

# Diabetes and Cognitive Decline: Causal Evidence under Positivity Violations

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## Background

Diabetes is highly prevalent among older adults and has long been linked to accelerated cognitive decline.

However, identifying its **causal effect** is challenging because diabetes onset is time-varying and highly correlated with health status, healthcare use, and survival.

Standard regression and marginal structural models often suffer from **extreme positivity violations**, making the “ever vs. never diabetes” contrast largely non-identifiable.

This study uses longitudinal data from the Health and Retirement Study and a design-based approach to estimate the cognitive effect of incident diabetes among comparable older adults.

## Research Question

Does **incident diabetes** ( $0 \rightarrow 1$ ) cause a change in cognitive function six years later among older adults in the Health and Retirement Study?

## Data & Variables

**Dataset.** We use the Health and Retirement Study (HRS), a nationally representative cohort of U.S. adults aged 50+, interviewed biennially from 1992–2020. All waves were first assembled into a person-wave panel for marginal structural models.

**Initial MSM Dataset.** The full longitudinal panel included:

- time-varying diabetes status,
- time-varying health, behavioral, socioeconomic covariates,
- repeated cognition scores.

MSM-based IPTW for the “ever vs. never” contrast showed severe **positivity violations**.

**Risk-Set Dataset (Final Analysis).** To obtain an identifiable contrast, we restricted to respondents who:

- had no diabetes at Wave 7, and
- had diabetes status observed at Wave 8.

Incident diabetes = transition  $0 \rightarrow 1$  (W7–W8). Outcome = cognition at Wave 10 (6-year follow-up).

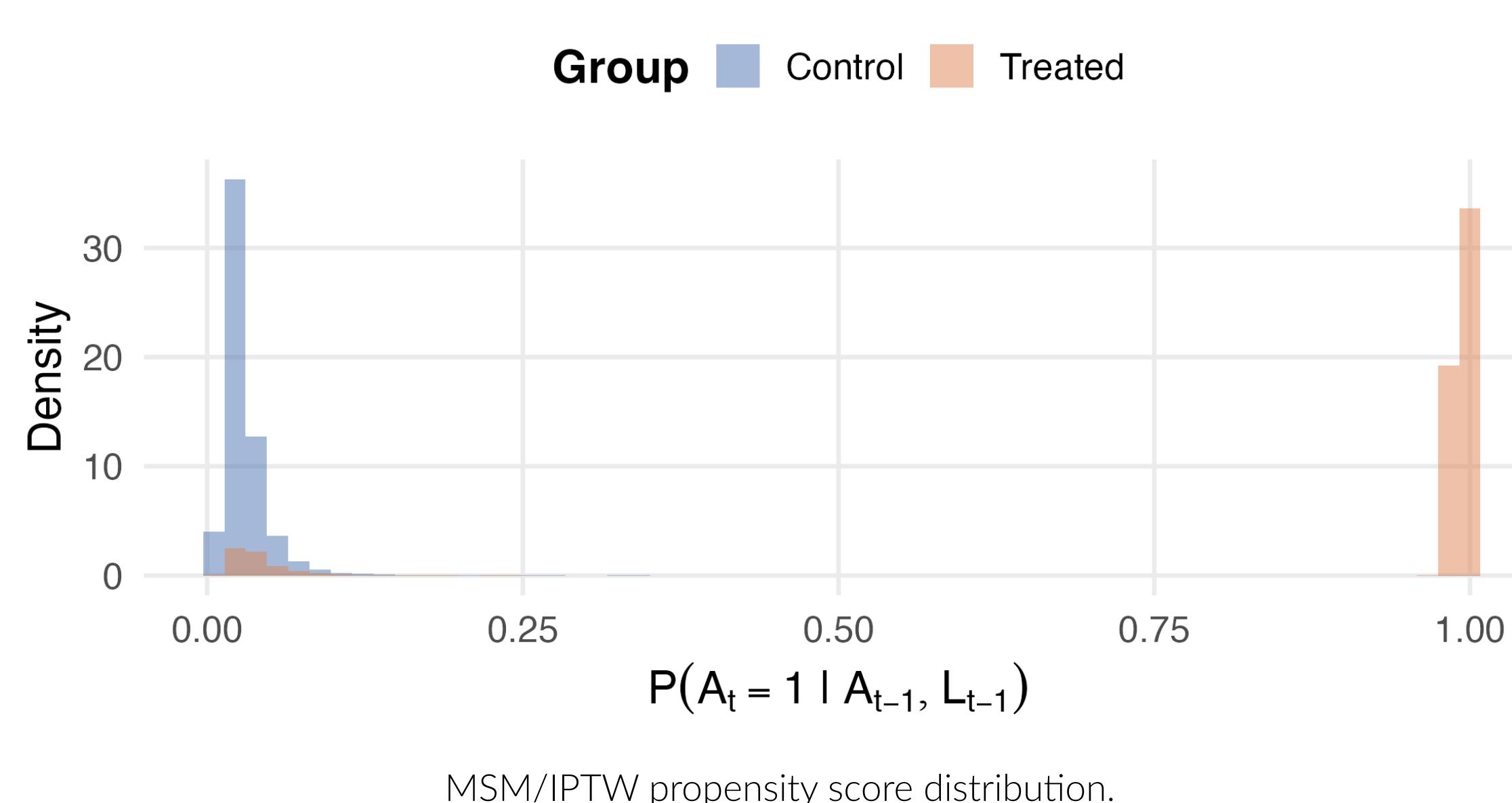
### Measures.

