A Tutorial of Casual Inference

Tinghua Chen

College of Information Sciences and Technology
Penn State University

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Outlines

Outlines

Simpson's Paradox

	Control Group		Treatment Group	
	Heart attack	No heart attack	Heart attack	No heart attack
Female	1	19	3	37
Male	12	28	8	12
Total	13	47	11	49

- $\frac{1}{20} = 5\%$ *vs* $\frac{3}{40} = 7.5\%$ The drug is bad for women
- $\frac{12}{40} = 30\%$ *vs* $\frac{8}{20} = 40\%$ The drug is bad for man

Simpson's Paradox

	Control Group		Treatment Group	
	Heart attack	No heart attack	Heart attack	No heart attack
Female	1	19	3	37
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Total	13	47	11	49

- $\frac{1}{20} = 5\%$ *vs* $\frac{3}{40} = 7.5\%$ The drug is bad for women
- $\frac{12}{40} = 30\%$ *vs* $\frac{8}{20} = 40\%$ The drug is bad for man
- $\frac{13}{60} = 21\%$ *vs* $\frac{11}{60} = 18\%$ The drug is GOOD for people????

Wait a min...

Correlation is not Causation

Sleeping with shoes on is strongly correlated with waking up with a headache

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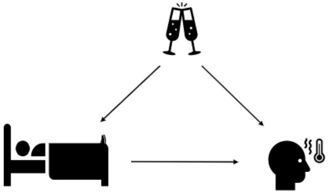
common cause : drinking the night before



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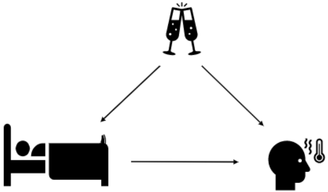
- Shoe sleepers differ from non-shoe sleeper in a key way



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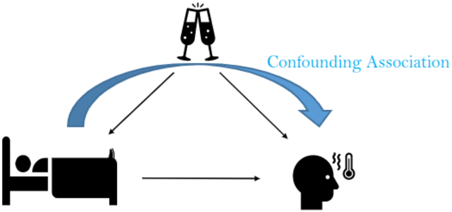

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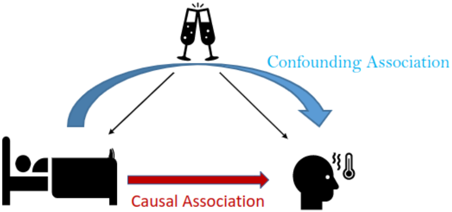
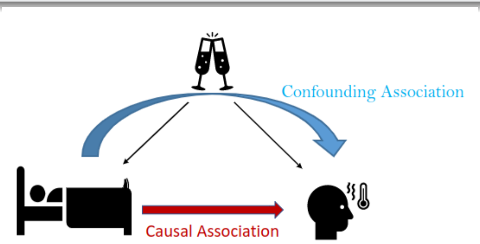


Correlation is not Causation

Note

Total association (e.g. correlation) is mixture of confounding and casual association



Outlines

Potential outcomes: intuition

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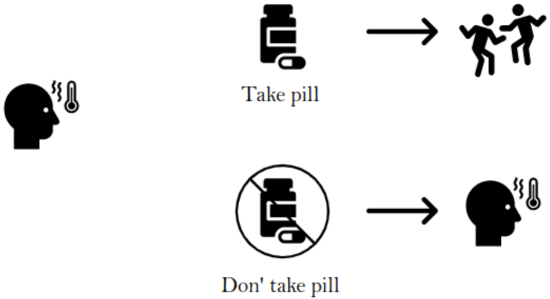
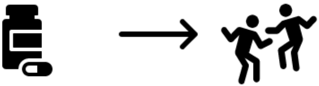


Potential outcomes: intuition

$\mathrm{do}(T = 1)$



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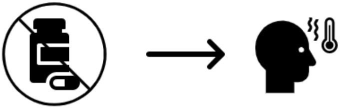

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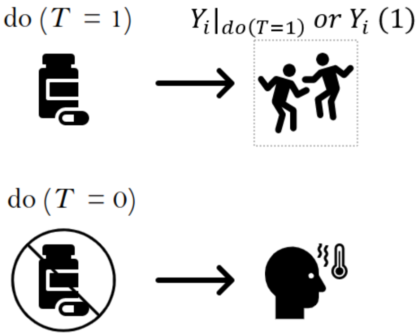
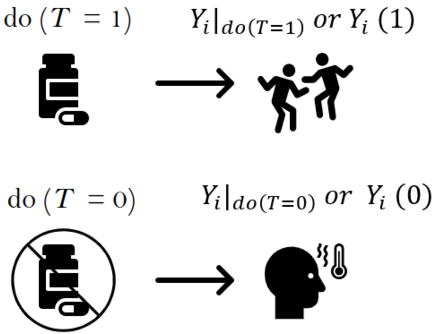


