بسم الله الرحمن الرحیم

FUNDAMENTALS OF PHYSICS

مریم بحرینی

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فهرست

FUNDAMENTALS OF PHYSICS

فیزیک ۱

Physics 1

فصل سوم:

بردارها

دانشگاه علم و صنعت ایران

CHAPTER 3

Vectors

3-1 VECTORS AND THEIR COMPONENTS

Learning Objectives

After reading this module, you should be able to . . .

3.01 Add vectors by drawing them in head-to-tail arrangements, applying the commutative and associative laws.

3.02 Subtract a vector from a second one.

3.03 Calculate the components of a vector on a given coordinate system, showing them in a drawing.

3.04 Given the components of a vector, draw the vector and determine its magnitude and orientation.

3.05 Convert angle measures between degrees and radians.

Key Ideas

- Scalars, such as temperature, have magnitude only. They are specified by a number with a unit (10°C) and obey the rules of arithmetic and ordinary algebra. Vectors, such as displacement, have both magnitude and direction (5 m, north) and obey the rules of vector algebra.

- Two vectors $\vec{a}$ and $\vec{b}$ may be added geometrically by drawing them to a common scale and placing them head to tail. The vector connecting the tail of the first to the head of the second is the vector sum $\vec{s}$. To subtract $\vec{b}$ from $\vec{a}$, reverse the direction of $\vec{b}$ to get $-\vec{b}$; then add $-\vec{b}$ to $\vec{a}$. Vector addition is commutative and obeys the associative law.

- The (scalar) components a_x and a_y of any two-dimensional vector $\vec{a}$ along the coordinate axes are found by dropping perpendicular lines from the ends of $\vec{a}$ onto the coordinate axes. The components are given by

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta,$$

where θ is the angle between the positive direction of the x axis and the direction of $\vec{a}$. The algebraic sign of a component indicates its direction along the associated axis. Given its components, we can find the magnitude and orientation of the vector $\vec{a}$ with

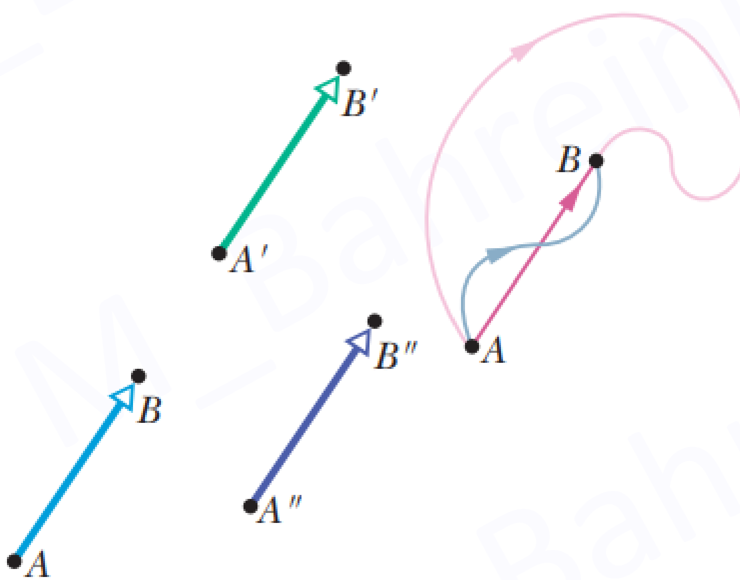
$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x}.$$

