OCR
Oxford Cambridge and RSA

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Practice Paper – Set 1

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, Variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Section A: Pure Mathematics

Answer **all** the questions

1 Simplify

(i) $\dfrac{(3x)^3 \times 2x^{-1}}{9x^2}$, **[2]**

(ii) $(49x^{-4})^{-\frac{1}{2}}$. **[2]**

2 **(i)** Find and simplify the first three terms in the expansion of $\left(3-\dfrac{x}{2}\right)^6$ in ascending powers of x. **[3]**

(ii) Explain how the result in part **(i)** can be used to give an approximation to the value of $(2.95)^6$. **[1]**

3 Find the set of values of x for which

$$x^2 < x+6 \quad \text{or} \quad 3x+2 \geqslant 20-x.$$

Give your answer in set notation. **[6]**

4 A curve has equation $y = \frac{1}{4}x^4 - x^3 - 2x^2$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. **[1]**

(ii) Hence sketch the gradient function for the curve. **[4]**

(iii) By considering the x-intercepts of the graph drawn in part **(ii)**, determine the coordinates of the maximum point on the curve with equation $y = \frac{1}{4}x^4 - x^3 - 2x^2$. **[2]**

5 **In this question you must show detailed reasoning.**

The cubic polynomial $2x^3 - x^2 + kx + 6$ is denoted by $\mathrm{f}(x)$. It is given that $(x+2)$ is a factor of $\mathrm{f}(x)$.

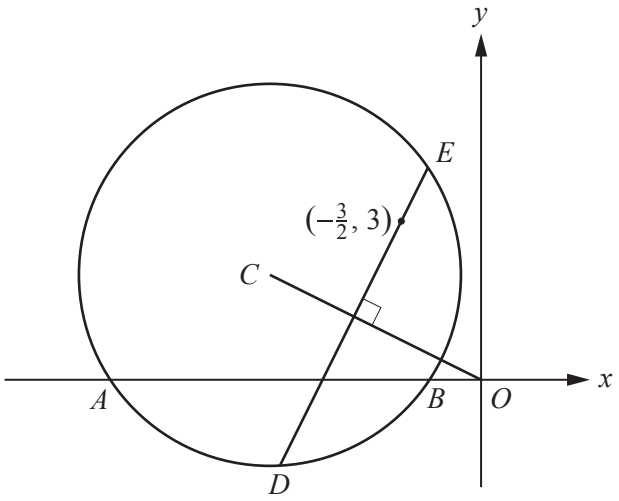
(i) Show that $k = -7$. **[2]**

(ii) Factorise $\mathrm{f}(x)$ completely. **[3]**

(iii) Hence solve the equation $2\mathrm{e}^{3y} - \mathrm{e}^{2y} - 7\mathrm{e}^{y} + 6 = 0$, giving each root in an exact form. **[4]**

6 Show that, for all values of k, the equation $x^2 + (k-5)x - 3k = 0$ has real roots. **[6]**

7



A circle with centre C has equation $x^2 + y^2 + 8x - 4y + 7 = 0$, as shown in the diagram. The circle meets the x-axis at A and B.

(i) Find

- the coordinates of C,
- the radius of the circle. **[3]**

(ii) Find the coordinates of the points A and B. **[2]**

The chord DE passes through the point $\left(-\frac{3}{2}, 3\right)$ and is perpendicular to OC, where O is the origin.

(iii) Find the coordinates of the points D and E. **[7]**

(iv) Hence find the area of the quadrilateral $BEAD$. **[2]**

Section B: Mechanics
Answer **all** the questions

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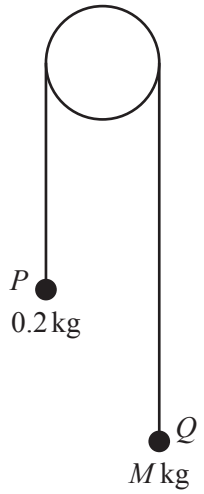

Particles P and Q, of masses 0.2 kg and M kg respectively, where $M > 0.2$, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley (see diagram). The system is in motion with the string taut and with each of the particles moving vertically.

