Department of Electronic and Telecommunication Engineering

University of Moratuwa

BM4152 - Biosignal Processing

Paper Implementation

A Wavelet Optimization Approach for ECG Signal Classification

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1 Introduction and background

In the domain of signal processing wavelets are highly effective for tasks such as feature extraction, signal denoising, and compression. However, among those applications, it has been proven that wavelets are especially useful for extracting distinguishing features that enable accurate classification of various biological signals.

However, it is obvious that the effectiveness of the features heavily depend on the mother-wavelet used for the feature extraction. In current signal processing world, the most commonly used wavelets include Daubechies, Symlet, Haar, Mexican Hat, and Morlet. Therefore, determining which wavelet provides the best classification accuracy—or, equivalently, which wavelet extracts the most optimal set of features—remains an open and significant research question.

The study "A Wavelet Optimization Approach for ECG Signal Classification" by Daamouche et al. (1) propose a novel method to derive an optimal wavelet that enhances classification accuracy for a specific task(ECG beat classification). This approach aims to fill the gap between wavelet selection and classification performance, offering a systematic way to improve results in ECG signal classification.

2 Overview

The research paper *"A Wavelet Optimization Approach for ECG Signal Classification"* (1) presents a novel method to enhance the classification accuracy of ECG signals by optimizing wavelet design specifically for this purpose. While traditional wavelets, such as Daubechies and Symlet, are widely used in ECG signal processing, the researchers believe that they do not provide the highest accuracy in classification tasks. Through this paper they addresses this limitation by proposing a new approach that integrates wavelet design with the classification process to achieve a better results.

The proposed method utilizes the polyphase representation of wavelet filter banks, enabling wavelet to design through a set of angular parameters. These parameters are optimized using a Particle Swarm Optimization (PSO) algorithm to maximize the accuracy of ECG beat classification. The classification process uses a Support Vector Machine(SVM). The study incorporates both morphological and temporal features extracted from ECG signals, for the classification.

The study validate their method using the MIT-BIH Arrhythmia Database, focusing on a subset of ECG recordings and classifying beats into six categories (Table 2).

Metrics such as Overall Accuracy (OA), Average Accuracy (AA), and individual class accuracies are used to evaluate performance. The experimental results show that the PSO-based wavelet outperforms standard wavelets in accuracy. For the best instance(wavelet length is 10 and decomposition level is 3), the PSO-based wavelet achieved an overall accuracy of **88.84%**, compared to **86.66%** for Daubechies and **86.83%** for Symlet.

3 Dataset

In this study, the authors utilize the widely recognized MIT-BIH Arrhythmia Dataset(2). This dataset comprises 48 ECG recordings, numbered from 100 to 234, and includes data from 47 subjects (25 men, 32 to 89 years old, and 22 women aged 23 to 89 years old). However, the study focused on a subset of 20 recordings, each representing a unique patient. The selected recordings are:

100, 102, 104, 105, 106, 107, 118, 119, 200, 201, 202, 203, 205, 208, 209, 212, 213, 214, 215, and 217.

Each record contains 30 minutes of Holter-monitor data captured from two leads:

- MLII (Modified Lead II)
- V1, or occasionally V2, V5, and in one case, V4.

For implementation purposes, the MLII signal was used, as the paper did not specify which lead was analyzed. All recordings are sampled at a rate of 360 Hz.

3.1 ECG Beat Classes

The complete dataset identifies 15 different types of ECG beats, as shown in Table 1.

However, for the purpose of their analysis, the authors grouped the beats into 6 key classes, as detailed in Table 2.

