SEP 28

The Binomial Theorem

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$$\binom{5}{0} \quad \binom{5}{1} \quad \binom{5}{2} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}$$

"5 choose 5"
find number of events
• choose 5 from 5

$$(x+y)^5$$

$$= \binom{5}{0}x^5y^0 + \binom{5}{1}x^4y^1 + \binom{5}{2}x^3y^2 + \binom{5}{3}x^2y^3 + \binom{5}{4}x^1y^4 + \binom{5}{5}x^0y^5$$

$$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

$$\lim_{h\to 0} \frac{(x+h)^5 - x^5}{h} \rightarrow \frac{d}{dx}x^5 \quad \text{OR} \quad \text{if } f(x) = x^5, \text{ determine } f'(x)$$

$$= \lim_{h\to 0} \frac{\cancel{x^5} + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5 - \cancel{x^5}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4)}{\cancel{h}}$$

$$= 5x^4$$

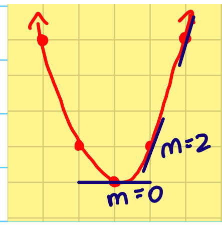
$$\overbrace{\lim_{h\to 0} \frac{(x+h)^{172} - x^{172}}{h}}^{\frac{d}{dx}x^{172}} = 172x^{171}$$

$$\lim_{h\to 0} \frac{(x+h)^a - x^a}{h} \quad \Big\{ \quad \boxed{\frac{d}{dx}x^a = ax^{a-1}}$$

"derivative" $\frac{d}{dx}$
slope of the tangent line at a point of a curve

m (slope)

$y = x^2$ $\qquad$ $y' = 2x$



slope of tangent line at $x = 0$

$$y'\Big|_{x=0} = 2(0) = 0$$

$$y'\Big|_{x=1} = 2(1) = 2$$

if

$$f(x) = x^3$$

$$f'(x) = 3x^2 \Rightarrow \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= 3x^2$$

find slope at $x = 1$

$$f'(1) = 3(1)^2 \Rightarrow \lim_{h\to 0} \frac{(1+h)^3 - 1}{h}$$

$$= 3 \qquad = \lim_{h\to 0} \frac{1 + 3h + 3h^2 + h^3 - 1}{h}$$

$$= 3$$

if $b \in \mathbb{R}$

$b > 0$

$b \neq 1$

$$\boxed{\frac{d}{dx} b^x = b^x \cdot \ln b}$$

DERIVATIVE : $\lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

if $f(x) = 2^x \rightarrow \lim_{h\to 0} \dfrac{2^{x+h} - 2^x}{h}$ THEN $\lim_{h\to 0} \dfrac{2^h - 1}{h}$

means $f'(0)$ of $f(x) = 2^x$

$$\frac{d}{dx} 2^x = 2^x \cdot \ln 2$$

$$= 2^0 \cdot \ln 2$$

$$= \ln 2$$