Notation

T : observed treatment
Y : observed outcome
i : denote a specific individual
$Y_i(1)$: potential outcome under treatment

Potential outcomes: intuition



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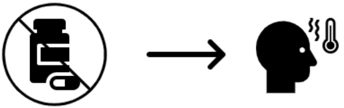
Potential outcomes: intuition

$\text{do}(T = 1)$ $\quad$ $Y_i|_{do(T=1)}$ or $Y_i(1)$

$\text{do}(T = 0)$ $\quad$ $Y_i|_{do(T=0)}$ or $Y_i(0)$



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Casusal effect

$Y_i(1) - Y_i(0)$

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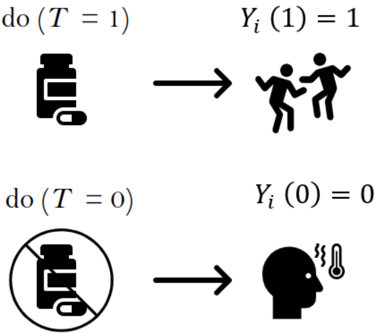

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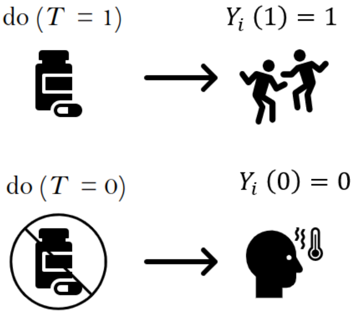
$$Y_i(1) - Y_i(0) = 1$$

Outlines

The fundamental problem of causal inference



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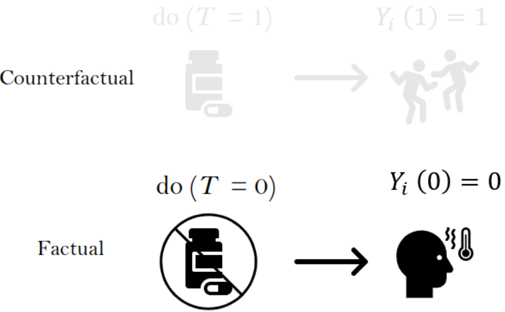

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Casusal effect

$$Y_i(1) - Y_i(0) = 1$$

Missing values perspective

i	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1	?	?
2	0	1	?	1	?
3	1	1	1	?	?
4	0	0	?	0	?
5	0	0	?	0	?
6	1	0	0	?	?

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Note

Missing the counterfactual outcomes is the fundamental problem of casual inference

Question

How could we get around it?

How could we get around them?

$Y_i(1) - Y_i(0)$

i	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1	?	?
2	0	1	?	1	?
3	1	1	1	?	?
4	0	0	?	0	?
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How could we get around them?

Individual treatment effect (ITE)
$Y_i(1) - Y_i(0)$

i	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1	?	?
2	0	1	?	1	?
3	1	1	1	?	?
4	0	0	?	0	?
5	0	0	?	0	?
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How could we get around them?

Average Treatment effect (ATE):
$\mathbb{E}[Y_i(1) - Y_i(0)]$

i	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1	?	?
2	0	1	?	1	?
3	1	1	1	?	?
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$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$
$= \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]$

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4	0	0		0	?
5	0	0		0	?
6	1	0	0		?
			$2/3$	$1/3$	

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2	0	1		1	?
3	1	1	1		?
4	0	0		0	?
5	0	0		0	?
6	1	0	0		?
			$2/3$ -	$1/3$	$= 1/3$

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How could we get around them?

Average Treatment effect (ATE):
$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$
$\neq \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]$

	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1		?
2	0	1		1	?
3	1	1	1		?
4	0	0		0	?
5	0	0		0	?
6	1	0	0		?
			2/3 -	1/3	= 1/3

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Why?

Recall: Correlation does not imply causation

How could we get around them?

Average Treatment effect (ATE):

$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$

$\neq \underbrace{\mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]}_{\text{total associational difference}}$

	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1		?
2	0	1		1	?
3	1	1	1		?
4	0	0		0	?
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How could we get around them?

