OCR
Oxford Cambridge and RSA

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Practice Paper – Set 2

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, Variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A: Pure Mathematics

Answer **all** the questions.

1 Show in a sketch the region of the x-y plane within which all three of the following inequalities are satisfied.

$$3y \geqslant 4x \qquad y - x \leqslant 1 \qquad y \geqslant (x-1)^2$$

You should indicate the region for which the inequalities hold by labelling the region R. **[4]**

2 The first term of a geometric progression is 12 and the second term is 9.

(i) Find the fifth term. **[3]**

The sum of the first N terms is denoted by S_N and the sum to infinity is denoted by S_∞. It is given that the difference between S_∞ and S_N is at most 0.0096.

(ii) Show that $\left(\frac{3}{4}\right)^N \leqslant 0.0002$. **[3]**

(iii) Use logarithms to find the smallest possible value of N. **[2]**

3 A sequence of three transformations maps the curve $y = \ln x$ to the curve $y = e^{3x} - 5$. Give details of these transformations. **[4]**

4 A curve is defined, for $t \geqslant 0$, by the parametric equations

$$x = t^2, \quad y = t^3.$$

(i) Show that the equation of the tangent at the point with parameter t is

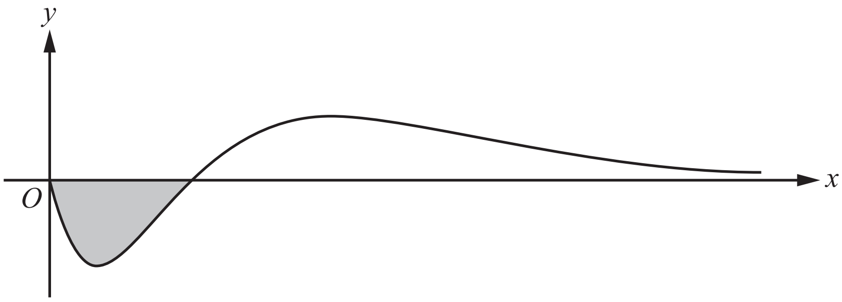
$$2y = 3tx - t^3.$$ **[4]**

(ii) In this question you must show detailed reasoning.

It is given that this tangent passes through the point $A\left(\frac{19}{12}, -\frac{15}{8}\right)$ and it meets the x-axis at the point B.

Find the area of triangle OAB, where O is the origin. **[7]**

5 In this question you must show detailed reasoning.



The function f is defined for the domain $x \geqslant 0$ by

$$f(x) = (2x^2 - 3x)e^{-x}.$$

The diagram shows the curve $y = f(x)$.

(i) Find the range of f. **[6]**

(ii) The function g is defined for the domain $x \geqslant k$ by

$$g(x) = (2x^2 - 3x)e^{-x}.$$

Given that g is a one-one function, state the least possible value of k. **[1]**

(iii) Find the exact area of the shaded region enclosed by the curve and the x-axis. **[7]**

6 **(i)** Determine the values of p and q for which

$$x^2 - 6x + 10 \equiv (x-p)^2 + q. \quad \textbf{[2]}$$

(ii) Use the substitution $x - p = \tan u$, where p takes the value found in part **(i)**, to evaluate

$$\int_3^4 \frac{1}{x^2 - 6x + 10}\,dx. \quad \textbf{[3]}$$

(iii) Determine the value of

$$\int_3^4 \frac{x}{x^2 - 6x + 10}\,dx,$$

giving your answer in the form $a + \ln b$, where a and b are constants to be determined. **[5]**

Section B: Mechanics

Answer **all** the questions.

7 Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle are given by

$$\mathbf{F}_1 = (3\mathbf{i} - 2a\mathbf{j})\,\mathrm{N}, \quad \mathbf{F}_2 = (2b\mathbf{i} + 3a\mathbf{j})\,\mathrm{N} \quad \text{and} \quad \mathbf{F}_3 = (-2\mathbf{i} + b\mathbf{j})\,\mathrm{N}.$$

The particle is in equilibrium under the action of these three forces.

