MTH712

Assignment 1

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I hereby declare that I am the sole author of this work.

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$$\text{Given,} \quad \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - rx, \quad 0 < x < L \quad t > 0$$
$$\text{and BC:} \quad \frac{\partial u}{\partial x}(0,t), \quad \frac{\partial u}{\partial x}(L,t) = \alpha,$$

Where, r and α are constants.

a)

The corresponding time independent problem is:

$$0 = \frac{\partial^2 u}{\partial x^2} - rx, \quad 0 < x < L$$
$$\text{with BC:} \quad \frac{\partial u}{\partial x}(0) = 0, \quad \frac{\partial u}{\partial x}(L) = \alpha$$

$$\text{Now,} \quad \int rx dx = \int \frac{\partial^2 u}{\partial x^2} dx$$
$$\frac{\partial u}{\partial x} = \frac{rx^2}{2} + K$$

$$\text{So,} \quad \frac{\partial u}{\partial x}(0) \implies \frac{r(0)^2}{2} + K = 0 \quad \text{ie, } K = 0$$
$$\text{And,} \quad \frac{\partial u}{\partial L} \implies \frac{rL^2}{2} = \alpha$$

Now, for a solution to exist, we must have $r = \dfrac{2\alpha}{L^2}$

Hence, the general solution is, $\dfrac{\partial u}{\partial x} = \dfrac{\alpha x^2}{L^2}$

$$\text{and, } U_E(X) = \int \frac{\partial u}{\partial x} dx = \int \frac{\alpha x^2}{L^2} dx = \frac{\alpha x^3}{3L^2} + K_2$$

b)

From the previous relationship, $U(x,o) = f(x), \quad 0 < x < L$

$$\int_0^L \frac{\partial u}{\partial t} dx = \int_0^L \frac{\partial^2 u}{\partial x^2} dx - \left.\frac{\alpha x^2}{L^2}\right|_0^L \quad \text{from } r = \frac{2\alpha}{L^2}$$

$$= \left(\frac{\partial u}{\partial x}(L,t) - \frac{\partial u}{\partial x}(0,t)\right) - \left(\frac{\alpha L^2}{L^2} - \frac{\alpha (0)^2}{L^2}\right)$$

$$= \alpha - 0 - \alpha - 0 = 0$$

Now, $\displaystyle \frac{d}{dt}\int_0^L U(x,t)dx = 0, \quad t > 0$

So, $\displaystyle \int_0^L U(x,t)dx = P$, where P is a constant

And, $\displaystyle \int_0^L U(x,t)dx = \int_0^L U_E(X)dx = \int_0^L \left(\frac{\alpha x^3}{3L^2} + K_2\right) dx$

$$= \left[\frac{\alpha x^4}{12L^2} + K_2 x\right]_0^L$$

Hence, $\displaystyle \int_0^1 f(x)dx = \frac{\alpha L^4}{12L^2} + K_2 L$

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$$\left|\frac{f(z)-f(z_0)-(-1)}{z-z_0}\right| = \left|\frac{\frac{1}{1+z}-\frac{1}{1+0}+1}{z-0}\right| = \left|\frac{2}{z+1}\right| < \epsilon$$
$$= 2\left|\frac{1}{z+1}\right|$$

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$$\begin{aligned}\cos(x_1)=\cos(x_2) &\iff \frac{\exp(iz_1)+\exp(-iz_1)}{2} = \frac{\exp(iz_2)+\exp(-iz_2)}{2}\\ &\iff \exp(iz_1)+\exp(-iz_1) = \exp(iz_2)+\exp(-iz_2)\\ &\iff [\exp(iz_1)-\exp(iz_2)]-[\exp(-iz_2)-\exp(-iz_1)] = 0\\ &\iff (\exp(iz_1)-\exp(iz_2)(1-\exp(-iz_2)-\exp(-iz_1))) = 0\end{aligned}$$