The tension in the string is 2.1 N.

(i) Show that the acceleration of P is $0.7\,\text{m s}^{-2}$. **[2]**

(ii) Find the value of M. **[2]**

(iii) At one instant P has speed $0.3\,\text{m s}^{-1}$ upwards. Find its speed 1.5 seconds later, assuming that it has not yet reached the pulley. **[1]**

9 In this question the horizontal unit vectors **i** and **j** are in the directions east and north respectively.

A toy car of mass 0.5 kg is moving so that its acceleration vector $\mathbf{a}\,\text{m s}^{-2}$ at time t seconds is given by $\mathbf{a} = 6t\mathbf{i} + (2 - 3t^2)\mathbf{j}$. When $t = 2$ the horizontal force acting on the car is $\mathbf{F}$ N.

Find

- the magnitude of **F**,
- the bearing of **F**. **[5]**

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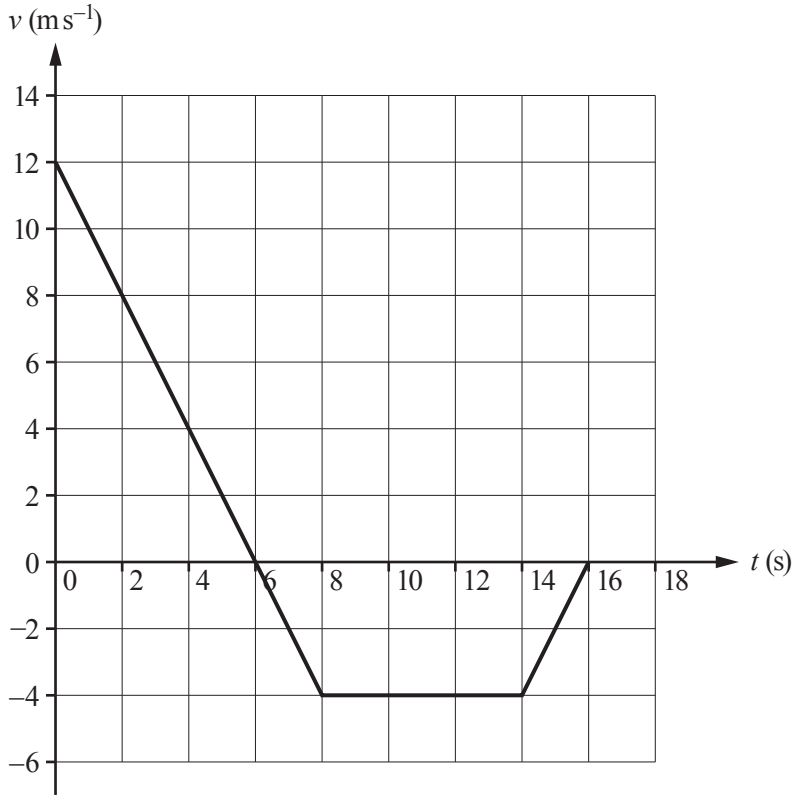

A particle is moving along a straight line. The motion of the particle is modelled by the velocity-time graph shown above, where v m s^{-1} is the velocity of the particle at time t s after it passes through a point A.

(i) Describe the motion of the particle between times $t = 0$ and $t = 8$. **[2]**

(ii) Calculate the acceleration of the particle at time $t = 3$. **[1]**

(iii) Find the displacement of the particle from A at time $t = 16$. **[3]**

A second model for the motion of the particle is given by $v = at^2 + bt + 12$, where a and b are constants. It is given that the two models agree on the value of v at times $t = 0$, $t = 6$ and $t = 16$.

(iv) Find the values of a and b. **[2]**

(v) Hence find, according to this second model,

- an expression in terms of t for the displacement of the particle from A,
- the distance travelled by the particle from its position when $t = 0$ to its position when $t = 16$. **[5]**

(vi) Calculate the time when the two models agree on the acceleration of the particle in the interval $0 \leqslant t \leqslant 8$. **[2]**

END OF QUESTION PAPER

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