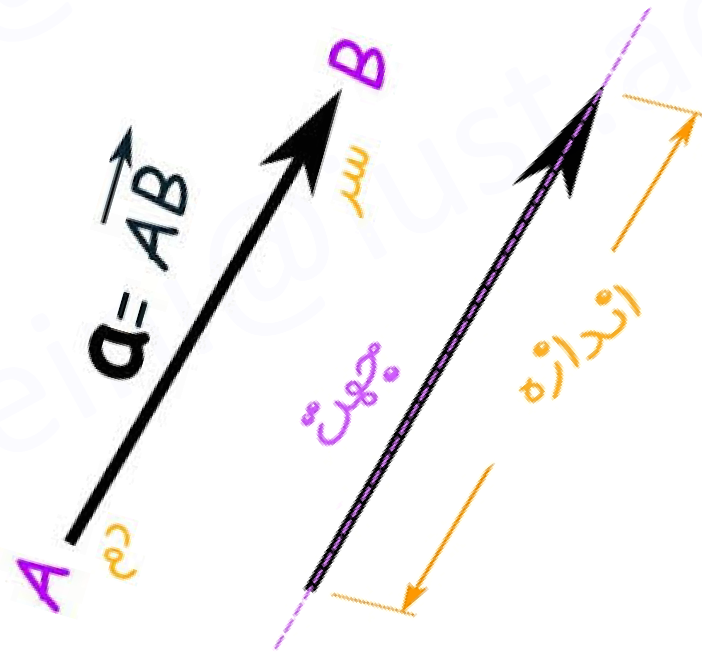
کمیت برداری: Vectors

دارای بزرگی و جهت است

و از قاعده های برداری معینی

پیروی می کند





کمیت نرده ای: Scalars

فقط دارای بزرگی است و جهت ندارد

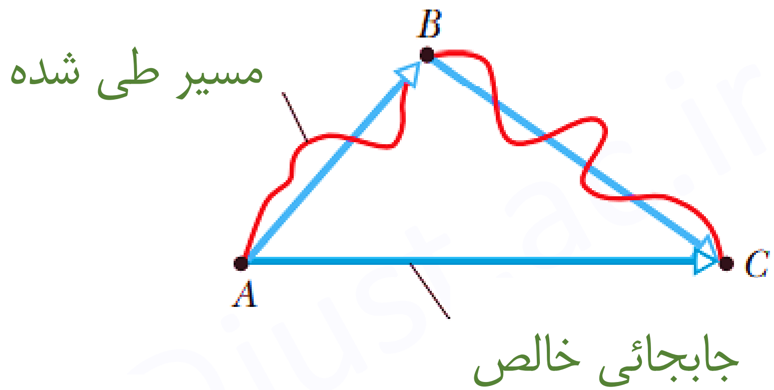
و از قاعده های جبری عادی پیروی

می کند

نمونه‌ای از کمیتهای برداری	نمونه‌ای از کمیتهای نرده‌ای
جابجایی	زمان
نیرو	جرم
سرعت	حجم
شتاب	فشار
میدان مغناطیسی	سطح

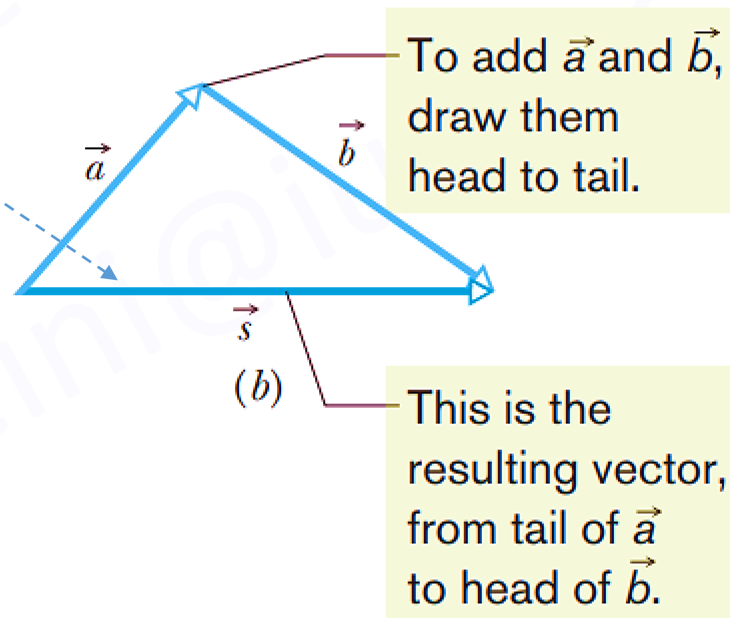
جمع کردن بردارها:

$$\vec{s} = \vec{a} + \vec{b}$$



مجموع برداری یا برآیند، برداری است که با کشیدن پیکانی

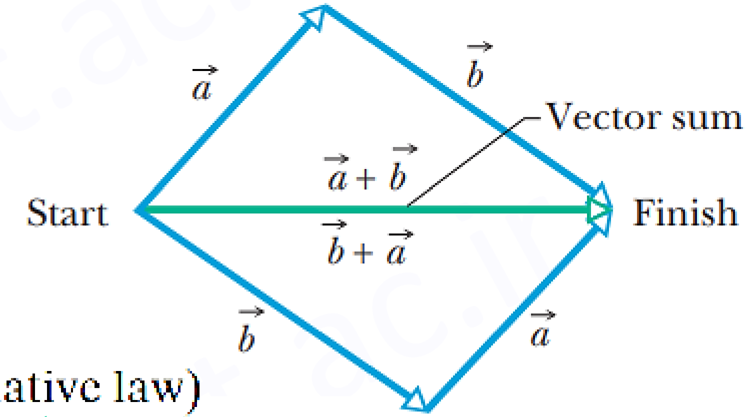
از ابتدای بردار اول به انتهای بردار دوم بدست می آید.



ویژگیهای جمع برداری:

