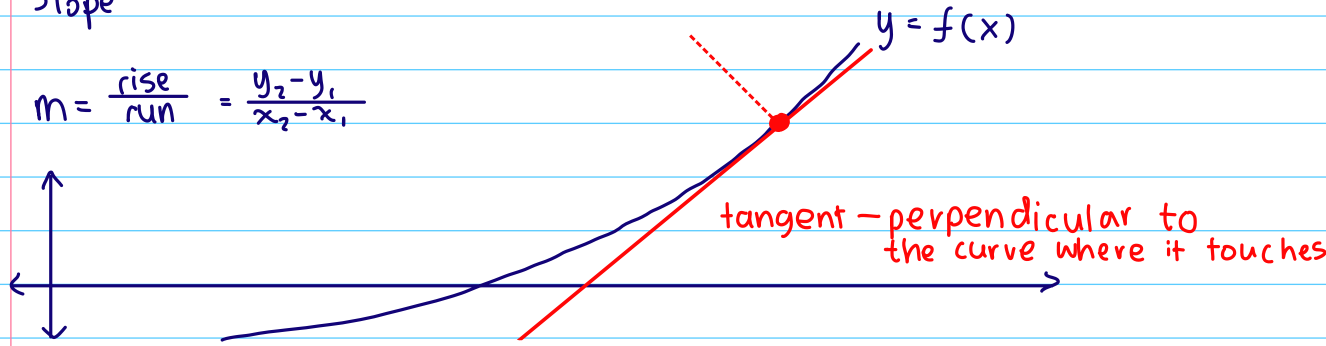
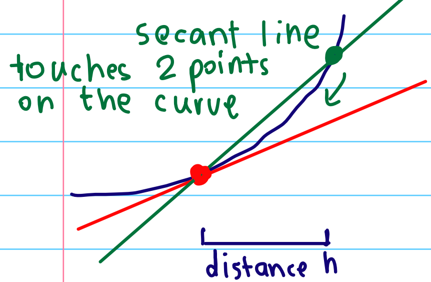
SEP 11 Slope

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$



Find the slope of the tangent line → draw a secant line



$$\text{slope of secant} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

slope of secant is closer to slope of tangent when ● moves closer to ●

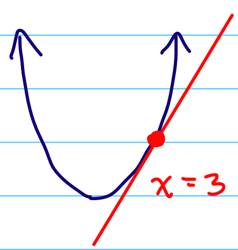
"limit"

THEN $\boxed{m_{tangent} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

less h = less distance
"as h approaches 0"

$f(x) = x^2 - 1$

find the tangent line slope at $x = 3$



$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ — plug x+h, x into f

$$= \lim_{h \to 0} \frac{(x+h)^2 - 1 - (x^2 - 1)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 1 - x^2 + 1}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2xh}{h}$$

$$= \lim_{h \to 0} h + 2x$$ h approaches 0

$$m = 2x$$

$$m\big|_{x=3} = 2(3)$$

$$= 6 \ \#$$

$e = 2.7182818\ldots$

$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$

$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

$e = \lim_{n \to 0} (1+n)^{\frac{1}{n}}$

[redacted]

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Sept 13, 2023

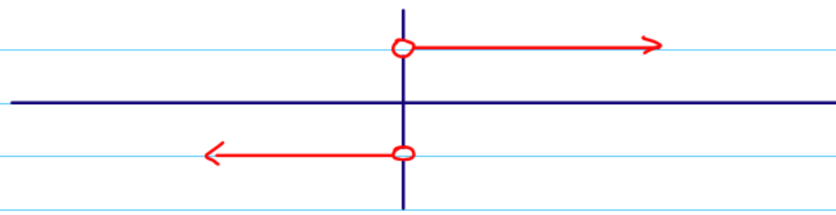
SEP 13 Absolute value in a limit

$$y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$\lim_{x \to 0} \frac{x}{|x|}$ → Limit = height of curve at x

