Oxford Cambridge and RSA

Practice Paper

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

Version **Pre SSU**
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This document consists of 14 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question includes the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			**DR**				
			$\frac{\mathrm{d}y}{\mathrm{d}x} = -6\sin 2x$	**M1**	**1.1**	For $k\sin 2x$	
				A1	**1.1**	For completely correct derivative	
			Substitute $x = \frac{1}{8}\pi$ in attempt at first derivative	**M1**	**1.1**		
			Obtain $-3\sqrt{2}$	**A1**	**1.1**	oe, e.g. $-\frac{6}{\sqrt{2}}$	
				[4]			
2	(i)		State $R = 5$	**B1**	**1.1**		
			Attempt to find value of α	**M1**	**1.1a**	May be implied by correct value or its complement	
			Obtain 36.9	**A1**	**1.1**	Accept $\tan^{-1}\left(\frac{3}{4}\right)$	
				[3]			
2	(ii)		Minimum temperature is 15 °C	**B1ft**	**3.4**	ft $20 - R$	
				[1]			
2	(iii)		Minimum occurs when $15t - \alpha = 180$	**M1**	**3.1a**		
			$t = 14.5$	**A1ft**	**1.1**	ft $(\alpha + 180) \div 15$	14.457993…
			Time is 2:27 am	**A1**	**3.2a**	oe, e.g. 0227	
				[3]			

Question			Answer	Marks	AOs	Guidance	
3	(i)		$\dfrac{AC}{\sin\frac{3}{4}\pi} = \dfrac{1}{\sin\left(\pi - \frac{3}{4}\pi - \theta\right)}$	M1	2.1	Attempt sine rule	
			$AC = \dfrac{\sin\frac{3}{4}\pi}{\sin\frac{1}{4}\pi\cos\theta - \cos\frac{1}{4}\pi\sin\theta}$	M1	2.1	For expanding $\sin\left(\frac{1}{4}\pi - \theta\right)$	
			$\sin\frac{3}{4}\pi = \sin\frac{1}{4}\pi = \cos\frac{1}{4}\pi$ so $AC = \dfrac{1}{\cos\theta - \sin\theta}$	E1	2.2a	**AG**, so must show sufficient working; e.g. stating $\sin\frac{3}{4}\pi = \sin\frac{1}{4}\pi = \cos\frac{1}{4}\pi$ or using $\dfrac{1}{\sqrt{2}}$ oe for each	
				[3]			
3	(ii)		$AC = \left(1 + (-\theta - \frac{1}{2}\theta^2)\right)^{-1}$	B1	1.1	Using both small angle approximations	
			$AC = 1 + (-1)\left(-\theta - \frac{1}{2}\theta^2\right) + \dfrac{(-1)(-2)}{2}\left(-\theta - \frac{1}{2}\theta^2\right)^2 + \dots$	M1	3.1a	Attempt binomial expansion of AC, with at least the first two terms present	
			$AC \approx 1 + \theta + \frac{3}{2}\theta^2$	A1	1.1	$p = 1$	
				A1	1.1	$q = \frac{3}{2}$	
				[4]			

Question			Answer	Marks	AOs	Guidance	
4	(i)		**DR**				
			State $-1<\frac{5}{3x-4}$ and/or $\frac{5}{3x-4}<1$	**B1**	**1.2**		
			Multiply by $(3x-4)^2$ and attempt to simplify	**M1**	**1.1a**		
			Obtain either $9x^2-9x-4>0$ or $3x^2-13x+12>0$	**A1**	**1.1**		
			Obtain critical values $\frac{4}{3},-\frac{1}{3}$ or $\frac{4}{3}, 3$	**A1**	**1.1**	BC	
			$\{x:x<-\frac{1}{3}\}\cup\{x:x>3\}$	**A1**	**2.5**		
			Alternative method				
			State $\left\lvert\frac{5}{3x-4}\right\rvert<1$	**B1**			
			Rewrite in the form $\lvert 3x-4\rvert>5$	**M1**			
			Obtain either $3x-4>5$ or $3x-4<-5$	**A1**		oe, eg $3x^2-8x-3>0$	
			Obtain both critical values 3 and $-\frac{1}{3}$	**A1**			
			$\{x:x<-\frac{1}{3}\}\cup\{x:x>3\}$	**A1**			
				[5]			
4	(ii)		**DR**				
			$S_\infty=\frac{1}{1-\frac{5}{3x-4}}$	**B1**	**1.1**	Correct use of sum to infinity formula	
			$\frac{3x-4}{3x-9}=\frac{2}{3}\Rightarrow x=\ldots$	**M1**	**1.1**	Equate to $\frac{2}{3}$ and attempt to solve for x	
			$x=-2$	**A1**	**1.1**		
				[3]			

