OCR
Oxford Cambridge and RSA

A Level Mathematics A

H240/01 Pure Mathematics

Practice Paper – Set 2

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k+\tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\sum(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\dfrac{\sum fx^2}{\sum f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, Variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

1 A circle with equation $x^2+y^2+6x-4y=k$ has a radius of 4.

(i) Find the coordinates of the centre of the circle. **[2]**

(ii) Find the value of the constant k. **[2]**

2 **(i)** Given that $|n|=5$, find the greatest value of $|2n-3|$, justifying your answer. **[3]**

(ii) Solve the equation $|3x-6|=|x-6|$. **[3]**

3 The equation $kx^2+(k-6)x+2=0$ has two distinct real roots. Find the set of possible values of the constant k, giving your answer in set notation. **[6]**

4 **(i)** Sketch the curves $y=\dfrac{3}{x^2}$ and $y=x^2-2$ on the axes provided in the Printed Answer Booklet. **[3]**

(ii) **In this question you must show detailed reasoning.**

Find the exact coordinates of the points of intersection of the curves $y=\dfrac{3}{x^2}$ and $y=x^2-2$. **[6]**

5 An ice cream seller expects that the number of sales will increase by the same amount every week from May onwards. 150 ice creams are sold in Week 1 and 166 ice creams are sold in Week 2. The ice cream seller makes a profit of £1.25 for each ice cream sold.

(i) Find the expected profit in Week 10. **[3]**

(ii) In which week will the total expected profits first exceed £5000? **[5]**

(iii) Give two reasons why this model may not be appropriate. **[2]**

6 Prove by contradiction that $\sqrt{7}$ is irrational. **[5]**

7 Two lifeboat stations, P and Q, are situated on the coastline with Q being due south of P. A stationary ship is at sea, at a distance of 4.8 km from P and a distance of 2.2 km from Q. The ship is on a bearing of 155° from P.

(i) Find any possible bearings of the ship from Q. **[4]**

(ii) Find the shortest distance from the ship to the line PQ. **[2]**

(iii) Give a reason why the actual distance from the ship to the coastline may be different to your answer to part **(ii)**. **[1]**

8 **(i)** Given that $y = \sec x$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \sec x \tan x$. **[3]**

(ii) **In this question you must show detailed reasoning.**

Find the exact value of $\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi} (\sec 2x + \tan 2x)^2 \, \mathrm{d}x$. **[6]**

9 **(i)** Express $\frac{5 + 4x - 3x^2}{(1-2x)(2+x)^2}$ in three partial fractions. **[5]**

(ii) Hence find the first three terms in the expansion of $\frac{5 + 4x - 3x^2}{(1-2x)(2+x)^2}$ in ascending powers of x. **[5]**

(iii) State the set of values for which the expansion in part **(ii)** is valid. **[1]**

10 **In this question you must show detailed reasoning.**

Show that the curve with equation $x^2 - 4xy + 8y^3 - 4 = 0$ has exactly one stationary point. **[10]**

11 The height, in metres, of the sea at a coastal town during a day may be modelled by the function

$$\mathrm{f}(t) = 1.7 + 0.8 \sin(30t)^\circ,$$

where t is the number of hours after midnight.

(i) **(a)** Find the maximum height of the sea as given by this model. **[1]**

(b) Find the time of day at which this maximum height first occurs. **[2]**

(ii) Determine the time when, according to this model, the height of the sea will first be 1.2 m. **[4]**

The height, in metres, at a different coastal town during a day may be modelled by the function

$$\mathrm{g}(t) = a + b \sin(ct + d)^\circ,$$

where t is the number of hours after midnight.

(iii) It is given that at this different coastal town the maximum height of the sea is 3.1 m, and this height occurs at 0500 and 1700. The minimum height of the sea is 0.7 m, and this height occurs at 1100 and 2300. Find the values of the constants a, b, c and d. **[4]**

(iv) It is instead given that the maximum height of the sea actually occurs at 0500 and 1709. State, with a reason, how this will affect the value of c found in part **(iii)**. **[1]**

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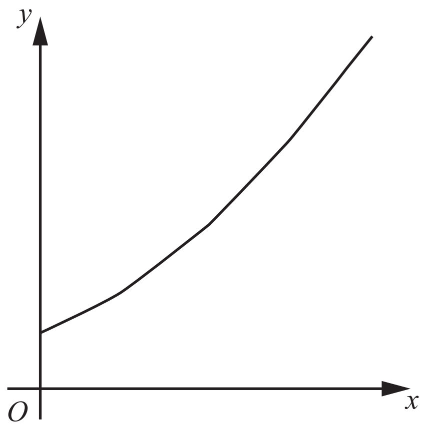

The diagram shows the curve $y = \mathrm{e}^{\sqrt{x+1}}$ for $x \geqslant 0$.

(i) Use the substitution $u^2 = x + 1$ to find $\int \mathrm{e}^{\sqrt{x+1}}\,\mathrm{d}x$. **[6]**

(ii) Make x the subject of the equation $y = \mathrm{e}^{\sqrt{x+1}}$. **[1]**

(iii) Hence show that $\int_{\mathrm{e}}^{\mathrm{e}^4} \left((\ln y)^2 - 1\right)\mathrm{d}y = 9\mathrm{e}^4$. **[4]**

END OF QUESTION PAPER

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