Table 1: All ECG Beat Classes in the MIT-BIH Dataset

Class Symbol	**Description**
N (·)	Normal beat
L	Left bundle branch block beat
R	Right bundle branch block beat
A	Atrial premature beat
a	Aberrated atrial premature beat
J	Nodal (junctional) premature beat
S	Supraventricular premature beat
V	Premature ventricular contraction
F	Fusion of ventricular and normal beat
e	Atrial escape beat
j	Nodal (junctional) escape beat
E	Ventricular escape beat
/	Paced beat
f	Fusion of paced and normal beat
Q	Unclassifiable beat

Table 2: ECG Beat Classes Used in the Study

Class Symbol	**Description**
N (·)	Normal beat
L	Left bundle branch block beat
R	Right bundle branch block beat
A	Atrial premature beat
V	Premature ventricular contraction
/	Paced beat

4 Method Description

The method aims to maximize ECG classification accuracy by optimizing wavelet shape (wavelet filter coefficients) using a PSO algorithm, the optimized wavelet filter is supposed to extract the most important features of the ECG signal. The fitness function (objective function for the optimization process) is based on the cross-validation accuracy (CVA) of an SVM classifier.

Particles in the PSO represent a particular wavelet (angular parameters of the wavelet filter's polyphase representation). These parameters are optimized to derive wavelet coefficients.

The section 4.1 explains the polyphase representation of the wavelet.

4.1 Polyphase Representation of the Wavelet Filters

The wavelet filters are represented using a polyphase structure, where the filter coefficients are defined by angular parameters. This representation is essential for systematically generating Low-pass filter coefficients through a recursive algorithm. And, High-pass filter coefficients by applying alternating flip construction to the low-pass coefficients.

4.1.1 Generation of Low-pass Coefficients

Sherlock and Monro have developed an polyphase method based on Vaidyanathan's factorization. Their algorithm enables the derivation of any orthonormal perfect-reconstruction FIR filter of arbitrary length. The algorithm ensures that filters align with the properties of wavelets.

Any low-pass FIR filter $H_0(z)$ of length $2N$ can be expressed as:

$$H_0(z) = \sum_{i=0}^{2N-1} h_i z^{-i}$$

And therefore we can decompose $H_0(z)$ into odd and even powers of z.

$$H_0(z) = \sum_{i=0}^{N-1} h_{2i} z^{-2i} + z^{-1} \sum_{i=0}^{N-1} h_{2i+1} z^{-2i}$$

According to the factorization algorithm proposed by the Vidyanthan we can write a polyphase matrix as follows.

$$\mathbf{H_p}(z) = \begin{bmatrix} H_{00}(z) & H_{01}(z) \\ H_{10}(z) & H_{11}(z) \end{bmatrix} = \begin{bmatrix} c_o & s_o \\ -s_0 & c_0 \end{bmatrix} \prod_{i=0}^{N-1} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} c_i & s_i \\ -s_i & c_i \end{bmatrix}$$

where,

$$H_{00}(z) = \sum_{i=0}^{N-1} h_{2i} z^{-2i}$$

and

$$H_{01}(z) = z^{-1} \sum_{i=0}^{N-1} h_{2i+1} z^{-2i}$$

whereas, $H_{10}(z)$ and $H_{11}(z)$ are those of the high-pass filter. The coefficients c_i and s_i are computed as $c_i = cos(\theta_i)$ and $s_i = sin(\theta_i)$.

Sherlock and Monro(3) developed a new formulation by rewriting the above factorization in a recursive form.

$$\mathbf{H_p^{(k+1)}}(z) = \mathbf{H_p^{(k)}}(z) \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} c_k & s_k \\ -s_k & c_k \end{bmatrix} ; with k = 1, 2, 3, ..., N$$

The superscript (k) refers to filters of length 2k and initial condition is given by,

$$\mathbf{H_p^{(0)}}(z) = \begin{bmatrix} c_0 & s_o \\ -s_0 & c_0 \end{bmatrix}$$

Using that knowledge even-numbered filter coefficients can be found using following set of equtions.

$$h_0^{(k+1)} = c_k h_0^{(k)}$$

$$h_{2i}^{(k+1)} = c_k h_{2i}^{(k)} - s_k h_{2i-1}^{(k)}$$

$$h_{2k}^{(k+1)} = -s_k h_{2k-1}^{(k)}$$

for $i = 1, 2, 3, ..., k - 1$

Also the odd-numbered filter coefficients can be found using following set of equtions.

