Department of Electronic & Telecommunication Engineering

University of Moratuwa

BM2102 - Modelling and Analysis of Physiological Systems

Assignment 2: Branched Cylinders

Dendritic Tree Approximations

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Question 1

Q(1)

General solution for the membrane potential,

$$V_1(x) = A_1 e^{-x} + B_1 e^{x} \quad 0 \le x \le L_1$$

$$V_{21}(x) = A_{21} e^{-x} + B_{21} e^{x} \quad L_1 \le x \le L_{21}$$

$$V_{22}(x) = A_{22} e^{-x} + B_{22} e^{x} \quad L_1 \le x \le L_{22}$$

(1) Let's consider boundary condition in equation (3)

$$V_1(x) = A_1 e^{-x} + B_1 e^{x} \quad 0 \le x \le L_1$$

$$\frac{d}{dx}(V_1(x)) = -A_1 e^{-x} + B_1 e^{x}$$

from equation (3)

$$\frac{d(V_1(x))}{dx} = -(r_1 \lambda)_1 I_{app}$$

When $x = 0$,

$$-A_1 + B_1 = -(r_i \lambda_c)_1 I_{app}$$

$$A_1 - B_1 = (r_i \lambda_c)_1 I_{app}$$

(2) $V_{21}(x) = A_{21} e^{-x} + B_{21} e^{x} \quad L_1 \le x \le L_{21}$

When $x = L_{21}$, daughter terminal is held at rest

$$V_{21}(L_{21}) = A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0$$

$$A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0$$

(3) $V_{22}(x) = A_{22} e^{-x} + B_{22} e^{x}$, $L_1 \le x \le L_{22}$

when $x = L_{22}$ daughter terminal is held at rest

$$V_{22}(L_{22}) = A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0$$

$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0$$

(4) Membrane potentials at the nodes must be continuous

$$V_1(L_1) = V_{21}(L_1)$$

$$A_1 e^{-L_1} + B_1 e^{L_1} = A_{21} e^{-L_1} + B_{21} e^{L_1}$$

$$A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{L_1} = 0$$

(5) Also, $V_{21}(L_1) = V_{22}(L_1)$

$$A_{21} e^{-L_1} + B_{21} e^{L_1} = A_{22} e^{-L_1} + B_{22} e^{L_1}$$

$$A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1} = 0$$

(6) By conservation of current

$$\frac{-1}{(r_i\lambda_c)_1}\left(-A_1 e^{-L_1} + B_1 e^{L_1}\right) = \frac{-1}{(r_i\lambda_c)_{21}}\left(-A_{21} e^{-L_1} + B_{21} e^{L_1}\right) + \frac{-1}{(r_i\lambda_c)_{22}}\left(-A_{22} e^{-L_1} + B_{22} e^{L_1}\right)$$

By solving the above equation we get.

$$\frac{-A_1 e^{-L_1}}{(r_i\lambda_c)_1} + \frac{B_1 e^{L_1}}{(r_i\lambda_c)_1} + \frac{A_{21} e^{-L_1}}{(r_i\lambda_c)_{21}} - \frac{B_{21} e^{L_1}}{(r_i\lambda_c)_{21}} + \frac{A_{22} e^{-L_1}}{(r_i\lambda_c)_{22}} - \frac{B_{22} e^{L_1}}{(r_i\lambda_c)_{22}} = 0$$

equation (7) is true

Question 02

$$Ax = b$$

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ \frac{-e^{-L_1}}{(r_i\lambda_c)_1} & \frac{e^{L_1}}{(r_i\lambda_c)_1} & \frac{e^{-L_1}}{(r_i\lambda_c)_{21}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{21}} & \frac{e^{-L_1}}{(r_i\lambda_c)_{22}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{22}} \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A_1 - B_1 = (r_i\lambda_c)_1 I_{app} \quad — ①$$

$$A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0 \quad — ②$$

$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0 \quad — ③$$

$$A_{22}e^{-L_2} + B_{22}e^{L_2} = 0$$

$$A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{-L_1} = 0 \quad — (4)$$

$$A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1} = 0 \quad — (5)$$

$$-\frac{A_1 e^{-L_1}}{(r_i \lambda_c)_1} + \frac{B_1 e^{L_1}}{(r_i \lambda_c)_1} + \frac{A_{21} e^{-L_1}}{(r_i \lambda_c)_{21}} - \frac{B_{21} e^{L_1}}{(r_i \lambda_c)_{21}} + \frac{A_{22} e^{-L_1}}{(r_i \lambda_c)_{22}} - \frac{B_{22} e^{L_1}}{(r_i \lambda_c)_{22}} = 0 \quad — (6)$$

from these (6) equations we can derive equation (7) as in the previous equation.