approaching from left: $\lim_{x \to 0^-} \frac{x}{-x} = -1$

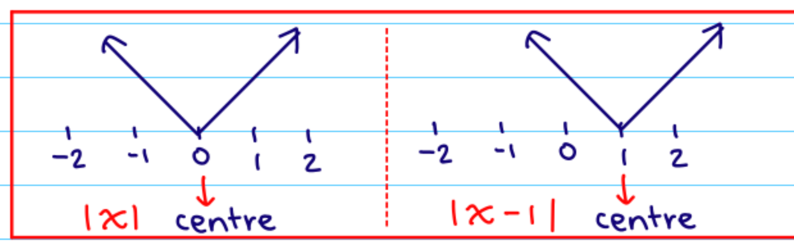
$\lim_{x \to 0^+} \frac{x}{x} = 1$ approaching from right



Limit from left MUST = to limit from right EVEN IF there is a hole

This limit therefore does NOT exist :(

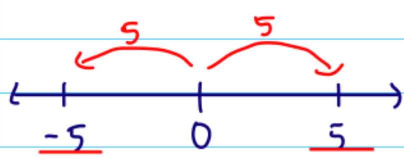
• Absolute
distance to the centre
of the absolute



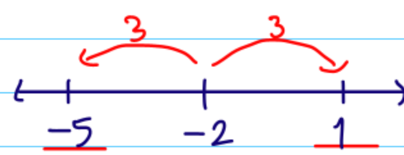
Centre is usually at the origin but can be different if there is a horizontal translation
(+/- at x)

$|xy| = |x| \cdot |y|$

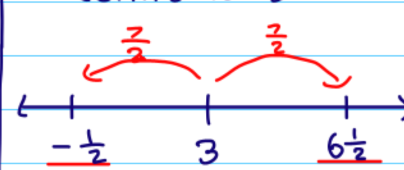
$|x| = 5$ → distance
centre is 0



$|\underbrace{x+2}_{\text{left } 2}| = 3$
centre is -2

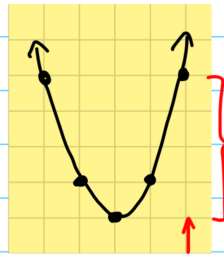


$|2x-6| = 7$
$|2(x-3)| = 7$
$|2| \cdot |x-3| = 7$
$|x-3| = \frac{7}{2}$
centre is 3



SEP 15 $\lim_{x \to a} f(x)$ → height of curve $f(x)$ as x approaches a

$\lim_{x \to 2} x^2$



height = 4

$x = 2$

Writing restrictions

$y = \dfrac{x(x-2)}{x-2}$

$y = x$

these 2 lines equal except when $x = 2$

$\boxed{x \neq 2}$ → because $(x-2)$ was cancelled, value was lost in the process of simplifying making a hole ← $\boxed{x \neq 2}$

$y = \dfrac{x(x-2)}{(x-1)(x-2)}$

$y = \dfrac{x}{x-1}$

$y = \dfrac{x(x-2)}{(x-2)^2}$

$y = \dfrac{x}{x-2}$

no restriction

unless the $(x-2)$ completely disappears from the denominator

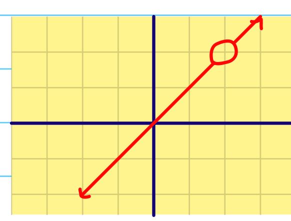
* ONLY write hole restrictions *

$\lim_{x \to 2} \dfrac{x^2 - 2x}{x-2}$

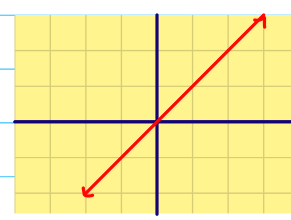
$= \lim_{x \to 2} x$

$= 2$

height of the hole ☆



$\dfrac{x^2 - 2x}{x-2}$



x

natural log

$\lim_{h \to 0} \dfrac{2^h - 1}{h} = \ln 2$ $\quad$ $\lim_{h \to 0} \dfrac{3^h - 1}{h} = \ln 3$ $\quad$ $\lim_{h \to 0} \dfrac{e^h - 1}{h} = 1$

$\log_{10} 10 = 1$