ECN627

Problem Set II

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I hereby declare that I am the sole author of this work.

3.3

$$X_i \sim Bin(400, \Pr) \implies \bar{X} \sim \mathcal{N}(\mu_x, \sigma_x^2)$$

a)

$$\hat{p} = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{215}{400} = 0.5375$$

b)

$$Var[\hat{p}] = \frac{\hat{p}(1-\hat{p})}{n} = \frac{(0.5375)(0.4625)}{400} \approx 0.000621$$

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.5375(0.4625)}{400}}$$
$$\approx 0.0249$$

c)

$$H_0 : p = 0.5 \quad H_1 : p \neq 0.5$$

$$t = \frac{\hat{p} - p_0}{SE(\hat{p})} \approx \frac{0.5375 - 0.5}{0.0249} \approx 1.504$$

P value $= 2\Phi(-|t|) \approx 2\Phi(-1.504)$
The two-tailed P value equals 0.1236

d)

$$H_0 : p = 0.5 \quad H_1 : p > 0.5$$

$$t = \frac{\hat{p} - p_0}{SE(\hat{p})} \approx \frac{0.5375 - 0.5}{0.0249} \approx 1.504$$

P value $= 1 - \Phi(t) \approx \Phi(1.504)$
The one-tailed P value equals 0.0618

e) The result differ because (c) tested under both tails of the normal distribution, whereas (d) tested under the tail of the right side of the normal distribution.

f) If the significance level was set at 5%, then there is no statistically significant difference.

3.4

a)

$$\begin{aligned} p &= \hat{p} \pm z_{95\%}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.5375 \pm 1.96\sqrt{\frac{0.5375(0.4625)}{400}} \\ &\approx 0.5375 \pm 0.0489 \end{aligned}$$

b)

$$\begin{aligned} p &= \hat{p} \pm z_{99\%}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.5375 \pm 2.575\sqrt{\frac{0.5375(0.4625)}{400}} \\ &\approx 0.5375 \pm 0.0652 \end{aligned}$$

b) The interval is wider in (b) than (a) because the critical value was larger, hence smaller tails under the normal distribution were tested.

3.12

$$\begin{aligned} &n_X = 100 \text{ (idd)} \implies \bar{X} \sim \mathcal{N}(\mu_X, \sigma_X^2/n_X) \\ &n_Y = 64 \text{ (idd)} \implies \bar{Y} \sim \mathcal{N}(\mu_Y, \sigma_Y^2/n_Y) \\ &\bar{X} - \bar{Y} \sim \mathcal{N}(\mu_X - \mu_y, \sigma_X^2/n_X + \sigma_Y^2/n_Y)) \end{aligned}$$

a)

$$H_0 : \mu_X - \mu_Y = 0 \quad H_1 : \mu_X - \mu_Y \neq 0$$

$$t = \frac{\bar{X} - \bar{Y}}{SE(\bar{X} - \bar{Y})} \qquad SE(\bar{X} - \bar{Y}) = \sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}} = \sqrt{\frac{40000}{100} + \frac{102400}{64}}$$

$$\approx 44.721$$

$$= \frac{200}{SE(\bar{X} - \bar{Y})}$$

$$\approx 4.472$$

P value $= 2\Phi(-|t|) \approx 2\Phi(-4.472)$

The two-tailed P value is less than 0.0001

We fail to reject H_0

b) There is statistically significant evidence that there exists a difference between the average salaries of the men and women at the firm.

3.16

$$n = 453 \text{ (idd)} \implies \bar{X} \sim \mathcal{N}(\mu_X, \sigma_X^2/n)$$

a)

$$\mu_X = \bar{X} \pm z_{95\%} SE(\bar{X}) = 1013 \pm 1.96 \frac{108}{\sqrt{453}}$$

$$\approx 1013 \pm 8.748$$

b)

$$H_0 : \bar{X} = 1000 \quad H_1 : \bar{X} \neq 1000$$

Significance level $= 1.96$

$$t = \left| \frac{1013 - 1000}{\frac{108}{\sqrt{453}}} \right|$$

$$\approx 2.561$$

$$
\begin{aligned}
&\text{Since } 2.651 > 1.96, \text{ reject } H_0 \\
&\text{p-value} = 2\Phi(-2.561) \approx 0.0108
\end{aligned}
$$

ci)

$$
\begin{aligned}
\mu_Y = \bar{Y} \pm z_{95\%} SE(\bar{Y}) &= 1019 \pm 1.96 \frac{95}{\sqrt{503}} \\
&\approx 1019 \pm 8.3022
\end{aligned}
$$

cii)

$$
H_0 : \mu_X - \mu_Y = 0 \quad H_1 : \mu_X - \mu_Y \neq 0
$$

$$
t = \frac{\bar{X} - \bar{Y}}{SE(\bar{X} - \bar{Y})} \qquad\qquad SE(\bar{X} - \bar{Y}) = \sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}} = \sqrt{\frac{11664}{453} + \frac{9025}{503}} \approx 6.610
$$

$$
\begin{aligned}
&= \frac{6}{SE(\bar{X} - \bar{Y})} \\
&\approx 0.908
\end{aligned}
$$

P value $= 2\Phi(-|t|) \approx 2\Phi(-0.908)$

The two-tailed P value equals 0.3639

We fail to reject H_0

Hence, there is no statistical evidence that the prep course helped.

di)

$$
\begin{aligned}
\mu_Z = \bar{Z} \pm z_{95\%} SE(\bar{Z}) &= 9 \pm 1.96 \frac{60}{\sqrt{453}} \\
&\approx 9 \pm 5.525
\end{aligned}
$$

dii)

$$H_0 : \bar{Z} = 0 \quad H_1 : \bar{Z} \neq 0$$

$$\text{Significance level} = 1.96$$

$$\begin{aligned} t &= \left| \frac{9 - 0}{\frac{60}{\sqrt{453}}} \right| \\ &\approx 3.193 \end{aligned}$$

Since $3.193 > 1.96$, reject H_0

Hence, there is statistical evidence that there is a difference in the test scores.

4.1

a)

$$\begin{aligned} \widehat{TestScores} &= 520.4 - 5.82 \times 22 \\ &= 392.36 \end{aligned}$$

b)

$$\begin{aligned} \Delta\widehat{TestScores} &= -5.82 \times 4 \\ &= -23.28 \end{aligned}$$

$$\begin{aligned} \widehat{TestScores}_{av} &= 520.4 - 5.82 \times 21.4 \\ &= 395.852 \end{aligned}$$