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QIC Final Project: Anisotropic Transmission of quantum information through quantum fields

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Abstract: In this letter, we briefly review the possible way to transmit the quantum information via quantum fields [1], and then we discuss

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1 Quantum Channel: Via Quantum Mechanics

In quantum information theory, the information is represented by a qubit, and it can be transformed, projected, and transmitted based on basic quantum mechanics postulates. In this letter, we focus on the transmission of a qubit from a spacetime emitter Alice A to a receiver Bob B.

There are various ways to transmit a qubit without contacting, which are based on the *resources* Alice and Bob share. For instance, if an entangled state is shared, they can transmit the qubit by Alice performing the Bell measurement and then send the result (a classical cbit) to Bob, which is the well-known *quantum teleportation*. Here, we simply consider transmission by a third quantum bit C, $\hat{\rho}_{C,0}$. Denote Alice's qubit as $\hat{\rho}_{A,0}$ and Bob's qubit $\hat{\rho}_{B,0}$; the transmission is done by performing SWAP between A and C, and then between C and B. The whole process is unitary and does not violate the non-cloning process because Alice's qubit becomes $\hat{\rho}_{C,0}$.

The SWAP operator can be derived by assuming $\hat{\rho}_{C,0} = |0\rangle\langle 0|$ and $\hat{\rho}_{A,0} = |a\rangle\langle a|$ with $\langle a|0\rangle \neq 0$, and $|a\rangle = \alpha|0\rangle + \beta|1\rangle$:

$$U\rho_{A,0} \otimes \rho_{C,0}U^\dagger = \rho_{C,0} \otimes \rho_{A,0} \tag{1.1}$$

The SWAP operator is:

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^* & \beta^* & 0 \\ 0 & \beta & -\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1.2}$$

Remark: The transmission of qubit described above is rather trivial; however, it is based on an important fact that the dimension of the Hilbert space of C is the same as those of the Hilbert space of A and B, so there is an isomorphism between the Hilbert spaces. As we will see in the next section, the Hilbert space (or more precisely, the Fock space) of quantum fields is infinite-dimensional, and therefore there is no isomorphism like SWAP gate in the quantum mechanic case.

2 Quantum Channel: Via Quantum Fields

In this section, we briefly review the idea of quantum transmission via quantum fields [1]. As we will see, quantum field theory generally provides a physical picture of transmission and is consistent with the principles of special relativity.

Brief Review on Quantum Field Theory

Many quantum field theory textbooks introduce the quantum field by analog of harmonic oscillators, and here we follow the same logic. The equation of motion (e.o.m) of harmonic oscillators in the configuration space:

$$\ddot{q}(t) + \omega^2 q(t) = 0 \tag{2.1}$$

If there is no specific boundary condition, the general solution of position $q(t)$ and the conjugate momentum $p(t)$is given by:

$$\begin{aligned} q(t) &= \sqrt{\frac{\hbar}{2\omega}} \left(a e^{-i\omega t} + a^* e^{i\omega t}\right) \\ p(t) &= -i\sqrt{\frac{\hbar\omega}{2}} \left(a e^{-i\omega t} - a^* e^{i\omega t}\right) \end{aligned} \tag{2.2}$$

The pre-factor is a convenient choice to canonical quantization:

$$[\hat{q}(t), \hat{p}(t)] = i\hbar, \ \ \left[\hat{a}, \hat{a}^\dagger\right] = 1 \tag{2.3}$$

$$\begin{aligned} \hat{q}(t) &= \sqrt{\frac{\hbar}{2\omega}} \left(\hat{a} e^{-i\omega t} + \hat{a}^\dagger e^{i\omega t}\right) \\ \hat{p}(t) &= -i\sqrt{\frac{\hbar\omega}{2}} \left(\hat{a} e^{-i\omega t} - \hat{a}^\dagger e^{i\omega t}\right) \end{aligned} \tag{2.4}$$

