

A Tutorial of Casual Inference

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Outlines

- ① Motivation
- ② What does imply Causation?
- ③ The fundamental problem of causal inference
- ④ Assumptions
- ⑤ Causal graph
- ⑥ Estimation
- ⑦ Example

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Simpson's Paradox

	Control Group		Treatment Group	
	Heart attack	No heart attack	Heart attack	No heart attack
Female	1	19	3	37
Male	12	28	8	12
Total	13	47	11	49

- $\frac{1}{20} = 5\%$ vs $\frac{3}{40} = 7.5\%$ The drug is bad for women
- $\frac{12}{40} = 30\%$ vs $\frac{8}{20} = 40\%$ The drug is bad for man

Simpson's Paradox

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Female	1	19	3	37
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- $\frac{1}{20} = 5\%$ vs $\frac{3}{40} = 7.5\%$ The drug is bad for women
- $\frac{12}{40} = 30\%$ vs $\frac{8}{20} = 40\%$ The drug is bad for man
- $\frac{13}{60} = 21\%$ vs $\frac{11}{60} = 18\%$ The drug is GOOD for people????

Wait a min...

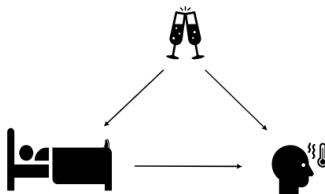
Correlation is not Causation

Sleeping with shoes on is strongly correlated with waking up with a headache



Correlation is not Causation

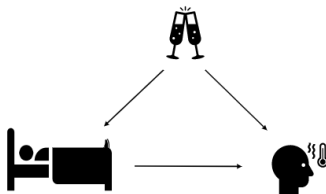
Sleeping with shoes on is strongly correlated with waking up with a headache
common cause : drinking the night before



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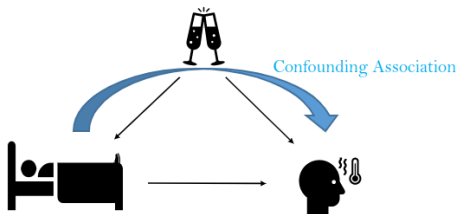
- Shoe sleepers differ from non-shoe sleeper in a key way



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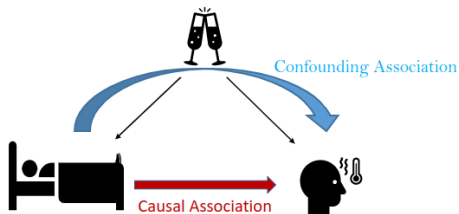
- Shoe sleepers differ from non-shoe sleeper in a key way
- Confounder



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Sleeping with shoes on is strongly correlated with waking up with a headache
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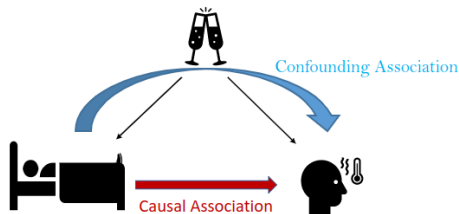
- Shoe sleepers differ from non-shoe sleeper in a key way
- Confounder



Correlation is not Causation

Note

Total association (e.g. correlation) is mixture of confounding and causal association



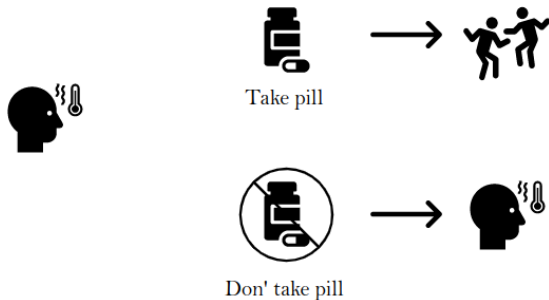
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Potential outcomes: intuition