- **Exposure:** incident diabetes ( $0 \rightarrow 1$ ).
- **Outcome:** cognition score (0–35).
- **Baseline covariates:** demographics; BMI, hypertension, heart disease, stroke; smoking, drinking, walking, sleep; income, work; baseline cognition.

## Initial Check: MSM + IPTW for Ever vs. Never Diabetes

We first attempted to estimate the average treatment effect of ever versus never having diabetes using a marginal structural model with stabilized inverse probability of treatment weights, based on the full longitudinal panel.

## Time-Varying Diabetes (MSM/ATE)



**Figure.** Estimated propensity scores for the “ever diabetes” MSM. Non-diabetic individuals have scores concentrated near 0, whereas those who ever develop diabetes have scores near 1, leaving almost no common support. This indicates severe **positivity violation**, making the ATE for ever vs. never diabetes non-identifiable under standard IPTW.

## Study Design

**Motivation.** The MSM/IPTW analysis for the ever vs. never diabetes contrast showed extreme positivity violations. This made the ATE non-identifiable.

**Risk-Set Construction.** To obtain an identifiable causal contrast, we focused on incident diabetes within a well-defined risk set. Respondents were included if they:

- had no diabetes at Wave 7 (baseline), and
- had observed diabetes status at Wave 8.

These individuals form a risk set where an **incident diabetes** transition is defined as:

$$0 \rightarrow 1 \text{ between Waves 7–8.}$$

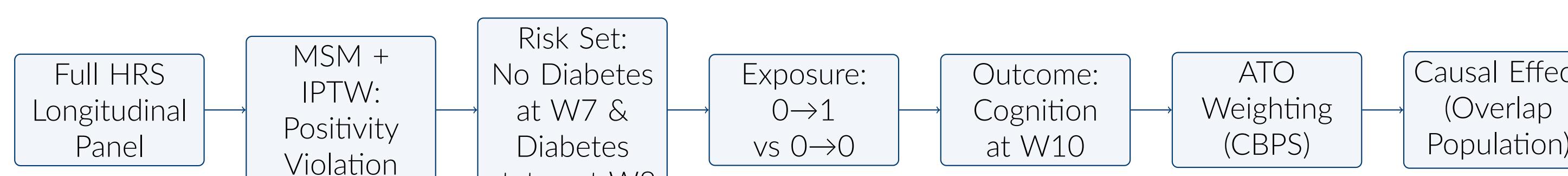
Cognition is measured 6 years later at Wave 10.

**Target Estimand.** Within the risk set, we estimate the causal contrast:

$$\mathbb{E}[Y(0 \rightarrow 1) - Y(0 \rightarrow 0)],$$

representing the effect of **incident diabetes** among individuals who were comparably at risk of developing diabetes.

