Quantum Field Theory Problems

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1 T P Symmetries

In S.Weinberg famous textbook about quantum field theory[3] section 2.6, he discusses parity$\mathcal{P}$ and time inversion$\mathcal{T}$. [1]

$$\begin{aligned} \mathcal{P}^{\mu}{}_{\nu} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \\ \mathcal{T}^{\mu}{}_{\nu} &= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{aligned} \tag{1.0.1}$$

The operators of $\mathcal{P}$ and $\mathcal{T}$ are believed to be

$$\begin{aligned} \mathrm{P} &\equiv U(\mathcal{P}, 0) \\ \mathrm{T} &\equiv U(\mathcal{T}, 0) \end{aligned} \tag{1.0.2}$$

He seems to define a new notation that corresponds to not only Lorentz transformation and translation but also parity and time inversion. The operators of

[1] I use different notation with Weinberg.

Poincaré algebra would transform according to the following law,

$$\begin{aligned} \mathrm{P}U(\Lambda,a)\mathrm{P}^{-1} &= U(\mathcal{P}\Lambda\mathcal{P}^{-1},\mathcal{P}a) \\ \mathrm{T}U(\Lambda,a)\mathrm{T}^{-1} &= U(\mathcal{T}\Lambda\mathcal{T}^{-1},\mathcal{T}a) \end{aligned} \tag{1.0.3}$$

I'm confused about what he means,

> These transformation rules incorporate most of what is meant when we say that P or T are 'conserved'.

Later, he points out that the above equations of P and T are merely approximation. These are provided by T. D. Lee, C. N. Yang and others works[2][1]. Before reading the references, I think that the problem arises from that equation (1.0.3) that physicists originally define is wrong, or merely an approximation. Still, I would like to know why and why it is regarded as approximation.

References

[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay. Evidence for the 2π decay of the k_2^0 meson. *Phys. Rev. Lett.*, 13:138–140, Jul 1964.

[2] T. D. Lee and C. N. Yang. Question of parity conservation in weak interactions. *Phys. Rev.*, 104:254–258, Oct 1956.

[3] Steven Weinberg. *RELATIVISTIC QUANTUM MECHANICS*, page 49–106. Cambridge University Press, 1995.