Average Treatment effect (ATE):

$$\underbrace{\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]}_{\text{casual difference}} \neq \underbrace{\mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]}_{\text{total associational difference}}$$

	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1		?
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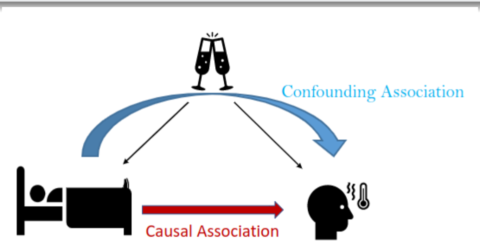
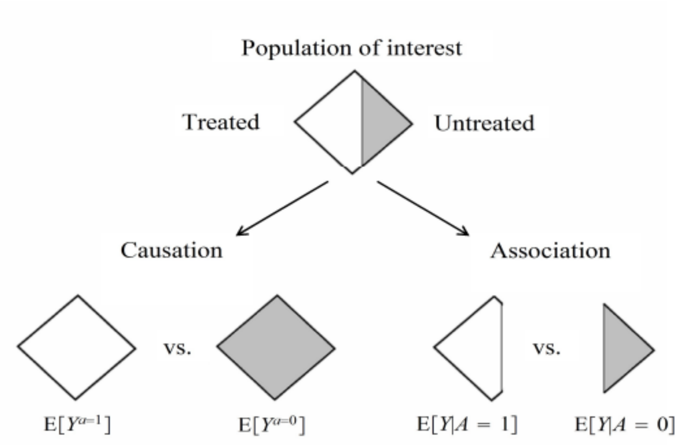
Recall

Note

Total association (e.g. correlation) is mixture of confounding and casual association



How could we get around them?



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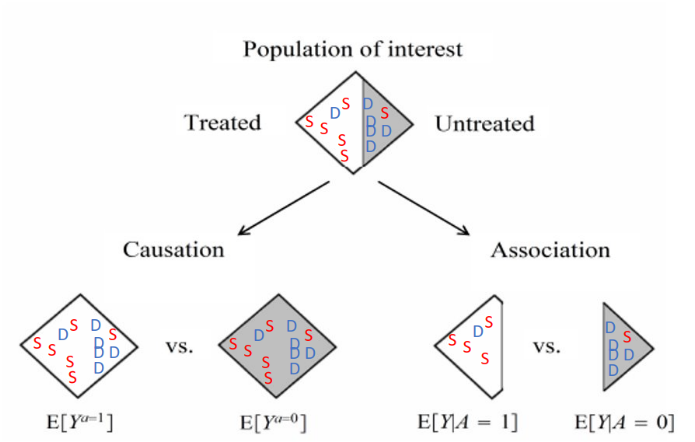


How could we get around them? - Right here!

Then, when casual difference equal to association difference?

$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] = \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]?$$

How could we get around them?- Right here!

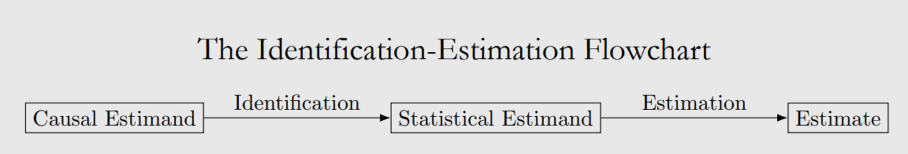
Then, when casual difference equal to association difference?

$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] = \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]?$$

Answer

When we DO NOT have confounding association, like Randomized control trials (RCTs)!!!

How could we get around them?



- Estimand - any quantity we want to estimate
  1. Causal estimand (e.g. $\mathbb{E}[Y(1) - Y(0)]$)
  2. Statistical estimand (e.g. $\mathbb{E}_X[\mathbb{E}[Y|T=1,X] - \mathbb{E}[Y|T=0,X]]$)
- Estimate: approximation of some estimand, using data
- Estimation: process for getting from data + estimand to estimate

Identifiablity

A causal quantity $\mathbb{E}[Y(t)]$ is identifiable if we can compute it from a purely statistical quantity $\mathbb{E}[Y|t]$.

How could we get around them?

What assumptions we need meet so that ATE equal to the associational difference?

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Assumptions

Ignorability: $(Y(1), Y(0)) \perp\!\!\!\perp T$

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$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] = \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]$(Ignorability)

Assumptions

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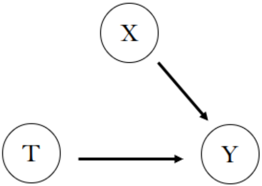

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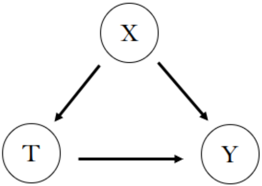


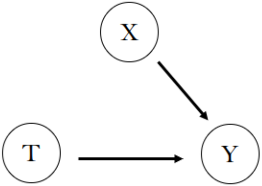
Assumptions

Conditional Ignorability: $(Y(1), Y(0)) \perp\!\!\!\perp T|X$

$\mathbb{E}[Y(1)|X] - \mathbb{E}[Y(0)|X] = \mathbb{E}[Y|T=1, X] - \mathbb{E}[Y|T=0, X]$

we also call this is a conditional average treatment effects (CATEs)

i	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1		?
2	0	1		1	?
3	1	1	1		?
4	0	0		0	?
5	0	0		0	?
6	1	0	0		?



