OCR
Oxford Cambridge and RSA

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Practice Paper – Set 1

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a\,(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots \qquad (|x| < 1,\ n \in \mathbb{N})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v}, \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1 - \tfrac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B|A) = \mathrm{P}(B)\mathrm{P}(A|B)$ **or** $\mathrm{P}(A|B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

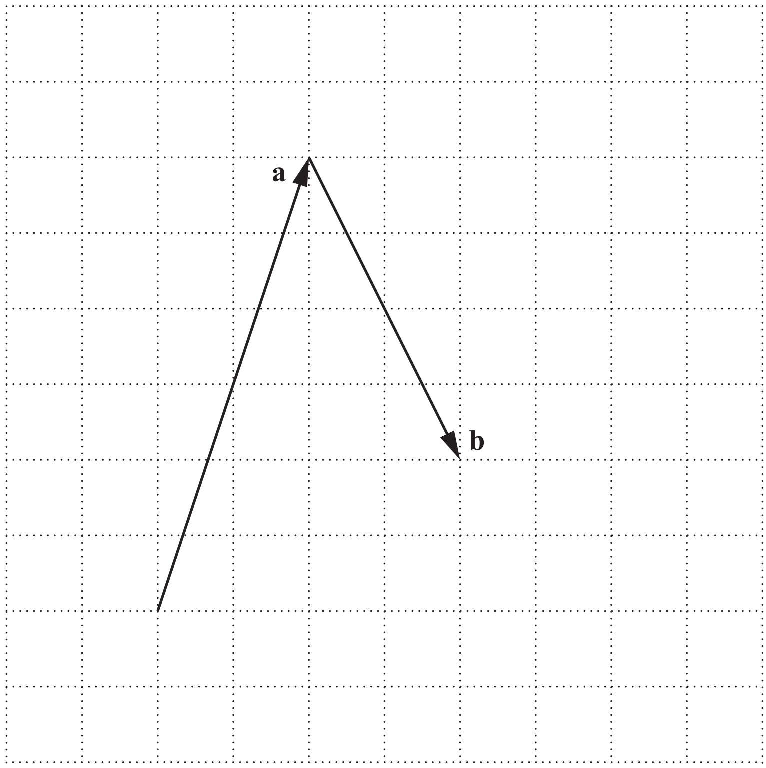
Section A: Pure Mathematics

Answer **all** the questions.

1 Vectors $\mathbf{a}$ and $\mathbf{b}$ are defined as follows: $\mathbf{a} = 2\mathbf{i} + 6\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - 4\mathbf{j}$.

(i) Given that $p\mathbf{a} + q\mathbf{b} = 6\mathbf{i} - 7\mathbf{j}$, find the values of the constants p and q. **[3]**

(ii) It is now given instead that $|\mathbf{a} + k\mathbf{b}| = 5$. Use the diagram in the Printed Answer Booklet to find the two possible values of the constant k. **[4]**



2 Find the area of the region enclosed by the curve $y = 5x - x^2$ and the line $y = 2x$. **[5]**

3 Differentiate $y = \cos x$ from first principles. **[6]**

4 It is given that n is an integer. Prove by contradiction the following statement.

n^2 is even $\Rightarrow$ n is even **[5]**

5 **In this question you must show detailed reasoning.**

(i) Solve the equation $\cos^2 x = 0.25$ for $0° \leqslant x < 180°$. **[3]**

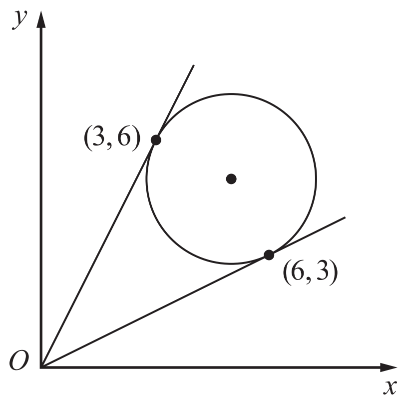
(ii) **(a)** Prove that $\dfrac{\cos\theta}{\cos\theta - \sin\theta} - \dfrac{\cos\theta}{\cos\theta + \sin\theta} \equiv \tan 2\theta$. **[3]**

(b) Hence or otherwise solve the equation

$$\frac{\cos\theta}{\cos\theta - \sin\theta} - \frac{\cos\theta}{\cos\theta + \sin\theta} = 1 \quad \text{for } 0° \leqslant \theta < 360°.$$ **[5]**

6 **In this question you must show detailed reasoning.**

A circle touches the lines $y = \frac{1}{2}x$ and $y = 2x$ at (6, 3) and (3, 6) respectively.



Find the equation of the circle. **[7]**

7 A new bird species is introduced into a region where it has previously been absent. Initially 20 birds are introduced. The rate of increase of the number N of birds after t years is modelled by

$$\frac{1}{1000}(10\,000 - N^2).$$

(i) Show that $N = \dfrac{100(3e^{0.2t} - 2)}{(3e^{0.2t} + 2)}$. **[7]**

(ii) Hence explain what will happen to the number of birds over a long period of time, as predicted by the model. **[2]**

(iii) State one limitation of the model. **[1]**

Section B: Statistics

Answer **all** the questions.