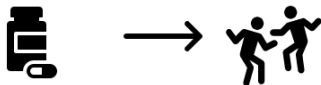


Potential outcomes: intuition

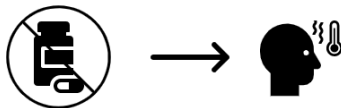


Potential outcomes: intuition

$\text{do}(T = 1)$



$\text{do}(T = 0)$



Potential outcomes: intuition

$\text{do}(T = 1)$ $Y_i|_{\text{do}(T=1)} \text{ or } Y_i(1)$



$\text{do}(T = 0)$



Notation

T : observed treatment

Y : observed outcome

i : denote a specific individual

$Y_i(1)$: potential outcome under treatment

Potential outcomes: intuition

$\text{do}(T = 1) \quad Y_i|_{\text{do}(T=1)} \text{ or } Y_i(1)$



$\text{do}(T = 0) \quad Y_i|_{\text{do}(T=0)} \text{ or } Y_i(0)$



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Casual effect

$$Y_i(1) - Y_i(0)$$

Potential outcomes: intuition

$\text{do}(T = 1)$ $Y_i(1) = 1$



$\text{do}(T = 0)$ $Y_i(0) = 0$



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The fundamental problem of causal inference

do ($T = 1$)



$Y_i(1) = 1$



do ($T = 0$)



$Y_i(0) = 0$



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Casual effect

$$Y_i(1) - Y_i(0) = 1$$

The fundamental problem of causal inference

Counterfactual

$\text{do}(T = 1)$



$Y_i(1) = 1$



Factual

$\text{do}(T = 0)$



$Y_i(0) = 0$



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Casual effect

$$Y_i(1) - Y_i(0) = 1$$

Missing values perspective

i	T	Y	$Y(1)$	$Y(0)$	$Y(1)-Y(0)$
1	1	1	1	?	?
2	0	1	?	1	?
3	1	1	1	?	?
4	0	0	?	0	?
5	0	0	?	0	?
6	1	0	0	?	?

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Note

Missing the counterfactual outcomes is the fundamental problem of causal inference

Question

How could we get around it?

How could we get around them?

$$Y_i(1) - Y_i(0)$$

i	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1	?	?
2	0	1	?	1	?
3	1	1	1	?	?
4	0	0	?	0	?
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How could we get around them?

Individual treatment effect (ITE)

$$Y_i(1) - Y_i(0)$$

i	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1	?	?
2	0	1	?	1	?
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How could we get around them?

Average Treatment effect (ATE):

$$\mathbb{E}[Y_i(1) - Y_i(0)]$$

i	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1	?	?
2	0	1	?	1	?
3	1	1	1	?	?
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$$\begin{aligned} & \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] \\ &= \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0] \end{aligned}$$

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3	1	1	1		?
4	0	0		0	?
5	0	0		0	?
6	1	0	0		?
			2/3	1/3	

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4	0	0		0	?
5	0	0		0	?
6	1	0	0		?
$2/3 - 1/3 = 1/3$					

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How could we get around them?

Average Treatment effect (ATE):

$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$

$$\neq \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]$$

	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1		?
2	0	1		1	?
3	1	1	1		?
4	0	0		0	?
5	0	0		0	?
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$$2/3 - 1/3 = 1/3$$

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Why?

Recall: Correlation does not imply causation

How could we get around them?

Average Treatment effect (ATE):

$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$

$$\neq \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]$$

total associational difference

	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1		?
2	0	1		1	?
3	1	1	1		?
4	0	0		0	?
5	0	0		0	?
6	1	0	0		?
			2/3 -	1/3	= 1/3

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How could we get around them?

