How to properly calculate sigma images for stacked frames in SDSS:

We want a way of taking multiple frames containing (at least part of) a target extended source, and obtaining an image and a sigma image in units of nanomaggies. Simply averaging sigma images of individual frames in quadrature does not properly account for possible covariances, proposed here is a potentially more thorough solution.

In Theory

For each pixel, we have

$$\frac{I}{C} = \frac{n}{g} - S,$$

where I represents the sky-subtracted, corrected image (nanomaggies), C reprents the calibration image, n is the number of electrons captured, g is the gain and S is the Sky value (data units).

Given Poisson error,

$$\sigma_n = \sqrt{n}.$$

If we stack images, given N images of a pixel

$$n_{\text{total}} = \sum_i n_i = \sum_i g_i \left(\frac{I_i}{C_i} + S_i \right),$$

$$\sigma^2_{n_{\text{total}}} = \sum_i \frac{g_i}{C_i} I_i + \sum_i g_i S_i.$$

Meaning

$$\sum_i \frac{g_i}{C_i} I_i = \sum_i n_i - \sum_i g_i S_i.$$

Assuming g_i and C_i are nearly constant gives

$$\frac{\langle g \rangle}{\langle C \rangle} \sum_i I_i = \sum_i n_i - \langle g \rangle \sum_i S_i,$$

meaning that the average image in nanomaggies is given by

$$\bar{I} = \frac{1}{N}\sum_i I_i = \frac{\langle C\rangle}{N\langle g\rangle}\sum_i n_i - \frac{\langle C\rangle}{N}\sum_i S_i.$$

The corresponding error is therefore given by

$$\sigma_I^2 = \sum_i \left(\frac{\mathrm{d}\bar{I}}{\mathrm{d}n_i}\right)^2 \sigma_{n_i}^2 + \sum_i \left(\frac{\mathrm{d}\bar{I}}{\mathrm{d}S_i}\right)^2 \sigma_{S_i}^2,$$

$$\sigma_I^2 = \sum_i \left(\frac{1}{N}\frac{\langle C\rangle}{\langle g\rangle}\right)^2 \sigma_{n_i}^2 + \sum_i \frac{\langle C\rangle^2}{N^2}\sigma_{S_i}^2.$$

Substituting $\sigma_{n_i}^2 = n_i$ as above, and $\sigma_{S_i}^2 = v_i$ for the dark-variance of an image

$$\sigma_I^2 = \frac{\langle C\rangle^2}{N^2}\left(\frac{1}{\langle g\rangle^2}\sum_i n_i - \sum_i v_i\right),$$

$$\sigma_I = \sqrt{\sum_i \frac{n_i}{\langle g\rangle^2} + \sum_i v_i}\ \frac{\langle C\rangle}{N}.$$

In Practise

- Our frames are not aligned.
- For each frame, create a slightly larger than required cutout of the Image, the Sky and the Calibration image
- Calculate n_i, the electron counts for each frame
- Use `reproject` to align to the WCS of the FITS header of the `Montage`-created image shown to volunteers.
- Proceed with the above
  - Note that N will not be the same for each pixel, as some regions of the image may be covered by different numbers of frames
- Once we have $\bar{I}$ and σ_I, perform a cutout of the required size for each and return!
- **MAKE LOTS OF SAVE POINTS** (i.e. write out the cutout + reprojected FITS files)