OCR
Oxford Cambridge and RSA

A Level Further Mathematics A

Y543/01 Mechanics

Practice Paper – Set 3

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 A particle, P, of mass 2 kg moves in two dimensions. Its initial velocity is $\begin{pmatrix} -19.5 \\ -60 \end{pmatrix}$ m s^{-1}.

(a) Calculate the initial kinetic energy of P. **[2]**

For $t \geqslant 0$, P is acted upon only by a variable force $\mathbf{F} = \begin{pmatrix} 4t \\ -2 \end{pmatrix}$ N, where t is the time in seconds.

(b) Find

- the velocity of P in terms of t,
- the times when the power generated by $\mathbf{F}$ is zero. **[6]**

2 A car of mass 800 kg is driven with its engine generating a power of 15 kW.

(a) The car is first driven along a straight horizontal road and accelerates from rest.

Assuming that there is no resistance to motion, find the speed of the car after 6 seconds. **[2]**

(b) The car is next driven at constant speed up a straight road inclined at an angle θ to the horizontal. The resistance to motion is now modelled as being constant with magnitude 150 N.

Given that $\sin\theta = \frac{1}{20}$, find the speed of the car. **[3]**

(c) The car is now driven at a constant speed of 30 m s^{-1} along the horizontal road pulling a trailer of mass 150 kg which is attached by means of a light rigid horizontal towbar.

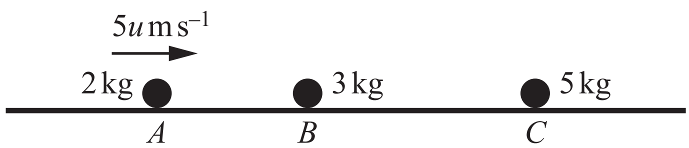
Assuming that the resistance to motion of the car is three times the resistance to motion of the trailer, find

- the resistance to motion of the car,
- the magnitude of the tension in the towbar. **[4]**

3 Three particles, A, B and C, of masses 2 kg, 3 kg and 5 kg respectively, are at rest in a straight line on a smooth horizontal plane with B between A and C.

Collisions between A and B are perfectly elastic. The coefficient of restitution for collisions between B and C is e.

A is projected towards B with a speed of $5u$ m s^{-1} (see diagram).



Show that only two collisions occur. **[8]**

4 A particle P of mass 8 kg moves in a straight line on a smooth horizontal plane. At time t s the displacement of P from a fixed point O on the line is x m and the velocity of P is $v\,\mathrm{m\,s^{-1}}$. Initially, P is at rest at O.

P is acted on by a horizontal force, directed along the line away from O, with magnitude proportional to $\sqrt{9+v^2}$. When $v = 1.25$, the magnitude of this force is 13 N.

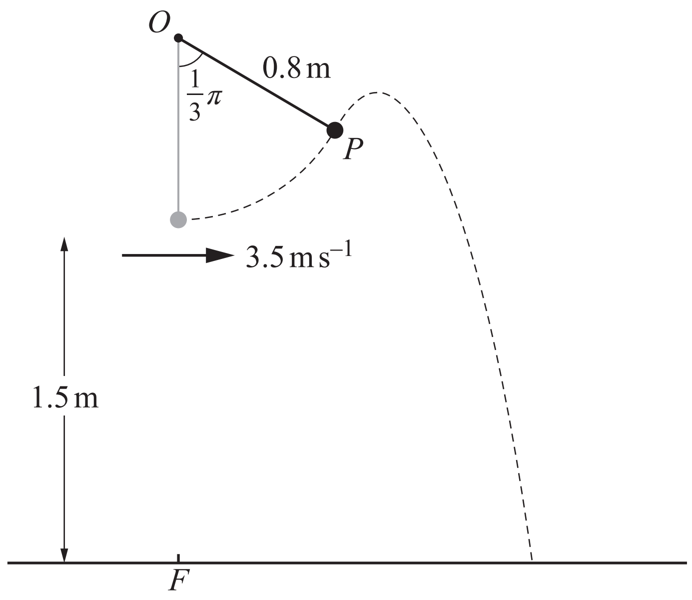
(a) Show that $\dfrac{1}{\sqrt{9+v^2}}\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{1}{2}$. **[3]**

(b) Find an expression for v in terms of t for $t \geqslant 0$. **[4]**

(c) Find an expression for x in terms of t for $t \geqslant 0$. **[2]**

5 One end of a light inextensible string of length 0.8 m is attached to a fixed point, O. The other end is attached to a particle P of mass 1.2 kg. P hangs in equilibrium at a distance of 1.5 m above a horizontal plane. The point on the plane directly below O is F.

P is projected horizontally with speed $3.5\,\mathrm{m\,s^{-1}}$. The string breaks when OP makes an angle of $\frac{1}{3}\pi$ radians with the downwards vertical through O (see diagram).