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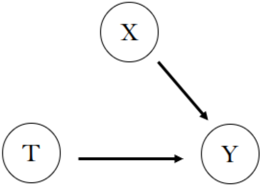

Then How could we get average treatment effect (ATE), $\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$?

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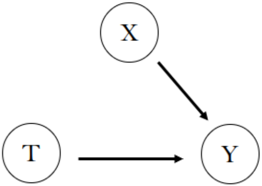
i	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1		?
2	0	1		1	?
3	1	1	1		?
4	0	0		0	?
5	0	0		0	?
6	1	0	0		?



Then what could we get average treatment effect (ATE), $\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$?

Marginalize over X

Another perspective

Exchangeability:
$\mathbb{E}[Y(1)|T=1] = \mathbb{E}[Y(1)|T=0]$

Another perspective

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Another perspective

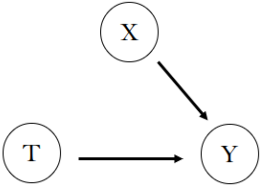
Exchangeability:
$\mathbb{E}[Y(1)|T=1] = \mathbb{E}[Y(1)|T=0] = \mathbb{E}[Y(1)]$
$\mathbb{E}[Y(0)|T=0] = \mathbb{E}[Y(0)|T=1] = \mathbb{E}[Y(0)]$

Let me try to summarize it

Ignorability (Exchangeability):

$$(Y(1), Y(0)) \perp\!\!\!\perp T$$

$$\begin{aligned} & \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] \\ = & \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0] \end{aligned}$$



Randomized Control Trails (RCTs)

Let me try to summarize it

Ignorability (Exchangeability):

$$(Y(1), Y(0)) \perp\!\!\!\perp T$$

$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$
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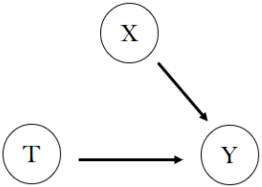
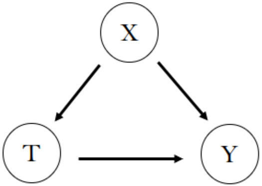


Randomized Control Trails (RCTs)



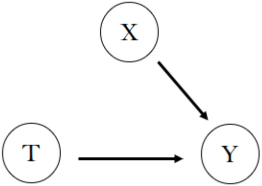
Observational study

Let me try to summarize it

Ignorability (Exchangeability):

$$(Y(1), Y(0)) \perp\!\!\!\perp T$$

$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$
$$= \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]$$

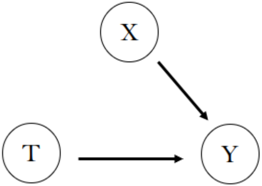


Randomized Control Trails (RCTs)

Conditional Ignorability (Conditional Exchangeability):

$$(Y(1), Y(0)) \perp\!\!\!\perp T|X$$

$$\mathbb{E}[Y(1)|X] - \mathbb{E}[Y(0)|X]$$
$$= \mathbb{E}[Y|T = 1, X] - \mathbb{E}[Y|T = 0, X]$$



Observational study

Different face

Unconfoundedness = conditional ignorability = conditional exchangeability

Assumptions

- Positivity
- Consistency

Assumptions

- Positivity : $0 < P(T = t|X = x) < 1$
- Consistency

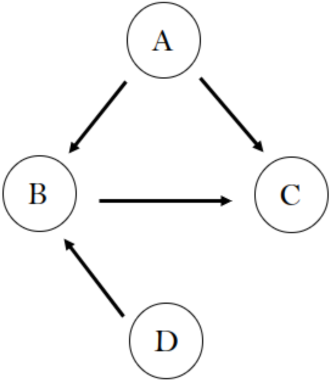
Assumptions

- Positivity : $0 < P(T = t|X = x) < 1$
- Consistency : $T = t \rightarrow Y = Y(t)$