Average Treatment effect (ATE):

$$\frac{\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]}{\text{casual difference}}$$

$$\neq \frac{\mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]}{\text{total associational difference}}$$

	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1		?
2	0	1		1	?
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$$2/3 - 1/3 = 1/3$$

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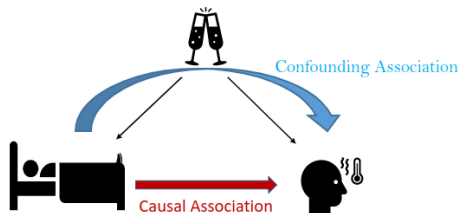
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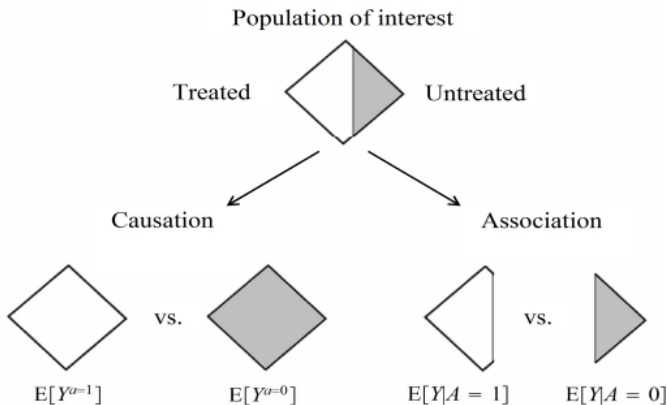
$Y_i(0)$: potential outcome under no treatment

Note

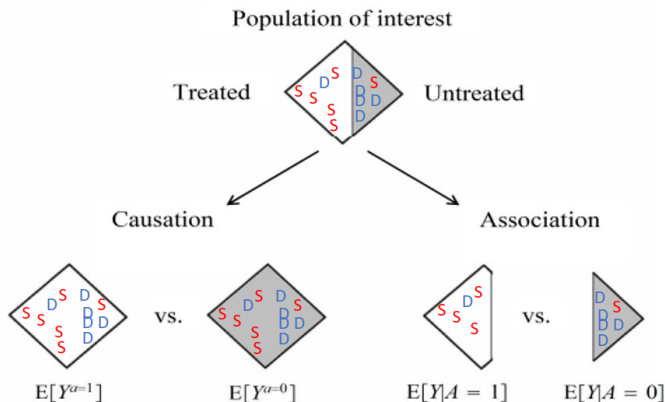
Total association (e.g. correlation) is mixture of confounding and causal association



How could we get around them?



How could we get around them?



How could we get around them? - Right here!

Then, when casual difference equal to association difference?

$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] = \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]?$$

How could we get around them?- Right here!

Then, when casual difference equal to association difference?

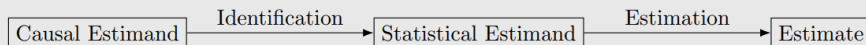
$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] = \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]?$$

Answer

When we DO NOT have confounding association, like Randomized control trials (RCTs)!!!

How could we get around them?

The Identification-Estimation Flowchart



- Estimand - any quantity we want to estimate
 - ① Causal estimand (e.g. $\mathbb{E}[Y(1) - Y(0)]$)
 - ② Statistical estimand (e.g. $\mathbb{E}_X[\mathbb{E}[Y|T = 1, X] - \mathbb{E}[Y|T = 0, X]]$)
- Estimate: approximation of some estimand, using data
- Estimation: process for getting from data + estimand to estimate

Identifiability

A causal quantity $\mathbb{E}[Y(t)]$ is identifiable if we can compute it from a purely statistical quantity $\mathbb{E}[Y|t]$.

How could we get around them?

What assumptions we need meet so that ATE equal to the associational difference?

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Assumptions

Ignorability: $(Y(1), Y(0)) \perp\!\!\!\perp T$

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$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] = \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0] \text{ (Ignorability)}$$

Assumptions

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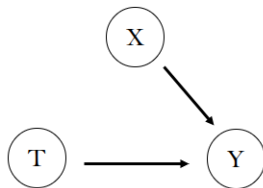
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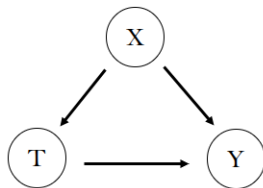


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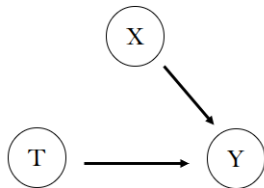
Assumptions

Conditional Ignorability: $(Y(1), Y(0)) \perp\!\!\!\perp T | X$

$$\mathbb{E}[Y(1)|X] - \mathbb{E}[Y(0)|X] = \mathbb{E}[Y|T = 1, X] - \mathbb{E}[Y|T = 0, X]$$

we also call this is a conditional average treatment effects (CATEs)

i	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	1	1	1		?
2	0	1		1	?
3	1	1	1		?
4	0	0		0	?
5	0	0		0	?
6	1	0	0		?

