OCR
Oxford Cambridge and RSA

# A Level Mathematics A

## H240/01 Pure Mathematics

# Practice Paper – Set 1

## Time allowed: 2 hours

**You must have:**
- Printed Answer Booklet

**You may use:**
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

**INFORMATION**
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

# Formulae
# A Level Mathematics A (H240)

**Arithmetic series**

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

**Geometric series**

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

**Binomial series**

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$

**Differentiation**

| $\mathrm{f}(x)$ | $\mathrm{f}'(x)$ |
|---|---|
| $\tan kx$ | $k\sec^2 kx$ |
| $\sec x$ | $\sec x \tan x$ |
| $\cot x$ | $-\mathrm{cosec}^2 x$ |
| $\mathrm{cosec}\, x$ | $-\mathrm{cosec}\, x \cot x$ |

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

**Differentiation from first principles**

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

**Integration**

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where $\theta$ is measured in radians

 

**Trigonometric identities**

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

**Numerical methods**

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

**Probability**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

**Standard deviation**

$\sqrt{\dfrac{\sum(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\dfrac{\sum fx^2}{\sum f} - \bar{x}^2}$

**The binomial distribution**

If $X \sim B(n, p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, Mean of $X$ is $np$, Variance of $X$ is $np(1-p)$

**Hypothesis test for the mean of a normal distribution**

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

**Percentage points of the normal distribution**

If $Z$ has a normal distribution with mean 0 and variance 1 then, for each value of $p$, the table gives the value of $z$ such that $P(Z \leqslant z) = p$.

| $p$ | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.9975 | 0.999 | 0.9995 |
|---|---|---|---|---|---|---|---|---|---|
| $z$ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807 | 3.090 | 3.291 |

**Kinematics**

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

**1** **In this question you must show detailed reasoning.**

Express each of the following in the form $a+b\sqrt{2}$, where $a$ and $b$ are integers.

**(i)** $\sqrt{3}\left(\sqrt{12}+\sqrt{54}\right)$ **[3]**

**(ii)** $\dfrac{6}{2+\sqrt{2}}$ **[3]**

**2** **In this question you must show detailed reasoning.**

Solve the equation $3\sin^2\theta - 2\cos\theta - 2 = 0$ for $0° \leqslant \theta \leqslant 360°$. **[5]**

**3** **(i)** Express $2x^2+4x+5$ in the form $p(x+q)^2+r$, where $p$, $q$ and $r$ are integers. **[4]**

**(ii)** State the coordinates of the turning point on the curve $y=2x^2+4x+5$. **[2]**

**(iii)** Given that the equation $2x^2+4x+5=k$ has no real roots, state the set of possible values of the constant $k$. **[1]**

**4** A sheet of metal is a square of side 21 cm. Equal squares of side $x$ cm are cut from each corner, and the sheet is then folded to make an open box with vertical sides.

**(i)** Use calculus to find the value of $x$ that maximises the volume of the box. Justify that the volume is a maximum. **[6]**

**(ii)** State an assumption that is needed when answering part **(i)**. **[1]**

**5** The cubic polynomial f($x$) is defined by $\mathrm{f}(x)=x^3+4x^2-7x-10$.

**(i)** Given that $\mathrm{f}(-1)=0$, express f($x$) in a fully factorised form. **[3]**

**(ii)** Show the equation $\mathrm{e}^{3y}+4\mathrm{e}^{2y}-7\mathrm{e}^{y}-10=0$ has exactly one real root. State the exact value of this root. **[3]**

 

**6** The population of fish, $P$, in a lake is recorded at 10 day intervals. The table below shows the data collected, where $t$ is the number of days since the population was first recorded.

| $t$ | 0 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| $P$ | 20 | 24 | 29 | 34 | 42 | 50 |

It is proposed the population can be modelled by the equation $P = ab^t$, where $a$ and $b$ are constants.

**(i)** Complete the table of values in the Printed Answer Booklet. Plot the final three values of $\log_{10}P$ against $t$ on the axes provided. **[1]**