Outlines

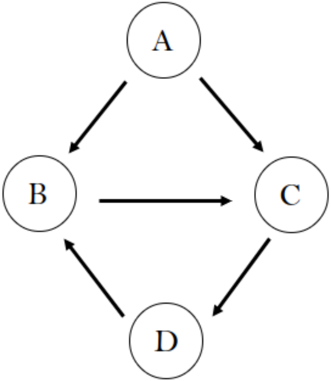
Causal graph

Directed Acyclic Graph (DAG)



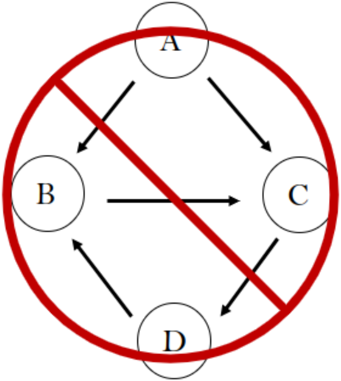
Causal graph

Directed Acyclic Graph (DAG)



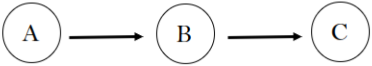
Causal graph

Directed Acyclic Graph (DAG)

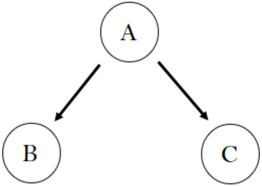


Three basic paths in causal graph

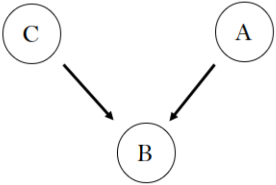
- Chain of Mediation
- Common Cause
- Collider



chain of mediation



common cause



collider

Three basic paths in causal graph

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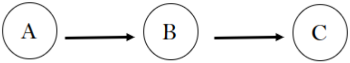

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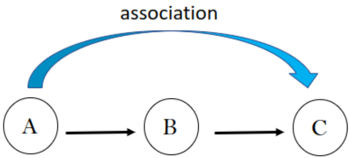
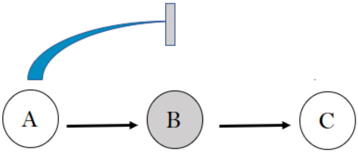


$$A \not\perp\!\!\!\perp C$$

Three basic paths in causal graph

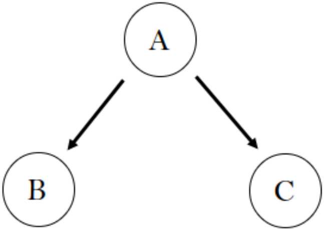
- Chain of Mediation
- Common Cause
- Collider



$$A \perp\!\!\!\perp C \mid B$$

Three basic paths in causal graph

- Chain of Mediation
- Common Cause
- Collider



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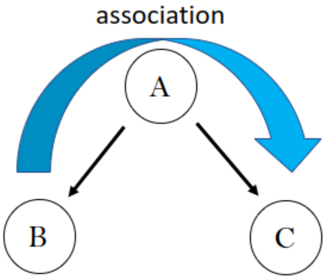

$B \not\perp\!\!\!\perp C$

Three basic paths in causal graph

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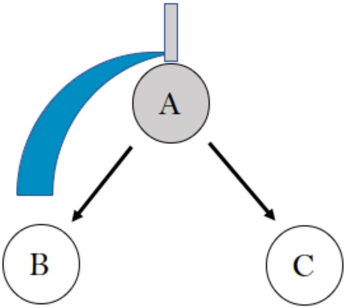

$B \perp\!\!\!\perp C | A$

Three basic paths in causal graph

- Chain of Mediation
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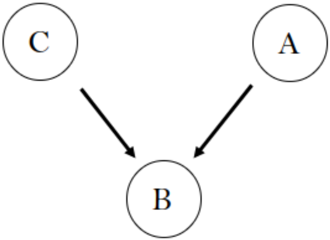

Three basic paths in causal graph

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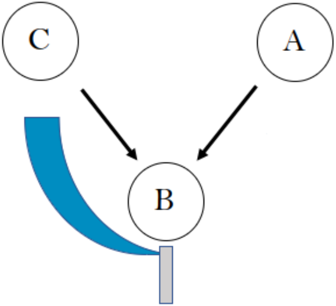

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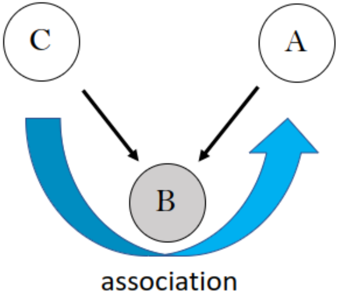

$$A \not\perp\!\!\!\perp C | B$$

Blocked path definition

A path between nodes T and Y is blocked by a (potentially empty) conditioning set w if either of the following is true:

- Along the path, there is a chain of mediation $\cdots \to X \to \ldots$ or a common cause $\cdots \leftarrow X \to \ldots$ where X is conditioned on $(X \in \ W)$.
- There is a collider X on the path that is not conditioned on $(X \notin \ W)$ and none of its descendants are conditioned on.

