ECN627

Problem Set IV

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I hereby declare that I am the sole author of this work.

7.1

Regressor	(1)	(2)	(3)
X_1	8.31 (0.23) **	8.32 (0.22) **	8.34 (0.22) **
X_2	-3.85 (0.23) **	-3.81 (0.22) **	-3.80 (0.22) **
X_3		0.51 (0.04) **	0.52 (0.04) **
X_4			0.18 (0.36) (Not statistically significant)
X_5			-1.23 (0.31) **
X_6			-0.43 (0.30) (Not statistically significant)
Intercept	17.02 (0.17) **	1.87 (1.18) (Not statistically significant)	2.05 (1.18) *

7.2

$$\begin{array}{llllll} \widehat{AHE} & = 8.31X_1 & -3.85X_2 & +17.02 & \text{SER} = 9.79 & \text{R}^2 = 0.162 \\ & (0.23) & (0.23) & (0.17) & & \end{array}$$

a)

$$\begin{aligned} &\text{Hold } X_2 \text{ constant} \\ &E[AHE_i|X_1 = 0] = -3.85X_2 + 17.02 \\ &E[AHE_i|X_1 = 1] = 8.31 - 3.85X_2 + 17.02 \end{aligned}$$

$$\begin{aligned} E[AHE_i|X_1 = 1] - E[AHE_i|X_1 = 0] &= (8.31 - 3.85X_2 + 17.02) - (-3.85X_2 + 17.02) \\ &= 8.31 \end{aligned}$$

Workers with a college degree earn \$8.31/hour more on average than workers with a high school degree.

$$
\begin{aligned}
t^{act} &= \frac{8.31 - 0}{0.23} \\
&\approx 36.13 \\
|t^{act}| &> 1.96
\end{aligned}
$$

95% confidence interval for $\beta_1 = \{8.31 \pm 0.45\}$
$7.86 \leq \beta_1 \leq 8.76$

b)

$$
\begin{aligned}
&\text{Hold } X_1 \text{ constant} \\
&E[AHE_i|X_2 = 0] = 8.31X_1 + 17.02 \\
&E[AHE_i|X_2 = 1] = 8.31X_1 - 3.85X + 17.02
\end{aligned}
$$

$$
\begin{aligned}
E[AHE_i|X_2 = 1] - E[AHE_i|X_2 = 0] &= (8.31X_1 - 3.85X + 17.02) - (8.31X_1 + 17.02) \\
&= -3.85
\end{aligned}
$$

Female workers earn \$8.31/hour less on average than male workers.

$$
\begin{aligned}
t^{act} &= \frac{-3.85 - 0}{0.23} \\
&\approx -16.74 \\
|t^{act}| &> 1.96
\end{aligned}
$$

95% confidence interval for $\beta_2 = \{-3.85 \pm 0.45\}$
$-4.30 \leq \beta_2 \leq -3.40$

7.4

a)

$$
\begin{aligned}
&H_0 : \beta_4 = 0 \text{ and } \beta_5 = 0 \text{ and } \beta_6 = 0 \\
&H_A : \beta_4 \neq 0 \text{ or } \beta_5 \neq 0 \text{ or } \beta_6 \neq 0
\end{aligned}
$$

Consider, $corr[t_1, t_2, t_3] = 0$, since the data is taken from different regions

$$\begin{aligned}
F_{3,\infty} &= \frac{1}{3}\left(t_1^2 + t_2^2 + t_3^2\right) \\
&= \frac{\left(\left(\frac{0.18-0}{0.36}\right)^2 + \left(\frac{-1.23-0}{0.31}\right)^2 + \left(\frac{-0.43-0}{0.30}\right)^2\right)}{3} \\
&\approx 6.01 \\
F_{3,\infty} &> \chi_3^2/3 = 2.60 \qquad \text{5\% significance}
\end{aligned}$$

Hence, the regional differences are jointly significant.

bi)

$$\widehat{Y} = 8.34X_1 - 3.80X_2 + 0.52X_3 + 0.18X_4 - 1.23X_5 - 0.43X_6 + 2.05$$

$$\begin{aligned}
Y_{Juanita} &= 8.34(1) - 3.80(1) + 0.52(28) + 0.18(0) - 1.23(0) - 0.43(1) + 2.05 \\
&= 20.72
\end{aligned}$$

$$\begin{aligned}
Y_{Molly} &= 8.34(1) - 3.80(1) + 0.52(28) + 0.18(0) - 1.23(0) - 0.43(0) + 2.05 \\
&= 21.15
\end{aligned}$$

$$\begin{aligned}
Y_{Molly} &= 8.34(1) - 3.80(1) + 0.52(28) + 0.18(0) - 1.23(1) - 0.43(0) + 2.05 \\
&= 19.92
\end{aligned}$$

The 95% confidence interval for the difference between Juanita and Molly is:
$-0.43 \pm 1.96 \times (0.30) = [-1.02, 0.16]$

bii) The estimated coefficient on X_6 would give the expected difference in earnings. Use $SE[\widehat{\beta}_6]$ to compute the confidence interval.

7.6

There are potentially omitted variables (education, years of experience, etc) in the regression that will lead to bias in the OLS coefficient estimator for Female. These effects need to be controlled to conclude wage discrimination.

7.9

a)

$$\begin{aligned} \text{estimate: } & Y_i = \beta_0 + \gamma X_{1,i} + \beta_2 \left(X_{1,i} + X_{2,i} \right) + u_i \\ \text{test: } & \gamma = 0 \end{aligned}$$

$$\begin{aligned} \text{estimate: } & Y_i = \beta_0 + \gamma X_{1,i} + \beta_2 \left(-\omega X_{1,i} + X_{2,i} \right) + u_i \\ \text{test: } & \gamma = 0 \end{aligned}$$

$$\begin{aligned} \text{estimate: } & Y_i - X_{1,i} = \beta_0 + \gamma X_{1,i} + \beta_2 \left(-X_{1,i} + X_{2,i} \right) + u_i \\ \text{test: } & \gamma = 0 \end{aligned}$$