**(ii)** By drawing an appropriate straight line on your graph, find the values of $a$ and $b$. **[3]**

**(iii)** Use the model to predict the population of fish when $t = 200$. **[1]**

**(iv)** Explain why this prediction may not be reliable. **[1]**

**7** Find $\int (2x+1)\ln x \, \mathrm{d}x$. **[5]**

**8** The function f is defined as $\mathrm{f}(x) = \dfrac{8}{x+2}$ for $x \geqslant 0$.

**(i)** State the range of f. **[1]**

**(ii)** Find an expression for $\mathrm{f}^{-1}(x)$. **[2]**

**(iii)** Solve the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. **[2]**

**9** Fig. 1 shows a garden that is to be designed to include a lawn and a flowerbed.

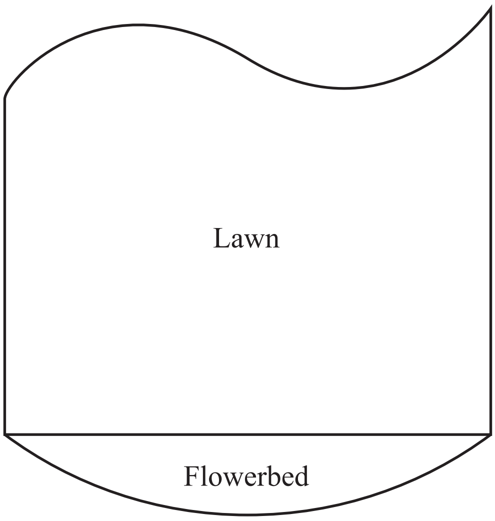


**Fig. 1**

The lawn can be modelled using four trapezia, as shown in Fig. 2. Each trapezium has a width of 1.5 m, and the lengths of the parallel sides are 8.0 m, 8.5 m, 8.2 m, 8.4 m and 8.6 m respectively.

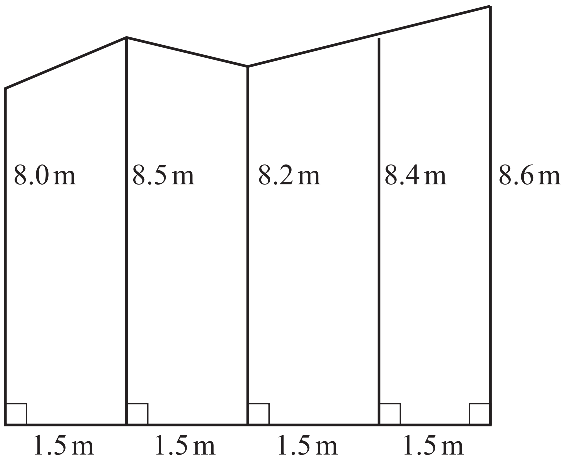


**Fig. 2**

**(i)** **(a)** Use the trapezium rule with 4 strips to estimate the area of the lawn. **[2]**

**(b)** Given that lawn seed costs £0.49 per square metre, estimate the total cost of the lawn seed required. **[1]**

**(ii)** Suggest two limitations of this model. **[2]**

**(iii)** Suggest one possible refinement of this model. **[1]**

The flowerbed can be modelled as the segment of a circle with radius 3.2 m. Fertiliser costs £0.17 per square metre.

**(iv)** Estimate the total cost of fertiliser required to cover the entire area of the flowerbed. **[5]**

**10** The first term in an arithmetic series is $(5t + 3)$, where $t$ is a positive integer. The last term is $(17t + 11)$ and the common difference is 4. Show that the sum of the series is divisible by 12 when, and only when, $t$ is odd. **[7]**

**11** **(i)** Find the first three terms in the expansion of $(1+2x)^{\frac{1}{2}}$ in ascending powers of $x$. **[3]**

**(ii)** Obtain an estimate of $\sqrt{3}$ by substituting $x = 0.04$ into your answer to part **(i)**. **[3]**

**(iii)** Explain why using $x = 1$ in the expansion would not give a valid estimate of $\sqrt{3}$. **[1]**

**12** **In this question you must show detailed reasoning.**

A curve has equation

$$x\sin y + \cos 2y = \frac{5}{2}$$

for $x \geqslant 0$ and $0 \leqslant y < 2\pi$.

Determine the exact coordinates of each point on the curve at which the tangent to the curve is parallel to the $y$-axis. **[9]**

**13** The equation $x^3 - x - 2 = 0$ has exactly one real root $\alpha$.

**(i)** Use the iterative formula $x_{n+1} = \sqrt[3]{x_n + 2}$ with $x_1 = 1$ to find $\alpha$ correct to 4 significant figures, showing the result of each iteration. **[3]**

**(ii)** An alternative iterative formula is $x_{n+1} = \mathrm{F}(x_n)$, where $\mathrm{F}(x_n) = \dfrac{x_n + 2}{x_n^2}$. By considering $\mathrm{F}'(\alpha)$, explain why this iterative process will not converge to $\alpha$. **[3]**

**14** A curve is defined by the parametric equations $x = \frac{2t}{1+t}$ and $y = \frac{t^2}{1+t}$, $t \neq -1$.

**(i)** **(a)** Show that the curve passes through the origin. **[1]**

**(b)** Find the $y$-coordinate when $x = 1$. **[1]**

**(ii)** Show that the area enclosed by the curve, the $x$-axis and the line $x = 1$ is given by

$$\int_0^1 \frac{2t^2}{(1+t)^3}\,\mathrm{d}t.$$

**[5]**

**(iii)** **In this question you must show detailed reasoning.**

Hence use an appropriate substitution to find the exact area enclosed by the curve, the $x$-axis and the line $x = 1$. **[6]**

**END OF QUESTION PAPER**