Oxford Cambridge and RSA

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Practice Paper – Set 2

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a\ (1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \dbinom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \cdots + \dfrac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots \quad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

f(x)	f$'(x)$
tan kx	k sec^2 kx
sec x	sec x tan x
cot x	–cosec^2 x
cosec x	–cosec x cot x

Quotient rule $y = \dfrac{u}{v}, \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Integration

$\displaystyle\int \frac{f'(x)}{f(x)}\,dx = 1n|f(x)| + c$

$\displaystyle\int f'(x)(f(x))^n\,dx = \frac{1}{n+1}(f(x))^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1 - \frac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k+\tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B|A) = \mathrm{P}(B)\mathrm{P}(A|B)$ **or** $\mathrm{P}(A|B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

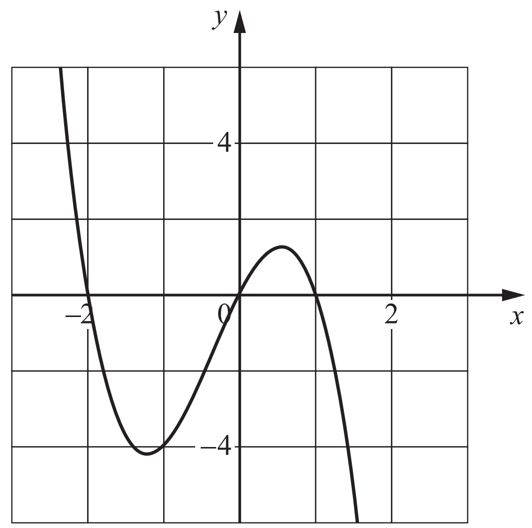
$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A: Pure Mathematics

Answer **all** the questions.

1 Part of the graph of $y = f(x)$ is shown below, where $f(x)$ is a cubic polynomial.



(i) Find $f(-1)$. **[1]**

(ii) Write down three linear factors of $f(x)$. **[1]**

It is given that $f(x) \equiv ax^3 + bx^2 + cx + d$.

(iii) Show that $a = -2$. **[3]**

(iv) Find b, c and d. **[1]**

2 Angela makes the following claim.

"n is an odd positive integer greater than 1 $\Rightarrow 2^n - 1$ is prime"

Prove that Angela's claim is false. **[4]**

3 On a particular voyage, a ship sails 500 km at a constant speed of v km/h. The cost for the voyage is £R per hour. The total cost of the voyage is £T.

(i) Show that $T = \dfrac{500R}{v}$. **[1]**

The running cost is modelled by the following formula.

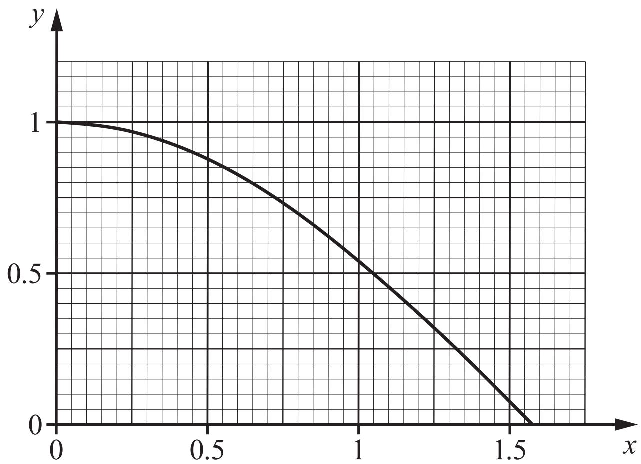
$$R = 270 + \frac{v^3}{200}$$

The ship's owner wishes to sail at a speed that will minimise the total cost for the voyage. It is given that the graph of T against v has exactly one stationary point, which is a minimum.

(ii) Find the speed that gives the minimum value of T. **[4]**

(iii) Find the minimum value of the total cost. **[2]**

4 The diagram shows part of the graph of $y = \cos x$, where x is measured in radians.



(i) Use the copy of this diagram in the Printed Answer Booklet to find an approximate solution to the equation $x = \cos x$. **[2]**

(ii) Use an iterative method to find the solution to the equation $x = \cos x$ correct to 3 significant figures. You should show your first, second and last two iterations, writing down all the figures on your calculator. **[3]**

5 Points A, B and C have position vectors $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix}$ respectively.

(i) Find the exact distance between the midpoint of AB and the midpoint of BC. **[4]**

Point D has position vector $\begin{pmatrix} x \\ -6 \\ z \end{pmatrix}$ and the line CD is parallel to the line AB.

