Quantum Field Theory Problems

Ting-Kai Hsu

February 23, 2024

Contents

1 T P Symmetries

In S.Weinberg famous textbook about quantum field theory[3] section 2.6, he discusses parity$\mathcal{P}$ and time inversion$\mathcal{T}$. [1]

$$\begin{aligned}\mathcal{P}^{\mu}{}_{\nu} &= \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}\\ \mathcal{T}^{\mu}{}_{\nu} &= \begin{pmatrix}-1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\end{aligned} \tag{1.0.1}$$

The operators of $\mathcal{P}$ and $\mathcal{T}$ are believed to be

$$\begin{aligned}\mathrm{P} &\equiv U(\mathcal{P}, 0)\\ \mathrm{T} &\equiv U(\mathcal{T}, 0)\end{aligned} \tag{1.0.2}$$

[1] I use different notation with Weinberg.

He seems to define a new notation that corresponds to not only Lorentz transformation and translation but also parity and time inversion. The operators of Poincaré algebra would transform according to the following law,

$$\begin{aligned} \mathrm{P}U(\Lambda, a)\mathrm{P}^{-1} &= U(\mathcal{P}\Lambda\mathcal{P}^{-1}, \mathcal{P}a) \\ \mathrm{T}U(\Lambda, a)\mathrm{T}^{-1} &= U(\mathcal{T}\Lambda\mathcal{T}^{-1}, \mathcal{T}a) \end{aligned} \tag{1.0.3}$$

I'm confused about what he means,

> These transformation rules incorporate most of what is meant when we say that P or T are 'conserved'.

Later, he points out that the above equations of P and T are merely approximation. These are provided by T. D. Lee, C. N. Yang and others works[2][1]. Before reading the references, I think that the problem arises from that equation (1.0.3) that physicists originally define is wrong, or merely an approximation. Still, I would like to know why and why it is regarded as approximation.

2 Lippmann-Schwinger equation

$$\Psi^{\text{in}}_{\mathbf{k}} = \Phi_{\mathbf{k}} + \left(E(|\mathbf{k}| - H_0 + i\epsilon)\right)^{-1} \Psi^{\text{in}}_{\mathbf{k}} \tag{2.0.1}$$

It seems Weinberg's reason of adding the positive infinitesimal parameter ϵ is weird, and it is unclear and unintuitive for giving meaning to operator.

3 Meaning and Function of Potential Operator

References

[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay. Evidence for the 2π decay of the k_2^0 meson. *Phys. Rev. Lett.*, 13:138–140, Jul 1964.

[2] T. D. Lee and C. N. Yang. Question of parity conservation in weak interactions. *Phys. Rev.*, 104:254–258, Oct 1956.

[3] Steven Weinberg. *RELATIVISTIC QUANTUM MECHANICS*, page 49–106. Cambridge University Press, 1995.