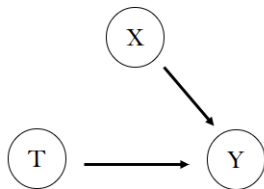


Assumptions

Conditional Ignorability: $(Y(1), Y(0)) \perp\!\!\!\perp T|X$

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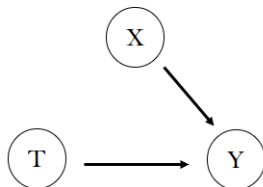
Then How could we get average treatment effect (ATE), $\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$?

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Then what could we get average treatment effect (ATE), $\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$?

Marginalize over X

Another perspective

Exchangeability:

$$\mathbb{E}[Y(1)|T = 1] = \mathbb{E}[Y(1)|T = 0]$$

Another perspective

Exchangeability:

$$\mathbb{E}[Y(1)|T = 1] = \mathbb{E}[Y(1)|T = 0] = \mathbb{E}[Y(1)]$$

Another perspective

Exchangeability:

$$\mathbb{E}[Y(1)|T = 1] = \mathbb{E}[Y(1)|T = 0] = \mathbb{E}[Y(1)]$$

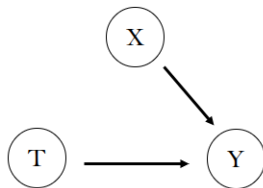
$$\mathbb{E}[Y(0)|T = 0] = \mathbb{E}[Y(0)|T = 1] = \mathbb{E}[Y(0)]$$

Let me try to summarize it

Ignorability (Exchangeability):

$$(Y(1), Y(0)) \perp\!\!\!\perp T$$

$$\begin{aligned} & \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] \\ = & \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0] \end{aligned}$$



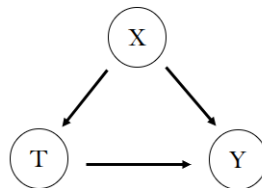
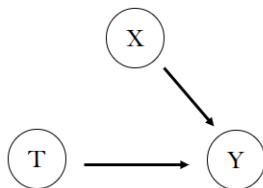
Randomized Control Trials (RCTs)

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Observational study

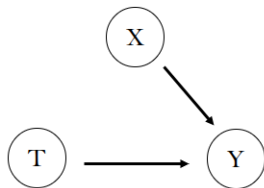
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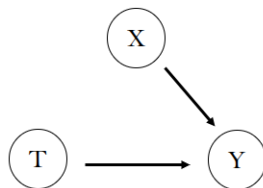
Randomized Control Trials (RCTs)

Conditional Ignorability

(Conditional Exchangeability):

$$(Y(1), Y(0)) \perp\!\!\!\perp T|X$$

$$\begin{aligned} & \mathbb{E}[Y(1)|X] - \mathbb{E}[Y(0)|X] \\ &= \mathbb{E}[Y|T=1, X] - \mathbb{E}[Y|T=0, X] \end{aligned}$$



Observational study

Unconfoundedness = conditional ignorability = conditional exchangeability

Assumptions

- Positivity
- Consistency

Assumptions

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- Consistency

Assumptions

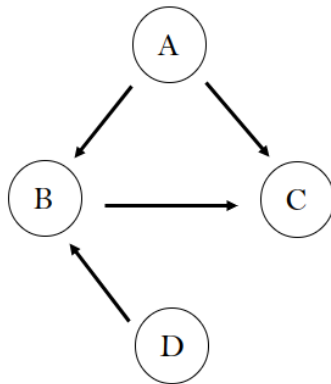
- Positivity : $0 < P(T = t|X = x) < 1$
- Consistency : $T = t \rightarrow Y = Y(t)$