$$h_1^{(k+1)} = s_k h_1^{(k)}$$

$$h_{2i+1}^{(k+1)} = s_k h_{2i}^{(k)} + c_k h_{2i-1}^{(k)}$$

$$h^{(k+1)}_{2k+1} = c_k h^{(k)}_{2k-1}$$

for $i = 1, 2, 3, ..., k-1$

Above equations express the low-pass coefficients $\{h_0, h_1, ..., h_{2N-1}\}$ in terms of N free chosen angular parameters $\{\theta_0, \theta_1, ..., \theta_{N-1}\}$ whose values are in the interval $[0, 2\pi)$.

4.1.2 Generation of High-pass Coefficients

The high-pass filter coefficients are found by alternating flip construction, that is

$$g_i = (-1)^{i+1} h_{2N-1-i}$$

In conclusion from N angular parameters $\{\theta_0, \theta_1, ..., \theta_{N-1}\}$ one can generate $2N$ low-pass filter coefficients and the corresponding high-pass filter coefficients ($2N$ length wavelet and a scaling function).

In PSO one set of N angular parameters (i.e. $\{\theta_0, \theta_1, ..., \theta_{N-1}\}$) is considered as a particle.

4.2 Particle Swarm Optimization(PSO) Process

After defining any wavelet and a scaling function using N angular parameters, the design of an optimal wavelet and a scaling function can be viewed as an optimization problem in the $\mathbb{R}^N$ space of the angular parameters θ_is.

This study uses the PSO method to find the optimal θ_is. The process involves initialization of the S number of particles (candidates for the optimal angular parameters). Let's say initialised particles are, $\mathbf{P_i}(o)$ for $i = 1, 2, ..., S$; where $\mathbf{P_i}(0) \in \mathbb{R}^N$. Then $\mathbf{Pi}(t)$ for each i are updated iteratively for each t according to the following algorithm.

$$\mathbf{V}_i(t+1) = w\mathbf{V}_i(t) + c_1 \cdot r_1(t)(\mathbf{P}_{bi}(t) - \mathbf{P}_i(t)) + c_2 \cdot r_2(t)(\mathbf{P}_g(t) - \mathbf{P}_i(t))$$

$$\mathbf{P}_i(t++1) = \mathbf{P}_i(t) + \mathbf{V}_i(t)$$

Where $\mathbf{V}_i(t)$ corresponds to the velocity of each particle $\mathbf{P}_i(t))$ at time iteration $t$and it is initialised to 0 at the beginning. $\mathbf{P}_{bi}(t)$ refers to the best local position of each particle Pi at each iteration, that is the angular parameters which gives the highest accuracy over all iterations. $\mathbf{P}_g(t)$ refers to the best global position at each iteration that is the angular parameters which gives the highest accuracy over all iterations and over all particles.

where $r1(t)$ and $r2(t)$ are random variables that are drawn from a uniform distribution in the range $[0, 1]$ to provide a stochastic weighting for components involved in the above algorithm. The constants c_1 and c_2 regulate the relative velocities with respect to the best local and global positions, respectively. The inertia weight w is used as a trade-off between global and local exploration capabilities of the swarm.

After a given number of iterations $\mathbf{P}_g(t)$ returns as the optimized angular parameters.

An illustration of the PSO search space for second-order filters (two angular parameters to be estimated) and its relationship with the wavelet filter design is given in the figure below.

