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QIC Final Project: Anisotropic Transmission of quantum information through quantum fields

T. Hsu

National Taiwan University,
Taipei, Taiwan

E-mail: b11901097@ntu.edu.tw

ABSTRACT: In this letter, we briefly review the possible way to transmit the quantum information via quantum fields [1], and then we discuss

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1 Quantum Channel: Via Quantum Mechanics

In quantum information theory, the information is represented by a qubit, and it can be transformed, projected, and transmitted based on basic quantum mechanics posulates. In this letter, we focus on the transmission of a qubit from a spacetime emitter Alice A to a receiver Bob B.

There are various ways to transmit a qubit without contacting, which are based on the *resources* Alice and Bob share. For instance, if an entagled state is shared, they can transmit the qubit by Alice performing the Bell measurement and then send the result (a classical cbit) to Bob, which is the well-known *quantum teleportation*. Here, we simply consider transmisstion by a third quantum bit C, $\hat{\rho}_{C,0}$. Denote Alice's qubit as $\hat{\rho}_{A,0}$ and Bob's qubit $\hat{\rho}_{B,0}$; the transmission is done by performing SWAP between A and C, and then between C and B. The whole process is unitary and does not violate the non-cloning process because Alice's qubit becomes $\hat{\rho}_{C,0}$.

The SWAP operator can be derived by assuming $\hat{\rho}_{C,0} = |0\rangle\langle 0|$ and $\hat{\rho}_{A,0} = |a\rangle\langle a|$ with $\langle a|0\rangle \neq 0$, and $|a\rangle = \alpha|0\rangle + \beta|1\rangle$:

$$U\rho_{A,0} \otimes \rho_{C,0} U^\dagger = \rho_{C,0} \otimes \rho_{A,0} \tag{1.1}$$

The SWAP operator is:

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^* & \beta^* & 0 \\ 0 & \beta & -\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1.2}$$

Remark: The transmission of qubit described above is rather trivial; however, it is based on an important fact that the dimension of the Hilbert space of C is the same as those of the Hilbert space of A and B, so there is an isomorphism between the Hilbert spaces. As we will see in the next section, the Hilbert space (or more precisely, the Fock space) of quantum fields is infinite-dimensional, and therefore there is no isomorphism like SWAP gate in the quantum mechanic case.

2 Quantum Channel: Via Quantum Fields

In this section, we breifly review the idea of quantum transmission via quantum fields [1]. As we will see, quantum field theory generally provides a physical picture of transmission and is consistent with the priciples of sepcial relativity.

2.1 Brief Review on Quantum Field Theory

Many quantum field theory textbooks introduce the quantum field by analog of harmonic oscillators, and here we follow the same logic. The equation of motion (e.o.m) of harmonic oscillators in the configuration space:

$$\ddot{q}(t) + \omega^2 q(t) = 0 \tag{2.1}$$

If there is no specific boundary condition, the general solution of position $q(t)$ and the conjugate momentum $p(t)$is given by:

$$\begin{aligned} q(t) &= \sqrt{\frac{\hbar}{2\omega}} \left(a e^{-i\omega t} + a^* e^{i\omega t}\right) \\ p(t) &= -i\sqrt{\frac{\hbar\omega}{2}} \left(a e^{-i\omega t} - a^* e^{i\omega t}\right) \end{aligned} \tag{2.2}$$

The pre-factor is a convenient choice to canonical quantization:

$$[\hat{q}(t), \hat{p}(t)] = i\hbar, \ \left[\hat{a}, \hat{a}^\dagger\right] = 1 \tag{2.3}$$

$$\begin{aligned} \hat{q}(t) &= \sqrt{\frac{\hbar}{2\omega}} \left(\hat{a} e^{-i\omega t} + \hat{a}^\dagger e^{i\omega t}\right) \\ \hat{p}(t) &= -i\sqrt{\frac{\hbar\omega}{2}} \left(\hat{a} e^{-i\omega t} - \hat{a}^\dagger e^{i\omega t}\right) \end{aligned} \tag{2.4}$$

The e.o.m, canonical quantization, and the Fourier modes of real scalar field are similar to the quantum oscillator, and we denote the conjugate momentum as $\pi(\mathbf{x}, t)$:

$$\begin{aligned} \ddot{\phi} + \nabla^2\phi + m^2\phi = 0 \\ \hat{\phi}(\mathbf{x}, t) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \left(\hat{a}(\mathbf{k}) e^{-i(E_k t - \mathbf{k}\cdot\mathbf{x})} + H.c.\right) \\ \hat{\pi}(\mathbf{x}, t) = \partial_t \hat{\phi} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \left(-iE_k \cdot \hat{a}(\mathbf{k}) e^{-i(E_k t - \mathbf{k}\cdot\mathbf{x})} + H.c.\right) \\ \left[\hat{\phi}(\mathbf{x}, t), \hat{\pi}(\mathbf{y}, t)\right] = i\delta^3(\mathbf{x} - \mathbf{y}), \ \left[\hat{a}(\mathbf{k}), \hat{a}^\dagger(\mathbf{k}')\right] = \delta^3(\mathbf{k} - \mathbf{k}') \end{aligned} \tag{2.5}$$

where $E_k = |\mathbf{k}|^2 + m^2$ is the energy.

2.2 Fock Space and Physical States

2.3 Unruh-DeWitt model

Acknowledgments

References

[1] Petar Simidzija, Aida Ahmadzadegan, Achim Kempf, and Eduardo Martín-Martínez. Transmission of quantum information through quantum fields. *Phys. Rev. D*, 101:036014, Feb 2020.