Find the value of a and the value of b. **[3]**

8 A jogger is running along a straight horizontal road. The jogger starts from rest and accelerates at a constant rate of $0.4\,\mathrm{m\,s^{-2}}$ until reaching a velocity of $V\,\mathrm{m\,s^{-1}}$. The jogger then runs at a constant velocity of $V\,\mathrm{m\,s^{-1}}$ before decelerating at a constant rate of $0.08\,\mathrm{m\,s^{-2}}$ back to rest. The jogger runs a total distance of $880\,\mathrm{m}$ in $250\,\mathrm{s}$.

(i) Sketch the velocity-time graph for the jogger's journey. **[2]**

(ii) Show that $3V^2 - 100V + 352 = 0$. **[6]**

(iii) Hence find the value of V, giving a reason for your answer. **[3]**

9 Two particles A and B have position vectors $\mathbf{r}_A$ metres and $\mathbf{r}_B$ metres at time t seconds, where

$$\mathbf{r}_A = t^2\mathbf{i} + (3t - 1)\mathbf{j} \quad \text{and} \quad \mathbf{r}_B = (1 - 2t^2)\mathbf{i} + (3t - 2t^2)\mathbf{j}, \quad \text{for } t \geqslant 0.$$

(i) Find the values of t when A and B are moving with the same speed. **[5]**

(ii) Show that the distance, d metres, between A and B at time t satisfies

$$d^2 = 13t^4 - 10t^2 + 2.$$ **[3]**

(iii) Hence find the shortest distance between A and B in the subsequent motion. **[6]**

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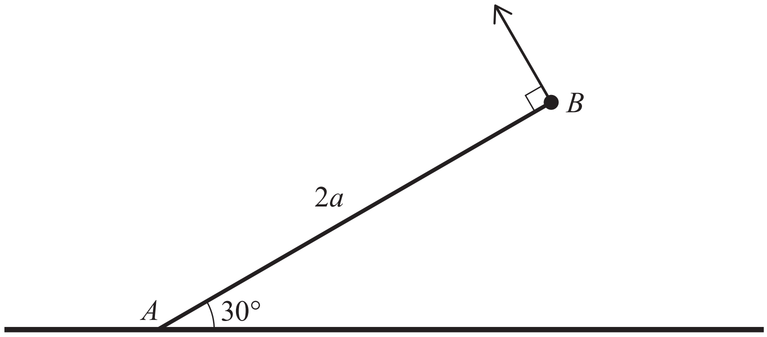
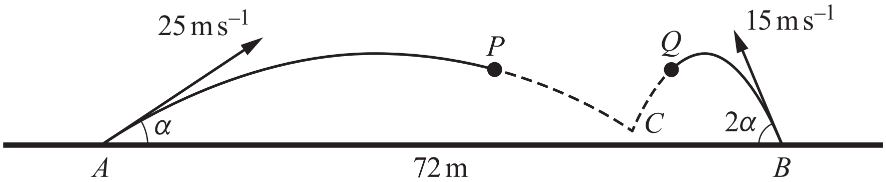


A uniform rod AB, of weight W N and length $2a$ m, rests with the end A on a rough horizontal table. A small object of weight $2W$ N is attached to the rod at B. The rod is maintained in equilibrium at an angle of 30° to the horizontal by a force acting at B in a direction perpendicular to the rod in the same vertical plane as the rod (see diagram).

(i) Find the least possible value of the coefficient of friction between the rod and the table. **[7]**

(ii) Given that the magnitude of the contact force at A is $\sqrt{39}$ N, find the value of W. **[2]**

11 **In this question you must show detailed reasoning.**



A football P is kicked with speed $25\,\text{m}\,\text{s}^{-1}$ at an angle of elevation α from a point A on horizontal ground. At the same instant a second football Q is kicked with speed $15\,\text{m}\,\text{s}^{-1}$ at an angle of elevation 2α from a point B on the same horizontal ground, where $AB = 72$ m. The footballs are modelled as particles moving freely under gravity in the same vertical plane and they collide with each other at the point C (see diagram).

(i) Calculate the height of C above the ground. **[7]**

(ii) Find the direction of motion of P at the moment of impact. **[4]**

(iii) Suggest one improvement that could be made to the model. **[1]**

END OF QUESTION PAPER