8 The variable X has the distribution $N(20, 4^2)$.

(i) Given that $P(X < a) = 0.1$, find a. **[1]**

(ii) Given that $P(b < X < c) = 0.95$, find a possible pair of values of b and c. **[2]**

9 Maria planned a statistical investigation into trees of a certain variety. She wished to test whether there is positive linear correlation between the height of a tree and the circumference of its trunk at the base.

(i) State, with a reason, whether a 1-tail or a 2-tail test is more appropriate. **[1]**

Maria recorded the height and circumference of a random sample of 10 trees of this variety in a wood near her home. She calculated the product-moment correlation coefficient for her sample and found that the value was 0.642.

(ii) Use the table below to carry out the test at the 2.5% significance level. **[5]**

(iii) Give two reasons why it would not be appropriate to use Maria's results to draw a conclusion about all trees of this variety. **[2]**

Critical values of Pearson's product-moment correlation coefficient.

	1-tail test	5%	2.5%	1%	0.5%
	2-tail test	10%	5%	2.5%	1%
n	9	0.5822	0.6664	0.7498	0.7977
	10	0.5494	0.6319	0.7155	0.7646
	11	0.5214	0.6021	0.6851	0.7348
	12	0.4973	0.5760	0.6581	0.7079

10 The heaviest 17% of rococo apples are classified as large, and the lightest 17% are classified as small. The remainder are classified as medium. The limits within which the masses of medium rococo apples lie are 96 g and 120 g. Stating a necessary assumption, estimate the mass of the heaviest rococo apple. **[4]**

11 On average, 40% of candidates pass a certain test on the first attempt.

Three candidates take the test. The number who pass on the first attempt is denoted by X.

(i) State an appropriate model for X, including the values of any parameters. **[1]**

(ii) State two necessary assumptions for your model to be valid. **[2]**

(iii) Suggest a reason why one of these assumptions might not be true in practice. **[1]**

You should now assume that both these assumptions are true.

(iv) Find the probability that exactly 2 of the 3 candidates pass the test. **[1]**

All candidates who fail the test take a re-test and, on average, 60% of these candidates pass. Assume that the same two assumptions are satisfied as for the original test.

(v) Find the probability that all three candidates pass, either on the test or on the re-test. **[3]**

12 The times taken by employees to complete a task are normally distributed with standard deviation 2.6 minutes. A manager claims that the mean time is 15.5 minutes but an employee suspects that the mean time is greater than this. He intends to carry out a hypothesis test at the 5% significance level to test this claim. He records the times taken by a random sample of 12 employees.

(i) Find the critical region for the test. **[3]**

(ii) The total of the times taken by the 12 employees was 202.1 minutes. Carry out the test. **[5]**

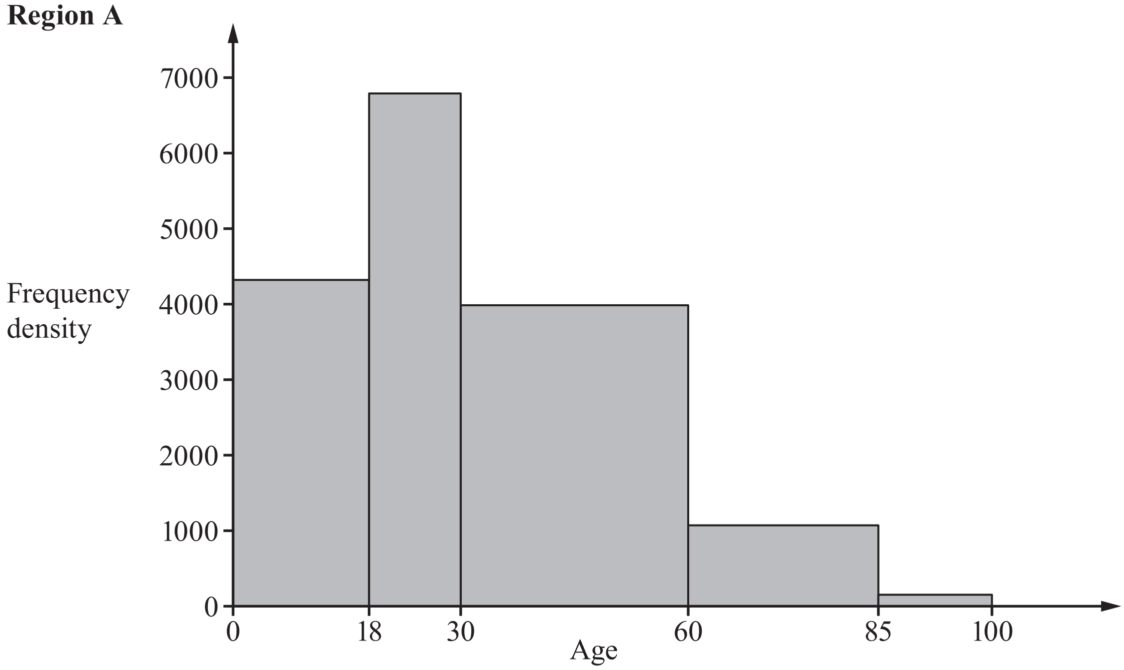
13 **(i)** Events A and B are independent, and $\mathrm{P}(A \cap B) = \frac{1}{24}$ and $\mathrm{P}(A \cup B) = \frac{3}{8}$.

