Assignment 3

Due at 6/12 5 p.m.

Q1. (40%) Consider an (r, Q) policy for continuous demands. Suppose the annual demand is distributed $\mathrm{N}(800, 40^2)$, the fixed cost is K = 50, and the holding and stockout costs are h = 3.1 and p = 45, respectively, per item per year. The lead time is 4 days. Find r and Q using each of the methods below.

(1) (10%) The EIL approximation

(2) (10%) The EOQB approximation

(3) (10%) The EOQ + SS approximation

(4) (10%) The loss-function approximation

For each method, report the values of r and Q you found, as well as the corresponding expected annual cost from (5.7) $\mathrm{g(r,Q)} = \frac{K\lambda + \int_r^{r+Q} g(y)dy}{Q}$.

Algorithm 5.2 Exact algorithm for continuous-review (r, Q) policy with continuous demand distribution

$Q \leftarrow Q_d^*$; $\overline{Q} \leftarrow Q_0$ from Theorem 5.5 ▷ Initialization
repeat ▷ Main loop
    $Q \leftarrow (\underline{Q} + \overline{Q})/2$ ▷ Candidate value for Q
    $r \leftarrow r(Q)$, where $r(Q)$ satisfies (5.9) ▷ Optimal r for Q
    $A \leftarrow A(Q)$ ▷ $A(Q)$
    if $A > K\lambda$ **then** $\overline{Q} \leftarrow Q$ ▷ Update bounds on Q
    else if $A < K\lambda$ **then** $\underline{Q} \leftarrow Q$
    end if
until $|A - K\lambda| \le \epsilon$ ▷ Termination check via Theorem 5.4
return (r, Q)

Q2. (30%) Consider an (r, Q) policy for discrete demands. Suppose the demand has a Poisson distribution with a mean of λ = 12 units/month, the fixed cost is K = 4, and the holding and stockout costs are h = 4 and p = 28, respectively, per item per month. The lead-time is 0.5 months.

(1) (10%) Find approximate values for r and Q by using the EOQB approximation described in Section 5.3.2 of our textbook, replacing g(y) with (4.32) $g(S) = \mathrm{h}\sum_{d=0}^{S}(S-d)f(d) + p\sum_{d=S}^{\infty}(d-S)f(d)$ when solving (5.9) $\mathrm{g(r)} = g(r+Q)$

(2) (20%) Find exact optimal values for r and Q using algorithm 5.3. (attached below)

For each method, report the values of r and Q you found, as well as the corresponding expected cost per week from (5.48) $\mathrm{g(r,Q)} = \frac{K\lambda + \sum_{y=r+1}^{r+Q} g(y)}{Q}$.

Algorithm 5.3 Exact algorithm for continuous-review (r, Q) policy with discrete demand distribution (Federgruen and Zheng 1992)

$Q \leftarrow 1$; $r(Q) \leftarrow S^* - 1$, where S^* minimizes $g(y)$ $\quad \triangleright$ Initialization
Calculate $g(r(Q), Q)$ from (5.48)
done $\leftarrow$ FALSE
while not done **do** $\quad \triangleright$ Main loop
    if $g(r(Q)) < g(r(Q) + Q + 1)$ **then** $\quad \triangleright$ Choose $r(Q+1)$
        $r(Q+1) \leftarrow r(Q) - 1$
    else
        $r(Q+1) \leftarrow r(Q)$
    end if
    Calculate $g(r(Q+1), Q+1)$ from (5.48)
    if $g(r(Q+1), Q+1) > g(r(Q), Q)$ **then** $\quad \triangleright$ Termination check
        done $\leftarrow$ TRUE
    else
        $Q \leftarrow Q + 1$ $\quad \triangleright$ Increment Q
    end if
end while
return $(r(Q), Q)$ $\quad \triangleright$ Q is optimal

Q3. (30%) Consider the EIL approximation in Section 5.3.1 of our textbook. Define a new type of service level as follows: $SL(a)$ is the percentage of order cycles during which there are at most a stockouts, for constant $a \geq 0$. Suppose that we wish to enforce a service level constraint that says $SL(a) \geq \gamma$, for fixed $0 \leq \gamma < 1$. What are the optimal values of r and Q for the problem with this service level constraint?

(20%) For Bonus, please write the Julia code to solve Q1 (3% for each sub-problem) and Q2 (4% for each sub-problem).