



Oxford Cambridge and RSA

# A Level Further Mathematics A

## Y541/01 Pure Core 2

### Practice Paper – Set 1

Time allowed: 1 hour 30 minutes

**You must have:**

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

#### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g\text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

#### INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1 Plane  $\Pi$  has equation  $3x - y + 2z = 33$ . Line  $l$  has the following vector equation.

$$l: \quad \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$$

- (i) Find the acute angle between  $\Pi$  and  $l$ . [3]
- (ii) Find the coordinates of the point of intersection of  $\Pi$  and  $l$ . [3]
- (iii)  $S$  is the point  $(4, 5, -5)$ . Find the shortest distance from  $S$  to  $\Pi$ . [2]

- 2 The complex number  $2 + i$  is denoted by  $z$ .

- (i) Show that  $z^2 = 3 + 4i$ . [2]
- (ii) Plot the following on the Argand diagram in the Printed Answer Booklet.
- $z$
  - $z^2$
- (iii) State the relationship between  $|z^2|$  and  $|z|$ . [1]
- (iv) State the relationship between  $\arg(z^2)$  and  $\arg(z)$ . [1]

- 3 In this question you must show detailed reasoning.

Use the formula  $\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$  to evaluate  $121^2 + 122^2 + 123^2 + \dots + 300^2$ . [3]

- 4 You are given that the cubic equation  $2x^3 - 3x^2 + x + 4 = 0$  has three roots,  $\alpha$ ,  $\beta$  and  $\gamma$ .

By making a suitable substitution to obtain a related cubic equation, determine the value of

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}. \quad [4]$$

**5 In this question you must show detailed reasoning.**

An ant starts from a fixed point  $O$  and walks in a straight line for 1.5 s. Its velocity,  $v \text{ cm s}^{-1}$ , can be modelled by  $v = \frac{1}{\sqrt{9-t^2}}$ .

By finding the mean value of  $v$  in  $0 \leq t \leq 1.5$ , deduce the average velocity of the ant. [5]

**6 In this question you must show detailed reasoning.**

(i) Find the coordinates of all stationary points on the graph of  $y = 6\sinh^2 x - 13\cosh x$ , giving your answers in an exact, simplified form. [9]

(ii) By finding the second derivative, classify the stationary points found in part (i). [3]

**7 In the following set of simultaneous equations,  $a$  and  $b$  are constants.**

$$\begin{aligned} 3x + 2y - z &= 5 \\ 2x - 4y + 7z &= 60 \\ ax + 20y - 25z &= b \end{aligned}$$

(i) In the case where  $a = 10$ , solve the simultaneous equations, giving your solution in terms of  $b$ . [3]

(ii) Determine the value of  $a$  for which there is **no** unique solution for  $x$ ,  $y$  and  $z$ . [3]

(iii) (a) Find the values of  $\alpha$  and  $\beta$  for which  $\alpha(2y - z) + \beta(-4y + 7z) = 20y - 25z$  for any  $y$  and  $z$ . [3]

(b) Hence, for the case where there is **no** unique solution for  $x$ ,  $y$  and  $z$ , determine the value of  $b$  for which there is an infinite number of solutions. [2]

(c) When  $a$  takes the value in part (ii) and  $b$  takes the value in part (iii)(b) describe the geometrical arrangement of the planes represented by the three equations. [1]

**8 In this question you must show detailed reasoning.**

Show that  $\int_0^2 \frac{2x^2 + 3x - 1}{x^3 - 3x^2 + 4x - 12} dx = \frac{3}{8}\pi - \ln 9$ . [12]

**9 In this question you must show detailed reasoning.**

(i) Show that  $e^{i\theta} - e^{-i\theta} = 2i \sin \theta$ . [1]

(ii) Hence, show that  $\frac{2}{e^{2i\theta} - 1} = -(1 + i \cot \theta)$ . [3]

(iii) Two series,  $C$  and  $S$ , are defined as follows.

$$C = 2 + 2 \cos \frac{\pi}{10} + 2 \cos \frac{\pi}{5} + 2 \cos \frac{3\pi}{10} + 2 \cos \frac{2\pi}{5}$$

$$S = 2 \sin \frac{\pi}{10} + 2 \sin \frac{\pi}{5} + 2 \sin \frac{3\pi}{10} + 2 \sin \frac{2\pi}{5}$$

By considering  $C + iS$ , find a simplified expression for  $C$  in terms of only integers and  $\cot \frac{\pi}{20}$ . [8]

(iv) Verify that  $S = C - 2$  and, by considering the series in their original form, explain why this is so. [2]

**END OF QUESTION PAPER**

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