(ii) Find all the possible pairs of x and z. **[4]**

6 **In this question you must show detailed reasoning.**

(i) Use the formula for $\tan(A - B)$ to show that $\tan\frac{\pi}{12} = 2 - \sqrt{3}$. **[4]**

(ii) Solve the equation $2\sqrt{3}\sin 3A - 2\cos 3A = 1$ for $0° \leqslant A < 180°$. **[7]**

7 A tank is shaped as a cuboid. The base has dimensions 10 cm by 10 cm. Initially the tank is empty. Water flows into the tank at $25\,\text{cm}^3$ per second. Water also leaks out of the tank at $4h^2\,\text{cm}^3$ per second, where h cm is the depth of the water after t seconds. Find the time taken for the water to reach a depth of 2 cm. **[9]**

Section B: Statistics

Answer **all** the questions.

8 The masses, X grams, of tomatoes are normally distributed. Half of the tomatoes have masses greater than 56.0 g and 70% of the tomatoes have masses greater than 53.0 g.

(i) Find the percentage of tomatoes with masses greater than 59.0 g. **[2]**

(ii) Find the percentage of tomatoes with masses greater than 65.0 g. **[4]**

(iii) Given that $P(a < X < 50) = 0.1$, find a. **[3]**

9 A bag contains 100 black discs and 200 white discs. Paula takes five discs at random, without replacement. She notes the number X of these discs that are black.

(i) Find $P(X = 3)$. **[2]**

Paula decides to use the binomial distribution as a model for the distribution of X.

(ii) Explain why this model will give probabilities that are approximately, but not exactly, correct. **[3]**

(iii) Paula uses the binomial model to find an approximate value for $P(X = 3)$. Calculate the percentage by which her answer will differ from the answer in part **(ii)**. **[2]**

Paula now assumes that the binomial distribution is a good model for X. She uses a computer simulation to generate 1000 values of X. The number of times that $X = 3$ occurs is denoted by Y.

(iv) Calculate estimates of the limits between which two thirds of the values of Y will lie. **[3]**

10 A researcher is investigating the actual lengths of time that patients spend at their appointments with the doctors at a certain clinic. There are 12 doctors at the clinic, and each doctor has 24 appointments per day. The researcher plans to choose a sample of 24 appointments on a particular day.

(i) The researcher considers the following two methods for choosing the sample.

Method A: Choose a random sample of 24 appointments from the 288 on that day.
Method B: Choose one doctor's 1st and 2nd appointments. Choose another doctor's 3rd and 4th appointments and so on until the last doctor's 23rd and 24th appointments.

For **each** of A and B state a disadvantage of using this method. **[2]**

Appointments are scheduled to last 10 minutes. The researcher suspects that the actual times that patients spend are more than 10 minutes on average. To test this suspicion, he uses method A, and takes a random sample of 24 appointments. He notes the actual time spent for each appointment and carries out a hypothesis test at the 1% significance level.

(ii) Explain why a 1-tail test is appropriate. **[1]**

The population mean of the actual times that patients spend at their appointments is denoted by μ minutes.

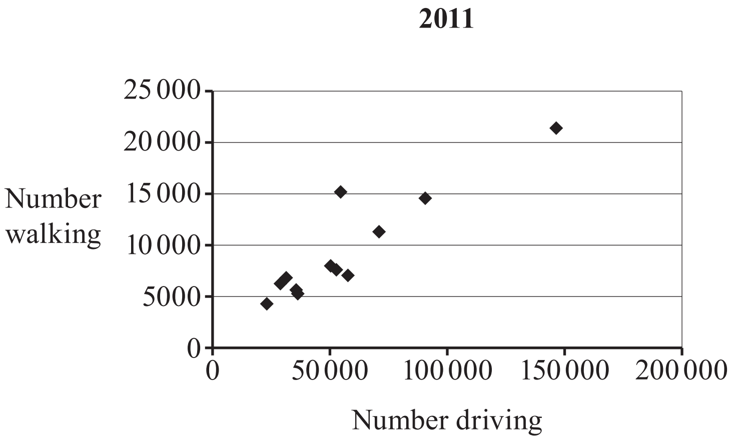
(iii) Assuming that $\mu = 10$, state the probability that the conclusion of the test will be that μ is not greater than 10. **[1]**

The actual lengths of time, in minutes, that patients spend for their appointments may be assumed to have a normal distribution with standard deviation 3.4.