Backdoor criterion and backdoor adjustment

A set of variables W satisfies the backdoor criterion relative to T and Y if the following are true:

- W blocks all backdoor paths from T to Y
- W does not contain any descendants of T

if we are able to find a set of W that satisfy the backdoor criterion, we can identify the causal effect of T on Y:

$$\begin{aligned} P(y|do(t)) &= \sum_x P(y|do(t), x)P(x|do(t)) \\ &= \sum_x P(y|t, x)P(x) \end{aligned} \quad (1)$$

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$$\begin{aligned} P(y|do(t)) &= \sum_{w} P(y|do(t), w) P(w|do(t)) \\ &= \sum_{w} P(y|t, w) P(w) \end{aligned} \tag{2}$$

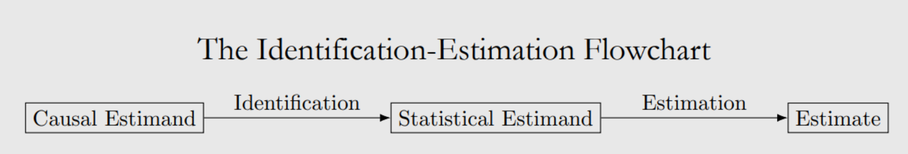
Recall

A causal quantity $\mathbb{E}[Y(t)]$ is identifiable if we can compute it from a purely statistical quantity $\mathbb{E}[Y|t]$.

Other important ways to do identification

- Frontdoor criterion and frontdoor adjustment
- do-calculus

Estimation


The Identification-Estimation Flowchart
Causal Estimand
Identification
Statistical Estimand
Estimation
Estimate

Outlines

Estimation

Recall: Conditional average treatment effects(CATEs):given X is a sufficient adjustment set

$$\mathbb{E}[Y(1) - Y(0)|X = x] \tag{3}$$

Estimation

Recall: Conditional average treatment effects(CATEs): given X is a sufficient adjustment set

$$\tau(x) \triangleq \mathbb{E}[Y(1) - Y(0)|X = x]$$

Estimation

Recall: Conditional average treatment effects(CATEs): given X is a sufficient adjustment set

$$
\begin{aligned}
\tau(x) &\overset{\Delta}{=} \mathbb{E}[Y(1) - Y(0) | X = x] \\
&= \mathbb{E}[Y | T = 1, X] - \mathbb{E}[Y | T = 0, X]
\end{aligned}
$$

(Always assuming unconfoundedness and positivity)

Estimation

Recall: Conditional average treatment effects(CATEs): given X is a sufficient adjustment set

$$\begin{aligned}\tau(x) &\triangleq \mathbb{E}[Y(1) - Y(0)|X = x] \\ &= \mathbb{E}[Y|T = 1, X] - \mathbb{E}[Y|T = 0, X]\end{aligned}$$

(Always assuming unconfoundedness and positivity)

And Average treatment effects(ATEs):

$$\tau = \mathbb{E}_x[\mathbb{E}[Y|T = 1, x] - \mathbb{E}[Y|T = 0, x]]$$

Conditional outcome modeling(COM)

Average treatment effect:

$$\tau = \mathbb{E}_x[\mathbb{E}[Y|T = 1, x] - \mathbb{E}[Y|T = 0, x]]$$

Conditional outcome modeling(COM)

Average treatment effect:

$$\tau = \mathbb{E}_x[\mathbb{E}[Y|T=1,x] - \mathbb{E}[Y|T=0,x]]$$
$$\tau = \mathbb{E}_X[\mu(1,X) - \mu(0,X)]$$

Conditional outcome modeling(COM)

ATE estimand:

$$\tau = \mathbb{E}_x[\mathbb{E}[Y|T = 1, x] - \mathbb{E}[Y|T = 0, x]]$$
$$\tau = \mathbb{E}_X[\mu(1, X) - \mu(0, X)]$$

ATE COM estimator:

$$\hat{\tau} = \mathbb{E}_X[\hat{\mu}(1, X) - \hat{\mu}(0, X)]$$

Conditional outcome modeling(COM)

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ATE COM estimator:

$$\hat{\tau} = \mathbb{E}_X[\hat{\mu}(1, X) - \hat{\mu}(0, X)]$$
$$\hat{\tau} = \frac{1}{n_x} \sum_i (\hat{\mu}(1, x_i) - \hat{\mu}(0, x_i))$$

Conditional outcome modeling(COM)

ATE estimand:

$$\tau = \mathbb{E}_x[\mathbb{E}[Y|T = 1, x] - \mathbb{E}[Y|T = 0, x]]$$
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What could go wrong here?