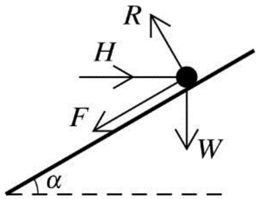
Question			Answer	Marks	AOs	Guidance	
5	(i)		Draw (more or less) correct sketch of $y = \frac{5}{x^2}$	B1	1.1	Must have both branches, and axes clearly shown as asymptotes	
			Draw (more or less) correct sketch of $y = \|2x - 4\|$	B1	1.1	V shape with vertex on the positive *x*-axis and intersecting the positive *y*-axis	*x*- and *y*-intercept values need not be stated
			Indicate two points of intersection	B1 [3]	2.2a	With both sketches correct	
5	(ii)		$\frac{5}{x^2} = -(2-4x) \Rightarrow \mathrm{f}(x) = 4x^3 - 2x^2 - 5$	B1 [1]	3.1a	**AG**	
5	(iii)		$x_{n+1} = x_n - \frac{4x_n^3 - 2x_n^2 - 5}{12x_n^2 - 4x_n}$	M1	1.1	Correct derivative seen and substituted into correct N-R formula	
			$= \frac{12x_n^3 - 4x_n^2 - 4x_n^3 + 2x_n^2 + 5}{12x_n^2 - 4x_n}$	M1	1.1	Combining terms; either one single fraction seen or two fractions with a common denominator	
			$= \frac{8x_n^3 - 2x_n^2 + 5}{12x_n^2 - 4x_n}$	E1 [3]	2.1	**AG**; working as in line above must be seen	Suffices must be present
5	(iv)		$x_2 = \frac{11}{8}$, $x_3 = 1.28090\ldots$	B1	1.1		
			$x_4 = 1.27232\ldots$, $x_5 = 1.27225\ldots$	B1	1.1		
			$\alpha = 1.272$	B1 [3]	1.1		
5	(v)		$y = 4x - 2$ was used to obtain the N-R formula and this line only intersects the 1st quadrant branch so can only give the positive root	B1 [1]	2.4		

Question			Answer	Marks	AOs	Guidance	
6	(i)		$\frac{dy}{dx} = \frac{1}{1+4x^2} \times 8x$	B1	1.1	For $\frac{1}{1+4x^2}$	
				B1	1.1	For $8x$	
			Attempt use of quotient rule or equivalent	M1*	3.1a	Condone only one slip in differentiating their 1st derivative, but if the quotient rule is used it must have subtraction in the numerator	Condone absence of necessary brackets
			$\frac{d^2y}{dx^2} = \frac{8-32x^2}{(1+4x^2)^2} = 0 \Rightarrow x = \ldots$	dep* M1	1.1	Equate 2nd derivative to 0 and attempt to solve for x	
			$x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}$	E1	2.1	**AG**	
			When $x = \pm\frac{1}{2}$, $\frac{dy}{dx} = \pm 2 \neq 0$ and there is a sign change in the second derivative on either side of x so these points are therefore non-stationary points of inflection	E1	2.4		
				[6]			
6	(ii)		$\text{Area} = 2\int_0^{\lambda} f(y)\,dy$	E1	2.1	Correct integral stated for required area	
			$y = \ln(1+4x^2) \Rightarrow 4x^2 = e^y - 1 \Rightarrow f(y) = \frac{1}{2}\sqrt{e^y - 1}$	E1	2.1	Sufficient working for $f(y) = \frac{1}{2}\sqrt{e^y - 1}$	
			$\lambda = \ln\left(1+4\left(\frac{1}{2}\right)^2\right) = \ln 2$	E1	2.1	Sufficient working for top limit of integral	
				[3]			
6	(iii)		$e^y = \sec^2\theta \Rightarrow dy = 2\tan\theta\,d\theta$	M1	3.1a	Allow for any genuine attempt to differentiate the given substitution and express integral entirely in terms of θ	Ignore limits for this mark
			$\text{Area} = 2\int_0^{\frac{1}{4}\pi} \sqrt{\sec^2\theta - 1}\,\tan\theta\,d\theta = 2\int_0^{\frac{1}{4}\pi} \tan^2\theta\,d\theta$	A1	2.2a	**AG**; must show evidence for change of limits	
				[2]			