(iv) Given that the total length of time spent for the 24 appointments is 285 minutes, carry out the test. **[7]**

(v) In part **(iv)** it was necessary to use the fact that the sample mean is normally distributed. Give a reason why you know that this is true in this case. **[1]**

11 The scatter diagram shows data, taken from the pre-release data set, for several Local Authorities in one region of the UK in 2011. The diagram shows, for each Local Authority, the number of workers who drove to work, and the number of workers who walked to work.



(i) Four students calculated the value of Pearson's product-moment correlation coefficient for the data in the diagram. Their answers were 0.913, 0.124, −0.913 and −0.124. One of these values is correct. Without calculation state, with a reason, which is the correct value. **[2]**

(ii) Sanjay makes the following statement.

"The diagram shows that, in **any** Local Authority, if there are a large number of people who drive to work there will be a large number who walk to work."

Give a reason why this statement is incorrect. **[2]**

(iii) Rosie makes the following statement.

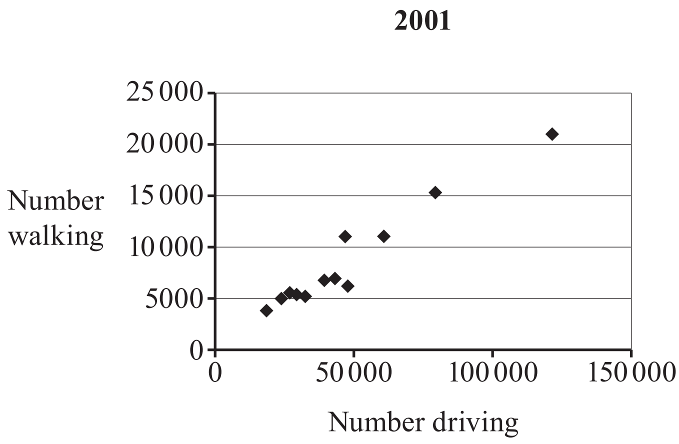
"The diagram must be wrong because it shows good positive correlation. If there are more people driving to work, there will be fewer people walking to work, so there would be negative correlation."

Explain briefly why Rosie's statement is incorrect. **[1]**

(iv) The diagram shows a fairly close relationship between the two variables. One point on the diagram represents a Local Authority where this relationship is less strong than for the others. On the diagram in the Printed Answer Booklet, label this point A. **[1]**

(v) Given that the point A represents a metropolitan borough, suggest a reason why the relationship is less strong for this Local Authority than for the others in the region. **[1]**

The scatter diagram below shows the corresponding data for the same region in 2001.



(vi) **(a)** State a change that has taken place in the metropolitan borough represented by the point A between 2001 and 2011. **[1]**

(b) Suggest a possible reason for this change. **[1]**

12 Rob has two six-sided dice, each with sides numbered 1, 2, 3, 4, 5, 6.
One dice is fair. The other dice is biased, with probabilities as shown in the table.

Biased die						
y	1	2	3	4	5	6
$\mathrm{P}(Y = y)$	0.3	0.25	0.2	0.14	0.1	0.01

Rob throws each dice once and notes the two scores, X on the fair dice and Y on the biased dice. He then calculates the value of the variable S which is defined as follows.

- If $X \leqslant 3$, then $S = X + 2Y$.
- If $X > 3$, then $S = X + Y$.

(i) **(a)** Draw up a sample space diagram showing all the possible outcomes and the corresponding values of S. **[2]**

(b) On your diagram, circle the four cells where the value $S = 10$ occurs. **[1]**

(ii) Explain the mistake in the following calculation.

$$\mathrm{P}(S = 10) = \frac{\text{Number of outcomes giving } S = 10}{\text{Total number of outcomes}} = \frac{4}{36} = \frac{1}{9}.$$ **[1]**

(iii) Find the correct value of $\mathrm{P}(S = 10)$. **[2]**

(iv) Given that $S = 10$, find the probability that the score on one of the dice is 4. **[3]**

(v) The events "$X = 1$ or 2" and "$S = n$" are mutually exclusive. Given that $\mathrm{P}(S = n) \neq 0$, find the value of n. **[1]**

END OF QUESTION PAPER

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