The e.o.m, canonical quantization, and the Fourier modes of real scalar field are similar to the quantum oscillator, and we denote the conjugate momentum as $\pi(\mathbf{x}, t)$:

$$\begin{gathered} \ddot{\phi} + \nabla^2\phi + m^2\phi = 0 \\ \hat{\phi}(\mathbf{x}, t) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \left(\hat{a}(\mathbf{k}) e^{-i(E_k t - \mathbf{k}\cdot\mathbf{x})} + H.c.\right) \\ \hat{\pi}(\mathbf{x}, t) = \partial_t\hat{\phi} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \left(-iE_k \cdot \hat{a}(\mathbf{k}) e^{-i(E_k t - \mathbf{k}\cdot\mathbf{x})} + H.c.\right) \\ \left[\hat{\phi}(\mathbf{x}, t), \hat{\pi}(\mathbf{y}, t)\right] = i\delta^3(\mathbf{x} - \mathbf{y}), \ \ \left[\hat{a}(\mathbf{k}), \hat{a}^\dagger(\mathbf{k}')\right] = \delta^3(\mathbf{k} - \mathbf{k}') \end{gathered} \tag{2.5}$$

where $E_k = |\mathbf{k}|^2 + m^2$ is the energy.

Fock Space and Physical States

Next, we focus on the quantum states built by the system, and see where is the difference between quantum mechanics and quantum field theory. Again, let's first start with the quantum oscillator, the Hamiltonian of this system can be obtain by:

$$\hat{H}(\hat{p},\hat{q}) := \hat{p}\dot{\hat{q}} - \hat{L} = \frac{1}{2}\left(\hat{p}^2 + \omega^2\hat{q}^2\right) \tag{2.6}$$

After some algebra and using the commutation relation of $\hat{a}$ and $\hat{a}^\dagger$, there is a simple relation between the Hamiltonian and the operator $\hat{a}$ and $\hat{a}^\dagger$:

$$\hat{H} = \frac{\hbar\omega}{2}\left(\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger\right) \equiv -\frac{\hbar\omega}{2} + \hbar\omega\hat{N} \tag{2.7}$$

Number operator $\hat{N} \equiv \hat{a}\hat{a}^\dagger$ is defined, and we see that it can be simultaneously diagonalized with the Hamiltonian. So consider the eigenstates of number operator, and assume there is no degeneracy, and now the operators $\hat{a}$ and $\hat{a}^\dagger$ are interpreted as *lowing* and *raising* operator.

$$\begin{aligned} \hat{N}|n\rangle &= n|n\rangle \\ \hat{a}|n\rangle &\propto |n-1\rangle \\ \hat{a}^\dagger|n\rangle &\propto |n+1\rangle \end{aligned} \tag{2.8}$$

Mathematically, there can be infinite number of eigenstates, but physically, we request the Hamiltonian is bounded below, and we define the state with lowest eigenvalue as *vacuum state*.

$$\begin{aligned} \hat{a}|0\rangle &= 0 \\ \hat{N}|0\rangle &= 0 \\ \langle 0|\hat{H}|0\rangle &= -\frac{\hbar\omega}{2} \end{aligned} \tag{2.9}$$

As for the real scalar field theory, the *lowering* and *raising* operators become particle *annihilator* and *creator*. The Hamiltonian of this system is

$$\begin{aligned} \hat{H} &= \int d^3\mathbf{x}\frac{1}{2}\left(\hat{\pi}^2 + (\nabla\hat{\phi})^2 + m^2\hat{\phi}^2\right) \\ &= \int \frac{d^3\mathbf{p}}{(2\pi)^3}E_p\left(\hat{a}^\dagger(\mathbf{p})\hat{a}(\mathbf{p}) + \frac{1}{2}\left[\hat{a}(\mathbf{p}),\hat{a}^\dagger(\mathbf{p})\right]\right) \end{aligned} \tag{2.10}$$

