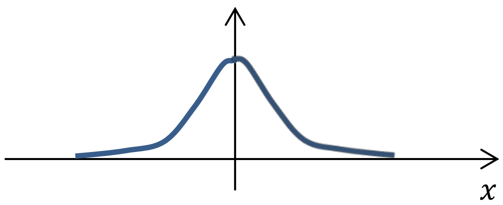
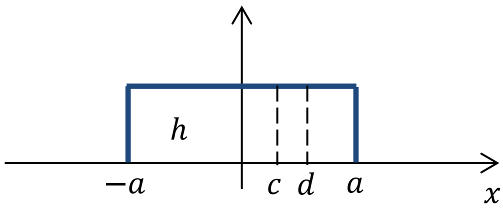
Homework Set 1

Problem 1 (Probability density function)



(A)



(B)

a) Figure (A) shows a Cauchy probability density function whose density is given by $\frac{1}{K}\frac{1}{1+x^2}$. What is the value of K for the density to be a probability density function? What are the mean and variance of the distribution?

b) Figure (B) shows a uniform probability density function. What is the height h of the density function? What are the mean and variance of the distribution? What is the probability that x lies in the interval between the vertical lines marked by c and d?

c) Consider two fair dice with six sides marked with the usual numbers 1 through 6. What is the probability that a throw of the dice results in a score of 7? What is the mean of the numbers that arise when two dice are thrown? What is the variance?

Problem 2 (Sample statistics and confidence intervals)
In characterizing the noise in an amplifier, which is normally distributed, we have the following noise voltages in micro Volts (μV):

-0.4326 -1.6656 0.1253 0.2877 -1.1465 1.1909 1.1892 -0.0376 0.3273 0.1746

a) Estimate the mean of the noise voltages and the variance of the mean.

b) Calculate the 95% and 99% confidence intervals of the mean (of noise voltages).

c) How confident are we that the noise voltage at any time lies between $1\mu V$ and $1.1\mu V$?

Problem 3 (Calculus)
Let $f(x,y) = 3x^2 + y^2 - xy - 11x$

a) Find $\frac{\partial f}{\partial x}$, the partial derivative of f with respect to x. Also find $\frac{\partial f}{\partial y}$.

b) Find the pair $(x,y) \in \Re^2$ that minimizes f.

c) Show that the pair (x,y) you found in b. is a minimizer instead of a maximizer.

Problem 4 (Vector Norms)
Compute the 0, 1, 2, and ∞ norms for $[3\ -1\ 3\ 5\ 0\ 2\ ]^T$.

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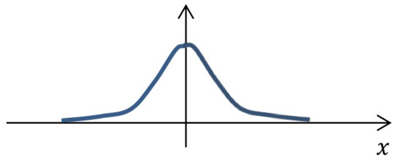
Problem 1 (Probability density function)



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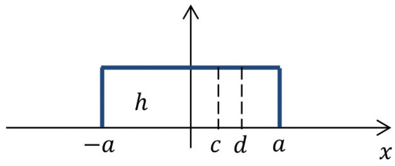

(B)

a) Figure (A) shows a Cauchy probability density function whose density is given by $\frac{1}{K}\frac{1}{1+x^2}$. What is the value of K for the density to be a probability density function? What are the mean and variance of the distribution?

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c) Consider two fair dice with six sides marked with the usual numbers 1 through 6. What is the probability that a throw of the dice results in a score of 7? What is the mean of the numbers that arise when two dice are thrown? What is the variance?

a) Cauchy distribution PDF: $\frac{1}{\pi r}\left[\frac{r^2}{x^2+r^2}\right] = \frac{1}{K}\,\frac{1}{1+x^2}$ $\sim r=1$

$\Rightarrow \frac{1}{\pi}\,\frac{r}{x^2+r^2} = \frac{1}{K}\,\frac{1}{1+x^2}$

$K = \pi$ #

Mean: undefined ∴ unknown #

Variance: undefined ∴ unknown #

b) $\int_{-a}^{a} hx\,dx = 1$

$\Rightarrow h = \frac{1}{2a}$ #

mean $= \frac{a+(-a)}{2} = 0$ #

variance $= \frac{[a-(-a)]^2}{12} = \frac{a^2}{3}$ #

∵ uniform distributed

∴ probability $= \frac{d-c}{2a}$

c) a score of 7: (1, 6)
(2, 5)
(3, 4)
(4, 3)
(5, 2)
(6, 1)