Question			Answer	Marks	AOs	Guidance	
6	(iv)		$\text{Area} = 2\int_0^{\frac{1}{4}\pi}(\sec^2\theta - 1)\,d\theta$	M1	3.1a	Reducing to form $\int(a\sec^2\theta + b)\,d\theta$	
			$= 2\left[\tan\theta - \theta\right]_0^{\frac{1}{4}\pi} = 2\left\{\left(\tan\frac{1}{4}\pi - \frac{1}{4}\pi\right) - \left(\tan 0 - 0\right)\right\}$	A1ft	1.1	Correctly integrating their $a\sec^2\theta + b$ with correct use of limits	
			$= 2\left(1 - \frac{1}{4}\pi\right)$	A1	1.1		
				[3]			
7	(i)		Correct trapezium shape	B1	3.1b	In 1st quadrant only; must start at origin	
			Correct values on axes	B1	3.1b	30 labelled on v axis; 180 labelled and 90 indicated on t axis	
				[2]			
7	(ii)		$\text{Distance} = \frac{1}{2}(180 + 90) \times 30$	M1	1.1	For complete attempt at area under their graph; may see two triangles (combined) plus rectangle	
			4050 m	A1	3.2a	oe, e.g. 4.05 km	
				[2]			
7	(iii)		$(t_{\text{acc}} =)\ \dfrac{30}{a}$ or $(t_{\text{dec}} =)\ \dfrac{30}{2a}$	B1	1.1	For a correct expression for either time	
			$\dfrac{30}{a} + \dfrac{30}{2a} = 90 \Rightarrow a = \ldots$	M1	1.1	Forming equation in a and attempt to solve	
			$a = 0.5$	A1	1.1		
			Alternative solution				
			State that $t_{\text{acc}} = 2t_{\text{dec}}$	B1			
			Obtain $t_{\text{acc}} = 60$ and use to evaluate a	M1			
			$a = 0.5$	A1			
				[3]			

Question			Answer	Marks	AOs	Guidance	
8	(i)		Use **F** = m**a** to obtain $\mathbf{a}=\begin{pmatrix}3\\-1\end{pmatrix}$	**B1**	**3.3**		
			$\mathbf{v}=\begin{pmatrix}1+3t\\-2-t\end{pmatrix}$	**B1ft**	**3.4**	For use of **v** = **u** + **a**t with their **a** (allow ft for this mark even if **F** used for **a**)	Or integrate and use initial conditions
				[2]			
8	(ii)		$(1+3t)^2+(-2-t)^2=25$	**M1**	**1.1**	Use of Pythagoras using their vector for **v**	
			$t^2+t-2=0 \Rightarrow t=\dots$	**M1**	**1.1**	Forming and attempting solution of 3-term quadratic for t	
			As t cannot be negative, t = 1 only	**A1**	**2.3**	BC; must explicitly reject t = –2	
				[3]			
8	(iii)		$\mathbf{s}=\begin{pmatrix}2+t+\frac{3}{2}t^2\\3-2t-\frac{1}{2}t^2\end{pmatrix}$	**M1**	**3.4**	For use of $\mathbf{s}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ with their **a**	Or integration of their **v** and use of initial conditions
			When t = 2, $\mathbf{s}=\begin{pmatrix}10\\-3\end{pmatrix}$ m	**A1**	**1.1**		
				[2]			

Question			Answer	Marks	AOs	Guidance	
9	(i)		$u_h = 15\text{ m s}^{-1}$	B1	1.1		
			$0 = 4u_v + \frac{1}{2}(-9.8)\times 4^2$	M1	3.3	Use of $s = ut + \frac{1}{2}at^2$ vertically, with $s = 0$	Or $v = u + at$, eg
			$u_v = 19.6\text{ m s}^{-1}$	A1	1.1		
				[3]			
9	(ii)		$u_h = 15 \Rightarrow t = 1$, so $h = 19.6\times 1 + \frac{1}{2}(-9.8)\times 1^2$	M1	3.4	Finding t and using in $s = ut + \frac{1}{2}at^2$	
			$h = 14.7$	A1	1.1	**AG**	
				[2]			
9	(iii)		$v_v^2 = 19.6^2 + 2(-9.8)\times 14.7$ (= 96.04)	M1	3.3	Use of $v^2 = u^2 + 2as$ vertically with their value of u_v from **(i)**	
			$v_h = 15$	B1ft	1.1	Their value of u_h from **(i)**	
			$v = \sqrt{96.04 + 15^2}$	M1	1.1	Use of Pythagoras to find the speed	
			17.9 m s^{-1}	A1	1.1		17.917589…
				[4]			
9	(iv)		Model takes no account of air resistance	E1	3.5b	Or any other reasonable comment, e.g. wind or rotation of the ball could affect the motion	
				[1]			
9	(v)		State larger, with suitable explanation	E1	3.5a	E.g. air resistance will slow the ball down so to achieve the given range (or time of flight, or height at the post) the initial speed would have to be higher	
				[1]			