Question 3

```
% Dimensions of compartments

d1 = 75e-4;             % cm
d21 = 30e-4;            % cm
d22 = 15e-4;            % cm
%d21 = 47.2470e-4;       % E9 cm
%d22 = d21;               % E9 cm

l1 = 1.5;           % dimensionless
l21 = 3.0;          % dimensionless
l22 = 3.0;          % dimensionless

% Electrical properties of compartments

Rm = 6e3;           % Ohms cm^2
Rc = 90;            % Ohms cm
Rs = 1e6;           % Ohms
```

Command Window

New to MATLAB? See resources for Getting Started.

```
X =

    0.0007
    0.0000
    0.0011
   -0.0000
    0.0011
   -0.0000
```

Therefore,

A1 = 0.0007

B1 = 0

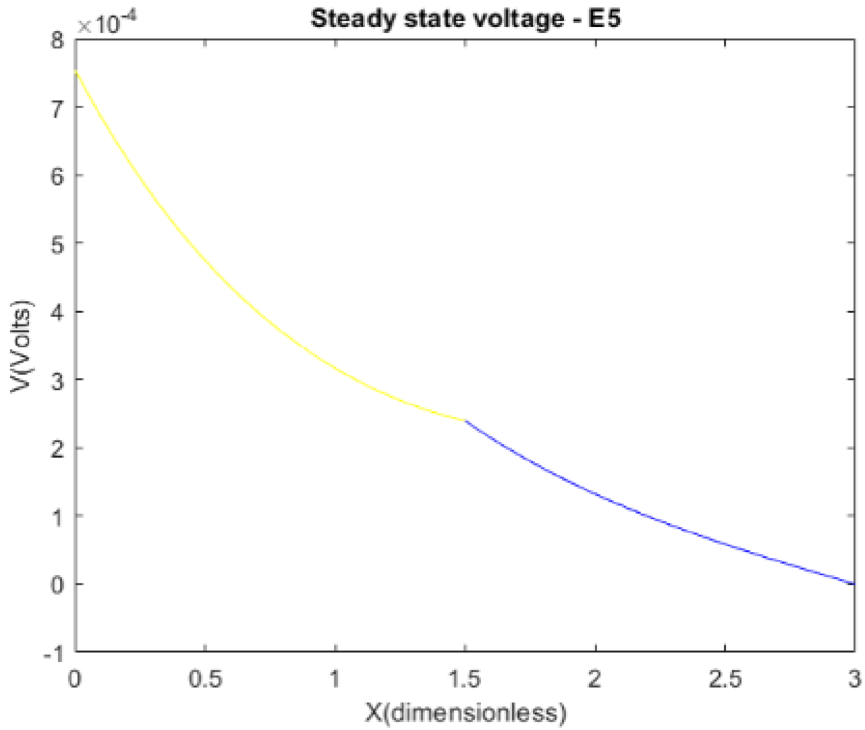
A21 = 0.0011

B21 = 0

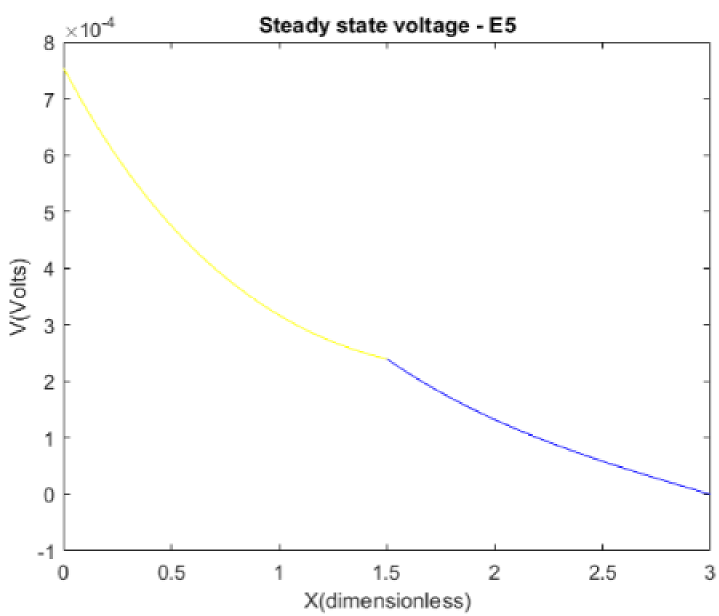
A22 = 0.0011

B22 = 0

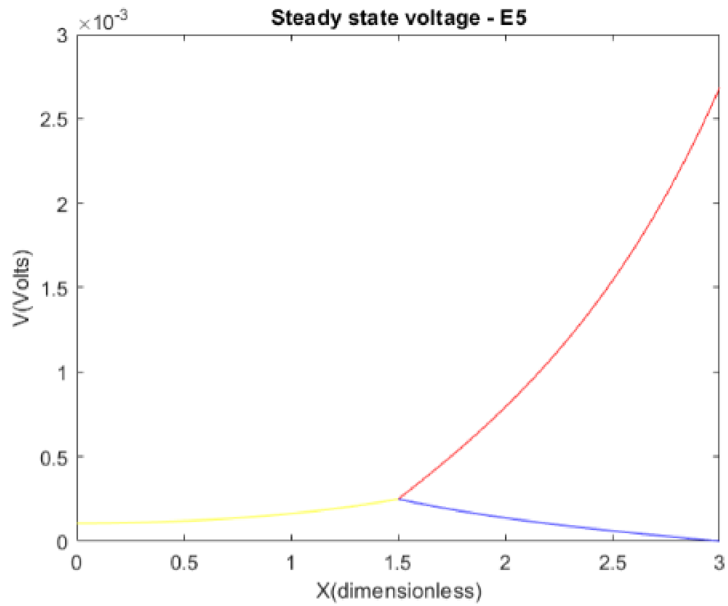
Question 4

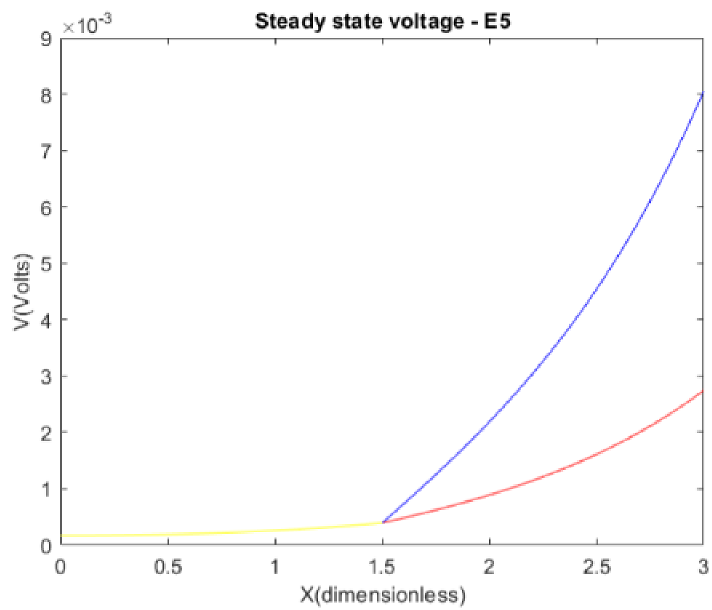


