

## Assignment 4

### CSC420 Introduction to Image Understanding

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**I Deep Learning** We have the computation graph, written symbolically (with the corresponding variable names in the code `q1.py`)

$$\text{sum1} = \Sigma_1 = w_1x_1 + w_2x_2 \quad (\text{I.1})$$

$$\text{sum2} = \Sigma_2 = w_3x_3 + w_4x_4 \quad (\text{I.2})$$

$$\text{sigma1} = \sigma_1 = \sigma(\Sigma_1) \quad (\text{I.3})$$

$$\text{sigma2} = \sigma_2 = \sigma(\Sigma_2) \quad (\text{I.4})$$

$$\text{sum3} = \Sigma_3 = w_5\sigma_1 + w_6\sigma_2 \quad (\text{I.5})$$

$$\text{sigma3} = \sigma_3 = \sigma(\Sigma_3) \quad (\text{I.6})$$

$$\text{yhat} = \hat{y} = \sigma_3 \quad (\text{I.7})$$

$$L = L = \|\hat{y} - y\|_2^2 \quad (\text{I.8})$$

In `q1.py`, I calculated the forward pass and then back propagated the error signal. It prints the following result, including intermediate values:

```
-> Initialization ... <-
x1 = 0.90000, x2 = -1.10000, x3 = -0.30000, x4 = 0.80000
w1 = 0.75000, w2 = -0.63000, w3 = 0.24000,
w4 = -1.70000, w5 = 0.80000, w6 = -0.20000
target y = 0.50000
-> Start Forward Pass <-
sum1 = 1.36800, sigma1 = 0.79706, sum2 = -1.43200
sigma2 = 0.19279, sum3 = 0.59909, sigma3 = 0.64545
yhat = 0.64545, L = 0.02116
-> Start Back Propagation <-
dLdL = 1.00000, dLdyhat = 0.29090, dLdsigma3 = 0.29090
dLdsum3 = 0.06657, dLdsigma2 = -0.01331, dLdsum2 = -0.00207
-> End Result: dLdw3 = 0.0006215780
```

Then, our desired final result is

$$\frac{\partial L}{\partial w_3} = .0006215780 \quad (\text{I.9})$$

## II Camera Models

**II.I Part 1** First we expand as hinted,

$$p = \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = KP = K \begin{bmatrix} X_0 + td_x \\ Y_0 + td_y \\ Z_0 + td_z \end{bmatrix} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_0 + td_x \\ Y_0 + td_y \\ Z_0 + td_z \end{bmatrix} \quad (\text{II.1})$$

$$= \begin{bmatrix} f(X_0 + td_x) + p_x(Z_0 + td_z) \\ f(Y_0 + td_y) + p_y(Z_0 + td_z) \\ Z_0 + td_z \end{bmatrix} \quad (\text{II.2})$$

then we can solve for  $x, y$  in where  $p = (wx, wy, w)^\top$ . We have

$$x = \frac{wx}{w} = \frac{f(X_0 + td_x) + p_x(Z_0 + td_z)}{Z_0 + td_z} \quad (\text{II.3})$$

and

$$y = \frac{wy}{w} = \frac{f(Y_0 + td_y) + p_y(Z_0 + td_z)}{Z_0 + td_z} \quad (\text{II.4})$$

which are (parametric) coordinates for the line on the image plane. To find the pixel coordinates of the vanishing point corresponding to the line, it suffices to take limit of  $t$  tends to infinity,

$$x_v = \lim_{t \rightarrow \infty} \frac{f(X_0 + td_x) + p_x(Z_0 + td_z)}{Z_0 + td_z} = \lim_{t \rightarrow \infty} \frac{ftd_x + p_xtd_z}{td_z} = \frac{fd_x + p_xd_z}{d_z} \quad (\text{II.5})$$

and

$$y_v = \lim_{t \rightarrow \infty} \frac{f(Y_0 + td_y) + p_y(Z_0 + td_z)}{Z_0 + td_z} = \lim_{t \rightarrow \infty} \frac{ftd_y + p_ytd_z}{td_z} = \frac{fd_y + p_yd_z}{d_z} \quad (\text{II.6})$$

Then, the vanishing point is, on the image plane,  $(x_v, y_v)$ , defined everywhere except for when  $d_z = 0$ , in which case vanishing point does not exist.

**II.II Part 2** What we have shown in Part 1 is true for all  $d$ . Now that we want to show for all lines that are on the plane, then it suffices to constraint  $d$  such that  $\langle n, d \rangle = 0$ , where  $n$  is the normal vector to the plane. This gives us an extra equation  $n_x d_x + n_y d_y + n_z d_z = 0$ . Our goal here is to show that  $\alpha x_v + \beta y_v = \text{constant}$  that is independent of  $d$  for some  $\alpha, \beta$ . (Notice that  $x_v, y_v$  is already independent of  $P$ , the starting point.) We can start with the inner product constraint, which is

$$n_x d_x + n_y d_y + n_z d_z = 0 \quad (\text{II.7})$$

then since  $n_z d_z \neq 0$  (otherwise there is no point in proving this) we have

$$n_x d_x + n_y d_y + n_z d_z = 0 \quad (\text{II.8})$$

$$\implies \frac{n_x d_x + n_y d_y}{n_z d_z} + 1 = 0 \quad (\text{II.9})$$

$$\implies \frac{n_x}{n_z} \frac{d_x}{d_z} + \frac{n_y}{n_z} \frac{d_y}{d_z} + 1 = 0 \quad (\text{II.10})$$

$$\implies \frac{d_x}{d_z} = -\frac{n_z}{n_x} - \frac{n_z n_y d_y}{n_x n_z d_z} \quad (\text{II.11})$$

$$\implies \frac{d_x}{d_z} = -\frac{n_z}{n_x} - \frac{n_y d_y}{n_z d_z} \quad (\text{II.12})$$

We can substitute this into what we had in part 1, i.e.

$$y_v = \frac{f d_y + p_y d_z}{d_z} = f \frac{d_y}{d_z} + p_y \quad (\text{II.13})$$

$$= f \left( -\frac{n_z}{n_x} - \frac{n_y d_y}{n_z d_z} \right) + p_y \quad (\text{II.14})$$

$$= -f \frac{n_z}{n_x} - f \frac{n_y d_y}{n_z d_z} + p_y = (\dagger) \quad (\text{II.15})$$

also,

$$x_v = \frac{f d_x + p_x d_z}{d_z} = f \frac{d_x}{d_z} + p_x \implies \frac{d_x}{d_z} = \frac{x_v - p_x}{f} \quad (\text{II.16})$$

Then,

$$y_v = (\dagger) = -f \frac{n_z}{n_x} - f \frac{n_y (x_v - p_x)}{n_z f} + p_y \quad (\text{II.17})$$

$$\implies -f^2 n_z^2 - f n_z n_y (x_v - p_x) + n_x n_z f p_y = n_x n_z f y_v \quad (\text{II.18})$$

$$\implies \underbrace{-f n_z n_y x_v}_{\alpha} + \underbrace{-f^2 n_z^2 + f n_z n_y p_x + n_x n_z f p_y}_{\text{constant independent of } d} = \underbrace{n_x n_z f}_{\beta} y_v \quad (\text{II.19})$$

which has exactly the form that we want in the first place, and it illustrates the linear relationship between  $x_v$  and  $y_v$ .