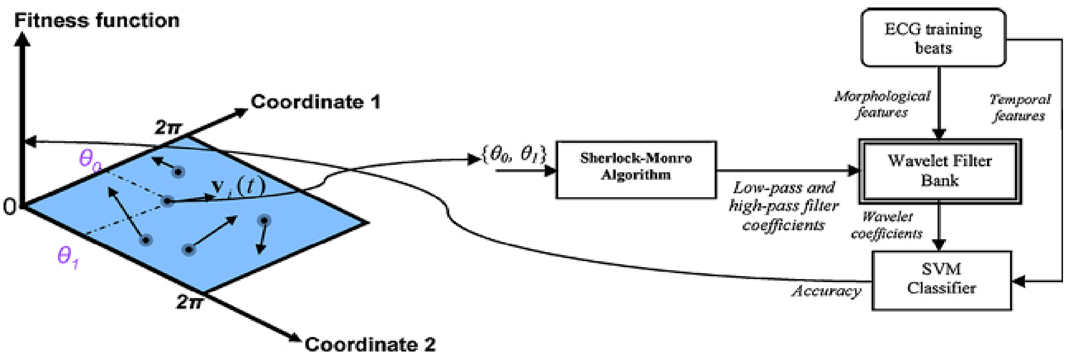


Figure 1: An illustration of the PSO search space for second-order filters (two angular parameters to be estimated) and its relationship with the wavelet filter design.

4.3 Fitness Function Value Obtained by the SVM classifier

The above PSO algorithm uses the objective function or a fitness function. This objective function is obtained using the classification accuracy of a SVM classifier.
The features to the SVM classifier are as follows,

- Features extracted by the low pass and high pass filters - (detailed coefficients of the selected level(scale)) for each ECG beat.

ECG Beat

$$onset = Rpeaklocation - \frac{1}{2} \times No.ofsamplesinpreRRinterval$$

$$offset = Rpeaklocation - \frac{1}{2} \times No.ofsamplesinpostRRinterval$$

Then the ECG beat is up-sampled or down-sampled accordingly to get the number of samples per ECG beat to 300(4).

$$x_i(j) = y_i(j^*) + (y_i(j^* + 1) - y_i(j^*))(r_j - j^*)$$

where $r_j = (j-1)(n^*-1)/(n-1)+1$ and j^* is the integral part of r_j; and x_i is the signal with 300 samples, y_i is the original signal, $n = 300$ and $n^* = length(y_i)$.

- The pre RR interval (the time span between two consecutive R points representing the distance between the QRS peaks of the present and previous beats)

- RR interval averaged over the ten last beats.

SVM classifier uses the Gaussian kerenel (RBM) with a certain γ value which is a hyper parameter also the training process involves the regularization parameter c as well which is also another hyper parameter. A grid serach process is used to find the γ and c value. Also for the formation of fitness function instead of just using the accuracy the study has used the Cross Validation accuracy for a better generalization.

5 Implementation Details

The section 5.1 explains the data pre-processing and preparing.

5.1 Preparing the Dataset

Beats are selected from 20 patient recordings as described in the section 3.

The selected data consists of 47,643 ECG beats corresponding to the classes N, A, V, /, R, and L. We divide the data in the selected records into two sets: the training set and the testing set. The training set contains a total of 125 beats, including 37 normal beats, 25 left bundle branch block beats, 24 right bundle branch block beats, 13 atrial premature beats, 13 premature ventricular contractions, and 13 paced beats. The remaining 42,767 beats are used for the testing set.

There are two main reasons to select only 125 ECG beats from the large dataset.

- The particle swarm optimization is not a gradient based optimization method it takes a lot of time per iteration with all the particles.
- A low amount of data allows testing the method in situations where a low amount of data is available.

Class	N	A	V	RB	/	LB	Total
Training beats	37	24	25	13	13	13	125
Test beats	26,570	578	3,969	3,591	6,240	1,819	42,767

Table 3: Distribution of training and testing beats for each class.

As the initial step in preprocessing the raw ECG signals, a bandpass filter is implemented and applied to remove frequencies outside the range of interest (0.5 Hz to 40 Hz). This process ensures the elimination of noise and irrelevant frequency components.

Features of the dataset were created under 2 main categories such as morphological features and temporal features as described in section 4.3 as morphologival features we used level 3 detailed coefficients.