(a) Find the magnitude of the tension in the string at the instant before the string breaks. **[4]**

(b) Find the distance between F and the point where P first hits the plane. **[7]**

6 This question is about modelling the relation between the pressure, P, volume, V, and temperature, θ, of a fixed amount of gas in a container whose volume can be varied.

The amount of gas is measured in moles; 1 mole is a dimensionless constant representing a fixed number of molecules of gas. Gas temperatures are measured on the Kelvin scale; the unit for temperature is denoted by K. You may assume that temperature is a dimensionless quantity.

A gas in a container will always exert an outwards force on the walls of the container. The pressure of the gas is defined to be the magnitude of this force per unit area of the walls, with P always positive.

An initial model of the relation is given by $P^{\alpha}V^{\beta} = nR\theta$, where n is the number of moles of gas present and R is a quantity called the Universal Gas Constant. The value of R, correct to 3 significant figures, is $8.31\,\mathrm{J\,K^{-1}}$.

(a) Show that $[P] = \mathrm{ML^{-1}T^{-2}}$ and $[R] = \mathrm{ML^{2}T^{-2}}$. **[4]**

(b) Hence show that $\alpha = 1$ and $\beta = 1$. **[3]**

5 moles of gas are present in the container which initially has volume $0.03\,\mathrm{m^3}$ and which is maintained at a temperature of 300 K.

(c) Find the pressure of the gas, as predicted by the model. **[2]**

An improved model of the relation is given by $\left(P+\frac{an^2}{V^2}\right)(V-nb) = nR\theta$, where a and b are constants.

(d) Determine the dimensions of b and a. **[3]**

The values of a and b (in appropriate units) are measured as being 0.14 and 3.2×10^{-5} respectively.

(e) Find the pressure of the gas as predicted by the improved model. **[2]**

Suppose that the volume of the container is now reduced to $1.5\times10^{-4}\,\mathrm{m^3}$ while keeping the temperature at 300 K.

(f) By considering the value of the pressure of the gas as predicted by the improved model, comment on the validity of this model in this situation. **[3]**

7 Particles A, B and C of masses 2 kg, 3 kg and 5 kg respectively are joined by light rigid rods to form a triangular frame. The frame is placed at rest on a horizontal plane with A at the point (0, 0), B at the point (0.6, 0) and C at the point (0.4, 0.2), where distances in the coordinate system are measured in metres (see Fig. 1).

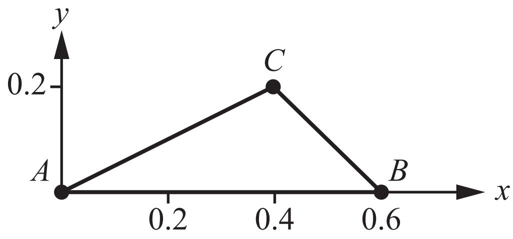


Fig. 1

G, which is the centre of mass of the frame, is at the point $(\bar{x}, \bar{y})$.

(a) • Show that $\bar{x} = 0.38$.

• Find $\bar{y}$. **[3]**

(b) Explain why it would be impossible for the frame to be in equilibrium in a horizontal plane supported at only one point. **[1]**

A rough plane, Π, is inclined at an angle θ to the horizontal where $\sin\theta = \frac{3}{5}$. The frame is placed on Π with AB vertical and B in contact with Π. C is in the same vertical plane as AB and a line of greatest slope of Π. C is on the down-slope side of AB.

The frame is kept in equilibrium by a horizontal light elastic string whose natural length is l m and whose modulus of elasticity is g N. The string is attached to A at one end and to a fixed point on Π at the other end (see Fig. 2).

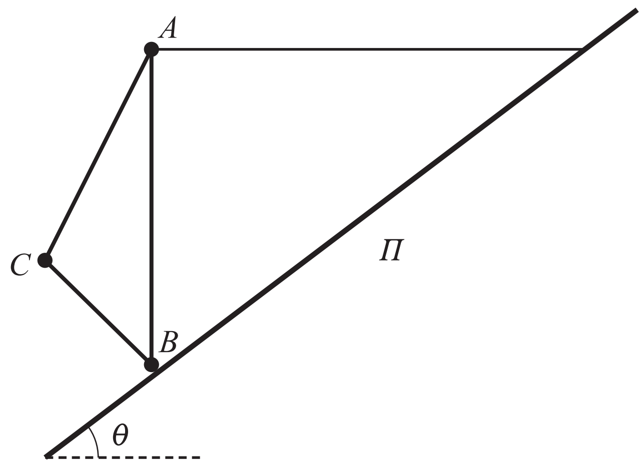


Fig. 2

The coefficient of friction between B and Π is μ.

(c) Show that $l = 0.3$. **[3]**

(d) Show that $\mu \geqslant \frac{14}{27}$. **[6]**

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