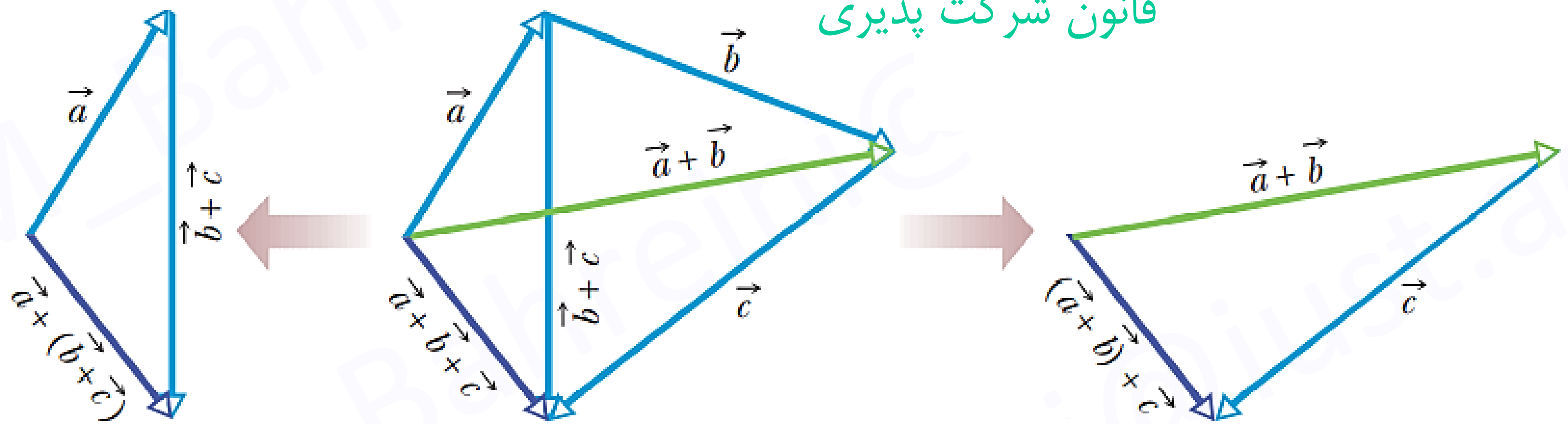
$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

قانون جابجائی
(commutative law)



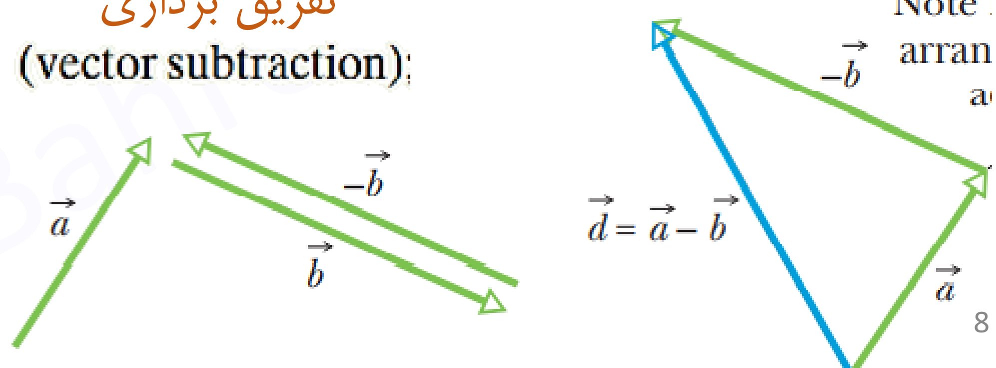
$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

(associative law)
قانون شرکت پذیری

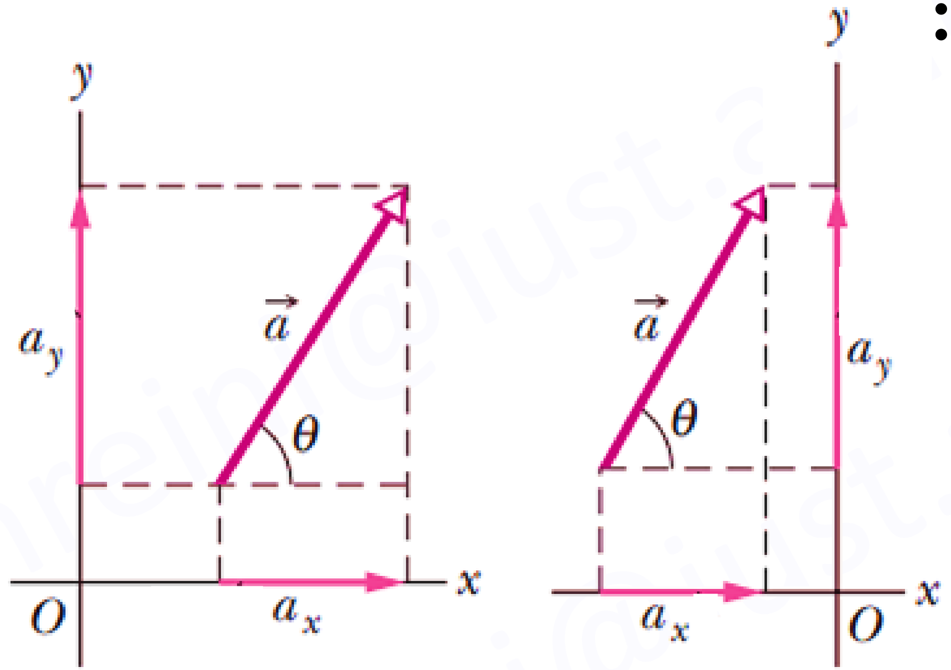


$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

تفریق برداری
(vector subtraction):

