Calibration Report Description

The report supplied with each calibration performed by Lake Shore Cryotronics consists of five parts: Certificate of Calibration, Data Plot, Test Data, Polynomial Equation, and Interpolation Table. A detailed discussion of each section follows.

Certificate of Calibration

This states the traceability of the calibrations performed by Lake Shore to international temperature scales and standards. Lake Shore calibrations are certified for one year.

Calibration Data

The measured test data (resistance or forward voltage) is graphed as a function of the temperature. A straight line interpolation is shown between the data points as a visual aid to the behavior of the sensor.

Calibration Data Plot

This table contains the actual calibration data recorded during the calibration of the temperature sensor. The indicated temperatures are those measured using the standard thermometers maintained by Lake Shore, while the voltage or resistance values are the measurements recorded on the device being calibrated. A description of the calibration system and the accuracy specifications for Lake Shore calibrations are described on the data sheet titled Low Temperature Calibration Service (Form F021-00-00).

Polynomial Equation

A polynomial equation based on the Chebychev polynomials has been fit to the calibration data. This equation is of the form:

$$T(x) = \sum_{i=0}^{n} a_i t_i(x) \qquad \text{Equation (1)}$$

where $T(x)$ represents the temperature in kelvin, $t_i(x)$ is a Chebychev polynomial and ai represents the Chebychev coefficients. The parameter x is a normalized variable given by:

$$x = \frac{(Z - ZL) - (ZU - Z)}{(ZU - ZL)} \qquad \text{Equation (2)}$$

For diodes, Z is simply the voltage V. For resistors, Z is either the resistance R or $Z = log_{10}(R)$. ZL and ZU designate the lower and upper limit of the variable Z over the fit range.

```
FUNCTION Chebychev (Z as double)as double
REM Evaluation of Chebychev series
   X=((Z-ZL)-(ZU-Z))/(ZU-ZL)
   Tc(0)=1
   Tc(1)=X
   T=A(0)+A(1)*X
   FOR I=2 to Ubound(A())
      Tc(I)=2*X*Tc(I-1)-Tc(I-2)
      T=T+A(I)*Tc(I)
   NEXT I
   Chebychev=T
END FUNCTION
```

Program 1. BASIC subroutine for evaluating the temperature T from the Chebychev series using Equations (1) and (3). An array $T_c(i_{degree})$ should be dimensioned. See text for details.

```
FUNCTION Chebychev (Z as double)as double
REM Evaluation of Chebychev series
    X=((Z-ZL)-(ZU-Z))/(ZU-ZL)
    T=0
    FOR I=0 to Ubound(A())
       T=T+A(I)*COS(I*ARCCOS(X))
    NEXT I
    Chebychev=T
END FUNCTION
```

NOTE: $\arccos(X) = \frac{\pi}{2} - \arctan\left[\frac{X}{\sqrt{1-X^2}}\right]$

Program 2. BASIC subroutine for evaluating the temperature T from the Chebychev series using Equations (1) and (4). Double precision calculations are recommended. See text for details.

The Chebychev polynomials can be generated from the recursion relation:

$$t_{i+1}(x) = 2x t_i(x) - t_{i-1}(x)$$
$$t_0(x) = 1, t_1(x) = x \qquad \text{Equation (3)}$$

Alternately, these polynomials are given by:

$$t_i(x) = \cos\left[i \times \arccos(x)\right] \qquad \text{Equation (4)}$$

All the necessary parameters for using Equations (1) thru (4) to calculate temperatures from either resistance or voltage are given in the calibration report. This includes the Chebychev coefficients, *ZL* and *ZU*, and also the definition of *Z*. Depending on the sensor being calibrated and the calibration range, several different fit ranges may be required to adequately span the full temperature range

The use of Chebychev polynomials is no more complicated than the use of the regular power series and they offer significant advantages in the actual fitting process. The first step is to transform the measured variable, either *R* or *V*, into the normalized variable using Equation (2). Equation (1) is then used in combination with Equation (3) or (4) to calculate the temperature. Programs 1 and 2 give sample BASIC subroutines which will take the parameter *Z* and return the temperature *T* calculated from the Chebychev fits. The subroutines assume the values *ZL* and *ZU* have been input along with the degree of the fit. The Chebychev coefficients are also assumed to be in an array $A(0), A(1), \ldots, A(i_{degree})$.

An interesting and useful property of the Chebychev fits is evident in the form of the Chebychev polynomial given in Equation (4). The cosine function means that $|t_n(x)| \leq 1$, and so no term in Equation (1) will be greater than the absolute value of the coefficient. This property makes it easy to determine the contribution of each term to the temperature calculation and where to truncate the series if the full accuracy of the fit is not required.

Along with each set of Chebychev coefficients, a deviation table is given to show how well the polynomial fits the measured test data. The table gives the measured resistance or voltage, the measured temperature and the temperature calculated from the fit equation. The last column gives the difference in millikelvin (0.001 K) between the measured value and the calculated value. A root mean square (RMS) deviation is given as an indication of the overall quality of the fit and as an indication of the accuracy with which the equation represents the calibration data.

Interpolation Table

A complete interpolation table is provided over the calibration range of the sensor. This table lists the temperature, the resistance (resistance sensors) or voltage (diode sensors), the slope (*dR/dT* or *dV/dT*), and in the case of resistors a normalized slope [*d(log R)/d(logT) = (T/R) (dR/dT)*]. Since the table generation requires *R(T)* or *V(T)*, the polynomials from the previous section which are *T(R)* or *T(V)* cannot be used. Instead, a cubic spline fitting technique is used for generating the interpolation table. Consequently, slight differences between the polynomial equations and the interpolation table are expected. These differences may be on the order of the RMS deviations for the polynomial fits.

Although the cubic spline technique does not yield a simple equation representing the calibration curve; the results are often a better representation of the data than the polynomial equation, particularly in the case of rapidly varying functions. This is true for silicon diodes near 20 K where the *V(T)* characteristics show a rapid change in behavior. For other temperature ranges and for resistance sensors in general, the differences between the spline technique and the polynomial fits will be considerably less than the measurement uncertainties.

For complete product description and detailed specifications on accessories and instruments, consult the Lake Shore Temperature Measurement and Control Catalog, call at (614) 891-2243, e-mail at sales@lakeshore.com, or visit our website at www.lakeshore.com.