We immediately see a problem of divergence from the second term. There are two divergences, one corresponds to the UV divergence of $\left[\hat{a}(\mathbf{p}),\hat{a}^\dagger(\mathbf{p})\right] = \delta(0)$, the other corresponds to the infinite volume in the momentum space over this commutator. The UV divergence can be dealt with a UV cutoff, and in field theory, we focus on the Hamiltonian density instead of Hamiltonian to remove the infinite volume. Again, we can simultaneously diagonalize the Hamiltonian and the number operator, and the eigenstates are called *particle states*:

$$\begin{aligned} \hat{a}(\mathbf{p})|\mathrm{VAC}\rangle &= 0 \\ \hat{a}^{\dagger}(\mathbf{p})|\mathrm{VAC}\rangle &= \sqrt{2E_p}|\mathbf{p}\rangle \\ \langle\mathbf{q}|\mathbf{p}\rangle &= \delta^3(\mathbf{q}-\mathbf{p}) \end{aligned} \tag{2.11}$$

The pre-factor is here to hold the Lorentz invariance of one particle state.

The non-trivial part of quantum field theory is the physical states, we have vacuum state, one-particle states, multi-particle states, and they do not correspond to a single Hilbert space, but a direct sum of Hilbert spaces, called Fock space. If a creator is acted on one-particle state, we expect it to be a two-particle state:

$$\hat{a}^{\dagger}(\mathbf{q})|\mathbf{p}\rangle = |\mathbf{q}\rangle \otimes |\mathbf{p}\rangle \tag{2.12}$$

It is well-known that in quantum mechanics, the one-particle state corresponds to a Hilbert space. In quantum field theory, the multiple particle states can be excited, and they are tensor products of one-particle state. For real scalar field theory, the Fock space can be written as the direct sum of Hilbert space:

$$F = \bigoplus_{n=0}^{\infty} H^{\otimes n} \tag{2.13}$$

Where $H^{\otimes 0}$ is defined as the complex space $\mathbb{C}$, which corresponds to the vacuum state. The transmission of quantum information via quantum fields is more complicated than that via quantum mechanics because of the mismatch of dimensionality between the dimension of quantum fields and the state owned by Alice and Bob.

Unruh-DeWitt model

In this section, we introduce the interaction of a quantum mechanics system and quantum fields, which is called Unruh-De Witt model. It is used to explain the Unruh-effect of horizon, and here it is used to construct the transmission system. Suppose the quantum detectors (it can be physically realized by atomic orbital of crystal) carried by Alice and Bob are localized at spacetime points, and they are coupled to the real scalar quantum field $\phi(\mathbf{x}, t)$. First, consider the realization of qubit in the quantum detector $\nu = \{A, B\}$, a convenient choice is the Pauli z-operator [1]. The qubits are realized by the eigenstates of the free Hamiltonian, which is $\hat{H}_\nu = \omega_\nu \hat{\sigma}_z$:

$$\hat{H}_\nu |\pm_{z,\nu}\rangle = \pm\omega_\nu |\pm_{z,\nu}\rangle \tag{2.14}$$

Then, the interaction between detectors and the field can be described by a local interaction:

$$\hat{H}_\nu(t) = \lambda\chi(t)\hat{m}_\nu(t) \otimes \hat{\mathcal{O}}_\nu(t) \tag{2.15}$$

λ is a coupling constant, $\chi(t)$ is an switching function that smoothly controls the time duration of interaction, and $\hat{m}_\nu(t)$ and $\hat{\mathcal{O}}_\nu(t)$ are detector and field observables.

The field observables should be local in spacetime, and it is realized by smearing function:

$$\hat{\mathcal{O}}_\nu(t) = \int d^3\mathbf{x} F_\nu(\mathbf{x}) \hat{\mathcal{O}}(\mathbf{x}, t) \tag{2.16}$$

The smearing function must have support in the region of space at time t where the observers are located. Note that we have to choose a picture to track the time-dependence of the quantum states and operators, see appendix for more information. The time-evolution operator of state by this interaction Hamiltonian is:

$$\hat{U}_\nu = \mathcal{T} \exp\left[-i \int_{-\infty}^{\infty} dt \hat{H}_\nu(t)\right] \tag{2.17}$$

where $\mathcal{T}$ denotes the time-ordering.