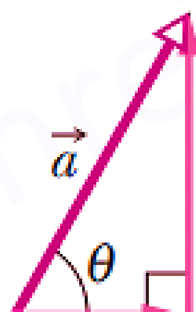


مولفه های بردار:



$$a = \sqrt{a_x^2 + a_y^2}$$

$$\tan\theta = \frac{a_y}{a_x}$$

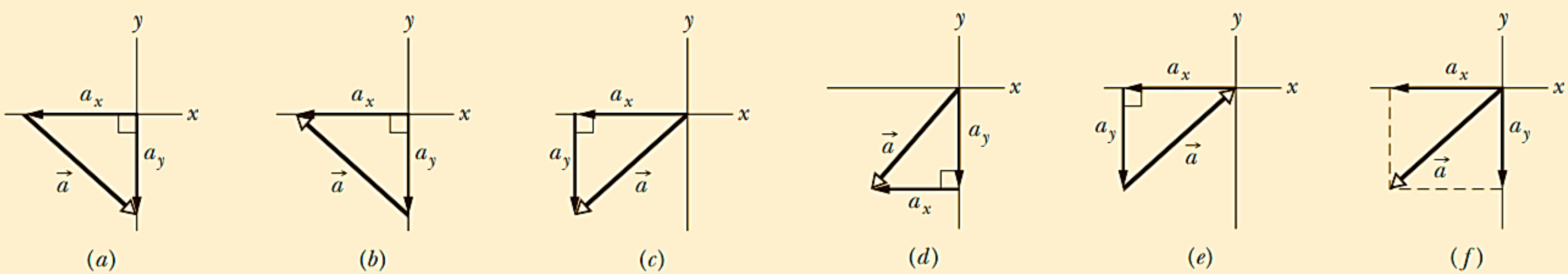


$$a_y = a\sin\theta$$

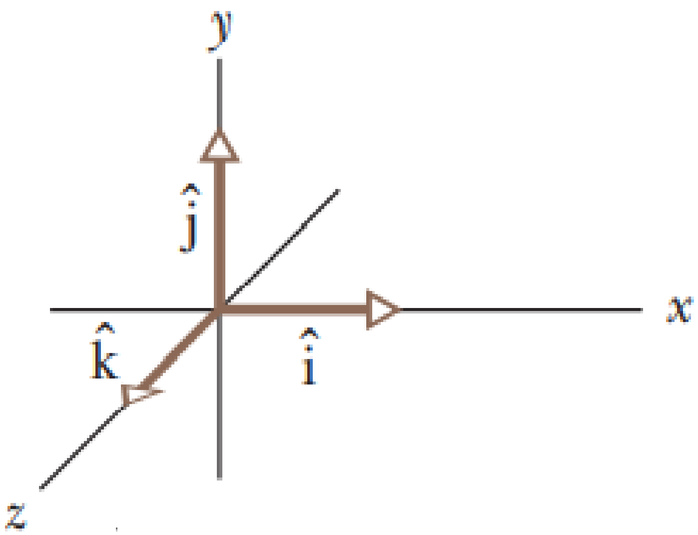
$$a_x = a\cos\theta$$

Checkpoint 2

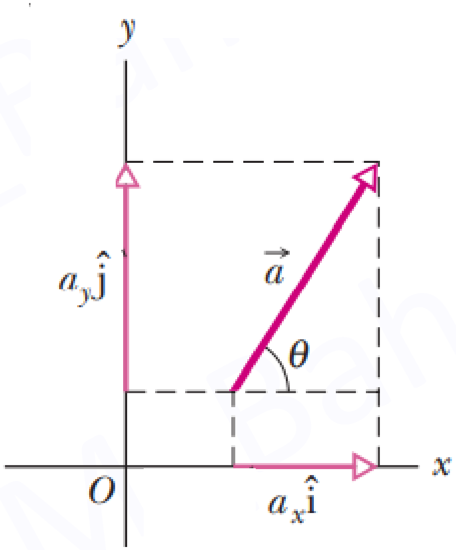
In the figure, which of the indicated methods for combining the x and y components of vector $\vec{a}$ are proper to determine that vector?

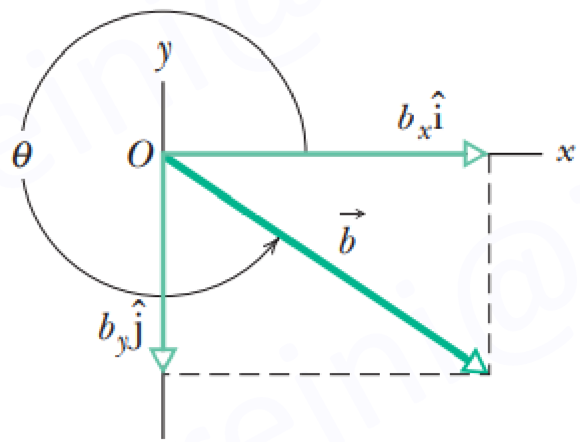


بردارهای یکه:



$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$
$$\vec{b} = b_x\hat{i} + b_y\hat{j}$$





