

Chapter 1 - Introduction

- An algorithm is a **sequence of unambiguous instructions** for **solving a problem** (obtaining output for any **legitimate input** in a ***finite amount of time***).

- > non-ambiguous
- > range of input has to be specified
- > same algorithm can be represented in many ways
- > there could exist several algorithm for a given problem,
- > different algorithms could solve a problem in dramatically different speeds

- Relationship among **algorithms**, **data structures**, and **programming languages**:

- > Algorithms: steps & planning
- > Data Structures: ingredients to use
- > Programming Languages: actual implementation

Analogy with building a house:

- > Alg: blue print; design efficiency (space, energy, etc.)
- > DS: bricks, doors, frames
- > PL: building the house; you can use different brands of products (different languages)

- Example: Compute the greatest common divisor $\text{gcd}(m, n)$

- Solution 1: Euclid's algorithm $\text{gcd}(m, n) = \text{gcd}(n, m \bmod n)$

Algorithm $\text{Euclid}(m, n)$

```
while n != 0:
    r = m % n
    m = n
    n = r
return m
```

How do we know this algorithm eventually stops?

The second int of the pair $(m \bmod n)$ gets smaller with each iteration and eventually becomes 0. Even if m end up being 1, all numbers mod 1 is 0, and the algorithm stops.

- Solution 2: Consecutive integer checking algorithm: finding the largest integer that divides both

Algorithm $\text{gcd}(m, n)$

```
t = min(m, n)
while t > 0:
    if m % t == 0 and n % t == 0:
        return t
    t -= 1
return -1 # when m or n == 0
```

- Solution 3: Middle-school procedure
 - > find prime factors of m
 - > find prime factors of n
 - > identify all common factors
 - > compute the product
- Performance Analysis
 - Solution 1 is the most efficient, as we get the solution with the least amount of steps
 - Solution 2 performs the worst. Worst case is if we have 2 very large prime numbers
 - Solution 3's performance depends on how we prime factor the numbers
- When asked to design an algorithm...
 - present key observations
 - describe algorithm
 - write code or pseudocode
 - explain why it works and check for correctness
 - analyze time complexity
- Important problem types:
 - sorting (key, stable, in-place, etc.)
 - searching
 - string processing (matching, etc.)
 - graph problems (traversal, shortest path, topological sort, TSP, graph-coloring, etc.)
 - combinatorial problems
 - geometric problems (convex-hull, closest pair, etc.)
 - numerical problems (equation-solving, etc.)