OCR
Oxford Cambridge and RSA

A Level Further Mathematics A

Y543/01 Mechanics

Practice Paper – Set 2

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

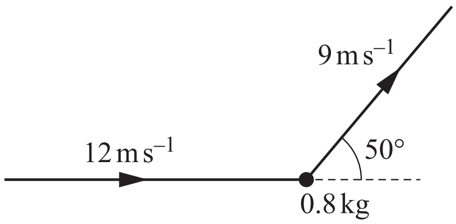
Answer **all** the questions.

1 A car of mass 850 kg is being driven uphill along a straight road inclined at 7° to the horizontal. The resistance to motion is modelled as a constant force of magnitude 140 N. At a certain instant the car's speed is $12\,\text{m}\,\text{s}^{-1}$ and its acceleration is $0.4\,\text{m}\,\text{s}^{-2}$.

(i) Calculate the power of the car's engine at this instant. **[3]**

(ii) Find the constant speed at which the car could travel up the hill with the engine generating this power. **[2]**

2 A particle of mass 0.8 kg is moving in a straight line on a smooth horizontal surface with constant speed $12\,\text{m}\,\text{s}^{-1}$ when it is struck by a horizontal impulse. Immediately after the impulse acts, the particle is moving with speed $9\,\text{m}\,\text{s}^{-1}$ at an angle of 50° to its original direction of motion (see diagram).



Find

(i) the magnitude of the impulse, **[3]**

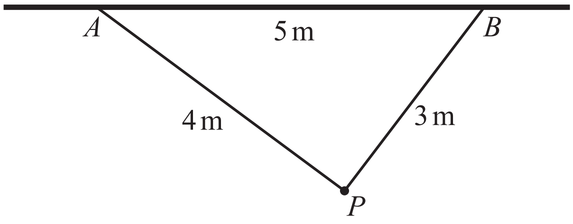
(ii) the angle that the impulse makes with the original direction of motion of the particle. **[3]**

3 Assume that the earth moves round the sun in a circle of radius 1.50×10^{8} km at constant speed, with one complete orbit taking 365 days. Given that the mass of the earth is 5.97×10^{24} kg,

(i) calculate the magnitude of the force exerted by the sun on the earth, giving your answer in newtons, **[5]**

(ii) state the direction in which this force acts. **[1]**

4 A and B are two points a distance of 5 m apart on a horizontal ceiling. A particle P of mass m kg is attached to A and B by light elastic strings. The particle hangs in equilibrium at a distance of 4 m from A and 3 m from B so that angle $APB = 90°$ (see diagram).



The string joining P to A has natural length 2 m and modulus of elasticity λ_A N. The string joining P to B also has natural length 2 m but has modulus of elasticity λ_B N.

(i) **(a)** Show that $\lambda_B = \frac{8}{3}\lambda_A$. **[4]**

(b) Find an expression for λ_A in terms of m and g. **[3]**

(ii) Find, in terms of m and g, the total elastic potential energy stored in the strings. **[2]**

The string joining P to A is detached from A and a second particle, Q, of mass $0.3m$ kg is attached to the free end of the string. Q is then gently lowered into a position where the system hangs vertically in equilibrium.

(iii) Find the distance of Q below B in this equilibrium position. **[4]**

5 One end of a non-uniform rod is freely hinged to a fixed point so that the rod can rotate about the point. When the rod rotates with angular velocity ω it can be shown that the kinetic energy E of the rod is given by $E = \frac{1}{2}I\omega^2$, where I is a quantity called the moment of inertia of the rod.

(i) Deduce the dimensions of I. **[3]**

(ii) Given that the rod has mass m and length r, suggest an expression for I, explaining any additional symbols that you use. **[3]**

A student notices that the formula $E = \frac{1}{2}I\omega^2$ looks similar to the formula $E = \frac{1}{2}mv^2$ for the kinetic energy of a particle, with angular velocity for the rod corresponding to velocity for the particle, and moment of inertia corresponding to mass. Assuming a similar correspondence between angular acceleration (i.e. $\frac{d\omega}{dt}$) and acceleration, the student thinks that an equation for angular motion of the rod corresponding to Newton's second law for the particle should be $F = I\alpha$, where F is the force applied to the rod and α is the resulting angular acceleration.

(iii) Use dimensional analysis to show that the student's suggestion is incorrect. **[2]**

(iv) State the dimensions of a quantity x for which the equation $Fx = I\alpha$ would be dimensionally consistent. **[1]**

(v) Explain why the fact that the equation in part **(iv)** is dimensionally consistent does not necessarily mean that it is correct. **[1]**

6 A particle P of mass m moves along the positive x-axis. When its displacement from the origin O is x its velocity is v, where $v \geqslant 0$. It is subject to two forces: a constant force T in the positive x direction, and a resistive force which is proportional to v^2.