When the coupling constant $\lambda << 1$ in the units of the characteristic length scale of this system, we can use the perturbation theory to approximate the evolution up to a desired order. However, as we shall see, this is not the case for transmission of quantum information, so we must deal with this evolution problem with non-perturbative technique. An simplification of interaction as an example, if the quantum detector only interacts with the quantum field at discrete instants in time, that is, the switching function of the quantum detector is $\chi(t) = \sum_{i=1}^{n} \delta(t - t_i)$ with $t_i < t_{i+1}$. Then the time evolution operator can be expressed as $\hat{U} = \hat{U}_n \cdots \hat{U}_1$:

$$\hat{U}_{\nu,i} = \exp\left[-i\lambda \hat{m}_\nu(t_i) \otimes \hat{\mathcal{O}}_\nu(t_i)\right] \tag{2.18}$$

This expression is exact analytical.

Strong Coupling Condition

The non-perturbative couplings between the detectors and the field is a necessary condition to construct a perfect quantum channel:

$$Q(\Xi) = 0 + \mathcal{O}(\lambda) \tag{2.19}$$

Qubit in a Field: Encoding

We call the gate encoding a qubit into a field an ENCODE gate, rather than a SWAP gate in the previous section, and similarly for the DECODE gate. Now, Alice's detector need to be coupled to the real scalar field at time t_A and with the spatial extent given by the support of the smearing function $F_A(\mathbf{x})$. Suppose, initially, the field is in the vacuum state $|\text{VAC}\rangle$, and Alice's qubit is $c_1|+_z\rangle + c_2|-_z\rangle$. The qubit-field interaction is given by:

$$\begin{aligned}\hat{H}_A(t) = \lambda_\phi \delta(t - t_A^-)\hat{\sigma}_z \otimes \int d^3\mathbf{x} F_A(\mathbf{x})\hat{\phi}(\mathbf{x}, t) \\ +\lambda_\pi \delta(t - t_A^+)\hat{\sigma}_z \otimes \int d^3\mathbf{x} F_A(\mathbf{x})\hat{\pi}(\mathbf{x}, t)\end{aligned} \tag{2.20}$$

where the times $t_A^\pm \approx t_A$ such that t_A^- is slightly smaller than t_A^+. Therefore, the time evolution operator is:

$$\hat{U}_A = \exp\left(i\hat{\sigma}_x \otimes \hat{\pi}_A\right) \exp\left(i\hat{\sigma}_z \otimes \hat{\phi}_A\right) \tag{2.21}$$

where $\hat{\phi}_A$ and $\hat{\pi}_A$ are smeared field observables:

$$\begin{aligned}\hat{\phi}_A &:= \lambda_\phi \int d^3\mathbf{x} F_A(\mathbf{x})\hat{\phi}(\mathbf{x}, t_A) \\ \hat{\pi}_A &:= \lambda_\pi \int d^3\mathbf{x} F_A(\mathbf{x})\hat{\pi}(\mathbf{x}, t_A)\end{aligned} \tag{2.22}$$

Acting on the initial state $(c_1|+_z\rangle + c_2|-_z\rangle) \otimes |\mathrm{VAC}\rangle$ with the rightmost exponential in $\hat{U}_A$ results in the state:

$$c_1|+_z\rangle \otimes |\alpha_A^+\rangle + c_2|-_z\rangle \otimes |\alpha_A^-\rangle \tag{2.23}$$

where coherent field state $|\alpha_A^\pm\rangle$ are defined by:

$$|\alpha_A^\pm\rangle := \exp\left(\pm i\hat{\phi}_A\right)|\mathrm{VAC}\rangle \tag{2.24}$$