جمع برداری:

$$\vec{r} = \vec{a} + \vec{b}$$

$$r_x = a_x + b_x$$

$$r_y = a_y + b_y$$

$$r_z = a_z + b_z$$

$$\vec{d} = \vec{a} - \vec{b}$$

$$\vec{d} = \vec{a} + (-\vec{b})$$

$$\vec{d} = d_x\hat{\mathrm{i}} + d_y\hat{\mathrm{j}} + d_z\hat{\mathrm{k}}$$

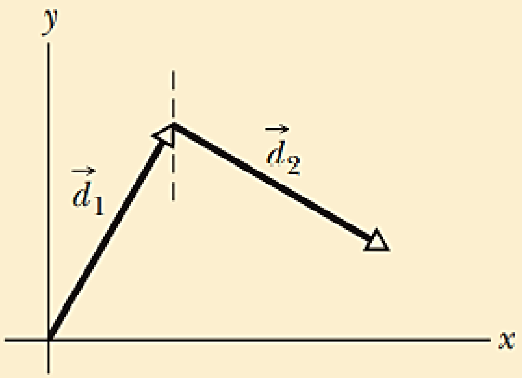
$$d_x = a_x - b_x$$

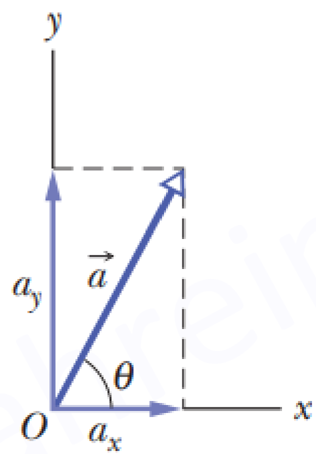
$$d_y = a_y - b_y$$

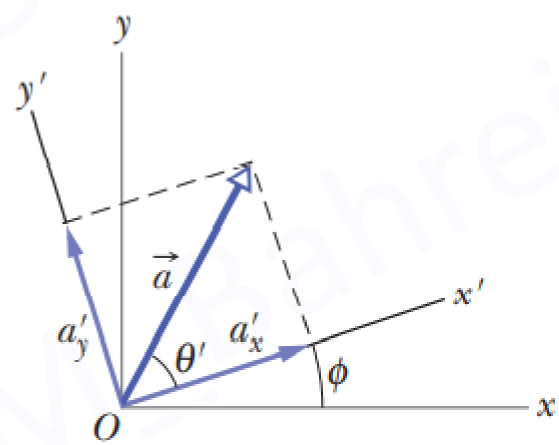
$$d_z = a_z - b_z$$

Checkpoint 3

(a) In the figure here, what are the signs of the x components of $\vec{d}_1$ and $\vec{d}_2$? (b) What are the signs of the y components of $\vec{d}_1$ and $\vec{d}_2$? (c) What are the signs of the x and y components of $\vec{d}_1 + \vec{d}_2$?







$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{a_x'^2 + a_y'^2}$$

$$\theta = \theta' + \phi.$$

ضرب کردن بردارها: ضرب نرده ای

The Scalar Product

dot product

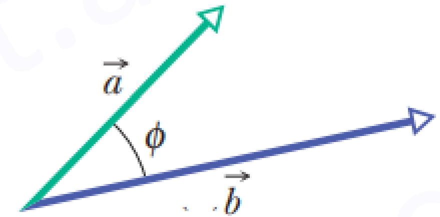
If the angle ϕ between two vectors is 0°, the component of one vector along the other is maximum, and so also is the dot product of the vectors. If, instead, ϕ is 90°, the component of one vector along the other is zero, and so is the dot product.

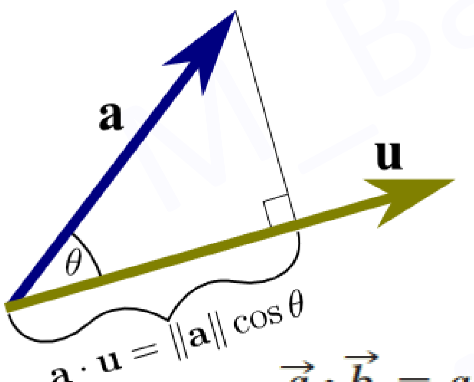
$$\vec{a}\cdot\vec{b} = (a\cos\phi)(b) = (a)(b\cos\phi)$$

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$$

$$\vec{a}\cdot\vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}})\cdot(b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}})$$

$$\vec{a}\cdot\vec{b} = a_xb_x + a_yb_y + a_zb_z$$

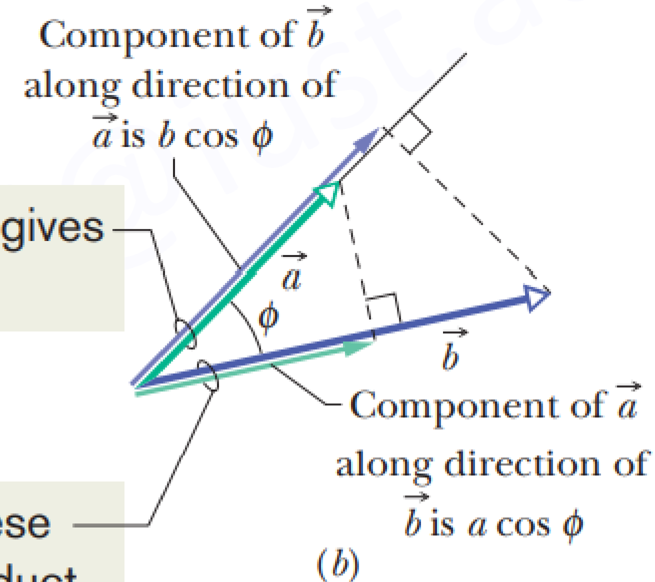




$$\vec{a}\cdot\vec{b} = ab\cos\phi$$

Multiplying these gives the dot product.