Outlines

- ① Motivation
- ② What does imply Causation?
- ③ The fundamental problem of causal inference
- ④ Assumptions
- ⑤ Causal graph
- ⑥ Estimation
- ⑦ Example

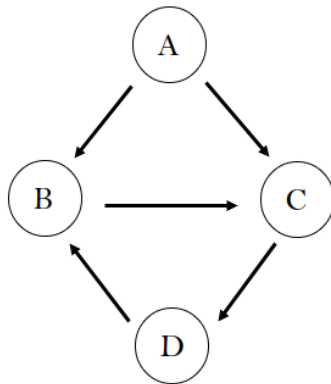
Causal graph

Directed Acyclic Graph (DAG)



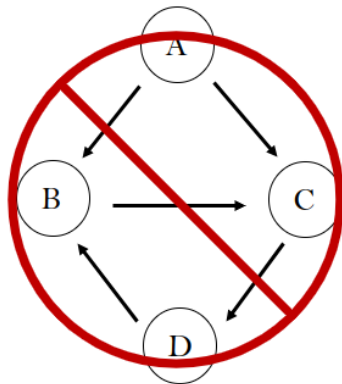
Causal graph

Directed Acyclic Graph (DAG)

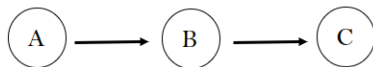


Causal graph

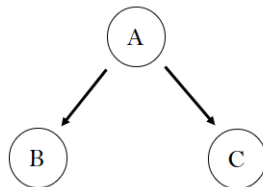
Directed Acyclic Graph (DAG)



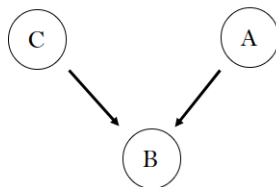
Three basic paths in causal graph



chain of mediation



common cause

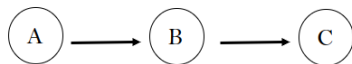


collider

- Chain of Mediation
- Common Cause
- Collider

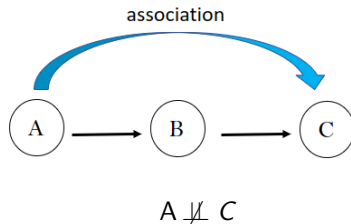
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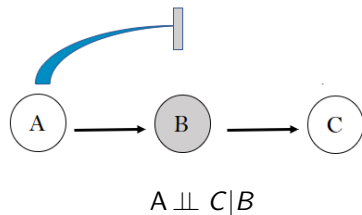
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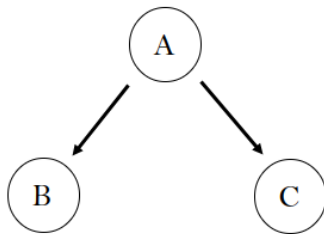
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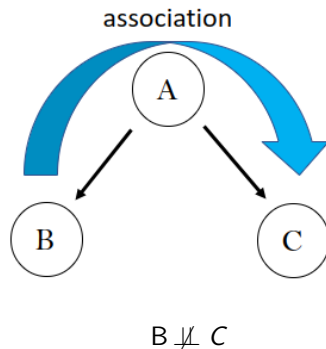
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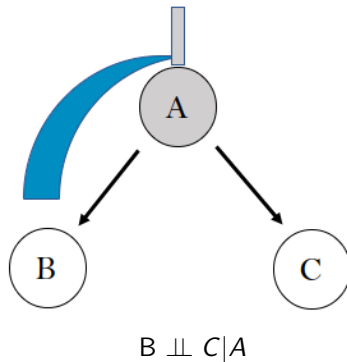
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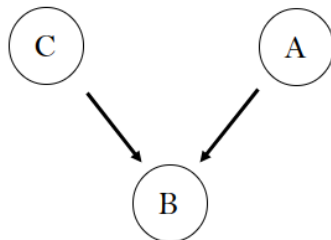
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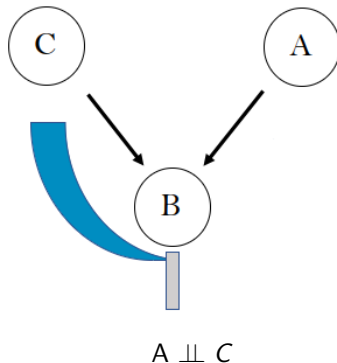
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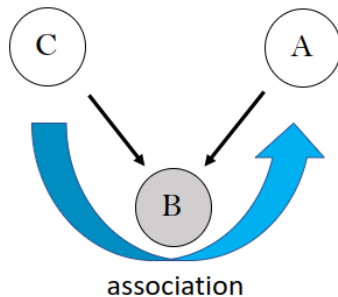
Three basic paths in causal graph