By mode expansion of real scalar field and the commutation relation of the creator and annihilator, the overlap between two coherent field states is:

$$\begin{aligned}|\langle\alpha_A^+|\alpha_A^-\rangle| &= |\langle\mathrm{VAC}|\exp\left(-i\hat{\phi}_A^\dagger\right)\exp\left(-i\hat{\phi}_A\right)|\mathrm{VAC}\rangle| \\ &= \exp\left[-\lambda_\phi^2 \int \frac{d^3\mathbf{k}}{2E_k}|\tilde{F}_A(\mathbf{k})|^2\right]\end{aligned} \tag{2.25}$$

where $\tilde{F}_A(\mathbf{k})$ is the Fourier transform of the smearing function.

Hence under the strong coupling condition $\lambda_\phi >> 1$, the coherent field states $|\alpha_A^\pm\rangle$ are almost orthogonal.

Then the operator $\exp\left(i\hat{\sigma}_x \otimes \hat{\pi}_A\right)$ is applied. First, consider the field observable $\hat{\pi}_A$ on the coherent field state, using the Baker-Campbell-Hausdorff formula[1]:

$$\hat{\pi}_A|\alpha_A^\pm\rangle = \pm\gamma_A|\alpha_A^\pm\rangle + \exp(\pm i\hat{\pi}_A)\hat{\pi}_A|\mathrm{VAC}\rangle \approx \pm\gamma_A|\alpha_A^\pm\rangle \tag{2.26}$$

where $\gamma_A := \lambda_\phi\lambda_\pi \int d^3\mathbf{k}|\tilde{F}_A(\mathbf{k})|^2$, and the second equality holds if $\gamma_A^2 >> \langle\mathrm{VAC}|\hat{\pi}_A|\mathrm{VAC}\rangle$, and the coherent field states are approximately eigenstates of the observables π_A with eigenvalues $\pm\gamma_A$.

If

$$\gamma_A = \frac{\pi}{4} \mod 2\pi, \tag{2.27}$$

then

$$\begin{aligned}
&\hat{U}_A \left(c_1|+_z\rangle + c_2|-_z\rangle\right) \otimes |\text{VAC}\rangle \\
&= \exp\left(i\hat{\sigma}_x\hat{\pi}_A\right)\left(c_1|+_z\rangle \otimes |\alpha_A^+\rangle + c_2|-_z\rangle \otimes |\alpha_A^-\rangle\right) \\
&\approx c_1 \exp\left(+i\frac{\pi}{4}\hat{\sigma}_x\right)|+_z\rangle \otimes |\alpha_A^+\rangle + c_2 \exp\left(-i\frac{\pi}{4}\hat{\sigma}_x\right)|-_z\rangle \otimes |\alpha_A^-\rangle \\
&= |+_y\rangle \otimes \left(c_1|\alpha_A^+\rangle - ic_2|\alpha_A^-\rangle\right)
\end{aligned} \tag{2.28}$$

In the second line, the identities $\exp\big(+i\frac{\pi}{4}\hat{\sigma}_x\big)|+_z\rangle = |+_y\rangle$ and $\exp\big(-i\frac{\pi}{4}\hat{\sigma}_x\big)|+_z\rangle = -i|-_y\rangle$ are used, which are the Bloch sphere rotations, from the eigenstates of $\hat{\sigma}_z$ into the positive eigenvalue eigenstate $|+_y\rangle$ of $\hat{\sigma}_y$ by applying rotation operator generated by $\hat{\sigma}_x$. Therefore the time evolution operator $\hat{U}_{\text{A}}$ encodes Alice's orthogonal qubit superposition $c_1|+_z\rangle + c_2|-_z\rangle$ into an (almost) orthogonal superposition of the coherent field states, $c_1|\alpha_{\text{A}}^+\rangle - ic_2|\alpha_{\text{A}}^-\rangle$.