Or multiplying these gives the dot product.



Checkpoint 4

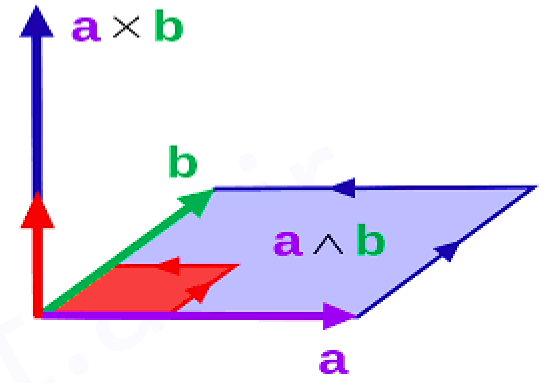
Vectors $\vec{C}$ and $\vec{D}$ have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of $\vec{C}$ and $\vec{D}$ if $\vec{C} \cdot \vec{D}$ equals (a) zero, (b) 12 units, and (c) -12 units?

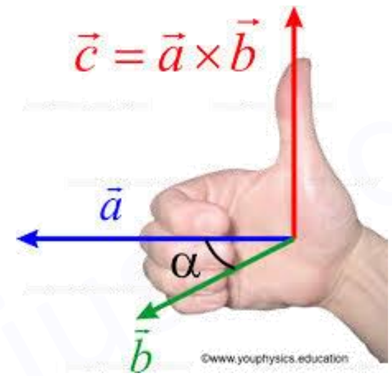
ضرب کردن بردارها: ضرب برداری

Vector Product
cross product

$$\vec{\mathbf{c}} = \vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =$$

$$= (a_2 b_3 - a_3 b_2)\vec{\mathbf{i}} + (a_3 b_1 - a_1 b_3)\vec{\mathbf{j}} + (a_1 b_2 - a_2 b_1)\vec{\mathbf{k}}$$



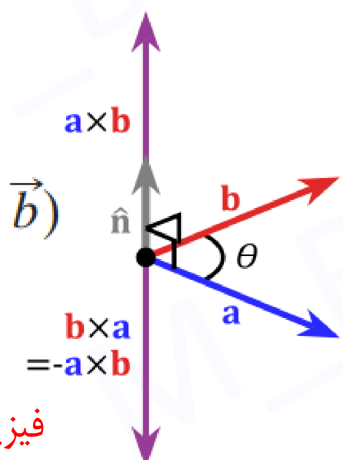


$$\vec{a} \times \vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}) \times (b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}})$$

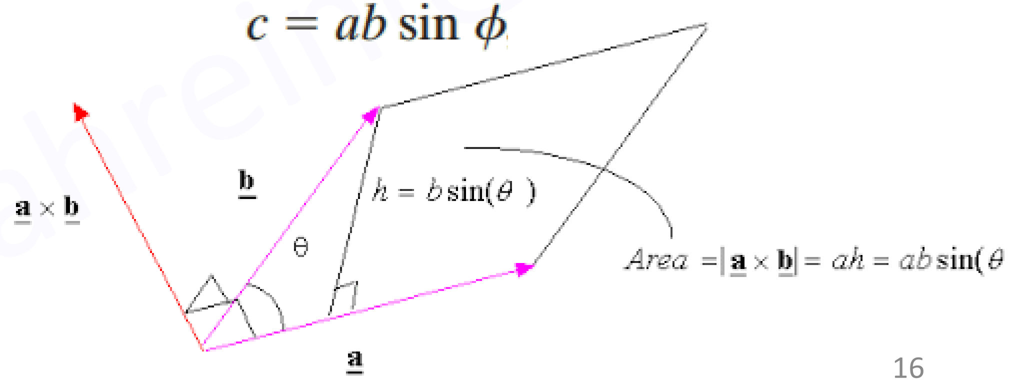
$$a_x\hat{\mathrm{i}} \times b_x\hat{\mathrm{i}} = a_x b_x(\hat{\mathrm{i}} \times \hat{\mathrm{i}}) = 0$$

$$a_x\hat{\mathrm{i}} \times b_y\hat{\mathrm{j}} = a_x b_y(\hat{\mathrm{i}} \times \hat{\mathrm{j}}) = a_x b_y\hat{\mathrm{k}}$$