Grouped COM (GCOM):

COM:

$$\hat{\tau} = \frac{1}{n_x} \sum_i (\hat{\mu}(1, x_i) - \hat{\mu}(0, x_i))$$

GCOM:

$$\hat{\tau} = \frac{1}{n_x} \sum_i (\hat{\mu_1}(x_i) - \hat{\mu_0}(x_i))$$

Separate data to two (treatment vs control) and train it using two model

Grouped COM (GCOM):

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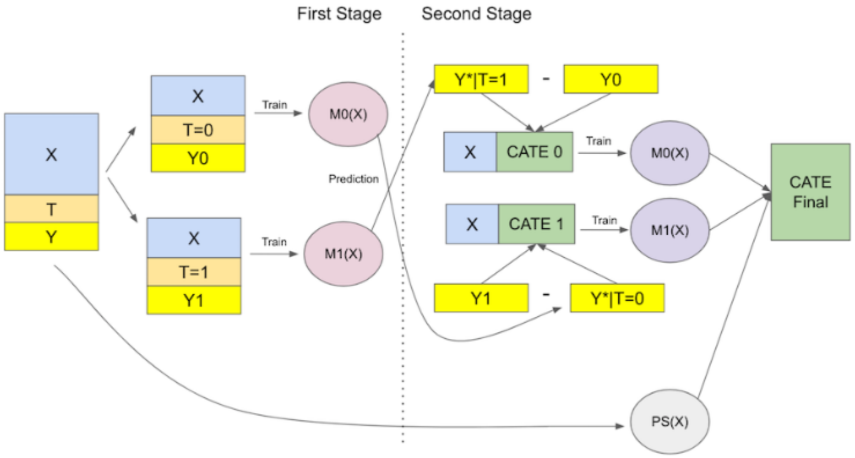
Separate data to two (treatment vs control) and train it using two model

But....

X-learner



First Stage
Second Stage
X
T
Y
X
T=0
Y0
Train
M0(X)
X
T=1
Y1
Train
M1(X)
Prediction
Y*|T=1
-
Y0
X
CATE 0
Train
M0(X)
X
CATE 1
Train
M1(X)
Y1
-
Y*|T=0
CATE
Final
PS(X)

Outlines

Example: effect of sodium intake on blood pressure

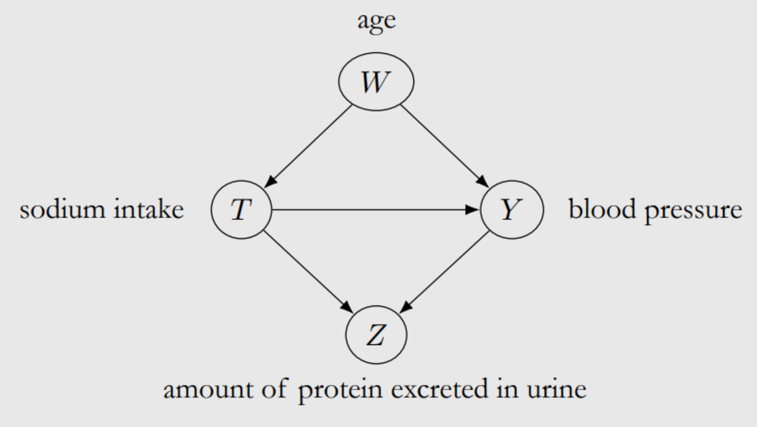
Motivation: 46% of Americans have high blood pressure and high blood pressure is associated with increased risk of mortality

Data:

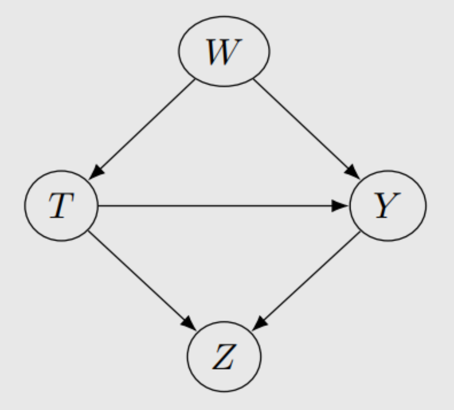
- Epidemiological example taken from Luque-Fernandez et al.(2018)
- Outcome Y: (systolic) blood pressure (continuous)
- Treatment T: sodium intake (1 if above 3.5 mg and 0 if below)
- Covariates
  1. W: age
  2. Z: amount of protein excreted in urine
- Simulation: so we know the "true" ATE is 1.05

