OCR
Oxford Cambridge and RSA

AS Level Further Mathematics A

Y531/01 Pure Core

Practice Paper – Set 1

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} -3 & 3 & 2 \\ 5 & -4 & -3 \\ -1 & 1 & 1 \end{pmatrix}$.

(i) Find $\mathbf{A}^{-1}$. **[1]**

(ii) Solve the simultaneous equations

$$\begin{aligned} -3x + 3y + 2z &= 12a \\ 5x - 4y - 3z &= -6 \\ -x + y + z &= 7 \end{aligned}$$

giving your solution in terms of a. **[3]**

2 The loci C_1 and C_2 are given by $|z-(3+2\mathrm{i})| = 2$ and $\arg(z-(3+2\mathrm{i})) = \frac{5\pi}{6}$ respectively.

(i) Sketch C_1 and C_2 on a single Argand diagram. **[4]**

(ii) Find, in surd form, the number represented by the point of intersection of C_1 and C_2. **[3]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z-(3+2\mathrm{i})| \leqslant 2 \text{ and } \frac{5\pi}{6} \leqslant \arg(z-(3+2\mathrm{i})) \leqslant \pi.$$ **[2]**

3 Two lines, l_1 and l_2, have the following equations.

$$l_1 : \mathbf{r} = \begin{pmatrix} -11 \\ 10 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

P is the point of intersection of l_1 and l_2.

(i) Find the position vector of P. **[3]**

(ii) Find, correct to 1 decimal place, the acute angle between l_1 and l_2. **[3]**

Q is a point on l_1 which is 12 metres away from P. R is the point on l_2 such that QR is perpendicular to l_1.

(iii) Determine the length QR. **[2]**

4 **In this question you must show detailed reasoning.**
The distinct numbers ω_1 and ω_2 both satisfy the quadratic equation $4x^2 + 4x + 17 = 0$.

(i) Write down the value of $\omega_1\omega_2$. **[1]**

(ii) A, B and C are the points on an Argand diagram which represent ω_1, ω_2 and $\omega_1\omega_2$.
Find the area of triangle ABC. **[6]**

5 **In this question you must show detailed reasoning.**
The equation $x^3 + 3x^2 - 2x + 4 = 0$ has roots α, β and γ.

(i) Using the identity $\alpha^3 + \beta^3 + \gamma^3 \equiv (\alpha+\beta+\gamma)^3 - 3(\alpha\beta+\beta\gamma+\gamma\alpha)(\alpha+\beta+\gamma) + 3\alpha\beta\gamma$ find the value of $\alpha^3 + \beta^3 + \gamma^3$. **[3]**

(ii) Given that $\alpha^3\beta^3 + \beta^3\gamma^3 + \gamma^3\alpha^3 = 112$ find a cubic equation whose roots are α^3, β^3 and γ^3. **[4]**

6 Prove by induction that $n! \geqslant 6n$ for $n \geqslant 4$. **[5]**

7 A transformation is equivalent to a shear parallel to the x-axis followed by a shear parallel to the y-axis and is represented by the matrix $\begin{pmatrix} 1 & s \\ t & 0 \end{pmatrix}$.

Find in terms of s the matrices which represent each of the shears. **[7]**

8 **(i)** **(a)** Find, in terms of x, a vector which is perpendicular to the vectors $\begin{pmatrix} x-2 \\ 5 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} x \\ 6 \\ 2 \end{pmatrix}$. **[2]**

(b) Find the shortest possible vector of the form $\begin{pmatrix} 1 \\ a \\ b \end{pmatrix}$ which is perpendicular to the vectors $\begin{pmatrix} x-2 \\ 5 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} x \\ 6 \\ 2 \end{pmatrix}$. **[5]**

(ii) Vector $\mathbf{v}$ is perpendicular to both $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ p \\ p^2 \end{pmatrix}$ where p is a real number. Show that it is impossible for $\mathbf{v}$ to be perpendicular to the vector $\begin{pmatrix} 1 \\ 1 \\ p-1 \end{pmatrix}$. **[6]**

END OF QUESTION PAPER