$$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b})$$



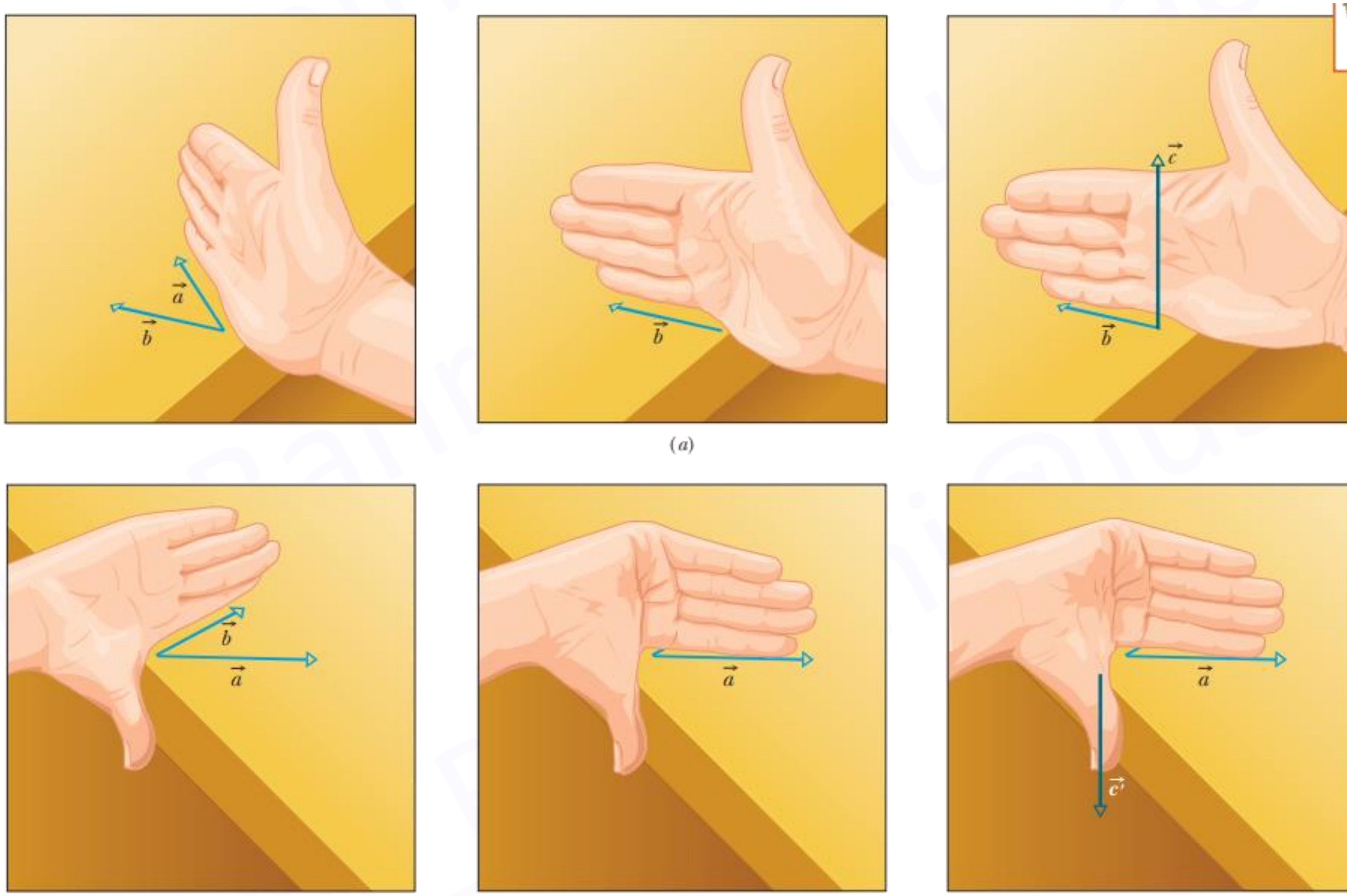
$$c = ab\sin\phi$$



ضرب کردن بردارها: ضرب برداری

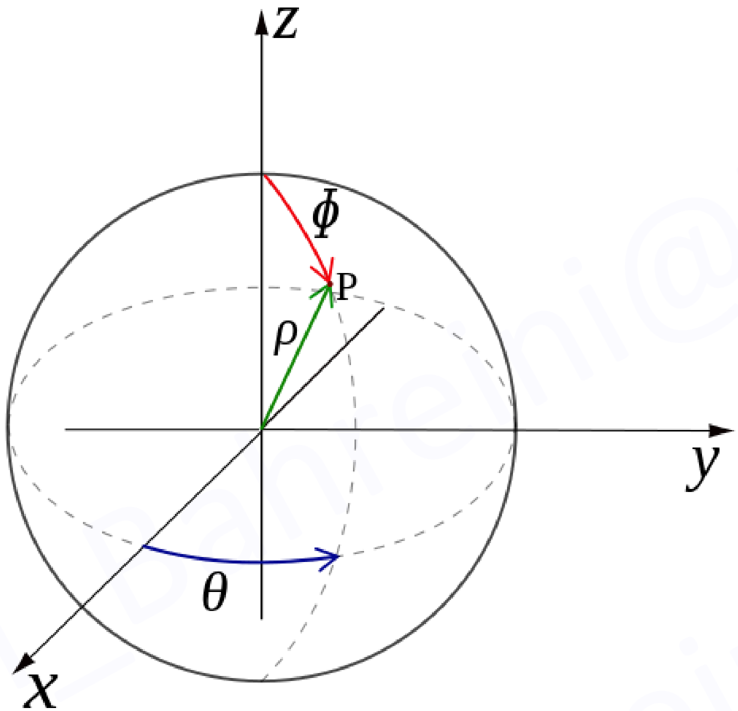
Checkpoint 5

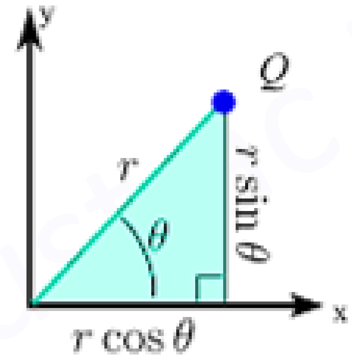
Vectors $\vec{C}$ and $\vec{D}$ have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of $\vec{C}$ and $\vec{D}$ if the magnitude of the vector product $\vec{C} \times \vec{D}$ is (a) zero and (b) 12 units?



(a)

