OCR
Oxford Cambridge and RSA

Accredited

A Level Further Mathematics A
Y540 Pure Core 1
Sample Question Paper Version 2

Date – Morning/Afternoon
Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

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INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 Show that $\dfrac{5}{2-4\mathrm{i}} = \frac{1}{2} + \mathrm{i}$. **[2]**

2 **In this question you must show detailed reasoning.**

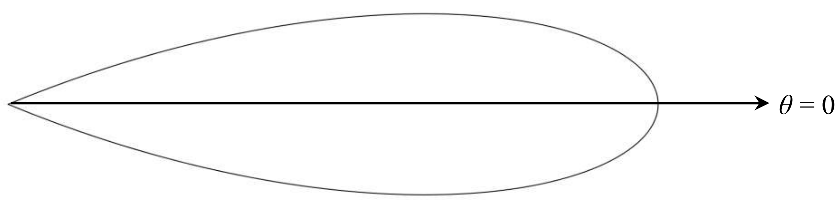
The equation $\mathrm{f}(x) = 0$, where $\mathrm{f}(x) = x^4 + 2x^3 + 2x^2 + 26x + 169$, has a root $x = 2 + 3\mathrm{i}$.

(i) Express $\mathrm{f}(x)$ as a product of two quadratic factors. **[4]**

(ii) Hence write down all the roots of the equation $\mathrm{f}(x) = 0$. **[1]**

3 **In this question you must show detailed reasoning.**

The diagram below shows the curve $r = 2\cos 4\theta$ for $-k\pi \le \theta \le k\pi$ where k is a constant to be determined.



Calculate the exact area enclosed by the curve. **[6]**

4 Draw the region in an Argand diagram for which $|z| \le 2$ and $|z| > |z - 3\mathrm{i}|$. **[3]**

5 **(i)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sinh^{-1}(2x)\right) = \dfrac{2}{\sqrt{4x^2+1}}$. **[2]**

(ii) Find $\displaystyle\int \frac{1}{\sqrt{2 - 2x + x^2}}\,\mathrm{d}x$. **[3]**

6 The equation $x^3 + 2x^2 + x + 3 = 0$ has roots α, β and γ.

The equation $x^3 + px^2 + qx + r = 0$ has roots $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$.

Find the values of p, q and r. **[5]**

7 The lines l_1 and l_2 have equations $\frac{x-3}{1}=\frac{y-5}{2}=\frac{z+2}{-3}$ and $\frac{x-4}{2}=\frac{y+2}{-1}=\frac{z-7}{4}$.

(i) Find the shortest distance between l_1 and l_2. **[5]**

(ii) Find a cartesian equation of the plane which contains l_1 and is parallel to l_2. **[2]**

8 **(i)** Find the solution to the following simultaneous equations.

$$\begin{aligned} x + \quad y + \quad z &= 3 \\ 2x + \quad 4y + \quad 5z &= 9 \\ 7x + \quad 11y + \quad 12z &= 20 \end{aligned}$$

[2]

(ii) Determine the values of p and k for which there are an infinity of solutions to the following simultaneous equations.

$$\begin{aligned} x + \quad y + \quad z &= 3 \\ 2x + \quad 4y + \quad 5z &= 9 \\ 7x + \quad 11y + \quad pz &= k \end{aligned}$$

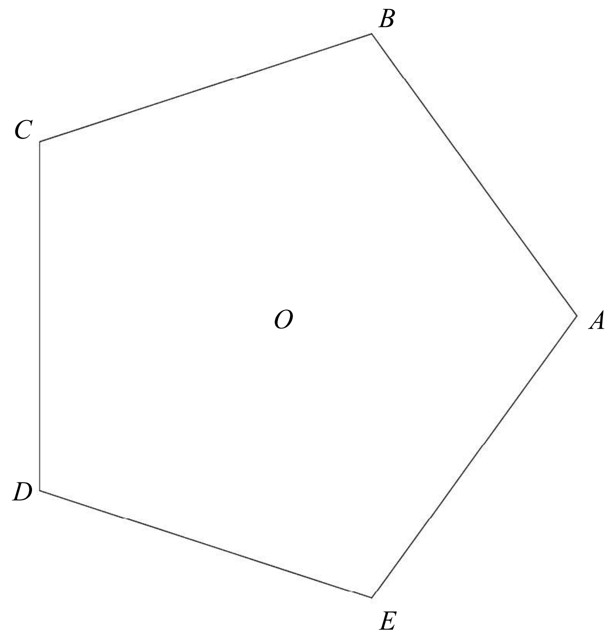
[6]

9 Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} \frac{5-4r}{5^r} = \frac{n}{5^n}.$$ **[5]**

10 The Argand diagram below shows the origin O and pentagon $ABCDE$, where A, B, C, D and E are the points that represent the complex numbers a, b, c, d and e, and where a is a positive real number.

You are given that these five complex numbers are the roots of the equation $z^5 - a^5 = 0$.



(i) Justify each of the following statements.

(a) A, B, C, D and E lie on a circle with centre O. **[1]**

(b) $ABCDE$ is a regular pentagon. **[2]**

(c) $b \times e^{\frac{2i\pi}{5}} = c$ **[1]**

(d) $b^* = e$ **[1]**

(e) $a + b + c + d + e = 0$ **[2]**

(ii) The midpoints of sides AB, BC, CD, DE and EA represent the complex numbers p, q, r, s and t. Determine a polynomial equation, with real coefficients, that has roots p, q, r, s and t. **[3]**

11 A company is required to weigh any goods before exporting them overseas. When a crate is placed on a set of weighing scales, the mass displayed takes time to settle down to its final value.

The company wishes to model the mass, m kg, which is displayed t seconds after a crate X is placed on the scales.

For the displayed mass it is assumed that the rate of change of the quantity $\left(0.5\frac{\mathrm{d}m}{\mathrm{d}t}+m\right)$ with respect to time is proportional to $(80-m)$.

(i) Show that $\frac{\mathrm{d}^2m}{\mathrm{d}t^2}+2\frac{\mathrm{d}m}{\mathrm{d}t}+2km=160k$, where k is a real constant. **[2]**

It is given that the complementary function for the differential equation in part **(i)** is $\mathrm{e}^{\lambda t}(A\cos 2t+B\sin 2t)$, where A and B are arbitrary constants.

(ii) Show that $k=\frac{5}{2}$ and state the value of the constant λ. **[4]**

When X is initially placed on the scales the displayed mass is zero and the rate of increase of the displayed mass is $160\,\mathrm{kg\,s^{-1}}$.

(iii) Find m in terms of t. **[7]**

(iv) Describe the long term behaviour of m. **[1]**

(v) With reference to your answer to part **(iv)**, comment on a limitation of the model. **[1]**

(vi) **(a)** Find the value of m that corresponds to the stationary point on the curve $m=\mathrm{f}(t)$ with the smallest positive value of t. **[2]**

(b) Interpret this value of m in the context of the model. **[1]**

(vii) Adapt the differential equation $\frac{\mathrm{d}^2m}{\mathrm{d}t^2}+2\frac{\mathrm{d}m}{\mathrm{d}t}+5m=400$ to model the mass displayed t seconds after a crate Y, of mass 100 kg, is placed on the scales. **[1]**

END OF QUESTION PAPER

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