Directions

1. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

a. Clearly define the decision variables

The definition of the problem given above indicates that the decisions to be made are the number of each type of backpacks to be produced per week so as to maximize their total profit. Therefore, to formulate the mathematical (linear programming) model for this problem, let

x1 = number of backpacks of Collegiate produced per week

x2 = number of backpacks of Mini produced per week

Z = total profit per week from producing these two products

Thus, x1 and x2 are the decision variables for the model.

b. What is the objective function?

The objective is to choose the values of x1 and x2 so as to maximize Z = 32x1 +24x2, subject to the restrictions imposed on their values by the limited production capacities

c. What are the constraints?

1. Material: The total square footage of nylon material used by Collegiate backpacks (3 square feet each) and Mini backpacks (2 square feet each) cannot exceed the 5000 square feet of material available per week. <u>3x1 + 2x2 <= 5000</u>

2. Production Capacity: The number of Collegiate backpacks produced (x1) cannot exceed 1000 units per week. The number of Mini backpacks produced (x2) cannot exceed 1200 units per week. <u>x1 <= 1000 and x2 <= 1200</u>

3. Labor Capacity: The total labor hours required for Collegiate backpacks (45 minutes each) and Mini backpacks (40 minutes each) cannot exceed the available labor capacity, which is 35 laborers working 40 hours per week. <u>0.75x1 + 0.67x2 ≤ 1400</u>

d. Write down the full mathematical formulation for this LP problem.

Maximize Z = 32x1 + 24x2

Subject to the constraints:

Material Constraint: 3x1 + 2x2 <= 5000

Production Capacity Constraints: x1 <= 1000 x2 <= 1200

Labor Capacity Constraint: 0.75x1 + 0.67x2 <= 1400
Non-negativity Constraints: x1 >= 0 x2 >= 0

2. The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.
The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.
Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.
At each plant, some employees will need to be laid off unless most of the plant’s excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their
excess capacity to produce the new product.
Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.
a. Define the decision variables
X1 number of units of the large size produced by plant 1
X2 number of units of the medium size produced by plant 1
X3 number of units of the small size produced by plant 1
Y1 number of units of the large size produced by plant 2
Y2 number of units of the medium size produced by plant 2
Y3 number of units of the small size produced by plant 2
Z1 number of units of the large size produced by plant 3
Z2 number of units of the medium size produced by plant 3
Z3 number of units of the small size produced by plant 3

b. Formulate a linear programming model for this problem.

Maximize Profit:

Capacity constrain-plant1: x1+x2+x3<=750

Capacity constrain-plant2: y1+y2+y3<=900

Capacity constrain-plant3: z1+z2+z3<=450

In process storage constrain-plant1: 20x1+15x2+12x3<=13000

In process storage constrain-plant2: 20y1+15y2+12y3<=12000

In process storage constrain-plant3: 20z1+15z2+12z3<=5000

Sales constrain-large: x1+y1+z1=900

Sales constrain-medium: x2+y2+z2=1200

Sales constrain-small: x3+y3+z3=750

Non-negativity constraint: x1,x2,x3,y1,y2,y3,z1,z2,z3>=0