OCR
Oxford Cambridge and RSA

A Level Further Mathematics A

Y541/01 Pure Core 2

Practice Paper – Set 1

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 Plane Π has equation $3x - y + 2z = 33$. Line l has the following vector equation.

$$l: \quad \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$$

(i) Find the acute angle between Π and l. **[3]**

(ii) Find the coordinates of the point of intersection of Π and l. **[3]**

(iii) S is the point (4, 5, −5). Find the shortest distance from S to Π. **[2]**

2 The complex number $2 + \mathrm{i}$ is denoted by z.

(i) Show that $z^2 = 3 + 4\mathrm{i}$. **[2]**

(ii) Plot the following on the Argand diagram in the Printed Answer Booklet.

- z
- z^2

[1]

(iii) State the relationship between $|z^2|$ and $|z|$. **[1]**

(iv) State the relationship between $\arg(z^2)$ and $\arg(z)$. **[1]**

3 **In this question you must show detailed reasoning.**

Use the formula $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ to evaluate $121^2 + 122^2 + 123^2 + \ldots + 300^2$. **[3]**

4 You are given that the cubic equation $2x^3 - 3x^2 + x + 4 = 0$ has three roots, α, β and γ.

By making a suitable substitution to obtain a related cubic equation, determine the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. **[4]**

5 **In this question you must show detailed reasoning.**

An ant starts from a fixed point O and walks in a straight line for 1.5 s. Its velocity, $v\,\text{cm}\,\text{s}^{-1}$, can be modelled by $v = \dfrac{1}{\sqrt{9-t^2}}$.

By finding the mean value of v in $0 \leqslant t \leqslant 1.5$, deduce the average velocity of the ant. **[5]**

6 **In this question you must show detailed reasoning.**

(i) Find the coordinates of all stationary points on the graph of $y = 6\sinh^2 x - 13\cosh x$, giving your answers in an exact, simplified form. **[9]**

(ii) By finding the second derivative, classify the stationary points found in part **(i)**. **[3]**

7 In the following set of simultaneous equations, a and b are constants.

$$\begin{aligned} 3x + 2y - z &= 5 \\ 2x - 4y + 7z &= 60 \\ ax + 20y - 25z &= b \end{aligned}$$

(i) In the case where $a = 10$, solve the simultaneous equations, giving your solution in terms of b. **[3]**

(ii) Determine the value of a for which there is **no** unique solution for x, y and z. **[3]**

(iii) **(a)** Find the values of α and β for which $\alpha(2y - z) + \beta(-4y + 7z) = 20y - 25z$ for any y and z. **[3]**

(b) Hence, for the case where there is **no** unique solution for x, y and z, determine the value of b for which there is an infinite number of solutions. **[2]**

(c) When a takes the value in part **(ii)** and b takes the value in part **(iii)(b)** describe the geometrical arrangement of the planes represented by the three equations. **[1]**

8 **In this question you must show detailed reasoning.**

Show that $\displaystyle\int_0^2 \frac{2x^2 + 3x - 1}{x^3 - 3x^2 + 4x - 12}\,\mathrm{d}x = \frac{3}{8}\pi - \ln 9$. **[12]**

9 In this question you must show detailed reasoning.

(i) Show that $e^{i\theta} - e^{-i\theta} = 2i\sin\theta$. **[1]**

(ii) Hence, show that $\dfrac{2}{e^{2i\theta} - 1} = -(1 + i\cot\theta)$. **[3]**

(iii) Two series, C and S, are defined as follows.

$$C = 2 + 2\cos\frac{\pi}{10} + 2\cos\frac{\pi}{5} + 2\cos\frac{3\pi}{10} + 2\cos\frac{2\pi}{5}$$

$$S = 2\sin\frac{\pi}{10} + 2\sin\frac{\pi}{5} + 2\sin\frac{3\pi}{10} + 2\sin\frac{2\pi}{5}$$

By considering $C + iS$, find a simplified expression for C in terms of only integers and $\cot\dfrac{\pi}{20}$. **[8]**

(iv) Verify that $S = C - 2$ and, by considering the series in their original form, explain why this is so. **[2]**

END OF QUESTION PAPER