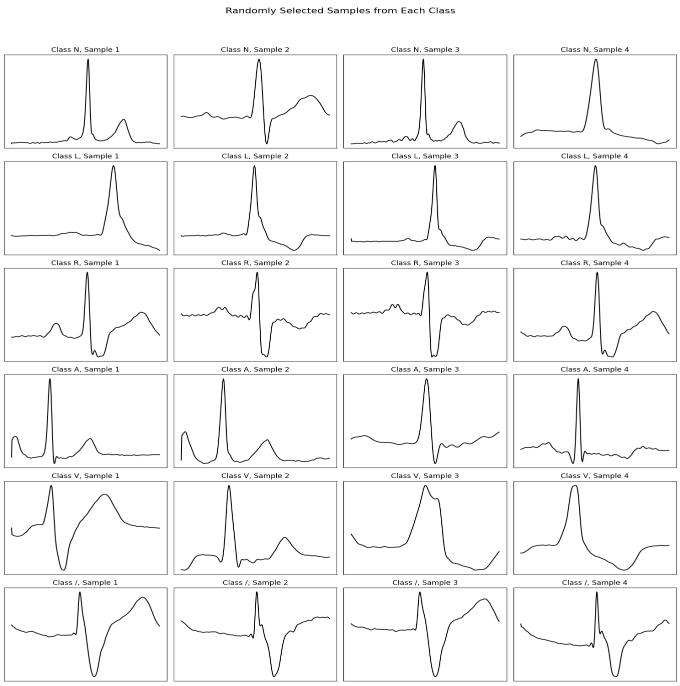


Figure 2: Random samples from the dataset showing representative ECG beats for different classes.

5.2 Generating the Wavelet bank and Optimization Algorithm

To obtain the wavelets for the filter bank through polyphase representation, the proposed Sherlock Monro iterative algorithm is used as in section 4.1. The wavelet parameters include the filter length $N = 20$ and the number of particles in the wavelet bank is initially set to $S = 20$.

The number of decomposition levels in the wavelet decomposition $L = 3$.

For optimizing the wavelets, the particle swarm method applies a search space dimension of 10 since length of the filter $N = 20$. The Particles initialize by sampling values between 0 and 2π. An SVM classifier is trained using the wavelet features(level 3 detailed coefficients) generated using the filter bank created using the Sherlock Monro algorithm.

The algorithm stores the best local position and the best global position at each iteration. The velocity vector and particle coordinates for each particle update iteratively using the specified equations as described in Section 3. The number of iteration is set to 50 and the stopping criterion is met if the number of iterations equals the user-defined maximum. Also, the algorithm sets the initial parameters as $c_1 = 1$, $c_2 = 1$, and an inertia weight $w = 0.75$.

After obtaining the particles related to the best global position with the highest fitness function value, we move on to our final target of building the ECG classification network to identify normal ECG beats (denoted in the following as 'N'), atrial premature beat ('A', irregular beat which starts in the atria, i.e., the upper two chambers of the heart), ventricular premature beat ('V', beat initiated by the heart ventricles rather than by the sinoatrial node), right bundle branch block ('RB', causes prolongation of the last part of the QRS complex and may shift the heart electrical axis to the right), left bundle branch block ('LB', widens the entire QRS and typically shifts the heart electrical axis to the left), and paced beat ('/') using an SVM classifier.

We initiate the SVM classifier with a radial basis function(RBF) kernel and a 5-fold cross-validation. The designed wavelet method applies on the level 3 detailed oefficients, while the 2 temporal features are added after the wavelet decomposition is performed as concatenation.

6 Implementation results

6.1 Testing Accuracy of the Particle Swarm Optimization

The testing accuracy of the PSO process progressively improves over 50 iterations. Starting from the initial random positions of the particles, the optimization process refines the wavelet coefficients by iteratively updating the velocity and position of each particle based on the fitness function values. The accuracy steadily increases as the particles converge toward the optimal solution as given in figure 3. However, over some periods, the overall accuracy does not change and stays steady. The reason for this is that, since the PSO algorithm is not a gradient-based approach, it does not always converge to the maximum fitness value. Therefore, the iteration continues until a new global maximum position is achieved.

