MTH719

Assignment 2

Saumitra Mazumder
500720916
Ryerson University
May 22, 2018

I hereby declare that I am the sole author of this work.

1.a

To show $\frac{A+A^T}{2}$ is symmetric, we need to show:
$\frac{A+A^T}{2} = \left(\frac{A+A^T}{2}\right)^T$

$$\text{Now,} \quad \left(\frac{A+A^T}{2}\right)^T = \frac{A^T+(A^T)^T}{2} = \frac{A^T+A}{2} = \frac{A+A^T}{2}$$
$$\text{Hence,} \quad \frac{A+A^T}{2} = \left(\frac{A+A^T}{2}\right)^T$$

Now, we want to show:

$$X^T\left(\frac{A+A^T}{2}\right)X = X^TAX$$

$$\text{We know,} \quad \frac{A+A^T}{2} = \left(\frac{A+A^T}{2}\right)^T$$

$$\begin{aligned}
\text{Then,} \quad X^T\left(\frac{A+A^T}{2}\right)X &= X^T\left(\frac{A+A^T}{2}\right)^T X \quad \text{by left and right matrix multiplication} \\
&= \left[\left(\frac{A+A^T}{2}\right)X\right]^T X \quad \text{by properties of transpose} \\
&= \left(\frac{AX+A^TX}{2}\right)^T X \quad \text{by distributive property} \\
&= \left(\frac{X^TA^T+X^TA}{2}\right)X \quad \text{by properties of transpose} \\
&= \frac{X^TA^TX+X^TAX}{2} \quad \text{by distributive property}
\end{aligned}$$

Now, if $A = A^T$ then:

$$\begin{aligned}
\frac{X^TA^TX+X^TAX}{2} &= \frac{X^TA^TX+X^TA^TX}{2} \\
&= \frac{2X^TA^TX}{2} = X^TA^TX
\end{aligned}$$

1.b

Suppose A is a positive definite matrix, then:

$$X^TAX = X^TLDUX = X^TLDL^TX = X^TU^TDUX$$

by definition of positive definite LDU factorization

$$= (UX)^TD(UX)$$

So, if the diagonal elements of D are positive numbers X^TAX is positive definite $\forall X$, since X^TAX also has a positive definite LDU factorization.

1.c

Using Taylor's theorem in $\mathbb{R}^n$ we know that $f(x) = f(x_0) + (x - x_0)^T g(x_0) + (x - x_0)^T$