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Three basic paths in causal graph

- Chain of Mediation
- Common Cause
- Collider



$$A \not\perp\!\!\!\perp C|B$$

Blocked path definition

A path between nodes T and Y is blocked by a (potentially empty) conditioning set w if either of the following is true:

- Along the path, there is a chain of mediation $\cdots \rightarrow X \rightarrow \cdots$ or a common cause $\cdots \leftarrow X \rightarrow \cdots$ where X is conditioned on ($X \in W$).
- There is a collider X on the path that is not conditioned on ($X \notin W$) and none of its descendants are conditioned on.

Backdoor criterion and backdoor adjustment

A set of variables W satisfies the backdoor criterion relative to T and Y if the following are true:

- W blocks all backdoor paths from T to Y
- W does not contain any descendants of T

if we are able to find a set of W that satisfy the backdoor criterion, we can identify the causal effect of T on Y :

$$\begin{aligned}P(y|do(t)) &= \sum_x P(y|do(t), x)P(x|do(t)) \\ &= \sum_x P(y|t, x)P(x)\end{aligned}\tag{1}$$

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$$\begin{aligned}P(y|do(t)) &= \sum_w P(y|do(t), w)P(w|do(t)) \\ &= \sum_w P(y|t, w)P(w)\end{aligned}\tag{2}$$

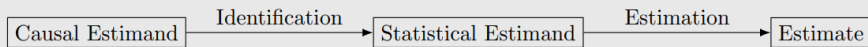
Recall

A causal quantity $\mathbb{E}[Y(t)]$ is identifiable if we can compute it from a purely statistical quantity $\mathbb{E}[Y|t]$.

Other important ways to do identification

- Frontdoor criterion and frontdoor adjustment
- do-calculus

The Identification-Estimation Flowchart



Outlines

- ① Motivation
- ② What does imply Causation?
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- ⑦ Example

Recall: Conditional average treatment effects(CATEs):given X is a sufficient adjustment set

$$\mathbb{E}[Y(1) - Y(0)|X = x] \quad (3)$$

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$$\tau(x) \triangleq \mathbb{E}[Y(1) - Y(0)|X = x]$$

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$$\begin{aligned}\tau(x) &\triangleq \mathbb{E}[Y(1) - Y(0)|X = x] \\ &= \mathbb{E}[Y|T = 1, X] - \mathbb{E}[Y|T = 0, X]\end{aligned}$$

(Always assuming unconfoundedness and positivity)

Estimation

Recall: Conditional average treatment effects(CATEs): given X is a sufficient adjustment set

$$\begin{aligned}\tau(x) &\triangleq \mathbb{E}[Y(1) - Y(0)|X = x] \\ &= \mathbb{E}[Y|T = 1, X] - \mathbb{E}[Y|T = 0, X]\end{aligned}$$

(Always assuming unconfoundedness and positivity)

And Average treatment effects(ATEs):

$$\tau = \mathbb{E}_x[\mathbb{E}[Y|T = 1, x] - \mathbb{E}[Y|T = 0, x]]$$

Conditional outcome modeling(COM)