**Why This Restores Identifiability.** Focusing on incident transitions avoids lifetime exposure separation. Overlap weighting (ATO) further reduces poorly comparable individuals, restoring meaningful common support.

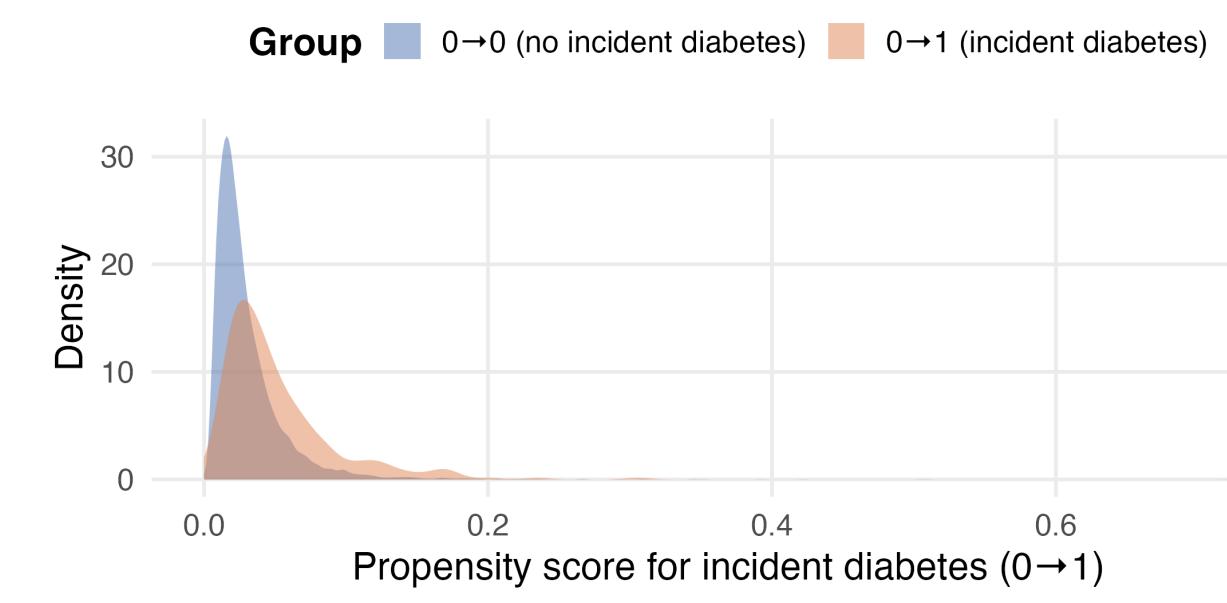


## Assumption Check

### 1. Positivity.

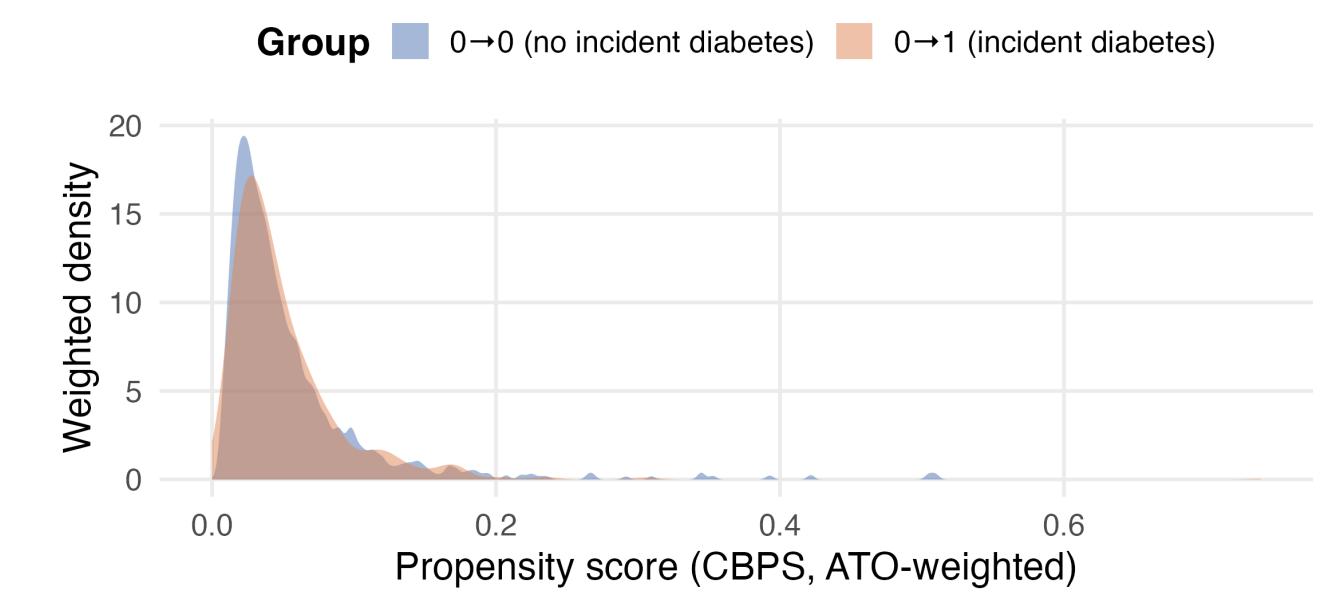
We evaluated whether individuals with incident diabetes ( $0 \rightarrow 1$ ) and those who remained non-diabetic ( $0 \rightarrow 0$ ) had comparable probabilities of developing diabetes, conditional on baseline covariates.