Question			Answer	Marks	AOs	Guidance	
10	(i)			B1 [1]	1.1	Correct four forces shown; F could be labelled as μR or $\frac{1}{2}R$	
10	(ii)		Resolve || to plane: $F + W\sin\alpha = H\cos\alpha$	M1	3.3	Three terms with resolving attempted	$\sin\alpha$ and $\cos\alpha$ may be numerical
			Resolve perp. to plane: $R = W\cos\alpha + H\sin\alpha$	M1	3.3	Three terms with resolving attempted	
			$H\cos\alpha - W\sin\alpha = \mu(W\cos\alpha + H\sin\alpha)$	M1	3.3	Use of $F = \mu R$; dep on first two M marks	
			$\frac{3}{5}H - \frac{4}{5}W = \frac{1}{2}\left(\frac{3}{5}W + \frac{4}{5}H\right)$	M1	2.1	oe; equation in H and W only, in any form	
			$\frac{1}{5}H = \frac{11}{10}W \Rightarrow H = \frac{11}{2}W$	A1	2.2a	**AG**; sufficient working must be shown	
			Alternative solution				
			Resolve vertically: $W + F\sin\alpha = R\cos\alpha$	M1			
			$W = \frac{3}{5}R - \frac{4}{5}\times\frac{1}{2}R = \frac{1}{5}R \Rightarrow R = 5W$	M1		Use $F = \mu R$ and obtain R in terms of W	
			Resolve horizontally: $H = F\cos\alpha + R\sin\alpha$	M1			
			Substitute: $H = \frac{1}{2}\times 5W\times\frac{3}{5} + 5W\times\frac{4}{5}$	M1			
			$H = \frac{11}{2}W$	A1		**AG**; sufficient working must be shown	
				[5]			
10	(iii)		$R = W\cos\alpha$	B1	1.1		Trig ratios may be numerical
			$W\sin\alpha - F = \frac{W}{g}a$	M1	3.3	For attempt at N2L || to plane, with F in the opposite direction to that seen in **(ii)**	Do not allow W for the mass
			$W\sin\alpha - \mu W\cos\alpha = \frac{W}{g}a$	M1	3.4	Using $F = \mu R$ and eliminating R and F	
			$a = \frac{1}{2}g$	A1 [4]	1.1		

Question			Answer	Marks	AOs	Guidance	
11	(i)	(a)	Moments @ A: $R_B a + 3aW\cos\theta = 2aR_C$	M1	3.3	Allow sign errors and sin/cos confusion	Or moments @ B
			Resolve vertically: $W + R_B\cos\theta = R_C\cos\theta$	M1	3.3	Allow sign errors and sin/cos confusion	Or resolve ‖ rod
				A1	1.1	For both equations correct	
			$W + (2R_C - 3W\cos\theta)\cos\theta = R_C\cos\theta$	M1	3.4	Attempt solution of simultaneous equations to find R_C in terms of W and θ	
			$R_C = W\left(\dfrac{3\cos^2\theta - 1}{\cos\theta}\right)$	A1	1.1	**AG**; sufficient working must be shown	
				[5]			
11	(i)	(b)	$R_B = W\left(\dfrac{3\cos^2\theta - 2}{\cos\theta}\right)$	B1	1.1	oe, e.g. $R_B = W(3\cos\theta - 2\sec\theta)$	
				[1]			
11	(ii)		For equilibrium, $R_B \geq 0$ and $R_C \geq 0$	M1	2.1	For either considered; allow = for ≥	
			Critical case is $R_B = 0$, as this gives lower limit for θ so $\cos^2\theta = \frac{2}{3} \Rightarrow \theta_{\max} = 35.3°$ (correct to 3sf)	E1	2.2a	**AG**; sufficient reasoning required	
				[2]			
11	(iii)		Resolve ‖ rod: $R_A\cos\theta = W\sin\theta$	M1	3.3	Allow sin/cos confusion	Or moments @ C
			Obtain $R_A = W\tan\theta$	A1	2.1	R_A in terms of W and θ correct in any form	
			$W\tan\theta = W\left(\dfrac{3\cos^2\theta - 1}{\cos\theta}\right)$	M1	2.1	Equate expressions for R_A and R_C	
			$3\sin^2\theta + \sin\theta - 2 = 0$	M1	2.2a	Use of trig identities to form 3-term quadratic equation in $\sin\theta$	
			$\sin\theta = \frac{2}{3}$ only, as $\sin\theta \neq -1$	A1	2.3	BC; the negative value must be seen and not given as a final answer	
			$\theta = 41.8°$, but as this is greater than 35.3° it is not possible that R_A and R_C are equal	E1	2.4	For correct argument justifying given result	
				[6]			