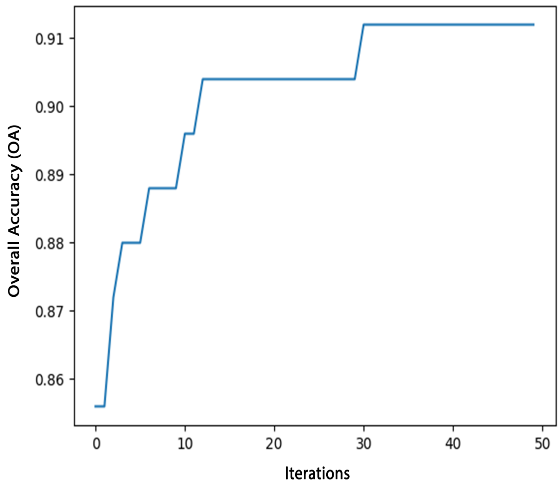
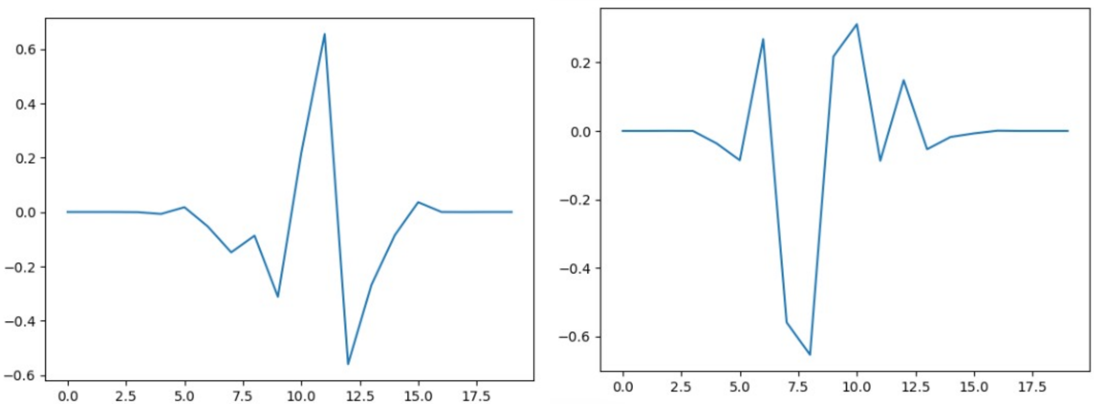


Figure 3: Variation of testing accuracy for 50 iterations in PSO algorithm.

6.2 Optimized Filters

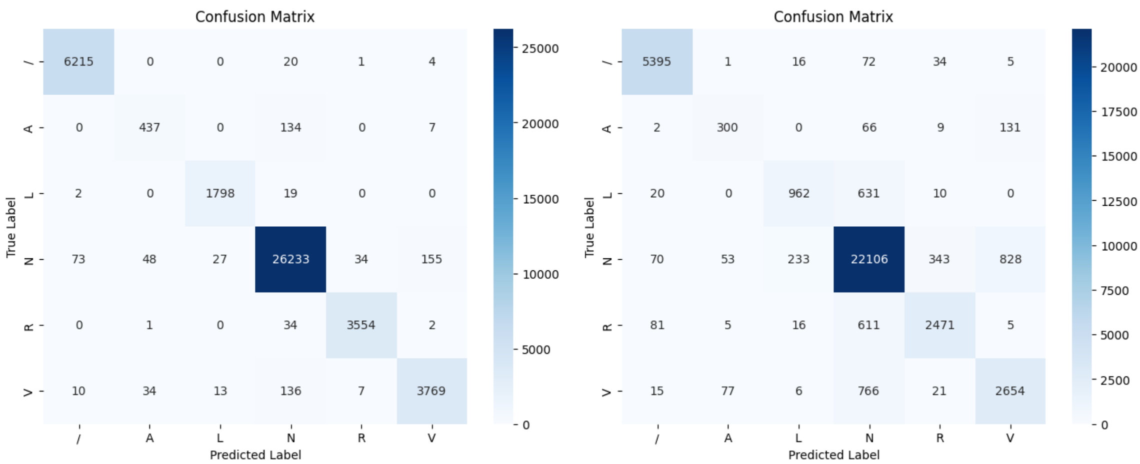
figure 4 demonstrate the graphical illustration of the mother wavelet and the respective scaling function for the particle with the best global position after 50 iterations. We can clearly see that the waveform of the wavelet significantly represents the graphical features of a typical ECG signal. Even so, we must mention that the graphical features of the optimal wavelet are highly dependent on the random initial position of the PSO algorithm, because they converge into different positions during different training processes due to the random initialization.



