Oxford Cambridge and RSA

AS Level Further Mathematics A

Y533/01 Mechanics

Practice Paper – Set 2

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 A particle P of mass 2.4 kg is attached to one end of a light inextensible string of length 1.4 m. The other end of the string is attached to a fixed point O on a smooth horizontal table. P moves on the table at constant speed along a circular path with O at its centre. The magnitude of the tension in the string is 21 N.

(i) **(a)** Find the magnitude of the acceleration of P. **[2]**

(b) State the direction of the acceleration of P. **[1]**

(ii) Find the speed of P. **[2]**

(iii) Find the time taken for P to complete a single revolution. **[2]**

2 A pump is pumping still water from the base of a well at a constant rate of 300 kg per minute. The well is 4.5 m deep and water is released from the pump at ground level in a horizontal jet with a speed of $6.2\,\mathrm{m\,s^{-1}}$.

Ignoring any energy losses due to resistance, calculate the power generated by the pump. **[5]**

3 A student is investigating fluid flowing through a pipe.

In her first model she assumes a relationship of the form $P = S\rho^{\alpha}g^{\beta}h^{\gamma}$ where ρ is the density of the fluid, h is the length of the pipe, P is the pressure difference between the ends of the pipe, g is the acceleration due to gravity and S is a dimensionless constant. You are given that ρ is measured in $\mathrm{kg\,m^{-3}}$.

(i) Use the fact that pressure is force per unit area to show that $[P] = \mathrm{ML^{-1}T^{-2}}$. **[2]**

(ii) Find the values of α, β and γ. **[6]**

The density of the fluid the student is using is $540\,\mathrm{kg\,m^{-3}}$. In her experiment she finds that when the length of the pipe is 1.40 m the pressure difference between the ends of the pipe is $3.25\,\mathrm{N\,m^{-2}}$.

(iii) Find the length of the pipe for which her first model would predict a pressure difference between the ends of the pipe of $4.65\,\mathrm{N\,m^{-2}}$. **[2]**

In an alternative model the student suggests a modified relationship of the form $P = S\rho^{\alpha}g^{\beta}h^{\gamma} + \frac{1}{2}hv^2$, where v is the average velocity of the fluid in the pipe.

(iv) Use dimensional analysis to assess the validity of her alternative model. **[2]**

4 A car has a mass of 850 kg and its engine can generate a maximum power of 35 kW. The total resistance to motion of the car is modelled as kv N where $v\,\mathrm{m\,s^{-1}}$ is the speed of the car and k is a constant.

When the car is moving in a straight line on a straight horizontal road, the greatest constant speed that it can attain is $25\,\mathrm{m\,s^{-1}}$.

(i) Show that $k = 56$. **[2]**

(ii) Find the greatest possible acceleration of the car on the road at an instant when it is moving with a speed of $15\,\mathrm{m\,s^{-1}}$. **[3]**

A trailer of mass 240 kg is attached to the car by means of a light inextensible tow bar which is parallel to the surface of the road. The resistance to motion of the trailer is modelled as a constant force of magnitude 350 N.

The car and trailer move on the horizontal road. At a certain instant the car's engine is working at a rate of 30 kW and the acceleration of the car is $0.2\,\mathrm{m\,s^{-2}}$.

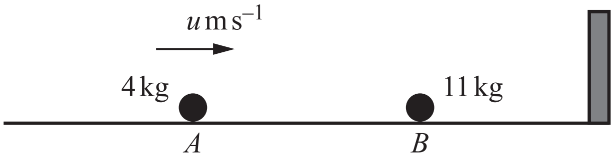
(iii) **(a)** Find the speed of the car at this instant. **[4]**

(b) Find the magnitude of the tension in the tow bar at this instant. **[2]**

The car and trailer now move in a straight line on a straight road inclined at 8° to the horizontal.

(iv) Find the difference between their greatest possible constant speed travelling up the slope and their greatest possible constant speed travelling down the slope. **[5]**

5 Two particles A and B are on a smooth horizontal floor with B between A and a vertical wall. The masses of A and B are 4 kg and 11 kg respectively. Initially, B is at rest and A is moving towards B with a speed of $u\,\mathrm{m\,s^{-1}}$ (see diagram). A collides directly with B. The coefficient of restitution between A and B is e.



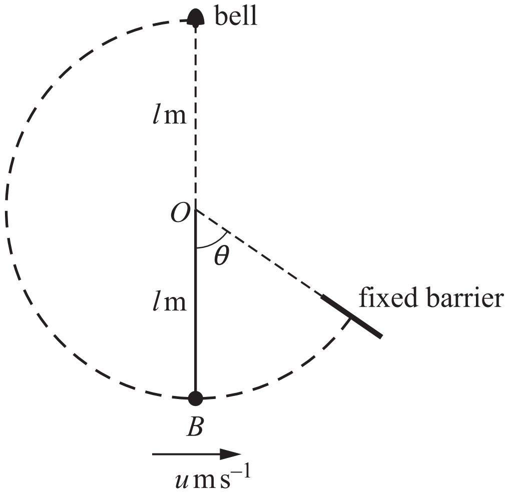
(i) Show that immediately after the collision the speed of B is $\frac{4}{15}u(1+e)$. **[4]**

After the collision between A and B the direction of motion of A is reversed. B subsequently collides directly with the vertical wall. The coefficient of restitution between B and the wall is $\frac{1}{2}e$.

(ii) Given that there is a second collision between A and B, find the range of possible values of e. **[7]**

6 A fairground game involves a player kicking a ball, B, from rest so as to project it with a horizontal velocity of magnitude $u\,\mathrm{m\,s^{-1}}$. The ball is attached to one end of a light rod of length l m. The other end of the rod is smoothly hinged at a fixed point O so that B can only move in the vertical plane which contains O, a fixed barrier and a bell which is fixed l m vertically above O.

Initially B is vertically below O. The barrier is positioned so that when B collides directly with the barrier, OB makes an angle θ with the downwards vertical through O (see diagram).



The coefficient of restitution between B and the barrier is e. B rebounds from the barrier, passes through its original position and continues on a circular path towards the bell. The bell will only ring if the ball strikes it with a speed of at least $V\,\mathrm{m\,s^{-1}}$. The player wins the game if the player causes the bell to ring having kicked B so that it first collides with the barrier. You may assume that B and the bell are small and that the barrier has negligible thickness.

Show that, whatever the position of the barrier, the player cannot win the game if $u^2 < 4gl + \dfrac{V^2}{e^2}$. **[9]**

END OF QUESTION PAPER