دستگاه مختصات: کروی



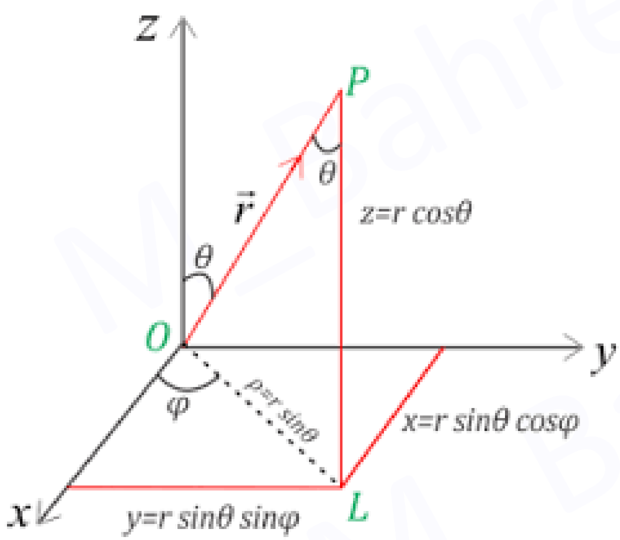


تبدیل مختصات قطبی به دکارتی

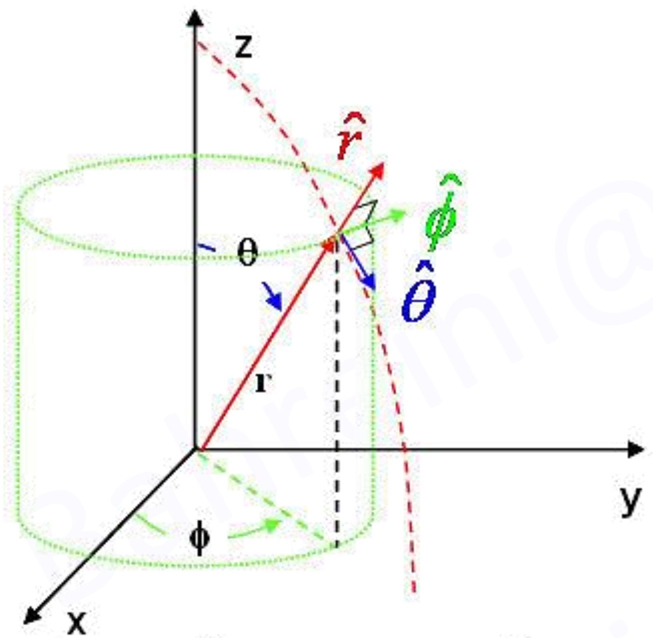
$$r^2 = x^2 + y^2 \ , \ \tan\theta = \frac{y}{x} \ , \ z = z$$

$$x = r\cos\theta \ , \ y = r\sin\theta \ , \ z = z$$

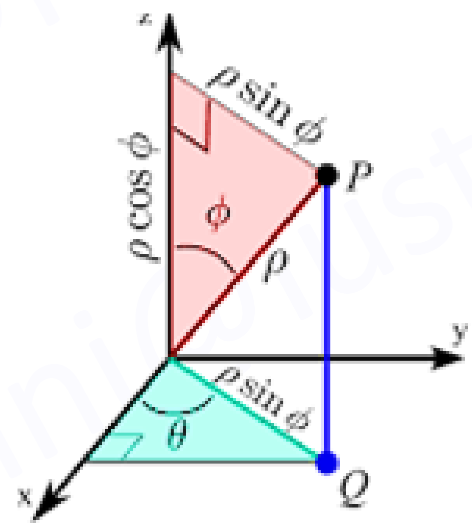
تبدیل مختصات دکارتی به کروی



دستگاه مختصات: استوانه ای



$$\vec{A} = A_r \hat{\boldsymbol{r}} + A_\theta \hat{\boldsymbol{\theta}} + A_\phi \hat{\boldsymbol{\phi}}$$



Problems