Discussions on CPT-symmetric Universe and Arrow of Time

Ting-Kai Hsu*
National Taiwan University

(Dated: December 9, 2023)

Abstract: This letter mainly discuss Bolye's series of papers regarding CPT-symmetric universe and the arrow of time[1] [2] [3]. In the early paper[2], Boyle solved the massive scalar field at Big Bang, and find out the solution would only be analytic on one side of the bang. This implies that the arrow of time should indeed point out of the bang. In his latter paper [1], imposing CPT-symmetry at the very beginning of Big Bang would enable us to think of the possiblity of anti-universe that is on the other side of the bang. The model is inspiring, for that it provides simpler explanations to odd phenomana in our universe that others so far. This letter doesn't go into details of derivation of the solution of massive scalar field but focuss on another thing that is still lingering question in physics but unsolved by allowing CPT-symmetry at the very beginning. That is, the missing of magnetic monopole. Its existence would be allowed if in the anti-universe, the field tensor would undergo duality transformation. The construction of CPT-universe adding duality transformation-invariant would be future work.

I. INTRODUCTION

I got interested in this topic when I saw the posts [4] and [5]. The determination of arrow of time in thermaldynamics is a lingering problem in physics. Although in relativity, only the spacetime difference of events is important, one still be interested in whether there is a way to give time absolute value at any given location in an inertial frame. If thinking a little bit more, the question would become whether there is an event allows observors in different location of universe to synchronize their clock? In fact, the Big Bang should make all observers agree that it happens at $t = 0$.

Recently, James Webber telescrope gave some inspiring observations which may imply there do is something *'before'* the Big Bang.

Boyle's paper [3] raises an interesting topic about what happens if CPT symmetry is preserved at the very beginning of the birth of universe. CPT symmetry is a famous discrete symmetry recognized to be a fundamental property of quantum physical laws. Recently, physicists have discussed the violation of CPT symmetry[6]. Althought current observations such as asymmetry of matter and antimatter imply CPT theorem needs modification, as Lehenrt's paper [6] discusses. And Boyle's paper [3] gives reasonalbe explanations that CPT-symmetric universe model is acceptable because it provides some simple explanations for observed phenomana.

In the longer companion paper [1], it has point out the CPT-symmetric universe model would provide a novel explanantion for the arrow of time, which is interesting.

In another Boyle's paper [2] has a mysterious comment about the arrow of time suggested by the solution of massive field. Therefore, the discussion of the arrow of time would be focused in this letter.

* b11901097@ntu.edu.tw

A. Mysterious Comment about Arrow of Time

The fact that basic considerations of symmetry and analyticity force all fields to satisfy a restrictive boundary condition at one end of spacetime (the bang) but not at the other end (the dS boundary) is striking. It seems to suggest a fundamental new explanation for why the thermodynamic arrow of time (i.e. the direction in which entropy increases) points away from the bang.

This is the original comment in [2]. Section II would introduce the arrow of time that related to entropy in thermodynamic and time reversal symmetry breaking, and section III would deal the solution solved in [2], and finally we would get back to this comment and discuss how the solution suggest.

II. CPT-SYMMETRY AND ENTROPY

A. Classical Mechanics

CPT-symmetry corresponds to three discrete reversals, that is, C for charge-conjugation, P for parity, and T for time-reversal. Interesting, only there operation performed together would this symmetry preserved. Others combination would be violation, like CP violation in weak interaction.

For instance, Lorentz's force law:

$$\frac{d\mathbf{p}}{dt} = q\left(\mathbf{E}(\mathbf{x}) + \mathbf{v} \times \mathbf{B}(\mathbf{x})\right) \tag{1}$$

We know that under CPT-reflection:

$$
\begin{aligned}
&\mathbf{x} \xrightarrow{T} \mathbf{x} \xrightarrow{P} -\mathbf{x} \xrightarrow{C} -\mathbf{x} \\
&\mathbf{p} \xrightarrow{T} -\mathbf{p} \xrightarrow{P} \mathbf{p} \xrightarrow{C} \mathbf{p} \\
\mathbf{E}(\mathbf{x}) = \frac{q\,(\mathbf{x}-\mathbf{x}')}{4\pi|\mathbf{x}-\mathbf{x}'|^3} \to \mathbf{E}(-\mathbf{x}) = &\frac{-q\cdot(-\mathbf{x}+\mathbf{x}')}{4\pi|\mathbf{x}-\mathbf{x}'|^3} \\
&\mathbf{v} \xrightarrow{T} -\mathbf{v} \xrightarrow{P} \mathbf{v} \xrightarrow{C} \mathbf{v} \\
\mathbf{B}(\mathbf{x}) = \frac{\mu_0 I}{2\pi|\mathbf{x}|}\hat{\phi} \to \mathbf{B}(-\mathbf{x}) = &\frac{\mu_0\cdot -I}{2\pi|-\mathbf{x}|}(-\hat{\phi}) \\
&q \xrightarrow{T} q \xrightarrow{P} q \xrightarrow{C} -q
\end{aligned} \tag{2}
$$