Average treatment effect:

$$\tau = \mathbb{E}_x[\mathbb{E}[Y|T = 1, x] - \mathbb{E}[Y|T = 0, x]]$$

Conditional outcome modeling(COM)

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$$\tau = \mathbb{E}_x[\mathbb{E}[Y|T=1, x] - \mathbb{E}[Y|T=0, x]]$$

$$\tau = \mathbb{E}_X[\mu(1, X) - \mu(0, X)]$$

Conditional outcome modeling(COM)

ATE estimand:

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ATE COM estimator:

$$\hat{\tau} = \mathbb{E}_X[\hat{\mu}(1, X) - \hat{\mu}(0, X)]$$

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ATE COM estimator:

$$\hat{\tau} = \mathbb{E}_X[\hat{\mu}(1, X) - \hat{\mu}(0, X)]$$

$$\hat{\tau} = \frac{1}{n_x} \sum_i (\hat{\mu}(1, x_i) - \hat{\mu}(0, x_i))$$

Conditional outcome modeling(COM)

ATE estimand:

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What could go wrong here?

Grouped COM (GCOM):

COM:

$$\hat{\tau} = \frac{1}{n_x} \sum_i (\hat{\mu}(1, x_i) - \hat{\mu}(0, x_i))$$

GCOM:

$$\hat{\tau} = \frac{1}{n_x} \sum_i (\hat{\mu}_1(x_i) - \hat{\mu}_0(x_i))$$

Separate data to two (treatment vs control) and train it using two model

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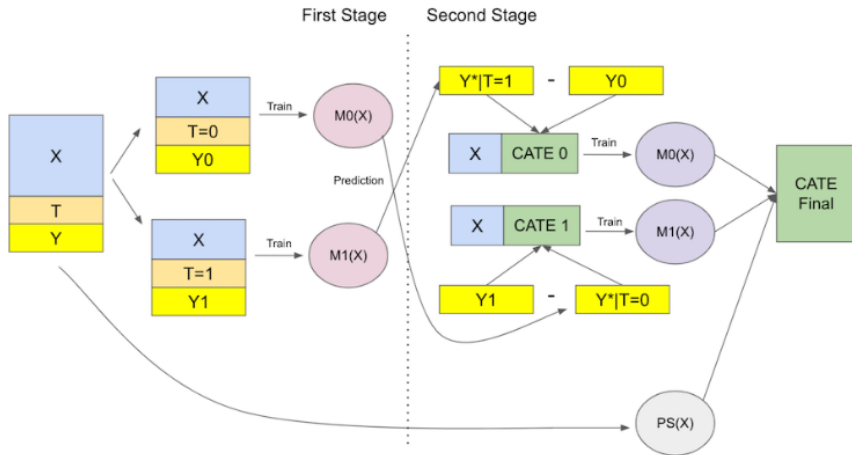
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But....

X-learner



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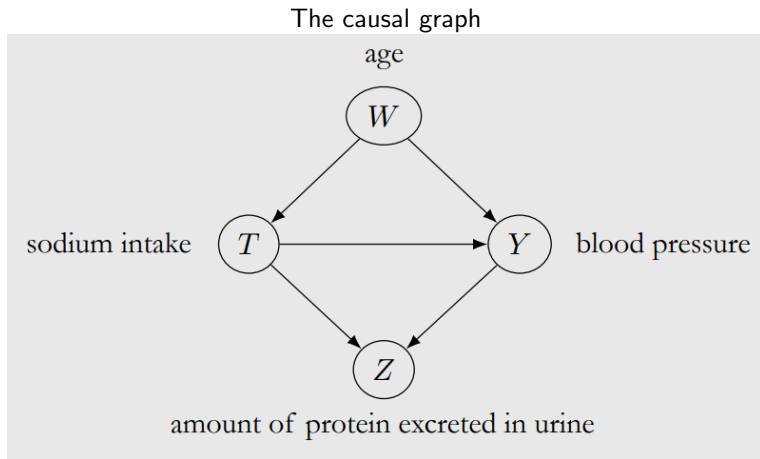
Example: effect of sodium intake on blood pressure