Find $\mathrm{P}(A)$ and $\mathrm{P}(B)$. **[5]**

(ii) Events C and D are such that $\mathrm{P}(C) = 0.6$, $\mathrm{P}(D) = 0.3$ and $\mathrm{P}(C \cup D) = 0.8$. Find $\mathrm{P}(D|C')$. **[4]**

14 John used data from the 2011 UK census to produce the following histogram for region A.

Region A



In the Census report, the age classes were given as follows.

0 to 4	**5 to 7**	**8 to 9**	**10 to 14**	**15**	**16 to 17**	**18 to 19**	**20 to 24**	**25 to 29**	**30 to 44**	**45 to 59**	**60 to 64**	**65 to 74**	**75 to 84**	**85 to 89**	**90 and over**

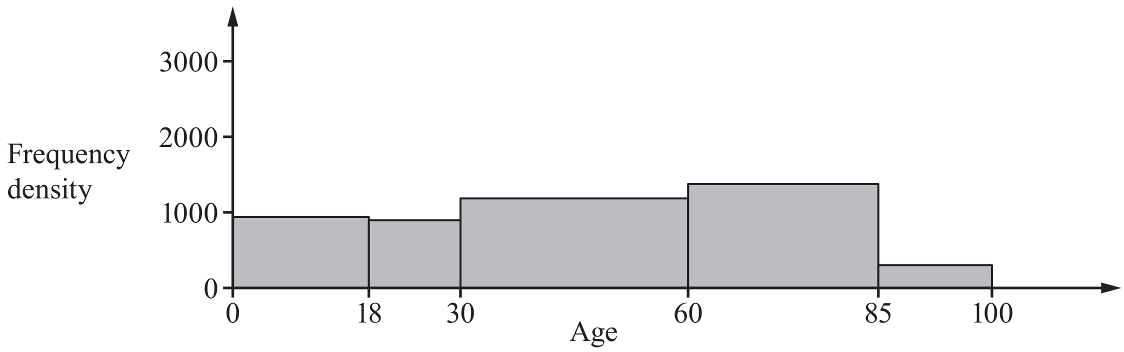
John combined classes to give the classes shown in the histogram.

(i) **(a)** Explain the reason for John's choice of upper class boundary for the first class. **[1]**

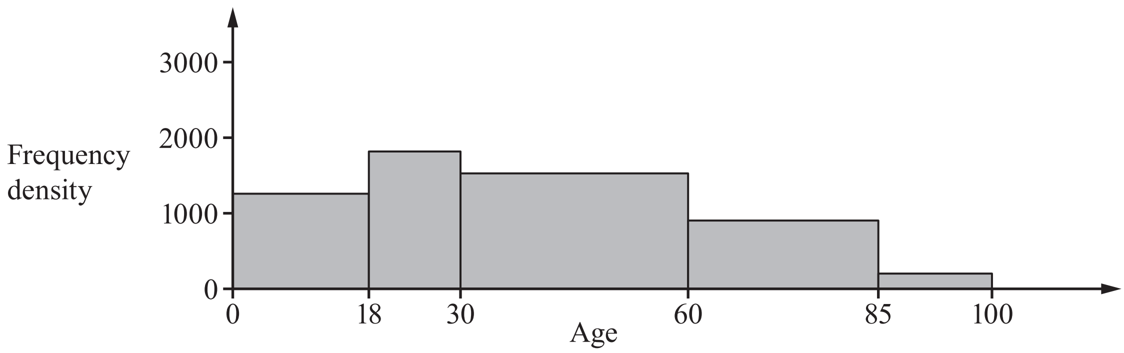
(b) Suggest a reason for John's choice of upper class boundary for the last class. **[1]**

John also produced similar histograms for two other UK regions, B and C.

Region B



Region C



(ii) Which of the three regions had the largest proportion of people aged 85 and over? Without detailed calculations, explain your answer. **[3]**

The mean ages, in years, of the populations in the three regions were 47.5, 39.5 and 31.5.

(iii) For each of these means, state the region to which it corresponds. Justify your answers. **[3]**

John made the following claim.

> "The histograms show that a child living in region B in 2011 could expect to live longer than a child living in region A in 2011."

(iv) Is this claim justified? Give a reason for your answer. **[1]**

END OF QUESTION PAPER

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