Lorentz's force equation becomes:

$$\frac{d\mathbf{p}}{d(-t)} = -q\,(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \tag{3}$$

Current is considered to have a minus sign under time-reversal because $\mathbf{j} = \rho\mathbf{v}$, where $\mathbf{v}$ is the velocity of charged particle. And the current vector would reverse under parity, and then changes sign again under charge-conjugation. Thus Eq(1) would be covariant under CPT-reflection. Others would be explained in [6].

B. Quantum Mechanics

Quantum-mechanically, we would have to consider Schrödinger's equation (Schrödinger's picture):

$$i\frac{\partial}{\partial t}\phi(\mathbf{x},t) = \hat{H}\phi(\mathbf{x},t) \tag{4}$$

Under time-reversal:

$$\phi(\mathbf{x},t) \to \phi'(\mathbf{x},t) = \phi^*(\mathbf{x},-t) \tag{5}$$

The complex conjugate is to make the Schrödinger's equation covariant under time-reversal, and it is assumed that Hamiltonian $\hat{H}$ is time-reversal invariant and real.

On the other hand, taking complex conjugate under time-reversal can be easily seen by considering commutation relation of position and momentum,

$$[x_i, p_j] = i\delta_{ij} \tag{6}$$

From Eq(2), it is easy to see:

$$[x_i, p_j] \to [x_i, -p_j] = -i\delta_{ij} \tag{7}$$

That is, taking complex conjugate. In fact, time reversal operator would be **unlinear** operator[7], more discussions in [8] and [6].

C. Thermodynamics

However, time-reversal apparently violates the second law of thermodynamics statistically. In fact, only when our system is in the highest possible state of disorder would the second law of thermodynamic allow the entropy to be conserved in either direction of time, that is, there would no longer be preferred direction of time. Moreover, it seems the universe had low entropy in the past which causes the distinction between past and future, and this is an unsolved problem in physics and beyond the scope of this letter.

However, if we consider CPT-invariant right after the bang, in other words, there would be another anti-universe that goes time-reversally, as illustrated in FIG.1.

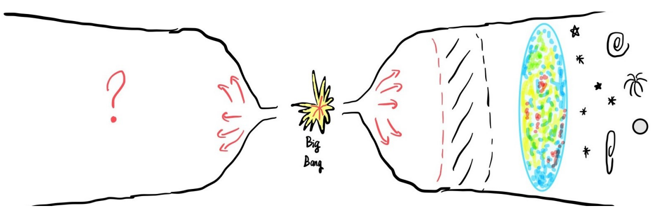

FIG. 1. The universe anti-iverse pair

The CPT reflection would immediately solved some confusing problems, that is, why we have more matter than anti-matter in present universe? By this model, we see that under C reflection, the anti-universe would have more anti-matter than matter. Therefore, if there really is a connection between our universe and the anti-universe, the abundance of matter and anti-matter may tie.

Moreover, in our universe, all three kinds of nuetrinos are mysteriously left-handed before they have mass. Under parity, the anti-unverse should have right-handed nuetrinos. And Boyle's paper [3] points these right-handed nuetrinos could be the dark matter candidate.

Remark: CPT model provides a simple explanation about the inequality of matter and anti-matter; moreover, it seems to be able to explain the nonexistence of *magnetic monopole* in our universe, that is, in anti-universe, we may observe the existence of magnetic monopole. There would be more discusstion about this in section IV.