Example: effect of sodium intake on blood pressure

The causal graph

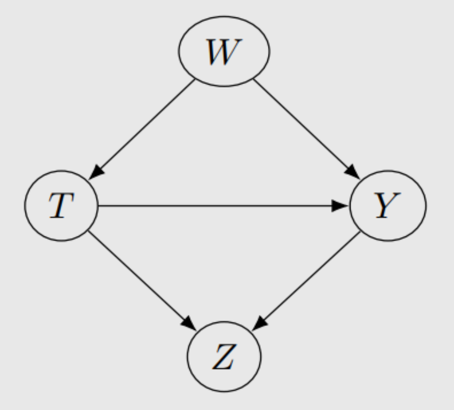


Example: effect of sodium intake on blood pressure



Identification

Example: effect of sodium intake on blood pressure

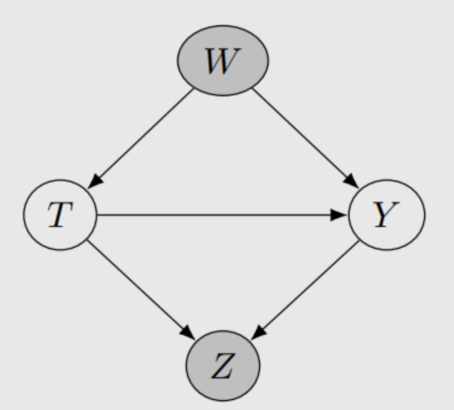


Identification

Causal estimand

$\mathbb{E}[Y(t)]$

Example: effect of sodium intake on blood pressure



Identification

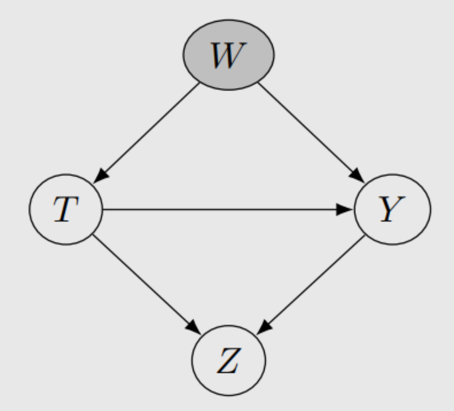
Causal estimand

$\mathbb{E}[Y(t)]$

Statistical estimand without causal graph:

$\mathbb{E}_{W,Z}\mathbb{E}[Y|t, W, Z]$

Example: effect of sodium intake on blood pressure



Identification

Causal estimand

$\mathbb{E}[Y(t)]$

Statistical estimand without causal graph:

$\mathbb{E}_{W,Z}\mathbb{E}[Y|t, W, Z]$

Statistical estimand with causal graph:

$\mathbb{E}_W\mathbb{E}[Y|t, W]$

Example: effect of sodium intake on blood pressure

Estimation of ATE

- True ATE: $\mathbb{E}[Y(1) - Y(0)] = 1.05$

Example: effect of sodium intake on blood pressure

Estimation of ATE

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Example: effect of sodium intake on blood pressure

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- COM Estimation: $\frac{1}{n}\sum_i \mathbb{E}([Y|T = 1, X = x_i] - \mathbb{E}[Y|T = 0, X = x_i])$

Example: effect of sodium intake on blood pressure

Estimation of ATE

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Example: effect of sodium intake on blood pressure

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- COM Estimation: $\frac{1}{n}\sum_i (\underline{\mathbb{E}[Y|T=1, X=x_i]} - \underline{\mathbb{E}[Y|T=0, X=x_i]})$

  Model (linear regression)

Estimate:

- $\mathrm{X} = \emptyset$ (naive) : 5.33 407% error

Example: effect of sodium intake on blood pressure

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Estimate:

- X = $\emptyset$ (naive) : 5.33 407% error
- X = W,Z (without graph):0.85 19% error

Example: effect of sodium intake on blood pressure

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Estimate:

- X = $\emptyset$(naive) : 5.33 — 407% error
- X = W,Z (without graph) : 0.85 — 19% error
- X = W (unbiased) : 1.0502 — 0.02% error

Topics I have not covered:

- D-separation
- Bayesian Networks
- Structural casual models
- Unobsereved confounding
- Instrumental Variables
- Mediation analysis
- Difference in Differences
- Transfer learning and transportability
- ...

Question?