(i) Show that $v^2 = \frac{1}{k}\left(T - A\mathrm{e}^{-\frac{2kx}{m}}\right)$ where A and k are constants. **[5]**

P starts from rest at O.

(ii) Find an expression for the work done against the resistance to motion as P moves from O to the point where $x = 1$. **[4]**

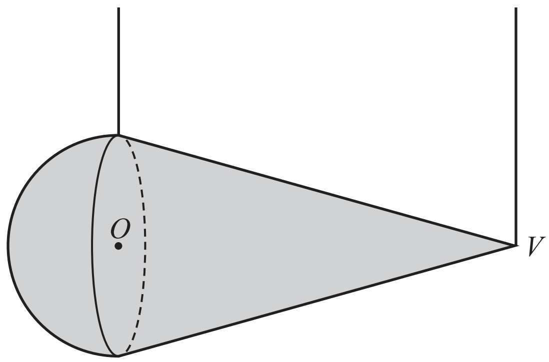
(iii) Find an expression for the limiting value of the velocity of P as x increases. **[1]**

7 A uniform solid hemisphere has radius 0.4 m. A uniform solid cone, made of the same material, has base radius 0.4 m and height 1.2 m. A solid, S, is formed by joining the hemisphere and the cone so that their circular faces coincide. O is the centre of the joint circular face and V is the vertex of the cone. G is the centre of mass of S.

(i) Explain briefly why G lies on the line through O and V. **[1]**

(ii) Show that the distance of G from O is 0.12 m.

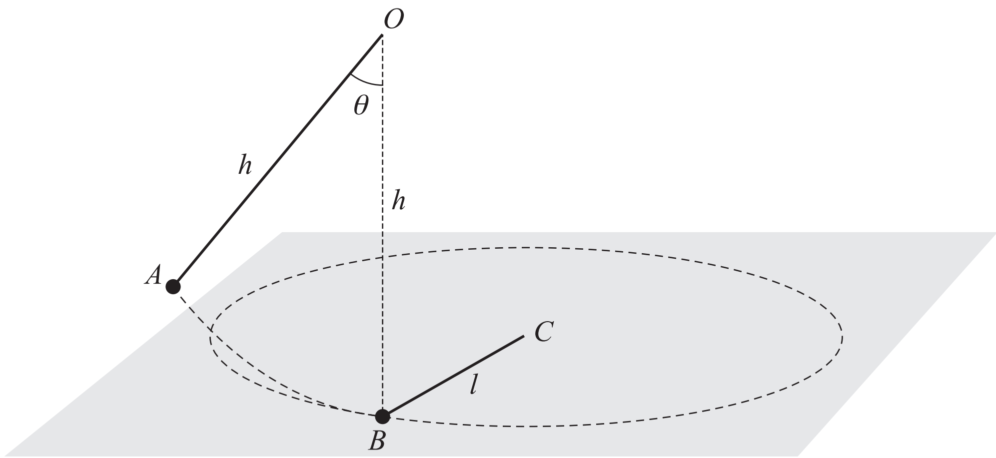
(The volumes of a hemisphere and cone are $\frac{2}{3}\pi r^3$ and $\frac{1}{3}\pi r^2 h$ respectively.) **[5]**



S is suspended from two light vertical strings, one attached to V and the other attached to a point on the circumference of the joint circular face, and hangs in equilibrium with OV horizontal (see diagram).

(iii) The weight of S is W. Find the magnitudes of the tensions in the strings in terms of W. **[3]**

8 A point O is situated a distance h above a smooth horizontal plane, and a particle A of mass m is attached to O by a light inextensible string of length h. A particle B of mass $2m$ is at rest on the plane, directly below O, and is attached to a point C on the plane, where $BC = l$, by a light inextensible string of length l. A is released from rest with the string OA taut and making an acute angle θ with the downward vertical (see diagram).



A moves in a vertical plane perpendicular to CB and collides directly with B. As a result of this collision, A is brought to rest and B moves on the plane in a horizontal circle with centre C. After B has made one complete revolution the particles collide again.

(i) Show that, on the next occasion that A comes to rest, the string OA makes an angle ϕ with the downward vertical through O, where $\cos\phi = \dfrac{3+\cos\theta}{4}$. **[9]**

A and B collide again when AO is next vertical.

(ii) Find the percentage of the original energy of the system that remains immediately after this collision. **[5]**

(iii) Explain why the total momentum of the particles immediately before the first collision is the same as the total momentum of the particles immediately after the second collision. **[1]**

(iv) Explain why the total momentum of the particles immediately before the first collision is different from the total momentum of the particles immediately after the third collision. **[1]**

END OF QUESTION PAPER

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