III. SOLUTION REVISIT

A. Massive Scalar Field

This section focuses on solution given by massive scalar field as Boyle' paper [2] shows, but this section goes into details [9]. Taking $\eta_{\mu\nu} = \{+,-,-,-\}$ convention. The e.o.m of massive scalar field without source would be,

$$\left(\partial_\mu^2 + m^2\right)\phi = 0 \tag{8}$$

However, there is a conformally coupling scalar term in the original paper [2].

$$\left(\partial_\mu^2 + m^2 + \xi R\right)\phi = 0 \tag{9}$$

B. Conformally Coupling Scalar

The original paper using the conformally coupling scalar suggested by Penrose and Roger's paper [10]. Usually the massless scalar field adding coupling scalar is conformal-invariant.

$$S = \frac{1}{2}\int d^4x\,\sqrt{|g|}(g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + \xi R\phi^2) \tag{10}$$

But because Eq(9) is a massive field, whether adding the conformally coupling scalar term would it becomes conformal-invariant needs verified. The post [11] has discussed similar question and then give a little proof that the following equation is conformally invairant:

$$\left(\partial_\mu^2 + m^2 + \xi R\right)\phi = 0 \tag{11}$$

with $\xi = 1/6$. It should be noted that the last two terms give mass to the field, and the last term would vanish in flat spacetime. Each term would be following under conformal transform $\tilde{g}_{\mu\nu} = a^{-2}g_{\mu\nu}$,

$$\begin{aligned} x &\to ax \\ \phi &\to a\phi \\ m &\to am \\ R &\to a^2 R \end{aligned} \tag{12}$$

The derivation of ξ and solution of it at band and dS boundary, however, is beyond the knowledge. Thus we consider result here but not derivation.

C. Result

Boyle's paper [2] finds out that only at bang is the massive field solvable, that is, would satisfy boundary condition and thus analytic. However, at another side of anti-universe, the fiedl seems to be not analytic, and Boyle just point out that this may imply the arrow of time should be pointing out bang but not inward, which is quite confusing. And then if we add more constrains on the original model in Boyle's paper [11], that is, adding CPT-symmetry at the very beginning of universe, as the latter paper [1] does. Then the arrow of time at both universe would be pointing outward the Big Bang.

IV. DISCUSSION

A. About CPT Model and Arrow of Time

Here we make some conclusions for the CPT-symmetry universe. (I)It seems that if the very beginning of bang preserve CPT-symmetry, the inequality of existence of antimatter and matter would immediately be solved. (II)Moreover, because parity symmery has been respected, too. All left-handed nuetrinos would become right-handed nuetrinos, and some would become dark-matter candidate. (III)It would become more natural to see the problem of arrow of time, though the problem why the non-analyticity at dS for field implies the direction of time needs more explanations. Here give some thoughts about the result. The non-analyticity might means that the consideration isn't sufficient and may cause the two-sheet model to be non-consistency, because the solution of field doesn't satisfy the boundary condition on other side of Big Bang. CPT model may solve the problem by consider the solution of field would be analytic if the arrow of time at the other side is oppisite.

B. Magnetic Monopole

Let's first see how Maxwell's equation would become under CPT-reflection.

$$\begin{aligned} F_{\mu\nu} &\equiv \partial_\mu A_\nu + \partial_\nu A_\mu \\ \partial_\rho F_{\mu\nu} &= 0 \\ \partial_{\{\rho}F_{\mu\nu\}} \equiv \partial_\rho F_{\mu\nu} + \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} &= 0 \end{aligned} \tag{13}$$

The second equation and the third equation give half of Maxwell equation respectively. From section II, we may expect to see the electric field and magnetic field transform like,

$$\begin{aligned} \mathbf{E}(\mathbf{x}) &\to \mathbf{E}(-\mathbf{x}) \\ \mathbf{B}(\mathbf{x}) &\to \mathbf{B}(-\mathbf{x}) \end{aligned} \tag{14}$$

Classically, we would see the 4-vector potential transform like,

$$A^\mu(x) \xrightarrow{CPT} -A^\mu(-x) \tag{15}$$

The minus sign is due to the source under charge conjugate reflection. Then the field tensor would transform like, [12]

$$F_{\mu\nu} \xrightarrow{CPT} -\frac{\partial A_\mu(-x)}{\partial x^\nu} + \frac{\partial A_\nu(-x)}{\partial x^\mu} = F_{\mu\nu}(-x) \tag{16}$$