#### Risk-Set: Incident Diabetes (Unweighted)



Unweighted propensity score overlap.

#### Risk-Set: Incident Diabetes (Overlap-Weighted, ATO)



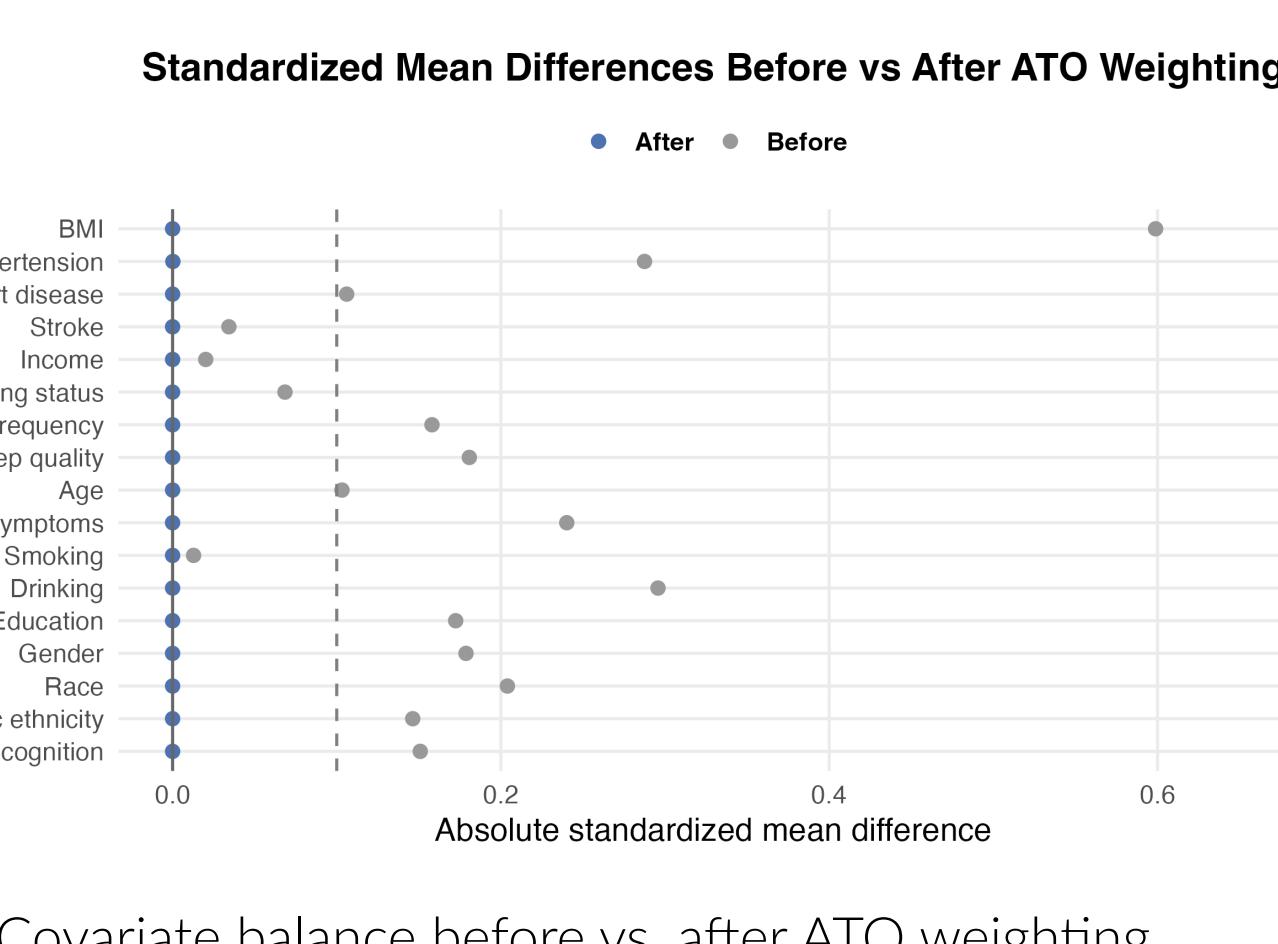
ATO-weighted propensity score overlap.

