MTH640

Exercises for MTH 640 II

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I hereby declare that I am the sole author of this work.

Sps, $\Omega = \{1,2,3\}$ and $\mathcal{F} = \{\varnothing, \Omega, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$, ie the power set of Ω. Sps further that $\mathcal{C} = \{\{1\}, \{2,3\}\}\} \subset \mathcal{F}$. Now,

$$\begin{aligned}\sigma(\mathcal{C}) = \{&\mathcal{C}, \\ &\mathcal{C}^c = \{\varnothing, \Omega, \{2\}, \{3\}, \{1,2\}, \{1,3\}\}, \\ &\mathcal{C} \cap \mathcal{C}^c = \varnothing, \\ &\mathcal{C} \cup \mathcal{C}^c = \mathcal{F}, \\ &\}\end{aligned}$$

I have failed to show $\mathcal{C} \subset \sigma(\mathcal{C})$, this is NOT a σ-algebra over $\mathcal{C}$ since a σ-algebra over $\mathcal{C}$ must contain $\mathcal{C}$. However, the following is a a σ-algebra over $\mathcal{C}$:

$$\begin{aligned}\sigma(\mathcal{C}) = \{&\overbrace{\{1\}, \{2,3\}}^{=\mathcal{C}}, \\ &\overbrace{\{1,2,3\}}^{\{1\}\cup\{2,3\}} = \Omega, \\ &\Omega^c = \varnothing, \}\end{aligned}$$

.

We notice that $\mathcal{C}$ is a subset of the above collection of sets. Furthermore, we find that the above collection of sets is a σ-algebra by Definition 1.1.