OCR
Oxford Cambridge and RSA

AS Level Further Mathematics A

Y531/01 Pure Core

Practice Paper – Set 2

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 **(i)** The complex number $3-4\mathrm{i}$ is denoted by z_1. Write z_1 in modulus-argument form, giving your angle in radians to 3 significant figures. **[3]**

(ii) The complex number z_2 has modulus 6 and argument -2.5 radians.
Express $z_1 z_2$ in modulus-argument form with the angle in radians correct to 3 significant figures. **[3]**

2 **In this question you must show detailed reasoning.**

The quadratic equation $3x^2 - 7x + 5 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. **[2]**

(ii) Hence find the values of the following expressions.

(a) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **[1]**

(b) $\alpha^2 + \beta^2$ **[2]**

3 l_1 and l_2 are two intersecting straight lines with the following equations.

$$l_1 : \mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} 1 \\ a \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix}$$

(i) Find the position vector of the point of intersection of l_1 and l_2. **[4]**

(ii) Determine the value of a. **[2]**

4 Find, in exact form, the area of the region on an Argand diagram which represents the locus of points for which $|z - 5 - 2\mathrm{i}| \leqslant \sqrt{32}$ and $\mathrm{Re}(z) \geqslant 9$. **[6]**

5 The matrix **A** is given by $\begin{pmatrix} 1 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and the matrix **B** is given by $\begin{pmatrix} 0.6 & b & 0 \\ -b & 0.6 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(i) **A** represents a reflection. Write down the value of det **A**. **[1]**

(ii) Hence find the possible values of a. **[2]**

(iii) **r** is the position vector of a point R. Given that $\mathbf{Ar} = \mathbf{r}$ describe the location of R. **[1]**

(iv) **B** represents a rotation. Write down the value of det **B**. **[1]**

(v) Hence find the possible values of b. **[2]**

6 The matrix **A** is given by $\begin{pmatrix} 1 & 2 \\ 1 & a \end{pmatrix}$ and the matrix **B** is given by $\begin{pmatrix} 2 & 1 \\ -1 & b \end{pmatrix}$.

(i) Find the matrix **AB**. **[2]**

(ii) State the conditions on a and b for **AB** to be a singular matrix. **[4]**

$PQRS$ is a quadrilateral and the vertices P, Q, R and S are in clockwise order. A transformation, T, is represented by the matrix **AB**.

(iii) State the effect on both the area and also the orientation of the image of $PQRS$ under T in each of the following cases.

(a) $a = 1$ and $b = 1$ **[2]**

(b) $a = 2$ and $b = 3$ **[2]**

7 **In this question you must show detailed reasoning.**

(i) Find the square roots of the number $528 + 46\mathrm{i}$ giving your answers in the form $a + b\mathrm{i}$. **[5]**

(ii) $3 + 2\mathrm{i}$ is a root of the equation $x^3 - ax + 78 = 0$, where a is a real number.

Find the value of a. **[4]**

8 In this question you must show detailed reasoning.

A sequence of vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, ... is defined by

- $\mathbf{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
- $\mathbf{a}_{n+1} = (\mathbf{a}_n \times \mathbf{b}) \times \mathbf{b}$, for integers $n \geqslant 1$, where $\mathbf{b}$ is the vector $\frac{1}{4}\begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$.

(i) Prove by induction that $\mathbf{a}_n = \left(-\frac{7}{8}\right)^{n-1}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. for all integers $n \geqslant 1$. **[8]**

(ii) Use an algebraic method to find the smallest value of n such that $|\mathbf{a}_n| < 0.001$. **[3]**

END OF QUESTION PAPER