MTH712

Assignment 1

Saumitra Mazumder
500720916
Ryerson University
October 25, 2018

I hereby declare that I am the sole author of this work.

1

$$\text{Given,} \quad \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - rx, \quad 0 < x < L \quad t > 0$$
$$\text{and BC:} \quad \frac{\partial u}{\partial x}(0,t), \quad \frac{\partial u}{\partial x}(L,t) = \alpha,$$

Where, r and α are constants.

a)

The corresponding time independent problem is:

$$0 = \frac{\partial^2 u}{\partial x^2} - rx, \quad 0 < x < L$$
$$\text{with BC:} \quad \frac{\partial u}{\partial x}(0) = 0, \quad \frac{\partial u}{\partial x}(L) = \alpha$$

$$\text{Now,} \quad \int rx dx = \int \frac{\partial^2 u}{\partial x^2} dx$$
$$\frac{\partial u}{\partial x} = \frac{rx^2}{2} + K$$

$$\text{So,} \quad \frac{\partial u}{\partial x}(0) \implies \frac{r(0)^2}{2} + K = 0 \quad \text{ie, } K = 0$$
$$\text{And,} \quad \frac{\partial u}{\partial L} \implies \frac{rL^2}{2} = \alpha$$

Now, for a solution to exist, we must have $r = \dfrac{2\alpha}{L^2}$

Hence, the general solution is, $\dfrac{\partial u}{\partial x} = \dfrac{\alpha x^2}{L^2}$

$$\text{and, } U_E(X) = \int \frac{\partial u}{\partial x} dx = \int \frac{\alpha x^2}{L^2} dx = \frac{\alpha x^3}{3L^2} + K_2$$

b)

From the previous relationship, $U(x,o) = f(x), \quad 0 < x < L$

$$\int_0^L \frac{\partial u}{\partial t} dx = \int_0^L \frac{\partial^2 u}{\partial x^2} dx - \left.\frac{\alpha x^2}{L^2}\right|_0^L \quad \text{from } r = \frac{2\alpha}{L^2}$$

$$= \left(\frac{\partial u}{\partial x}(L,t) - \frac{\partial u}{\partial x}(0,t)\right) - \left(\frac{\alpha L^2}{L^2} - \frac{\alpha(0)^2}{L^2}\right)$$

$$= \alpha - 0 - \alpha - 0 = 0$$

Now, $\dfrac{d}{dt}\displaystyle\int_0^L U(x,t)dx = 0, \quad t > 0$

So, $\displaystyle\int_0^L U(x,t)dx = P$, where P is a constant

And, $\displaystyle\int_0^L U(x,t)dx = \int_0^L U_E(X)dx = \int_0^L \left(\frac{\alpha x^3}{3L^2} + K_2\right) dx$

$$= \left[\frac{\alpha x^4}{12L^2} + K_2 x\right]_0^L$$

Hence, $\displaystyle\int_0^1 f(x)dx = \frac{\alpha L^4}{12L^2} + K_2 L$

And the steady-state solution is $U_E(x) = \dfrac{\alpha x^3}{3L^2} + \dfrac{1}{L}\displaystyle\int_0^L f(x)dx - \frac{\alpha}{12}$

2

$$f(n) = \begin{cases} x & \text{if} \quad -2 < x < 0 \\ 1 + x & \text{if} \quad 0 < x < 2 \end{cases}$$

4

$$g(y) \approx a_0 + \sum_{n=1}^{\infty} (a_n \cos \lambda_n(y) + b_n \sin \lambda_n(y))$$

Let $f(x)$ denote $g(y)$ in terms of x and, $y = \dfrac{a+b}{2} + \dfrac{b-a}{2L}x$

So, $f(x) \approx a_0 + \displaystyle\sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right)\right)$

$$\begin{aligned}
\text{Now,} \quad a_0 &= \frac{1}{2L}\int_{-L}^{-L} f(x)dx = \frac{1}{2L}\int_{-L}^{L}\left(\frac{a+b}{2} + \frac{b-a}{2L}x\right)dx \\
&= \frac{1}{2L}\left[\frac{a+b}{2}x + \frac{b-a}{4L}x^2\right]_{x=-L}^{x=L} \\
&= \frac{1}{2L}L(a+b) = \frac{a+b}{2} \\
\text{and,} \quad a_n &= \frac{1}{L}\int_{-L}^{L}\left(\frac{a+b}{2} + \frac{b-a}{2L}x\right)\cos\left(\frac{n\pi}{L}x\right)dx \\
&=
\end{aligned}$$