CS 344: Homework #4

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Professor Bahman Kalantari Section #1

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Problem 1

Using shortest path algorithm determine of the following system of inequalities has a feasible solution.

$x_4 - x_2 \leq 2$
$x_1 - x_3 \leq 3$
$x_5 - x_4 \leq -4$
$x_3 - x_2 \leq 2$
$x_2 - x_1 \leq 8$
$x_4 - x_3 \leq 1$
$x_1 - x_5 \leq 3$

If it has no solution can you change 2 in the first inequality to a number so that the system will be feasible? And what would be the smallest such value to change it to so that the system becomes feasible.

Based on these equations, we can determine the vertices and their distances in the initial graph:

u	$->$	**v**	**d(u,v)**
x_1	$->$	x_2	8
x_2	$->$	x_3	2
x_3	$->$	x_4	1
x_2	$->$	x_4	-2
x_4	$->$	x_5	-4
x_3	$->$	x_1	3
x_5	$->$	x_1	-3

From here, we can use Bellman-Ford to figure out if the system is feasible. Perform $n-1$ iterations of Bellman Ford where n is the number of vertices. In this case, we know that there are 5 vertices x_1,x_2,x_3,x_4, and x_5. Therefore, we need to perform 4 iterations.

First Iteration

				Final distances	
$x_1:$	0	$x_1:$	0	$x_1:$	-1
$x_2:$	∞	$x_2:$	8	$x_2:$	8
$x_3:$	∞	$x_3:$	10	$x_3:$	10
$x_4:$	∞	$x_4:$	6	$x_4:$	6
$x_5:$	∞	$x_5:$	2	$x_5:$	2

Second Iteration

		Final distances	
$x_1:$	-1	$x_1:$	-2
$x_2:$	8	$x_2:$	7
$x_3:$	10	$x_3:$	9
$x_4:$	6	$x_4:$	5
$x_5:$	2	$x_5:$	1

Third Iteration

		Final distances	
$x_1:$	-2	$x_1:$	-3
$x_2:$	7	$x_2:$	6
$x_3:$	9	$x_3:$	8
$x_4:$	5	$x_4:$	4
$x_5:$	1	$x_5:$	0

Fourth Iteration

		Final distances	
$x_1:$	-3	$x_1:$	-4
$x_2:$	6	$x_2:$	5
$x_3:$	8	$x_3:$	7
$x_4:$	4	$x_4:$	3
$x_5:$	0	$x_5:$	-1

We can conclude that the system is infeasible because there is a negative cycle with edges x_1 to x_2 to x_4 to x_5 back to x_1. The total sum of these edges is -1, so if we changed the -2 in the first equation to -1 ($x_4 - x_2 \leq -1$).

Problem 2

Given 4 matrices, $A_1, ..., A_4$, where A_i is $m_{i-1}m_i$ matrix, $i = 1, ..., 4$, with $m_0 = 50, m_1 = 10, m_2 = 30, m_3 = 20, m_4 = 100$. Compute $c(i, j)$'s by following the matrix chain multiplication algorithm

$(A_1$x$A_2)$x$(A_3$x$A_4)$	$(A_1$x$(A_2$x$A_3))$xA_4	A_1x$((A_2$x$A_3)$x$A_4)$
A_1x$A_2 = 15000$ mult.	A_2x$A_3 = 6000$ mult.	A_2x$A_3 = 6000$
A_3x$A_4 = 60000$ mult.	x$A_1 = 10000$ mult.	x$A_4 = 20000$
Last combination: 150000 mult.	x$A_4 = 100000$ mult.	x$A_1 = 50000$
Total: 225000 mult.	Total: 116000	Total: 76000

Optimal $C(i, j) = 76000$

Problem 3