How to properly calculate sigma images for stacked frames in SDSS:

We want a way of taking multiple frames containing (at least part of) a target extended source, and obtaining an image and a sigma image in units of nanomaggies. Simply averaging sigma images of individual frames in quadrature does not properly account for possible covariances, proposed here is a potentially more thorough solution.

In Theory

For each pixel, we have

$$\frac{I}{C} = \frac{n}{g} - S + V,$$

where I represents the sky-subtracted, corrected image (nanomaggies), C reprents the calibration image, n is the number of electrons captured, g is the gain, S is the Sky value (data units) and V is the dark current, $V = 0 \pm \sqrt{v}$ (v being the dark variance).

> Coleman: *SDSS lumps in the read-out noise (the thermal noise in the wires of the electronics on the telescope) with the dark variance… [this is a good reference textbook](). Bias is a zero sec exposure, dark is a 0 light exposure of the same length as the observation. These are combined with the "read noise" to form the dark variance provided by SDSS*

Given Poisson error,

$$\sigma_n = \sqrt{n}.$$

If we stack images, given N images of a pixel

$$\begin{aligned} n_{\text{total}} &= \sum_i n_i = \sum_i g_i \left(\frac{I_i}{C_i} + S_i - V_i \right), \\ &= \sum_i \frac{g_i}{C_i} I_i + \sum_i g_i \left(S_i - V_i \right) = \sigma^2_{n_{\text{total}}} . \end{aligned}$$

This is ideal, and is the level that many fitting software packages work at, we, however, want to return to working in units of nanomaggies on a stacked image, and so further calculation is needed:

$$I = \frac{1}{N} \sum_i I_i,$$

$$I = \frac{1}{N} \sum_i C_i \left(\frac{n_i}{g_i} - S_i + V_i \right),$$

And so

$$\sigma_I^2 = \frac{1}{N^2} \sum_i \frac{C_i^2}{g_i^2} \sigma_{n_i}^2 + \frac{1}{N^2} \sum_i C_i^2 \sigma_{S_i}^2 + \frac{1}{N^2} \sum_i C_i^2 \sigma_{V_i}^2.$$

We treat the sky value as a constant, such that $\sigma_{S_i}^2 = 0$. Substituting $\sigma_{n_i}^2 = n_i$ as above gives

$$\sigma_I^2 = \frac{1}{N^2} \sum_i \frac{C_i^2}{g_i^2} n_i + \frac{1}{N^2} \sum_i C_i^2 v_i.$$

$$\sigma_I = \frac{1}{N} \sqrt{\sum_i C_i^2 \left(\frac{n_i}{g_i^2} + v_i \right)}.$$

Note that this is identical to saying

$$\sigma_I^2 = \frac{1}{N^2} \sum_i \sigma_{I_i}^2.$$

Weighted stacking

A (potentially) better way of stacking images would be to use a weighted average for pixel values, in which case

$$I = \frac{\sum_i \sigma_{I_i}^{-2} I_i}{\sum_i \sigma_{I_i}^{-2}}$$

The standard error of the weighted mean is thus

$$\sigma_I = \sqrt{\frac{1}{\sum_i \sigma_{I_i}^{-2}}}$$

$$\sigma_I = \left[\sum_i C_i^2 \left(\frac{n_i}{g_i^2} + v_i \right) \right]^{-\frac{1}{2}}$$

(this has not been implemented)

In Practise

- Our frames are not aligned.
- Calculate n_i, the electron counts for each frame
- For each frame, create a slightly larger than required cutout of nelec and the Calibration image
- Use `reproject` to align the electron counts and the calibration images of each frame to the WCS of the FITS header of the `Montage`-created image (which is what volunteer models were drawn on).
- Proceed with the above calculation
  - Note that N will not be the same for each pixel, as some regions of the image may be covered by different numbers of frames

- Once we have $\bar{I}$ and σ_I, perform a cutout of the required size for each and return!