As expected. Explicitly we show in deed the Maxwell's equation would convariant,

$$\begin{aligned} \nabla_{\mathbf{x}} \cdot \mathbf{E}(\mathbf{x}) = \frac{\rho(\mathbf{x}')}{\epsilon_0} &\to \nabla_{\mathbf{x}} \cdot \mathbf{E}(-\mathbf{x}) = \frac{-\rho(-\mathbf{x}')}{\epsilon_0} \\ &\Rightarrow \nabla_{-\mathbf{x}} \cdot \mathbf{E}(-\mathbf{x}) = \frac{\rho(-\mathbf{x}')}{\epsilon_0} \\ \nabla_{\mathbf{x}} \cdot \mathbf{B}(\mathbf{x}) = 0 &\to \nabla_{\mathbf{x}} \cdot \mathbf{B}(-\mathbf{x}) = 0 \\ &\Rightarrow \nabla_{-\mathbf{x}} \cdot \mathbf{B}(-\mathbf{x}) = 0 \end{aligned} \tag{17}$$

$$\begin{aligned}\nabla_{\mathbf{x}} \times \mathbf{E}(\mathbf{x}) = -\frac{\partial \mathbf{B}(\mathbf{x})}{\partial t} \rightarrow \nabla_{\mathbf{x}} \times \mathbf{E}(-\mathbf{x}) &= -\frac{\partial \mathbf{B}(-\mathbf{x})}{\partial(-t)} \\ \Rightarrow \nabla_{-\mathbf{x}} \mathbf{E}(-\mathbf{x}) &= -\frac{\partial \mathbf{B}(-\mathbf{x})}{\partial t} \\ \nabla_{\mathbf{x}} \times \mathbf{B}(\mathbf{x}) &= \mu_0 \mathbf{J}(\mathbf{x}) + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}(\mathbf{x})}{\partial t} \\ \rightarrow \nabla_{\mathbf{x}} \times \mathbf{B}(-\mathbf{x}) &= -\mu_0 \mathbf{J}(-\mathbf{x}) + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}(-\mathbf{x})}{\partial(-t)} \\ \Rightarrow \nabla_{-\mathbf{x}} \times \mathbf{B}(-\mathbf{x}) &= \mu_0 \mathbf{J}(-\mathbf{x}) + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}(-\mathbf{x})}{\partial t}\end{aligned} \tag{18}$$

There is discussion about magnetic monopole classically in Jackson's book [13] and quantum-mechanically in Sakurai's book [8]. However, if we would like to see the magnetic monopole in anti-universe, it is expected to see the field tensor transform to *dual field tensor*, which is defined as,

$$\mathcal{F}^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta} F_{\gamma\delta} \tag{19}$$

This kind of transformation is recognized as duality transformation,

$$\begin{aligned}\mathbf{E} &\rightarrow \mathbf{B} \\ \mathbf{B} &\rightarrow -\mathbf{E}\end{aligned} \tag{20}$$

The duality transform isn't respected in CPT-symmetric universe model. There should be more discussions about whether considering duality transformation would make sensible because it is a unusal symmetry and needs to be dealt carefully. Construction of duality-CPT-symmetric universe is left for future work.

[1] L. Boyle, K. Finn, and N. Turok, The big bang, cpt, and neutrino dark matter, Annals of Physics **438**, 168767 (2022).

[2] L. Boyle and N. Turok, Two-sheeted universe, analyticity and the arrow of time, arXiv preprint arXiv:2109.06204 (2021).

[3] L. Boyle, K. Finn, and N. Turok, C p t-symmetric universe, Physical review letters **121**, 251301 (2018).

[4] J. Smithrs, Why did the big bang happen first?, Physics Stack Exchange.

[5] user218912, One-way speed of light and the big bang, Physics Stack Exchange.

[6] R. Lehnert, Cpt symmetry and its violation, Symmetry **8**, 10.3390/sym8110114 (2016).

[7] C and P operation would be linear operator.

[8] J. J. Sakurai, *Modern quantum mechanics; rev. ed.* (Addison-Wesley, Reading, MA, 1994).

[9] The symbol in this letter would be slightly different with the original paper [2].

[10] R. Penrose, Republication of: Conformal treatment of infinity, General Relativity and Gravitation **43**, 901 (2011).

[11] Octavius, Conformal coupling of massive klein-gordon equation?, Physics Stack Exchange.

[12] ∂_μ is invariant under CPT-reflection by convention, and we we may think of that observing the spacetime change rate of a quantity should be invariant under transformation.

[13] J. D. Jackson, *Classical electrodynamics*, 3rd ed. (Wiley, New York, NY, 1999).