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$$\int_\gamma \mathrm{Arg}(z)dz \quad \gamma(t)=\exp(it) \quad \gamma'(t)=\exp(it)$$

$$\begin{aligned}\int_0^{2\pi} ti\exp(it))dt &= t\exp(it)\Big|_0^{2\pi} + i[\exp(it)]\Big|_0^{2\pi} \quad \text{IBP}\\ &= 2\pi\end{aligned}$$

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$$\begin{aligned}\frac{d}{dt}Log(-z) &= \frac{-1}{1-z}\\ &= -\sum z^n\end{aligned}$$

$$\lim_{n\to\infty}$$

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a)

$$
\begin{aligned}
A^b &= \exp(BLog(a)) \\
\text{Let } f(x) &= (1+2)^a \\
\text{so, } \exp(Log(f(x)) &= \exp(Log(1+z)^a) \\
&= \exp(aLog(1+z)) \\
\text{Hence, } \frac{d}{dz}\exp(Log(1+z)^a) &= \frac{a}{1+z}\exp(aLog(1+z)) \\
&= a(1+z)^(a-1)
\end{aligned}
$$

b)

$$
\begin{aligned}
f^n(z) &= \left(\prod_{n=1}^{\infty}(a-n+1)\right) \times (1+z)^{a-n} \\
\frac{f^n(z)}{n!} &= \frac{\prod_{n=1}^{\infty}(a-n+1)}{n!} \\
\text{so, } (1+z)^a &= 1 + \sum_{n=1}^{\infty}\frac{\prod_{n=1}^{\infty}(a-n+1)}{n!} \times z^n \quad |z| < 1
\end{aligned}
$$

$$
\begin{aligned}
c_n &= \frac{\prod_{n=1}^{\infty}(a-n+1)}{n!} \\
\text{So, } \lim_{k\to\infty}&\left(\frac{\prod_{n=1}^{\infty}(a-n+1)}{n!}\right)^{\frac{1}{k}}
\end{aligned}
$$

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$\exp(z) = \sum_{n=0}^{\infty}\frac{z^n}{n!}$

$$
\begin{aligned}
\text{So, } \exp(z^2) &= \sum_{n=0}^{\infty}\frac{z^{2n}}{n!} \\
\text{Now, } c_n &= \frac{1}{n!}
\end{aligned}
$$

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b)

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$$
\begin{aligned}
\frac{1}{1+z^2} &= \frac{1}{1-(-z^2)} \\
&= \sum_{i=0}^{n} (-1)^k (z)^{2k} \quad |z| < 1 \\
&= \sum \frac{(-1)^k}{2k+1} \times z^{2k+1} + A \\
\text{Now, } \arctan(z) &= 0 \\
\text{So, } A &= 0 \\
\text{Hence, } \arctan(z) &= \sum_{i=0}^{n} (-1)^k (z)^{2k}
\end{aligned}
$$

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$$
\begin{aligned}
\int_\gamma (f(\gamma(t))\gamma'(t)dt &= \int_0^{\frac{\pi}{2}} ((2\exp(it))^2 + 3(2\exp(it)))2i\exp(it)dt \\
&= \int_0^{\frac{\pi}{2}} 8i\exp(3it)dt + \int_0^{\frac{\pi}{2}} 12i\exp(2it)dt \\
&= \frac{8}{3}\left[\exp(3it)\right]\Big|_0^{\frac{\pi}{2}} + 6\left[\exp(2it)\right]\Big|_0^{\frac{\pi}{2}} \\
&= \frac{8}{3}\left[\text{cis}\left(\frac{3\pi}{2}\right) - 1\right] + 6\left[\text{cis}\left(\pi\right) - 1\right] \\
&= \frac{-28}{3} - \frac{8i}{3}
\end{aligned}
$$

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S5:34

S5:38

b)

S5:49

S5:56

a)

S5:80

a)

S5:82

S5:89

S6:45

S6:52

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