Steady state voltage - E5
×10^-4
V(Volts)
X(dimensionless)
8
7
6
5
4
3
2
1
0
-1
0
0.5
1
1.5
2
2.5
3



Steady state voltage - E5
×10^-4
V(Volts)
X(dimensionless)
8
7
6
5
4
3
2
1
0
-1
0
0.5
1
1.5
2
2.5
3





Question 5

The positive right-hand sides represent impulses flowing from ends of daughter branches to starting of the month

The electrical impulse is received in the initial node of the parent cell, leading to a negative voltage gradient. Conversely, the terminal nodes of the two daughter branches *transmit the electrical impulse to other neurons or branches, resulting in a positive voltage gradient.*

Question 6

Recalculated coefficients,

$A_1 = 0.7189$

$B_1 = -0.0014$

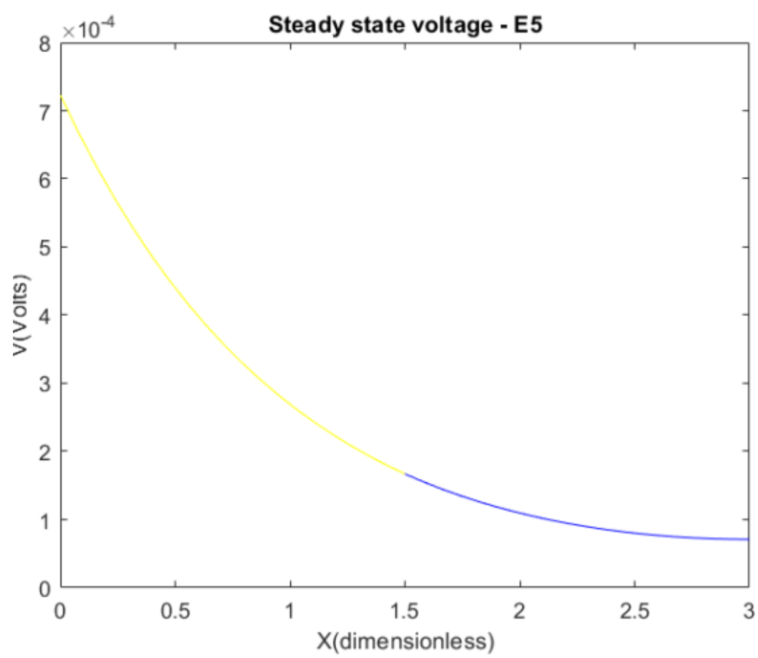
$A_{21} = 0.7275$

$B_{21} = -0.0018$

$A_{22} = 0.7275$

$B_{22} = -0.0018$

Steady state voltage of (b)



Steady state voltage profile of (d)