As long as the strong coupling condition and the $\gamma_\pm = \pi/4$ are satisfied, the encoding can be carried out by the interaction $\hat{U}_A = \exp\left(i\hat{\sigma}_x\hat{\pi}_A\right)\exp\big(i\hat{\sigma}_z\hat{\phi}_A\big)$. In the above example, the rightmost operator $\exp\Big(i\hat{\sigma}_z \otimes \hat{\phi}_A\Big)$ will partially entangle (depending on the superposition of Alice's initial state) Alice's qubit with the field's state. Following this, the leftmost operator $\exp\left(i\hat{\sigma}_x \otimes \hat{\pi}_A\right)$ will then use the state of the field to perform a controlled rotation in the Bloch sphere of the qubit, thus leaving the field into an superposition of coherent field states, where information of Alice initial qubit is stored. In other words, the qubit-field interaction $\hat{U}_{\text{A}}$ encodes Alice's qubit into the field's state.

Qubit out of a Field: Decoding

Having understood how Alice can ENCODE her qubit of information into the field, the final step in constructing the field-mediated quantum channel from Alice to Bob is to construct the DECODE gate that allows Bob to recover Alice's message from the field. The most straightforward way to proceed is to note that the DECODE gate should simply be the inverse of the ENCODE gate. Thus, since we know the unitary $\hat{U}_A = \exp\left(i\hat{\sigma}_x\hat{\pi}_A\right)\exp\big(i\hat{\sigma}_z\hat{\phi}_A\big)$ implementing the encode gate, we also know that the inverse unitary $\hat{U}_A^{-1} = \hat{U}_A^\dagger = \exp\big(-i\hat{\sigma}_z\hat{\phi}_A\big)\exp\big(-i\hat{\sigma}_x\hat{\pi}_A\big)$ will implement the DECODE gate. We can now simply set the unitary $\hat{U}_B$, which acts on detector B and the field, to be the unitary $\hat{U}_A^\dagger$ with the understanding that the qubit observables $\hat{\sigma}_x$ and $\hat{\sigma}_z$ now act on the Hilbert space $\mathcal{H}_{\text{B}}$ rather than $\mathcal{H}_{\text{A}}$.

Note however that there is a problem with this construction of the decoding unitary $\hat{U}_B$. Namely, while we have modified the qubit observables in $\hat{U}_B$ from the ones in $\hat{U}_A^\dagger$ so that now they act on $\mathcal{H}_{\text{B}}$ rather than $\mathcal{H}_{\text{A}}$, the field observables ϕ_A and π_A appearing in $\hat{U}_B$ are still defined at the time t_{A} (c.f. Eqs. (**??**) and (**??**)). But in order for Bob to implement $\hat{U}_B$ at a later time $t_{\rm B}$, he needs to couple his qubit to field observables defined at the time $t_{\rm B}$, not at $t_{\rm A}$.

We will now solve this problem by proving a mathematical result which expresses the field observables ϕ_A and π_A as observables at time $t_{\rm B}$. Fundamentally, this result arises due to the fact that the field $\hat{\phi}(\mathbf{x},t)$ is by definition a solution to the wave equation, which, being a hyperbolic PDE, has a well defined initial value formulation that allows solutions at time $t_{\rm A}$ to be propagated to solutions at time $t_{\rm B}$.

With this mathematical result at hand, we can now write the unitary $\hat{U}_B$ — which decodes Alice's qubit out of the field and onto Bob's detector — in terms of field observables at the time t_B. Namely, the theorem allows us to write the field observables $\hat{\phi}_A$ and $\hat{\pi}_A$ as

$$\begin{aligned}\phi_A &= \lambda_\phi \hat{\phi}[F_{\rm B2}](t_B) + \lambda_\phi \hat{\pi}[F_{\rm B1}](t_B),\\ \pi_A &= \lambda_\pi \hat{\phi}[F_{\rm B3}](t_B) + \lambda_\pi \hat{\pi}[F_{\rm B2}](t_B),\end{aligned}$$