### 2. Covariate Balance.

Standardized mean differences (SMDs) were examined to assess covariate balance before and after weighting.

- **Before weighting:** several covariates showed moderate imbalance ( $>0.1$ ).
- **After ATO weighting:** all covariates achieved excellent balance ( $SMD \approx 0$ ).

**Interpretation of the ATO population.** ATO weighting focuses inference on individuals with moderate diabetes risk, reducing the weight of those at extremely low or high risk and producing a more comparable overlap population.



Covariate balance before vs. after ATO weighting.

## Statistical Model, Results, & Interpretation

**Statistical Model.** Within the risk-set population, we estimate the causal effect of incident diabetes on 6-year cognition using a weighted linear model:

$$Y_{i,6\text{yr}} = \beta_0 + \beta_1 D_i + \varepsilon_i,$$

where

- $D_i = 1$  if respondent transitioned  $0 \rightarrow 1$  (incident diabetes) between Waves 7–8;
- $Y_{i,6\text{yr}}$  is the global cognition score at Wave 10;
- weights = overlap weights (ATO) from CBPS, defining the overlap population of comparable individuals.

We report two estimates for comparison:

1. **Unweighted model** — naive comparison (confounding not addressed).
2. **ATO-weighted model** — causal contrast in the overlap population.

**Results.** Table displays regression estimates and 95% confidence intervals:

Model	Estimate	95% CI
Unweighted	$\hat{\beta}_1 = 0.46$	[-0.43, 1.34]
ATO-weighted (CBPS)	$\hat{\beta}_1 = 1.15$	[0.37, 1.93]

The weighted estimate corresponds to the causal contrast

$$\mathbb{E}[Y(0 \rightarrow 1) - Y(0 \rightarrow 0)]_{ATO},$$

representing the effect of incident diabetes among individuals who were comparably at risk of developing diabetes.

**Interpretation.** The unweighted model shows no clear causal effect between incident diabetes and later cognition. After applying overlap weighting, which creates a balanced comparison group, incident diabetes causally lowers 6-year cognition by **1.15-point lower** (95% CI: 0.37–1.93). This suggests a meaningful negative cognitive impact of new-onset diabetes among older adults who were comparably at risk.

## Conclusion & Limitations

### Conclusion

- MSM/IPTW estimation for the ever/never diabetes contrast failed due to severe positivity violations.
- A risk-set design targeting **incident diabetes** yields an identifiable and meaningful causal contrast.
- Overlap weighting (ATO) restores covariate balance and restricts inference to comparable individuals.
- Incident diabetes causally lowers 6-year cognition by about 1.15 points in the overlap population.

### Limitations

- Diabetes and covariates rely on self-reported measures, which may introduce misclassification.
- Overlap weighting balances observed covariates but does not eliminate potential unmeasured confounding.

## References

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