(a) Scaling function (b) Mother wavelet

Figure 4: Scaling function and mother wavelet for the particle with the best global position.

6.3 Comparison with DB10 Wavelet



(a) confusion matrix for custom created wavelet (b) confusion matrix for db10 wavelet

Figure 5: Confusion matrix for testing the classification using custom created wavelet and db10 wavelet

Class	N	LB	RB	A	V	/
Custom Wavelet	98.70	99.01	98.94	77.86	95.29	99.55
DB10 Wavelet	93.54	59.27	77.49	59.06	74.99	97.68

Table 4: Recall percentage for each arrhythmia detection using an SVM classifier trained separately with features extracted from custom and DB10 wavelets. Recall is reported for different arrhythmia types: Normal (N), Left Bundle Branch Block (LB), Right Bundle Branch Block (RB), Atrial Premature Beat (A), Ventricular Premature Beat (V), and paced beats (/). The custom wavelet achieved higher recall values across all categories.

The comparison between the SVM classification accuracy using the custom-designed wavelet and the DB10 wavelet appears in figure 5. The custom-designed wavelet demonstrates superior performance by achieving a higher overall accuracy of 0.982 compared to the generic DB10 wavelet, which achieves an overall accuracy of 0.89. This improvement shows that the tailored wavelet better captures the distinguishing features of ECG signals, such as variations in beat morphology and rhythm, resulting in more accurate identification of different cardiac conditions.

The confusion matrices 5 and the recall table 4 clearly show that both wavelet methods fail to identify atrial premature beats as accurately as the other five labels. Therefore, further improvements are necessary for the classifier to enhance accuracy in atrial premature beat classification. Apart from this, identifying other labels, such as normal sinus rhythm ('N'), ventricular premature beat ('V'), right bundle branch block ('RB'), left bundle branch block ('LB'), and paced beat ('/'), shows significant improvement with the custom wavelet approach compared to the DB10 wavelet.

7 Conclusion

This project proposes a custom wavelet optimization in enhancing the accuracy of ECG signal classification. By employing the Sherlock Monro algorithm, a custom-designed mother wavelet is generated to extract features that better represent the morphological and temporal characteristics of ECG signals. The optimized wavelet is obtained using particle Swarm Optimization with SVM accuracy as the fitness function. Integrating the generated wavelet achieves superior classification performance compared to traditional wavelets like DB10, as evidenced by the higher overall testing accuracy of 98.2

When we use a custom wavelet which is optimized for a given classification task which may give good results than using the conventional well defined wavelets.

7.1 Possible Improvements

- Investigate better alternative methods for classification like neural networks, which could provide a better accuracy.
- Interpret the generated wavelets in relation to the characterization of ECG waveforms, aiming to better understand their relevance to specific cardiac features.

- Extend the classification framework to identify a larger set of cardiac arrhythmia, improving its diagnostic applicability in clinical settings.
- Explore the use of more advanced optimization algorithms beyond PSO, to potentially achieve better performance and convergence.

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