where Bob's smearing functions $F_{{\rm B}i}(\mathbf{x})$ are defined in terms of Alice's smearing $F_{\rm A}$ through their Fourier transforms,

$$\begin{aligned}\tilde{F}_{\rm B1}(\mathbf{k}) &= \tilde{F}_{\rm A}(\mathbf{k})\mathrm{sinc}(\Delta E_k)(-\Delta),\\ \tilde{F}_{\rm B2}(\mathbf{k}) &= \tilde{F}_{\rm A}(\mathbf{k})\cos(\Delta E_k),\\ \tilde{F}_{\rm B3}(\mathbf{k}) &= \tilde{F}_{\rm A}(\mathbf{k})\sin(\Delta E_k)E_k.\end{aligned} \tag{2.29}$$

Hence the unitary $\hat{U}_B$, defined by

$$\hat{U}_B = \exp\big(-i\hat{\sigma}_z\phi_A\big)\exp\big(-i\hat{\sigma}_x\pi_A\big), \tag{2.30}$$

can now be alternatively defined in terms of field observables at time t_B, namely

$$\begin{aligned}\hat{U}_B = \exp&\left[-i\lambda_\phi\hat{\sigma}_z\left(\hat{\phi}[F_{\rm B2}](t_B) + \hat{\pi}[F_{\rm B1}](t_B)\right)\right]\\ \times\exp&\left[-i\lambda_\pi\hat{\sigma}_x\left(\hat{\phi}[F_{\rm B3}](t_B) + \hat{\pi}[F_{\rm B2}](t_B)\right)\right].\end{aligned}$$

In summary, we have succeeded in constructing the quantum channel shown in Fig., which allows Alice to perfectly transmit a qubit through a quantum field to Bob. The quantum channel consists of two steps:

- First, at time $t = t_A$, Alice encodes her qubit state in a spatial region of the field characterized by $F_{\rm A}(\mathbf{x})$ by implementing the unitary $\hat{U}_A$ given in .

- Then, at a later time $t = t_B$, Bob decodes the qubit from the field by coupling with the unitary $\hat{U}_B$ given in Eq. In order for Bob to be able to implement this unitary, his detector must be smeared in a spatial region that contains the supports of the functions $F_{\rm B1}(\mathbf{x})$, $F_{\rm B2}(\mathbf{x})$, and $F_{\rm B3}(\mathbf{x})$ defined by Eqs.

Additionally, in order for the channel to succeed, the conditions on the coupling strengths λ_ϕ and λ_π must be satisfied. Physically, Eq. (**??**) is a strong-coupling condition which ensures that Alice's qubit first gets maximally entangled with orthogonal coherent field states, while Eq. (**??**) is a fine-tuning condition which ensures that Alice's qubit is then rotated by the right amount in the Bloch sphere so that it gets completely unentangled from the field. Together, these conditions ensure that the encoding gate (and hence the decoding gate, which is just the inverse encoding gate) are implemented successfully. In particular we note that, as was discussed above, a strong (i.e. non-perturbative) coupling of detectors to the field is necessary in order for the field-mediated quantum channel from Alice to Bob to have maximal quantum channel capacity.

Despite our successes so far, there still remain two pertinent issues that must be addressed before one can be fully satisfied with our construction of a perfect, field-mediated quantum channel from Alice to Bob. First, it should be verified, without the use of any approximations (such as the one in Eq. (**??**)), that our supposedly perfect quantum channel Ξ indeed has a maximal quantum channel capacity of $\mathcal{Q}(\Xi) = 1$. And second, the smearing functions $F_{\text{B}i}(\mathbf{x})$ are defined in terms of their Fourier transforms, and hence it is presently not clear where in space Bob needs to be located in order to receive Alice's quantum message, which is crucial for our study. We will successively address these two remaining issues in Sec. **??** and Sec. **??**.

3 Broadcasting Quantum Information

Isotropic Smearing Function

Anisotropic Smearing Function

Acknowledgments

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