Total possibilities: 36

∴ probability $= \frac{6}{36} = \frac{1}{6}$

同理结果为 2的机率 $= \frac{1}{36} =$ 12的机率

3 〃 $= \frac{2}{36} =$ 11 〃

4 〃 $= \frac{3}{36} =$ 10 〃

5 〃 $= \frac{4}{36} =$ 9 〃

6 〃 $= \frac{5}{36} =$ 8 〃

∴ Mean $= \frac{1}{36}\times(2+12) + \frac{2}{36}\times(3+11) + \frac{3}{36}\times(4+10) + \frac{4}{36}\times(5+9) + \frac{5}{36}\times(6+8) + \frac{6}{36}\times 7$

$= \frac{252}{36} = 7$ #

∴ Variance $= \frac{1}{36-1}\times(5^2+5^2) + \frac{2}{36-1}\times(4^2+4^2) + \frac{3}{36-1}\times(3^2+3^2) + \frac{4}{36-1}\times(2^2+2^2) + \frac{5}{36-1}\times(1^2+1^2)$

$= \frac{210}{35} = 6$ #

Problem 2 (Sample statistics and confidence intervals)

In characterizing the noise in an amplifier, which is normally distributed, we have the following noise voltages in micro Volts (μV):

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a) Estimate the mean of the noise voltages and the variance of the mean.

b) Calculate the 95% and 99% confidence intervals of the mean (of noise voltages).

c) How confident are we that the noise voltage at any time lies between $1\mu V$ and $1.1\mu V$?

a) $\frac{\text{Sum up}}{10} = \frac{0.0127}{10} = \underline{\underline{0.00127 = \text{mean}}}$

$$\text{Variance} = \frac{0.43387^2 + 1.66687^2 + 0.12403^2 + 0.28643^2 + 1.14777^2 + 1.18963^2 + 1.18793^2 + 0.03887^2 + 0.32603^2 + 0.17333^2}{10-1}$$

$$= \frac{7.3457473}{9} = \underline{\underline{0.81619}}$$

b) $95\% = \left(0.00127 - 1.96 \times \sqrt{\frac{0.8162}{10}},\ 0.00127 + 1.96 \times \sqrt{\frac{0.8162}{10}}\right)$

$= \underline{(-0.5587, 0.5612)}$

$99\% = \left(0.00127 - 2.576 \times \sqrt{\frac{0.8162}{10}},\ 0.00127 + 2.576 \times \sqrt{\frac{0.8162}{10}}\right)$

$= \underline{(-0.734473, 0.737213)}$

c) $Z_1 = \frac{1 - 0.00127}{\sqrt{\frac{0.8162}{10}}} = 3.4958 \approx 3.50 \Rightarrow P = 0.99977$

$Z_{1.1} = \frac{1.1 - 0.00127}{\sqrt{\frac{0.8162}{10}}} = 3.8459 \approx 3.85 \Rightarrow P = 0.99994$

$\Rightarrow P = 0.99994 - 0.99977 = \underline{\underline{0.017\%}}$

Problem 3 (Calculus)

Let $f(x, y) = 3x^2 + y^2 - xy - 11x$

a) Find $\frac{\partial f}{\partial x}$, the partial derivative of f with respect to x. Also find $\frac{\partial f}{\partial y}$.

b) Find the pair $(x, y) \in \Re^2$ that minimizes f.

c) Show that the pair (x, y) you found in b. is a minimizer instead of a maximizer.

a) $\underline{\underline{\frac{\partial f}{\partial x} = 6x - y - 11 \quad \frac{\partial f}{\partial y} = 2y - x}}$

b) $\begin{cases} 6x - y - 11 = 0 \\ 2y - x = 0 \end{cases} \Rightarrow \begin{cases} x = 2y \\ 6x - y = 11 \end{cases} \Rightarrow y = 1 \quad x = 2 \quad \underline{(x, y) = (2, 1)}$

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c) $f_{xx}(2,1) = 6$

$f_{yy}(2,1) = 2$

$f_{xy}(2,1) = -1$

$d = f_{xx}(2,1) f_{yy}(2,1) - [f_{xy}(2,1)]^2$

$= 6 \times 2 - (-1)^2$

$= 11$

$\because d > 0$ & $f_{xx}(2,1) > 0$ $\therefore$ 相对极小值 Q.E.D

If $f_{xx} < 0 \rightarrow$ 相对极大值

Problem 4 (Vector Norms)
Compute the 0, 1, 2, and ∞ norms for $[3\ -1\ 3\ 5\ 0\ 2\ ]^T$.

L_0: 非0个数 $= \underline{\underline{5}}$

$L_1 = |3| + |-1| + |3| + |5| + 0 + |2| = \underline{\underline{14}}$

$L_2 = \sqrt{3^2 + (-1)^2 + (3)^2 + (5)^2 + (2)^2} = 4\sqrt{3} = \underline{\underline{6.928}}$

$L_\infty = (|3|, |-1|, |3|, |5|, 0, |2|) = \underline{\underline{5}}$