Motivation: 46% of Americans have high blood pressure and high blood pressure is associated with increased risk of mortality

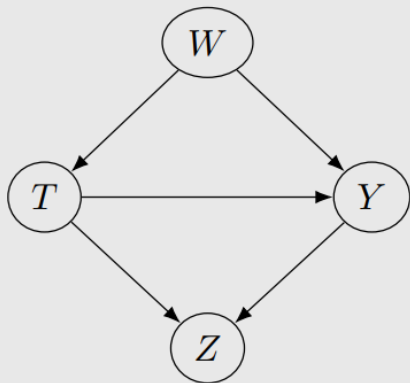
Data:

- Epidemiological example taken from Luque-Fernandez et al.(2018)
- Outcome Y: (systolic) blood pressure (continuous)
- Treatment T: sodium intake (1 if above 3.5 mg and 0 if below)
- Covariates
 - ① W: age
 - ② Z: amount of protein excreted in urine
- Simulation: so we know the "true" ATE is 1.05

Example: effect of sodium intake on blood pressure

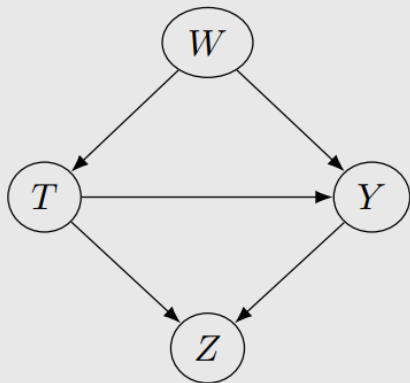


Example: effect of sodium intake on blood pressure



Identification

Example: effect of sodium intake on blood pressure

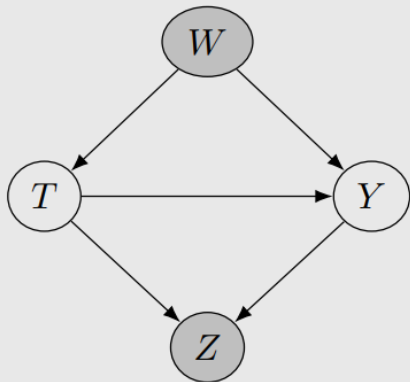


Identification

Causal estimand

$$\mathbb{E}[Y(t)]$$

Example: effect of sodium intake on blood pressure



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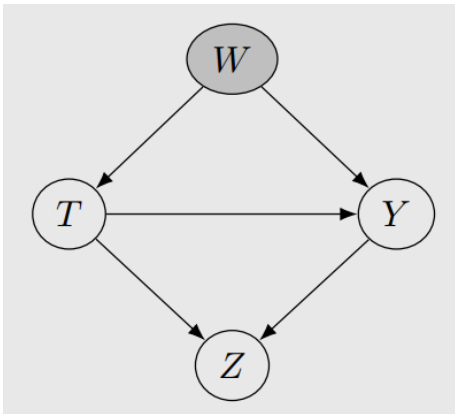
Causal estimand

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Statistical estimand without causal graph:

$$\mathbb{E}_{W,Z} \mathbb{E}[Y|t, W, Z]$$

Example: effect of sodium intake on blood pressure



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- True ATE: $\mathbb{E}[Y(1) - Y(0)] = 1.05$

Example: effect of sodium intake on blood pressure

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Model (linear regression)

Estimate:

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Estimate:

- $X = \emptyset$ (naive) : 5.33 407% error
- $X = W, Z$ (without graph) : 0.85 19% error
- $X = W$ (unbiased) : 1.0502 0.02% error

Topics I have not covered:

- D-separation
- Bayesian Networks
- Structural casual models
- Unobserved confounding
- Instrumental Variables
- Mediation analysis
- Difference in Differences
- Transfer learning and transportability
- ...

Question?